4

Metaphysics Between Reductionism and a Non-Reductionist Ontology

D.F.M. Strauss
Faculty of Humanities, University of the Free State, South Africa

1. Introduction

Philosophy and all the academic disciplines are sensitive to the aim of *sound reasoning* – except for the dialectical tradition which sanctions contradictions and antinomies (Heraclitus, Nicolas of Cusa, Hegel, Marx, Vaihinger, Simmel, Rex, and Dahrendorf). A brief overview is presented of conflicting theoretical stances within the various academic disciplines *before* an assessment is given of the positive and negative meaning of 'reductionism' and the implications of a non-reductionist ontology. These implications are explained by distinguishing between different fundamental irreducible *modes of being*, such as the numerical, spatial, kinematic, physical, biotic, sensitive, logical-analytical, cultural-historical, lingual, social, economic, aesthetic, jural, moral and certitudinal aspects of reality. When these original *modal functions* are not acknowledged theoretical thought entangles itself in insurmountable *antinomies*. Every single academic discipline therefore has to employ such basic (and irreducible) concepts. Precisely because these concepts are *basic* they cannot be *defined*. Various disciplines acknowledge this state of affairs by explicitly introducing "primitive terms." Furthermore, when it is attempted to reduce what is irreducible the antinomy involved at once expresses itself as a logical contradiction. We shall argue that an antinomy as such is inter-modal (such as when Zeno attempts to reduce motion to static positions in space), while a contradiction is intra-modal (for example when a triangle and a circle is confused). A clear example of the irony of an *ismic ortientation* will be discussend in sub-paragraph 10 when the impasse of historicism is discussed.

Against the background of historical lines of development the multiple terms employed in this context are mentioned and eventually positioned within the context of the normativity holding for logical thinking. It is argued that the logical contrary between *logical* and *illogical* serves as the foundation of other normative contraries, such as *polite* – *impolite*, *legal* – *illegal* and *moral* – *immoral*.

It will be shown that through the discovery of irrational numbers the initial Pythagorean conviction that everything *is* number reverted to a geometrical perspective that generated a static metaphysics of being which challenged the ideas of *plurality* and *motion*. This development uncovered the well-known problem of *primitive terms* in scientific discourse as an alternative for those metaphysical attempts aimed at *reducing* whatever there is to one single *mode of explanation*. Zeno's paradoxes are used to demonstrate an alternative understanding of the difference between the *potential* and the *actual infinite* as well as the nature of (theoretical) *antinomies*. It is argued that genuine antinomies are *inter-modal* in nature (such as is found in the attempt to reduce movement to static positions in space) and therefore differ from a *logical contradiction* (such as a 'square circle' which merely confuses two figures within one modal aspect). Although every antinomy does entail logical contradictions, the latter do not necessarily presuppose an antinomy. The implication is that logic itself has an *ontic* foundation – as is seen from the nature of the *principle of sufficient reason* (ground) and the *principle of the excluded antinomy* – and therefore only acquires meaning on the basis of a non-reductionist ontology. When the method of *immanent critique* unveils genuine antinomies, the way is opened for meaningful intellectual interaction between different philosophical stances. In distinguishing between contradiction and antinomy philosophers are actually challenged to contemplate the implications of a non-reductionist ontology as an alternative to all metaphysical attempts to over-emphasize one or another aspect of empirical reality.[1]

2. Setting the stage: Unity and diversity

Our human experience of reality is embedded in an awareness of *unity and diversity*. For that reason we have to discern, that is, we have to *identify* and *distinguish*. Whereas it is quite natural and meaningful to articulate differences between *distinct* (kinds of) entities in our everyday life, it is equally natural that we are (analytically) sensitive to a *confusion* of what is distinct. It is a standard practice amongst philosophers and logicians to designate this kind of confusion by using the terms *contradiction* and *antinomy* – which are normally taken to be *synonymous*.

Concurrent with the rise of philosophy and some of the disciplines in ancient Greece (such as mathematics, astronomy and the medical sciences) an awareness of the *logical-analytical* capacities of human beings surfaced. Early Greek philosophy also witnessed the emergence of *dialectical* conceptions in which it was attempted to accommodate (and sanction) *contradictions*. A later disciple of Heraclitus said:

> For all things are alike in that they differ, all harmonize with one another in that they conflict with one another, all converse in that they do not converse, all are rational in being irrational; individual things are by nature contrary, because they mutually agree. For rational world-order [*nomos*] and nature [*physis*], by means of which we accomplish all things, do not agree in that they agree.[2]

In the course of the intellectual tradition of the West academic disciplines more and more acquired the status of independent special sciences, since they also increasingly adhered to principles for logical reasoning. Yet a first glance at the history of the various disciplines

[1] I want to thank Dr. Darrell Patrick Rowbottom, University of Durham, England, for many valuable comments and suggestions that are incorporated in the final version of this article. The same applies to Prof. Hubertus Bargenda (a mathematician from the University of the Free State). The underlying perspective of this contribution draws upon my work *Philosophy*: *Discipline of Disciplines* (2009) as well as Strauss 2006.

[2] These words, expressed by a later disciple of Heraclitus, were erroneously ascribed to Hippocrates' writing, *Peri diatès*, I, xi, 6.

shows that within each of them alternative, often conflicting orientations developed – a state of affairs that cannot be explained on purely *logical* grounds. This predicament rather suggests that theoretical (i.e., scientific) thought cannot escape from considerations *exceeding* the boundaries of logicality.[3]

In order to substantiate this suggestion, a distinction will be introduced between an antinomy and a contradiction. It may turn out that this distinction is closely connected to the above-mentioned issue of unity and diversity. The problem of the one and the many, alongside others, such as the relationship between universality and individuality, constancy and dynamics, the finite and the infinite, what is necessary and contingent and what is considered to be knowable and unknowable, co-determined the development of philosophy and the disciplines.

From the history of these disciplines we learn that the *foundational* problems within the (natural and social) sciences are indeed *philosophical* in nature. This explains the mentioned *historical fact* that in their development all the academic disciplines reflect divergent philosophical schools of thought and it urges us to ask how this situation ought to be assessed in terms of the requirements for logical thinking and sound reasoning. The following (incomplete) overview may help to portray the background picture of our subsequent discussion of the distinction between contradiction and antinomy. This succinct overview merely provides a *glimpse* and not a detailed exposition – for that would require more than an article on each mentioned discipline. The *rationale* for providing this glimpse is to allude to the widespread reality of opposing *ismic* positions – with a view to the fact that such positions mostly entail *antinomies* since they are normally *reductionist* (the connection between *reductionism* and *antinomies* will be explained below). The brief overview below first mentions the name of prominent trends (schools of thought) within various academic disciplines and then mention key scholars who adhered to this theoretical stance within these special sciences.

- Mathematics: Axiomatic *formalism* (Hilbert), *logicism* (Russell, Frege) and *intuitionism* (Brouwer, Heyting, Troelstra, Dummett) in modern mathematics;[4]

[3] I do not intend to view the relationship between "purely logical" and what exceeds the logical aspect in terms of the distinction between statements within an object language and statements belonging to a meta-language (utens/docens). The original distinction – probably going back to the 13th century – continued an old question regarding *logic* as a scientific part of philosophy or merely as an *instrument* of philosophy. The *dialectica utens* was viewed as treating arguments within all disciplines, whereas the *dialectica docens* was seen as a special science (*scientia specialis*) focused upon the dialectical syllogism or with the secondary intentions connected to dialectical conclusions.

[4] Salmon merely refers to "intuitionistic philosophers of mathematics" – without acknowledging the truly *mathematical character* of this trend in 20th century mathematics (see Salmon, 2001:23 – he refers to Körner's work *The Philosophy of Mathematics*, 1968). By contrast, Stegmüller remarks: "The special character of intuitionistic mathematics is expressed in a series of theorems that contradict the classical results. For instance, while in classical mathematics only a small part of the real functions are uniformly continuous, in intuitionistic mathematics the principle holds that any function that is definable at all is uniformly continuous" (1970:331). Beth also highlights this point: "It is clear that intuitionistic mathematics is not merely that part of classical mathematics which would remain if one removed certain methods not acceptable to the intuitionists. On the contrary, intuitionistic mathematics replaces those methods by other ones that lead to results which find no counterpart in classical mathematics" (1965:89).

- Physics: Classical determinism (Einstein, Schrödinger, Bohm and the school of De Broglie) and the mechanistic main tendency of classical physics (last representative Heinrich Hertz)[5] versus the Kopenhagen interpretation of quantum mechanics (Bohr and Heisenberg); the contemporary ideal to develop "a theory of everything" (Hawking and super string theory: Greene).
- Biology: Mechanistic orientation (Eisenstein), physicalistic approach (neo-Darwinism), neo-vitalism (Driesch, Sinnott, Rainer-Schubert Soldern, Haas, Heitler), holism (Adolf Meyer-Abich), emergence evolutionism (Lloyd-Morgan, Woltereck, Bavinck, Polanyi) and pan-psychism (Teilhard de Chardin, Bernard Rensch);
- Psychology: The initial atomistic association psychology (Herbart), the stimulus-response approach, Gestalt-psychology [the Leipzig school (Krüger and Volkelt) and the Berlin school (Koffka and Köhler)], depth psychology (Freud, Adler, Jung), the logo-therapy of Frankl, phenomenological psychology, contemporary system theoretical approaches (under the influence of von Bertalanffy).
- The science of history: Compare the conflict between *linear* and *cyclical* conceptions of history, the Enlightenment ideal of linear accumulative growth, the recurrence of the Greek conviction that history is *eternally recurrent* in the thought of Vico, Herder, Hegel, Goethe, Daniliwski, Nietzsche, Spengler and to a certain degree also Toynbee;
- Linguistics: Two lines of thought dominated the 19th century – Rousseau, Herder, Romanticism, von Humboldt and the rationalistic trend running from Bopp, Schleicher, and 'Jung-Grammatici' to Paul (with his historicistic conception of language-in-development). Cassirer, by contrast, developed his neo-Kantian theory of language (in which *language* is a thought-form imprinted upon reality), Bühler pursued the stimulus of behaviorism in his theory of signs, at the beginning of the 20th century Wundt dominated the scene, De Saussure contributed to the development of a *structuralist* understanding (followed by Geckeler, Coseriu and others), Reichling explored elements of Gestalt-psychology in his emphasis on the *word* as the core unit of language, Chomsky revived the doctrine of the *a priori* within the context of his transformative generative grammar, more recently the manifestation of systems theory within general and applied linguistics;
- Sociology: The initial organicistic orientation (Comte, Spencer) was continually opposed by mechanistic and physicalistic approaches (cf. L.F. Ward – late 19th century – and in the second half of the 20th century W.R. Catton), the dialectical heritage of Hegel permeated Georg Simmel's formalistic sociology with its individualistic neo-Kantian focus (Park and Burgess explored this direction in the USA), Max Weber developed the sociological and economic implications of the neo-Kantian Baden school of thought, Talcott Parsons made the systems model (based upon von Bertalanffy's generalization of the second main law of thermodynamics) fruitful for sociological thinking, opposed by conflict sociology (Dahrendorf, C. Wright Mills and Rex and by the Frankfurt school of neo-Marxism), a systems theoretical approach was recently revived by J.C. Alexander, A. Giddens developed his structuration theory – and during the past two decades J. Habermas elaborated his theory of communicative actions;

---

[5] Mario Bunge says: "It is now generally understood that mechanics is only a part of physics, whence it is impossible to reduce everything to mechanics, even to quantum mechanics." Although he holds that the physicalism of the Vienna Circle and the Encyclopedia of Unified Science is dead, "the sharp decline of physicalism has not been the end of reductionism" (see his "The Power and Limits of Reduction" in Agazzi, 1991:33).

- Economics: The classical school of Adam Smith, the neoclassical approach (from Cournot and Dupuit to Menger, Jevons, Walras and Pareto), the marginalism of Marshall, Keynes's 'General Theory,' alternative approaches to competition (Chamberlin and Robinson);
- The science of law: The historicistic orientation of von Savigny – followed by the Romanist (von Jhering) and Germanistic (von Gierke) schools, neo-Hegelianism (Binder), neo-Kantianism (Stammler, Radbruch, Kelsen), the revival of natural law theories after the second world war, and legal positivism (which seems to remain alive amongst legal scholars);
- Theology: Dialectical theology (Barth, Gogarten, Brunner) in its dependence upon Kierkegaard and Jaspers, Bultmann (dependent on Heidegger), theology of hope (Moltmann – dependent upon the neo-Marxism of Ernst Bloch), the historicistic design of Pannenberg (dependent upon Dilthey and Troeltsch), the 'atheistic' theology of Altizer and Cox (influenced by neo-positivism), existentialist-hermeneutical trends (Fuchs, Ebeling, Steiger), theology of liberation (influenced by neo-Marxism).

What is particularly striking regarding these (philosophically founded) schools of thought within the disciplines is that many of them are entangled in what should be labeled *reductionism* in a pejorative sense.[6] Surely there are also positive and largely unrelated connotations attached to the term *reduction* in different special sciences. For example, mathematicians may speak about the construction of numbers from sets and then designate it as "reduction". Separating chemical compounds into their simpler constituents is also known as "reduction", and so on.

By designating more problematic situations the term reductionism emerged by the middle of the 20th century. In 1953 Quine used it in his discussion of "The Verification Theory and Reductionism" (see Quine, 1953:37 ff.) and in the early seventies the work "Beyond Reductionism" appeared (see Smythies & Koestler, 1972). Smith considers Polanyi to be "perhaps the severest and most comprehensive critic of reductionism" because he "was a major scientist of this century and was drawn into philosophical debate primarily because of the threat to scientific freedom, political democracy, and to humane values that he saw in reductionism". To this he adds the remark: "His works *The Contempt of Freedom, The Logic of Liberty, Science Faith and Society, Personal Knowledge,* and *The Tacit Dimension* have as a common theme the criticism of reductionism in all its scientific, cultural and moral forms."[7]

---

[6] Popper states: "As a philosophy, reductionism is a failure" (Popper, 1974:269). And Goodfield remarks: "Reductionist methodology may have been extremely successful, but the history of science abounds with examples where forms of explanation, successful in one field, have turned out to be disastrous when imported into another" (Goodfield, 1974:86). A *positive* appreciation of reductionism is, for example, found in the thought of Dawkins and Dennett (see Dennet, 1995:80 ff.).

[7] See Smith, G.L., 1994. *On Reductionism*. Sewanee, Tennessee – available on the WEB at: http://smith2.sewanee.edu/texts/Ecology/OnReductionism.html (accessed on 22-01-2005). Putnam holds that scientism and relativism are *reductionist* theories (Putnam, 1982:126). In respect of 'phenomenalism' he remarks: "the idea that the statements of science are translatable one by one into statements about what experiences we will have if we perform certain actions has now been given up as an unacceptable kind of reductionism" (Putnam, 1982:187).

Our approach in what follows will be to investigate the limits of logical discernment (identification and distinguishing) in order to account for the real antinomies arising from the attempt to reduce what is truly *irreducible*. This approach is similar to the strategy defended by the physicist Henry Margenau (in following some ideas of Mario Bunge). He takes this to be "the strategy consisting of reducing whatever can be reduced without however ignoring emergence or persisting in reducing the irreducible" (cf. Margenau, 1982:187, 196-197). Once we have assessed the systematic distinction between antinomy and contradiction its implication for understanding the nature of the various "ismic" orientations within the disciplines will briefly be highlighted.

3. Brief historical contours

Since both academics and non-academics enjoy the fun of wrestling with "logical" problems we proceed by mentioning the liar-paradox attributed by Diels and Kranz to Epimenides (5th Century B.C.) – where it is asserted that one of the Cretans, their own prophet, said all Cretans are liars. In the account of Titus 1:12-13 it is reported that the Apostle Paul holds that this testimony is true. How can such a statement, made by a liar, be true without being false at the same time?[8]

The mere statement of this contradiction shows that ancient Greek thought already wrestled with the above-mentioned basic logical ability of humans to *identify* and to *distinguish*.[9] The school of Parmenides postulated the primordial nature of *being* and even *identified* it with *thought*.[10] But in Plato's dialogue *Parmenides* one finds a *negative* argument concerning the *mutuality* (relatedness) of *identification* and *distinguishing*, ultimately also highlighting the limits of concept formation, for conceiving the *One* (and the *Many*) in an *absolute* sense, escapes the grip of logical concept formation.[11] In the *Sophist* it is consequently acknowledged that trying to know what *being* and *non-being* in themselves are present thought with an *aporia*, i.e., an unresolved problem.[12] Yet, whenever *being* is thought *non-being* is thought as well.[13] In other words, *identification* refers to what is *distinct* from it.

[8] In this formulation an escape route is given by observing that normally a liar is a person who *sometimes* tells a lie, but not *always*.

[9] Derrida applies the mutuality of identity and difference to language: "The identity of a language can only affirm itself as identity to itself by opening itself to the hospitality of a difference from itself or of a difference with itself" (Derrida, 1993:10).

[10] Diels-Kranz I, 231; Parmenides, B. Fr. 3: "For thinking and being are the same."

[11] The final conclusion to the four paths of the dialectical argument regarding the *One* and the *Many* reads: "Therefore, if the One is, it is everything and nothing, in relation to itself and to the many" (Parmenides 160b1-3).

[12] Logic eventually used the term "aporia" in connection with the theoretical truth of a statement where there are grounds *for* and *against* it. In Latin *aporia* turned into *dubitation* and *question* (see Waldenfels, 1971:448).

[13] Spivak explains Derrida's view of deconstruction in similar terms: "Deconstruction, as it emerged in Derrida's early writings, examined how texts of philosophy, when they established definitions as starting points, did not attend to the fact that all such gestures involved setting each defined item off from all that it was not" (Spivak, 1999:426).

Whereas Plato therefore already had a clear understanding of the meaning of the logical principles of *identity* and *contradiction*,[14] Aristotle, in addition, already understood the meaning of the principle of the *excluded middle* (see *Metaph*. 1057a).

During the middle ages, alongside the continuation of Aristotle's predicate logic, a notable dialectical tradition, proceeding from Heraclitus and the dialectical logic of Plato, remained in force. This so-called *via negativa* of neo-Platonism (Pseudo-Dionysius, Plotinus) eventually brought Nicholas of Cusa to his notion of the coincidence of opposites (*coincidentia oppositorum*). Nicholas of Cusa explored the so-called *actual infinite* in terms of which he claimed that God, as the actual infinite, is at once the largest and the smallest (*De Docta Ignorantia*, I,5), i.e. the *coincidentia oppositorum* (see *De Docta Ignorantia*, I,22). A remarkable analysis eventually came from Georg Cantor, the founder of modern set theory and the theory of transfinite numbers, who demonstrated something similar about the smallest transfinite ordinal number *omega* (ω), because this number is both even and uneven and at the same time neither even nor uneven.

Perhaps the most significant elaboration of the dialectical tradition on the one hand is found in the thought of Hegel, Marx and those sociologists of the 20th century who are known as *conflict theorists* (Simmel, Rex and Dahrendorf), and in the philosophy of "As If" of Vaihinger on the other. The significance of the latter is linked to its relevance for various academic disciplines (such as mathematics, physics, linguistics, economics, and the science of law – to name some of them). Vaihinger claims that the use of *inherently antinomic* constructions (designated as *fictions*) may serve human (scientific) thought in surprisingly efficient ways. For example, he characterizes mathematical constructs such as *negative numbers*, *fractions*, *irrational* and *imaginary numbers* as "fictional constructs" (that are *not* hypotheses) with a "great value for the advancement of science and the generalization of its results in spite of the crass contradictions which they contain" (Vaihinger, 1949:57). In general Vaihinger aims at providing an explanation of "the riddle that by means of such illogical, indeed senseless concepts, correct results are obtained" (Vaihinger, 1949:240). His answer is given in what he terms to be "the general law of fictions," i.e., in the "correction of the errors that have been committed" or in a procedure called "the *method of antithetic error*" (Vaihinger, 1949:109). However, since this method holds that thought "progresses by means of antithetic operations," and since including under one concept antithetic operations creates fictions viewed as merely the symbol of "such antithetic operations and antithetic errors" (see Vaihinger, 1949:119-120), it is clear that his "*method of antithetic error*" simply duplicates the initial problematic construction of internally antinomic or illogical fictions – as if two logical errors in practice will generate what is right. The coherence of what is irreducible within reality requires an alternative approach, namely that what we have called a non-*reductionist ontology*.

At this point the above-mentioned problem of unity and diversity comes to mind again – not only with respect to the various ismic positions within the disciplines, but also regarding the opposing and oftentimes contradicting philosophical schools of thought – including the

[14] The following phrase highlights both principles: "No objection of that sort, then, will disconcert us or make us believe that the same thing can ever act or be acted upon in two opposite ways, or be two opposite things, at the same time, in respect of the same part of itself, and in relation to the same object" (*Politeia*, Book IV, Ch.XIII, 436 – translation by Cornford 1966:130).

dialectical tradition that affirms contradictions. But what is meant when different terms are employed alongside the term 'contradiction'? This question calls for a clarification of the terminological problem regarding multiple terms and for an investigation of the question whether or not there is more at stake than purely *logical* distinctions. We therefore proceed first of all by looking at the multiple terms that are employed in this context.

4. Multiple terms

Both in scholarly and within everyday contexts we hear about *contradictions*, *antinomies*, *paradoxes*, *riddles*, *dilemmas* and even *puzzles*. Particularly since Immanuel Kant explained apparently stringent proofs, in the *Transcendental Dialectics* (second Book, second Chapter) of his *Critique of Pure Reason* (CPR), for a set of four *theses* and *antitheses*, under the category of *antinomies*, the latter term became common knowledge in subsequent philosophical literature and reflection.

The different terms used in this context are seen in the following arbitrarily selected references. In 1849 a posthumously published book from the German philosopher-mathematician, Bernard Bolzano, appeared with the title: *Pardoxien des Unendlichen* (*Paradoxes of the Infinite*). Russell speaks about contradiction, paradox and antinomy (Russell, 1956:144, 190 ff.). E. Teensma published a book with the title: *The Paradoxes* (1969), while E.P. Northrop wrote a work on *Riddles in Mathematics* (1944 – Penguin version 1964). In a work with the title "Dilemmas" Gilbert Ryle discusses Achilles and the tortoise as an example (Ryle, 1953:50-69). Sometimes the word "puzzles" is used – Martin Gardner employed it in a (Pelican) book on *Mathematical Puzzles and Diversions* (1968). Recently Michael Clark published another work: *Paradoxes from A to Z* (2002).

By and large the legacy of classical and modern logic as well as philosophy in general did not develop a *systematic analysis* of the *differences* between these diverse designations. In general *contradictions*, *antinomies* and *paradoxes* are used interchangeably. For example, when Fraenkel et al discuss the known contradictions and paradoxes they are called *antinomies*.[15]

We commence by considering the nature of *normative contraries* in order to highlight the *normativity* of *logicality*.

5. Normative contraries

The scope of the logical principles of identity and (non-)contradiction applies to the human ability to *conceive* and to *argue*. Copi states a generally accepted conviction when he says that the "principle of contradiction asserts that *no statement can be both true and false*" (Copi, 1994:372).

The classical example of an *illogical concept* stems from Immanuel Kant and concerns a "square circle" (see Kant 1783:341; § 52b). Establishing that this concept is *illogical* entails that a *normative standard* has been applied and that the said concept does not *conform* to the requirement of *ought to be* inherent in this normative standard. It is contradictory not to distinguish between a square and a circle, or, to put it differently, confusing two *spatial*

[15] He distinguishes between logical antinomies (those of Russell, Cantor and Burali-Forty) and semantic antinomies (those of Richard, Grelling and The Liar – 1973:5-12).

figures violates the demands for *identifying* and *distinguishing* properly: a square is a square (logically correct *identification*) and a square is not a non-square (such as a circle – logically correct *distinguishing*).

Thinking in a logically antinormative way, i.e., thinking *illogically*, remains bound to the structure of *logicality* and does not turn into something *a-logical* (non-logical), such as the economic, the moral or the jural. These (non-logical) facets of our experience are said to be *a-logical* – but they are not *illogical*. In fact they also have room for contraries *similar* (or analogous) to the contrary between logical and illogical, namely *economic* and *uneconomic*, *moral* and *immoral* and *legal* and *illegal*. Although the history of humankind tells the story of different assessments of what may count as economically, morally or legally proper behaviour, one can hardly deny the normativity inherent in these dimensions as it is manifested in the mentioned contraries. The logical contrary actually lies at the *foundation* of all these other instances of normative contraries – the latter *analogically reflect* within their own domains the meaning of logical analysis (identification and distinguishing).

Yet the phenomenon of contradictions does not tell the full story. Let us return to the confusion of spatial figures present in the illogical concept of a *square circle* and compare it with something more drastic, namely the attempt to explain whatever there is purely in *spatial terms*. This happened in Greek philosophy after the discovery of *irrational numbers* – an event that led to the *geometrization* of Greek mathematics (after its initial Pythagorean *arithmetization*). This alteration within mathematics inspired the development of a speculative metaphysics in which material entities were exclusively characterized in terms of their *spatiality*. The result was that the Greeks did not contemplate an *empty space*. According to their mature understanding space does not exist, only *place*. Place is a property exclusively attributed to a concrete body. In the absence of a body, there is no subject for the predicate *place*. From this it naturally follows that an "empty place" is the place of nothing – in other words, it is no place at all! But what then do we have to say about the *movement* of a material body? Will it be possible to assert that motion is a "change of place"? Surely, given the identification of a *body* with its *place*, motion would then be an impossibility – at least when a body is supposed to be the subject of motion – for a *change of place* will amount to a change of "essence"! In terms of such a metaphysics of space the introduction (or: "definition") of motion yields to the *contradiction* that a body can move if and only if it cannot move – which actually approximates the arguments of Zeno against multiplicity and movement alluded to above.

The attempt to explain whatever there is exclusively in *spatial terms*, is nothing but pursuing the aim of *reducing* everything to space (similar to the Pythagorean assertion "everything is number"). But, as we have noted, the first "victim" of such a spatially oriented reductionism is found in the function of *motion* (the school of Parmenides). In order to acquire a better handle on this problem we first have to pay attention to the underlying problem of the "coherence of irreducibles" – which is just a different formulation of the basic philosophical problem of *unity* and *diversity*. Russell refers to Hegel in respect of the difference between a continuous magnitude (wholeness) and a discrete magnitude – as "*different*" instances of the "class-concept" and then proceeds by saying that he "strongly" holds "that this opposition of identity and diversity in a collection constitutes a fundamental problem of Logic – perhaps even *the* fundamental problem of philosophy" (Russell, 1956:346).

6. Uniqueness and coherence

The claim that a *reduction* is unwarranted implicitly presupposes the conviction that there exist "*irreducibles*" (and: *primitives*).[16] Typical (reductionsistic) **all**-claims, such as the mentioned Pythagorean conviction that *everything is number*, the statement that *everything is physical* (materialism) or that *everything is interpretation* (postmodernism), challenge the idea of *uniqueness* and *irreducibility*.[17] All-claims like these are mainly *monistic* in nature – in the sense that they want to find *one* single, all-encompassing perspective or principle of explanation capable of accounting for the entire diversity manifest in our experience of the universe.[18] An argument in favour of the acknowledgement of irreducibility – as one side of the coin (with the mutual coherence of what is unique as the other side) – ought to show that an unwarranted *reductionism* gets entangled in unsolved problems (normally referred to as contradictions, paradoxes or antinomies).[19]

Ernst Cassirer, the philosopher from the neo-Kantian Marburg school (perhaps best known for his *Philosophy of Symbolic Forms*), is also quite explicit in this regard when he claims that a critical analysis of knowledge, in order to side-step a *regressus in infinitum*, has to accept certain *basic functions* which are not capable of being "deduced" and which are not in need of a deduction (Cassirer, 1957:73).

Every single academic discipline has to employ such basic (and irreducible) concepts. Precisely because these concepts are *basic* they cannot be *defined*. Various disciplines acknowledge this state of affairs by explicitly introducing "primitive terms."

For example, in Zermelo-Fraenkel set theory first order predicate calculus is assumed and on that basis it introduces as an *undefined term* the specific set-theoretical primitive binary predicate ε which is called the *membership relation* (Fraenkel et al., 1973:23).[20] Bertrand Russell states: "The relation of whole and part is, it would seem, an indefinable and ultimate relation" (Russell, 1956:138). For the sake of an economy of primitive terms even the term

---

[16] Salmon refers to "primitive terms" in "pure mathematics" (Salmon, 2001:32).

[17] P. Hoyningen-Huene writes about irreducibility in the context of *complementarity*: "But this property is just *identical with the epistemological non-reducibility* of these features. In other words: in order to establish that in a certain situation complementarity prevails, it has to be shown that the features involved are irreducible to each other" (see his *Theory of Antireductionist Arguments: The Bohr Case Study*, in: Agazzi, 1991:67). Weingartner refers to *primitive terms*: "Term (concept, idea) *t is* scientifically analyzable iff it is reducible to primitive terms. t is reducible to primitive terms iff t is itself a primitive term or it can be traced back to primitive terms by a chain of definitions" (see his article on *Reductionism and Reduction in Logic and in Mathematics*, in: Agazzi, 1991:124).

[18] With reference to Einstein's thirty year search for a unified field theory, Brian Greene, a specialist in the theory of super strings, believes that physicists will find a framework fitting their insights into a "seamless whole," a "single theory that, in principle, is capable of describing all phenomena" (Greene, 2003:viii). He introduces Super String theory as the "Unified Theory of Everything" (Greene, 2003:15; see also pp.364-370, 385-386).

[19] Compare, for example, the remark of Weingartner regarding the failure of logicism: "Logicism is an example of reduction which was as a whole unsuccessful" (Weingartner, 1991:130).

[20] This approach follows a general pattern: an axiomatic theory (axiomatic theories of logic excluded) "is constructed by adding to a certain basic discipline – usually some system of logic (with or without a set theory) but sometimes also a system of arithmetic – new terms and axioms, the specific undefined terms and axioms under consideration" (Fraenkel et al., 1973:18).

identity need not to be taken as primitive, since in the approach to axiomatic set theory explained by Lemmon it could be defined by the use of the axiom of extensionality (see Lemmon, 1968:124). In general linguistics the term "meaning" is primitive; in kinematics the term constancy ("invariance" – normally associated with a *uniform movement*) is primitive; in the discipline of law the term "retribution" is primitive. When Russell discusses the mathematical meaning of constants and variables he says that "constancy of form must be taken as a primitive idea" (Russell, 1956:89) – and so on.

The upshot of this is that the acquisition of *concepts* and the formulation of *definitions* ultimately rest upon *primitive terms* – they are not defined and they cannot be defined.

The question *how does one know these indefinable* (*primitive*) *terms*? is an *epistemological issue* which is rooted in *philosophical assumptions* about the world in which we live and therefore it involves *ontological commitments*.[21] However, since a discussion of this issue exceeds our present context, we return to the *space metaphysics* of the school of Parmenides.

7. Zeno's paradoxes – A different understanding of antinomies

In the school of Parmenides Zeno argued against *multiplicity* and *movement* by assuming an absolutely static *being*. The well-known reasoning regarding the *flying arrow*, *Achilles and the tortoise* as well as what is known as the *dichotomy paradox* is reported by Aristotle in his *Physics* (239 b 5 ff.). Aristotle's own approach proceeds from the assumption that "it is impossible for anything continuous to be composed of indivisible parts" (*Phys*. 232 a 23 ff.) and that "everything continuous is divisible into an infinite number of parts" (*Phys*. 238 a 22). The basis of the first paradox is found in *divisibility* and that of the third in the successive addition of two distinct series of *diminishing magnitudes* both converging to the same limit – but given the different points of departure the first one is *nested* within the second one. It looks as if the Aristotelian account of paradoxes one and three collapsed movement *ab initio* into an issue of *spatial divisibility* and the addition of *diminishing magnitudes* (therefore both cases are related to what modern mathematics calls the *density* of spatial continuity), whereas the account of the paradox of the *flying arrow* seems to *allow* for movement to begin with and then "freezes" it into distinct "moments" of time – as if something moving from "moment" to "moment" has a *definitive place* in space.[22]

It may be worthwhile to mention the fourth *Fragment* of Zeno known to us, for it explicitly starts by *granting* the reality of movement and then it proceeds with an argument launched from the perspective of the static nature of space in order to rule out the possibility of movement: "That which moves neither moves in the space it occupies, nor in the space it does not occupy" (Diels-Kranz B Fr.4). This certainly explains why Grünbaum distinguishes between the "paradoxes of *extension*" and the "paradoxes of *motion*" (Grünbaum, 1967:3) –

[21] Contemporary formal ontologies intersect with certain basic ideas of a non-reductionist ontology but unfortunately did not develop a theory inter-modal coherences between the various modal aspects of reality, causing this trend also to by-pass the importance of *modal universality*. However these issues cannot be treated in this context.

[22] If "being at one place" means "being at rest," and if this is "every moment" the case with the "flying arrow," then the arrow is actually only "at rest" – i.e., it is not moving at all. Of course, modern kinematics holds that "rest" is a (relative) state of motion. But without reference to some or other system one cannot speak about the motion of a specific kinematical subject (see Stafleu, 1980:81, 83-84).

but he explicitly distances himself from the authenticity of the historical sources by restricting himself to the legacy of Zeno in "the present-day philosophy of science" (Grünbaum, 1967:4).[23]

Unfortunately, in his encompassing treatment of Zeno's paradoxes, Grünbaum does not anywhere in his analysis pay attention to the problem of *uniqueness* and *irreducibility* (with the accompanying problem of *primitive terms* and *indefinability*). Yet this does not mean that in his mode of argumentation there is not an *implicit* acceptance of the uniqueness of the core meaning of motion. In his work on space and time, for example, he discusses Einstein's "principle of the constancy of the speed of light" (Grünbaum, 1974:376) and points out that it concerns an upper limit that is only realized in a vacuum (Grünbaum, 1974:377). Einstein's theory of relativity proceeds from the hypothesis that one singular light signal has a constant velocity (in respect of all possible moving systems) without necessarily claiming that such a signal actually exists. Stafleu remarks: "The empirically established fact that the velocity of light satisfies the hypothesis is comparatively irrelevant" (Stafleu, 1980:89).

The postulate concerning the *constancy* of the velocity of light explores an insight already advanced by Galileo and his predecessors in the meaning of *inertia*. Galileo reversed the Aristotelian view that whatever moves requires a *causing force* in order to continue its movement. He did that with the aid of a thought-experiment concerning a body that is in motion on a plane which is extended into the infinite – and from this experiment he derives the law of inertia. The question is whether or not the meaning of uniform motion is *primitive* and *unique* in the sense that it ought to be distinguished both from the static meaning of *space* and the dynamic meaning of *physical energy-operation* (*causes* and *effects*)? Since physics always deals with *dynamic forces* operative in the interplay of energy transformation, and since a (constant) *uniform motion* can indeed be envisaged without making an appeal to a *cause* (causal *force*), it is clear that something unique and irreducible is here at stake. It entails that in a *functional* sense movement is something *original*. Whatever moves will continue its (uniform) motion endlessly. Motion is not in need of a cause – only a *change* of motion needs a cause. Both *acceleration* and *deceleration* require an energy-input (i.e., a physical cause).

Although modern physics was dominated by a mechanistic inclination until the end of the 19th century, it eventually realized that a purely *kinematical* explanation of physical phenomena is untenable. A consistent mechanistic approach, such as that still found in the posthumously published work by Heinrich Hertz (the German physicist who did experimental work about electromagnetic waves more than a hundred years ago) on "The Principles of Mechanics developed in a New Context," demonstrates the dilemma of reductionism, for his aim to restrict physics to *number*, *space* and *movement* only (re-presented by the concepts *mass*, *space* and *time*), led him to reject the (physical) concept *force*. He claimed that the concept of force is something *inherently antinomic* (cf. Katscher, 1970:329).

[23] This position closely imitates a similar disclaimer found in Russell's treatment: "Not being a Greek scholar, I pretend to no first-hand authority as to what Zeno really did say or mean. The form of his four arguments which I shall employ is derived from the interesting article of M. Noël, 'Le mouvemont et les arguments de Zénon d'Elée,' *Revue de Métaphisique et de Morale*, Vol.I, pp.107-125. These arguments are in any case well worthy of consideration, and as they are, to me, merely a text for discussion, their historical correctness is of little importance" (Russell, 1956:348, note).

But as soon as the dynamic physical sense of *force* is acknowledged, as it has been done by 20th century physics, what Hertz deemed to be an antinomy turns out to require an acknowledgement of another unique and irreducible *functional mode*[24] of reality (in addition to number, space and movement), namely the *physical*.[25]

8. The inter-modal meaning of an "antinomy"

But the kinematic function of uniform motion also differs from the functional modes of number and space. The above-mentioned B Fragment 4 of Zeno actually demonstrates that the unique and irreducible meaning of uniform flow (motion) cannot be captured purely in spatial terms, except in an *antinomic* way. In order to explain what this means we first have to alter the meaning of the term "antinomy." The most obvious way to accomplish this is to allude to its literal sense, which is intended to designate a *clash of laws*: *anti* = against, and *nomos* = law. The attempt to explain movement in terms of space results in a (theoretical) conflict between *kinematic laws of motion* and *spatial laws*.

Such a conflict or clash between distinct functional (modal) laws indeed demonstrates the nature of a *theoretical antinomy*. After all, in the actual world these two modes of being are unique *and* are mutually cohering.[26] Yet the attempt to *reduce* one unique mode to another one invariably results in genuine (theoretical) *antinomies*.

> In this sense antinomies therefore concern an *inter-modal* confusion, i.e., a lack of distinguishing properly between *different* modes, functions or aspects of reality.

Furthermore, an *antinomy* always entails a *logical contradiction*, whereas a contradiction does not *necessarily* presuppose an antinomy. The above-mentioned illogical concept of a "square circle" exemplifies an instance where two *spatial* figures are not properly identified and distinguished. In other words, a *contradiction* such as this one has an *intra-modal* character since its confusion relates to givens *within* the modal-functional boundaries of one aspect or function only.

---

[24] The aspects of reality are not brought to sight by asking questions about the concrete *what* of entities and processes, for these aspects represent the *way* (*manner*) in which such entities and processes *function* – i.e., they relate to the *how* of concrete entities and processes. From Latin we have inherited expressions such as *modus operandi* and *modus vivendi* in which the *how* is represented by the term 'modus'. An aspect is therefore to be seen as a specific (unique) *mode* which, in a general sense, is a *modus quo*, a *mode of being*. As an equivalent for referring to *facets*, *aspects* or *functions*, one can therefore also speak about *modalities* or *modal aspects* or *modal functions*. Already in 1910 Cassirer highlighted the importance of this distinction between *entity* ('substance') and *function* (see Cassirer, 1953). When entities and processes are resolved into functions we meet *functionalism*; and when modal functions are treated as entities they are *reified*. An in-depth analysis of the decisive role of functionalism in the development of the modern natural sciences is found in an important work of Rombach (see Rombach, 1965-66).

[25] It is therefore understandable that Janich distinguishes between *phoronomic* and *dynamic* statements (Janich, 1975:68-69). Also Einstein highlights the difference between the mechanical point of view (where all processes are reversible) and thermodynamics (the most general physical discipline where courses of events are irreversible) (Einstein, 1959:42-43).

[26] The "path" of a movement highlights the undeniable interconnection between motion and space. The serial order of events reveals a connection with the numerical meaning of succession. We shall briefly return to this perspective below.

Within the dialectical tradition of Marxism this difference between an (inter-modal) antinomy and an (intra-modal) contradiction surfaced strikingly. In following the dialectical-materialistic conception of Engels the Marxist physicist Hörz talks about a "dialectical *Widerspruch* (antinomy)." One can say that a moving body (i.e., a body involved in a change of place) at the same time is and is not at a specific place. According to him this is the "dialectical antinomy (*Widerspruch*)" of *change of place*. But a formulation precluding every *logical contradiction* runs as follows: "as the result of movement a body finds itself at a specific place and with regard to the movement itself the body does not find itself at a specific place" (Hörz, 1967:58). Hörz's designation of this situation as the "dialectical antinomy (*Widerspruch*)" of change of place is also found in the thought of Hegel:

> When we speak about movement as such, we say: a body is at a specific place and then moves to a different place. While it is moving, it is no longer at the first place, but also not yet at the second. When it is at any one of the two it is at rest. When it is said that it is between both, this is not said for between both there is also a place and therefore the same problem occurs. But movement means: to be at this place and at the same time not be there; this is the continuity of space and time and this is it that makes possible motion (Hegel, 1833:337 ff.).[27]

Hegel and Hörz distinguish the aspect of movement (when a body does *not find itself at a specific place*) and the spatial aspect (the position of a body, when *it has a definite place*). In other words, they are actually making an appeal to *two different aspects* in order to side-step the accusation of a *logical contradiction* but at once they appreciate the (inter-modal) distinction that they make as a "dialectical Widerspruch."

9. Different aspects involved in Zeno's 'paradoxes'

In the case of Zeno's arguments different *modal aspects* are at stake. The theoretical attempt to reduce the meaning of movement to that of space is *antinomic* – but this antinomy shows itself in an implied *logical contradiction*: For example, as we have seen, in his fourth Fragment Zeno grants movement to begin with, but then concludes that movement is impossible. Something therefore can move if and only if it cannot move. This logical *contradiction* is the outcome of his *antinomic* attempt to reduce the original (primitive and indefinable meaning of) motion to *static spatial extension*. In other words, since an antinomy results from an attempt to reduce what is irreducible, it is always *inter-modal* in nature and *simultaneously* it expresses itself (intra-modally) within the logical mode as a logical *contradiction*.[28]

Of course this perspective does not eliminate a meaningful analysis of the numerical and spatial *aspects* of a (concretely) moving body (compare many of the arguments found in Grünbaum, 1967). Material (physical) entities and processes display various functional

---

[27] "Wenn wir von der Bewegung überhaupt sprechen, so sagen wir: Der Körper ist an einem Orte, und dann geht er an einen anderen Ort. Indem er sich bewegt, ist er nicht mehr am ersten, aber auch noch nicht am zweiten; ist er an einem von beiden, so ruht er. Sagt man, er sey zwischen beiden, so ist dieß nicht gesagt; denn zwischen beiden ist er auch an einem Orte, es ist also diesselbe Schwierigkeit hier vorhanden. Bewegen heist aber: an diesem Orte seyn, und zugliech nicht; dies ist die Kontinuität des Raums und der Zeit, und diese ist es, welche die Bewegung erst möglich macht."

[28] Stafleu is correct in suggesting that one can interpret Zeno's arguments "against" motion as a demonstration that motion cannot be explained by *numerical* and *spatial relations* (see Stafleu, 1987:61).

properties without being *exhausted* in their concrete many-sided existence by any single one of these modes of being (which are at once *modes of explanation*).

The solution of Zeno's problem of Achilles and the tortoise is certainly *not* given by the view that Zeno understood the metaphor of the "moving observer" in a *literal* way, as it is claimed by Lakoff and Johnson (see Lakoff and Johnson, 1999:157-158), since what is ultimately shown by this 'antinomy' is that it is impossible to *define* uniform motion (exhaustively or exclusively) in *spatial terms*.

The acknowledgement of *irreducible* modes (functions or aspects) of reality is intimately connected with the idea of the *identity* of an entity which has a concrete and many-sided functioning within each one of these aspects without ever totally being *absorbed* by any one of them. Consider the four most basic functions of an atom. Besides the arithmetic function which an atom has (think about the *atomic number*), it also clearly has a spatial function since it is characterized by a particular *spatial configuration* – the *nucleus* of an atom with *peripheral* electron systems. According to wave mechanics, we find quantified wave movements around the nucleus of the atom – the *kinematic* function of the atom. Already in 1911, in Rutherford's atomic theory, the hypothesis was posed that atoms consist of a positively charged nucleus and negatively charged particles which move around it (a view which was inspired by the nature of a planetary system). In the following year (1912), Niels Bohr set up a new theory which contained two important ideas: (i) the electrons move only in a limited number of discrete orbits around the nucleus and (ii) when an electron moves from an orbit with a high energy content to one with a low energy content, electromagnetic radiation occurs. Therefore an atom is a micro-totality is qualified by its physical function of energy-operation.

The (relative) motion of a material entity concerns the *ontic* functioning of such an entity within the *kinematical aspect of reality*. But the motion of a physical entity pre-supposes the *spatial* function of physical entities – just think about the *path* of movement – as well as the *numerical* function – normally evinced when the *measure* of motion acquires a numerical specification (designated by establishing its *speed*).[29] Although Salmon is correct in stating that Zeno, "[I]n his attempt to demonstrate the impossibility of plurality, motion, and change" points at "problems lying at the very heart of our concepts of space, time, motion, continuity, and infinity" (2001:5), none of the selections contained in the work edited by him on *Zeno's Paradoxes* enters into a discussion of the mutual *ireducibility* of these functions of reality (namely the numerical, spatial, kinematical and the physical). Neither does any one of them

---

[29] The notion of 'speed' in phoronomy is similar to the notion of 'magnitude' in metric spaces. The classical 'definition' of a *line* as the *shortest distance* between two points is mistaken. Hilbert rather speaks about the straight line as the shortest *connection* between two points (Hilbert, 1970:302 – problem 4 of his classical 23 mathematical problems presented at the International Congress held in Paris in 1900). In this work *Grundlagen der Geometrie* (1899), Hilbert abstracts from the contents of his axioms and proceeds upon the basis of three undefined terms: "point," "lies on," and "line." Within the functional structure of (a metrical) space *distance* (i.e., one dimensional extension) is the (numerical) *measure* of the extension of a line, the continuous extension of the line itself is *primitive*, just as specifying the *speed* of a moving body requires a *measure* of movement while movement itself remains a primitive. In both cases we may speak of the fact that the quantitative meaning of number is *analogically* reflected within the aspects of space and movement. Physics designates the numerical analogy within the function of energy-operation with the term *mass*.

consider the *scope* and *limitations* of *different* modes of explanation in respect of the *uniqueness* (primitive meaning) of *motion*. However, it does sometimes happen that an author highlights different aspects of an event. For example, when Max Black summarizes his argument by saying: "But Achilles is not called upon to do the logically impossible; the illusion that he must do so is created by our failure to hold separate the finite number of real things that the runner has to accomplish and the infinite series of numbers by which we describe what he actually does" (Black, in Salmon, 2001:80), then he actually distinguishes between different aspects through which we can approach such an event – namely the numerical and the physical.

Dividing Zeno's arguments into (i) the paradox of plurality and (ii) the paradoxes of motion, may seem to juxtapose two *disconnected* areas of reflection, but, as Salmon correctly remarks, they are not unrelated (Salmon, 2001:vi). He is also justified to hold at the same time that (i) is more basic than (ii). He writes: "we shall see that the paradox of plurality is logically more basic than the paradoxes of motion" (Salmon, 2001:7). However, we may want to expand his qualifier "logically" to read "*onto*-logically" – because numerical considerations in an ontic sense are *foundational* to an understanding of the meaning of *space* and *motion*.[30]

Even in respect of (ii) most philosophers and mathematicians focus on the problem of an *infinity* of points or intervals that ought to be passed / traversed in a finite time. A constantly recurring consideration concerns the "logical impossibility" to "complete" an endless (infinite) series. The "mathematical solution" of this problem is apparently found in the observation that, in terms of an arithmetical perspective (mode of explanation), that the successive partial sums of the series $1 + 1/2 + 1/2^2 + 1/2^3 + \ldots 1 - 1/2^n$ ($n = 1, 2, 3, \ldots$) do not grow beyond all limits but converge towards 1 (see Weyl, 1966:61). But Weyl immediately adds the remark that if the stretch of length 1 really consists of infinitely many partial stretches of length 1/2, 1/4, 1/8, … as *separated* wholes, then it would contradict the essence of infinity as the "Unvollendbaren" (what *cannot be completed*) that Achilles finally had to pass through (Weyl, 1966:61).[31] In other words, as soon as the idea of a *completed totality* is combined with the infinite – for instance in speaking about an *infinite whole* or an *infinite totality* – then the true nature of *infinity* is contradicted (in this case in the claim that Achilles in the end completely passed through that which cannot be completed).

Interestingly Max Black argues from an understanding of infinity as "uncompleted" when he says that what is meant "by the assertion that the sum of the infinite series 100 + 10 + 1 + 1/10 + 1/100 + is 111 1/9" and that this "does not mean, as the naive might suppose, that mathematicians have succeeded in adding together an infinite number of terms" (Black, in Salmon, 2001:70). Black and Wisdom both criticize the "mathematical solution" (see Wisdom

---

30 In passing it should be mentioned that prominent 20th century mathematicians, such as Gödel and Bernays, argued for the ontic status of the numerical aspect of reality (see Wang, 1988:304 and Bernays, 1976:45, 122).

31 "Die Unmöglichkeit, das Kontinuum als ein starres Sein zu fassen, kann nicht prägnanter formuliert werden als durch das bekannte Paradoxon des Zenon von dem Wettlauf zwischen Achilleus und der Schildkröte. Der Hinweis darauf, daß die sukzessiven Partialsummen der Reihe $1/2 + 1/2^2 + 1/2^3 + \ldots$, $1 - 1/2^n$ ($n = 1, 2, 3, \ldots$) nicht über alle Grenzen wachsen, sondern gegen 1 konvergieren, durch den man heute das Paradoxon zu erledigen meint, ist gewiß eine wichtige, zur Sache gehörige und aufklärende Bemerkung. Wenn aber die Strecke von der Länge 1 wirklich aus unendlich vielen Teilstrecken von der Länge 1/2, 1/4, 1/8, … als 'abgehackten' Ganzen besteht, so widerstreitet es dem Wesen des Unendlichen, des 'Unvollendbaren', daß Achilleus sie alle schließlich durchlaufen hat."

in Salmon, 2001:83). Wisdom concludes: “The idea that the limit of an infinite series is attainable is a mistake. If a physical action is interpreted by means of an infinite series, then the completion of the action is self-contradictory” (Wisdom in Salmon, 2001:87).[32]

Owen points out that one “beneficial result” of Zeno's “arguments (on this familiar account),” was “to compel mathematicians to distinguish arithmetic from geometry” (Owen in Salmon, 2001:139). But Owen questions the idea of an *infinite divisibility* by posing the question whether or not such a division ever could be (could have been) *completed*?[33]

Likewise, the arguments found in Ryle's *Dilemmas* are based on the same assumption of the “uncompleted infinite,” although in addition he does introduce into his discussion the *whole-parts relation* (with reference to the classical slogan that the “the whole is more than the sum of its parts”). He says that the question “how many parts have been cut off from an object?” must be distinguished from the question “in how many parts did you divide it?” (Ryle, 1977:61). The first point proceeds from a notion of wholeness containing all its (finite) parts, whereas the second point reverts to the perspective and explores an on-going process of *division*.

This distinction actually imitates B Fragment 3 of Zeno in which he argues as follows (in the translation of Guthrie): “if there is a plurality, it must contain both a finite and an infinite number of components: finite, because they must be neither more nor less than they are; infinite, because if they are separate at all, then however close together they are, there will always be others between them, and yet others between those, *ad infinitum*” (Guthrie, 1980:90-91). Therefore assuming a plurality leads to the contradictory conclusion that it contains “a finite and an infinite number of components.” But given the fact that Parmenides and his school, as an effect of the discovery of irrational numbers, switched from an *arithmetical* mode of explanation to a *spatial* one, we may look at the *spatial* whole-parts relation in order to understand what is here at stake.[34] If the plurality of the first argument refers to a perspective from the parts to the whole, then the number of these parts must be limited while at once they constitute the world as a whole (the universe). By contrast, if the argument proceeds from the whole to the parts, the *infinite divisibility* operative in this move entails that “there will always be others between them” and so on indefinitely. Fränkel explicitly employs the whole-parts relation to explain the meaning of B Fragment 3 of Zeno (see Fränkel, 1968:430).[35] Perhaps Zeno's B Fragment 3 could be seen as the first ‘two-directional’ discussion of the spatial whole-parts relation.[36]

---

[32] The standard mathematical formulation of the nature of a limit does not hold “that the limit of an infinite series is attainable” (see note 36 below).

[33] “For suppose we ask whether such a division could be (theoretically, at least) continued indefinitely: whether any division can be followed by a sub-division, and so on, through an infinite number of steps. Let us say, to begin with, (A) that it does have an infinite number of steps. Then could such a division nevertheless ever be (or ever have been) completed?” (Owen, in Salmon, 2001:142).

[34] Whatever is continuously extended is a coherent whole in the sense that all its parts are connected – therefore the terms *coherence* and *connectedness* are mere synonyms for the terms *wholeness*, *totality* and *continuity*. A whole (or totality) contains all its parts. Paul Bernays affirms that ‘wholeness’, i.e., the *totality-character* of spatial continuity, will resist a “perfect arithmetization of the continuum” (see Bernays, 1976:74).

[35] Guthrie has a positive appreciation of this article of Fränkel. He refers to an English translation of it: *Zeno of Elea's Attacks on Plurality* (see Guthrie, 1980:88 ff., 512).

[36] In response to the phrase “If they are just as many as they are, they will be finite in number” Russell states: “This phrase is not very clear, but it is plain that it assumes the impossibility of definite infinite

10. Confusing the nature of infinity

The mere fact that so many scholars involved in discussing Zeno's paradoxes argue in terms of the 'uncompleted' nature of infinity demonstrates that they probably do not have a clear understanding of the difference between what traditionally is called the *potential infinite* and the *actual infinite*. Even when a competent scholar like Russell – who certainly has a sound understanding of the difference between these two kinds of infinity – sets out to present a historical presentation of the problem of infinity (see Salmon, 2001:45-68), one does not encounter an exploration of these two kinds of infinity.

Before Gregor of Rimini lectured on the *Sentences* at Paris in 1344, the objections to the actual infinite formulated by Fitzralph resulted in a rejection of both the potential and the actual infinite. In his argumentation he speaks about the *simultaneous infinite* (*simultanes infinitum*). In their discussion of the problem of infinity (during the 14th century) Henry of Harclay and others contemplated the difference between what was labeled the *infinitum successivum* and *infinitum simultaneum* (cf. Maier, 1964:77-79). These designations make and appeal to our most basic intuitions of number (succession) and space (at once). Therefore, it is recommendable rather to distinguish between the *successive infinite* and the *at once infinite* than between the potential infinite and the actual infinite.

The successive infinite brings to expression the most basic meaning of infinity – in the literal sense of without an end.[37] It is determined by the primitive quantitative meaning of succession: one, another one, and so on indefinitely. The reason why the phrase actual infinity ought to be replaced by the expression *the at once infinite* is found in the inter-connection between number and space. The awareness of *simultaneity* (*at once*) is the correlate of any spatially extended figure (subject) for if the different *parts* of a spatial figure (as a whole) are not given *at once* the spatial figure itself is not given. Any understanding of infinity in terms of the idea of *infinite wholes* or *infinite totalities* is therefore dependent upon 'borrowing' a crucial element of space. In fact it seems to be impossible to develop *set theory* without "borrowing" key-elements from our basic intuition of space, in particular the mentioned spatial order of *at once* and its correlate: *wholeness* or *totality*. Hao Wang mentions that Gödel speaks about sets as being 'quasi-spatial' and then says: "I am not sure whether he would say the same thing about numbers" (Wang, 1988:202).

There is no *constructive transition* between the successive infinite and the at once infinite (see Wolff, 1971). But a full-blown treatment of the real numbers, transcending the approximative approach found in intuitionistic mathematics based upon the *denseness* of the rational numbers, does require the employment of the at once infinite. Even though it seems as if the limit concept can be formulated merely in terms of the "endlessness" of the successive infinite, the requirement of the numerical nature of an arbitrary limit is dependent upon the use of the real numbers while an account of the real numbers, in turn, requires the use of the at once infinite. It is not possible to define (irrational) real numbers with the aid of converging sequences of rational numbers, because the classical definition of a *limit* (stemming from Cantor and Heine) presupposes that whatever functions as a

---

numbers" (Russell, in Salmon, 2001:47). Clearly Russell did not explore a "two-directional" use of the spatial whole-parts relation suggested above.

[37] Russell holds that an *endless series* neither has a beginning nor an end (Russell, 1956:297).

limit in advance already must be a *number* – explaining why it cannot be 'created' through a 'convergence process.' In 1883 Cantor expressly rejected this circle in the definition of irrational real numbers (going mainly back to Cauchy in 1921 – see Cantor, 1962:187). The eventual description of a limit still found in textbooks today was only given in 1872 by E. Heine, who was a student of Karl Weierstrass.[38] In 1887 Cantor pointed out that the core of the ideas in Heine's article was borrowed from him (Cantor, 1962:385).

What is important to realize is that as soon as the *successive infinite meaning* of the numerical order of succession is deepened through the spatial order of *at once*, the meaning of successive infinity is enriched through its connection with the meaning of *simultaneity*. Any *successive* sequence of numbers could then, under the guidance of this deepened hypothesis, be viewed *as if* its elements are given *all at once*. The deepened and disclosed meaning of the infinite (under the guidance of an insight into the *spatial* meaning of *simultaneity*) encountered here, justifies our choice to designate it as the *at once infinite*. Under the guidance of this hypothesis the initial successive infinite sequences of natural numbers, integers and rational numbers could be viewed as actual infinities, i.e., as *infinite totalities given at once*.[39]

Both Salmon and Grünbaum are sufficiently acquainted with (Cantor's) Set Theory (see the Appendix in Salmon, 2001:251-268; and Grünbaum, 1967)[40] and they are justified in employing set theory in an understanding of the numerical side of physical movement (even though they do not realize that set theory is not a purely numerical theory but a *spatially deepened* arithmetical theory).[41]

Similar to those who argue against the "completed" infinite by applying the standard of the successive infinite, already Immanuel Kant thought that he can bring his first antinomy[42] to a 'solution' by taking recourse merely to 'endlessness' (i.e., the *successive infinite*). In his remarks about the first thesis of the second antinomy[43] Kant states that space is not a

---

[38] In general a number l is called the limit of the sequence (x_n), when for an arbitrary $0 < \varepsilon$ a natural number n_0 exists such that $|x_n - l| < \varepsilon$ for all $n \geq n_0$. (See Heine, 1872:178,182).

[39] Lorenzen, although rejecting the at once infinite in his constructive logic and mathematics, does provide a lucid description of the classical theory of real numbers in its dependence upon the use of the *at once infinite*: "One imagines much rather the real numbers as all at once actually present – even every real number is thus represented as an infinite decimal fraction, as if the infinitely many figures (Ziffern) existed all at once (alle auf einmal existierten)" (Lorenzen, 1972:163).

[40] That they did not discern the *circularity* present in Cantor's attempt to develop a supposedly purely arithmetical understanding of a continuum of points cannot be analyzed in the present context.

[41] Grünbaum's sympathetic quotation from Weyl (in Salmon, 2001:175) raises the suspicion that he is not aware of the fact that Weyl, in rejecting the at once infinite, also rejects the Cantorean view employed by him in his attempt to develop a "consistent conception of the extended linear continuum as an aggregate of unextended elements" (see Grünbaum, 1952). Just compare the following words of Weyl: "In agreement with intuition Brouwer sees the essence of the continuum not in the relation of the element to the set, but in that of the part to the whole" (Weyl, 1966:74).

[42] "The world has a beginning in time, and is limited also with regard to space" versus "The world has no beginning and no limits in space, but is infinite, in respect both to time and space" (cf. Kant, 1787:454).

[43] The two sides of this second 'antinomy' corresponds to the "two-directional" nature of the spatial whole-parts relation referred to above – Zeno B Fragment 3 and Ryle (in connection with the antithesis Kant raises the issue of *infinite divisibility*): "Every compound substance in the world consists of simple parts, and nothing exists anywhere but the simple, or what is composed of it" versus "No compound thing in the world consists of simple parts, and there exists nowhere in the world anything simple" (Kant, 1787:462 ff., 467).

compositum (in reaction to atomistic views of space), since in determining its parts space is a *totum*.[44] He therefore does have an eye for the totality character of spatial continuity.

Clearly those who attempt to liberate themselves from the impasse in Zeno's arguments by pointing at the 'impossibility' of the 'completion' of endlessness (including the untenability of "infinity machines"),[45] have to account for the difference between the potential and actual infinite (the successive and the at once infinite). The assumption in Zeno's bisection paradox seems to be that it is *logically absurd* to argue that *all* of an infinite number of tasks have been (can be) completed (in a finite time). Many authors did not come to terms with the endlessness of the successive infinite since they constantly alluded to a "last element." Likewise it is self-contradictory to speak of *an infinite division* of a continuum if this entails a "last division" – though it is meaningful to speak about infinite *divisibility*.[46] Yet, when the distinction between the successive and the at once infinite is applied, a deepened arithmetical perspective does allow for the acknowledgement of an infinite totality where any given succession can be viewed as being *given at once*, as an infinite whole. The fact that the at once infinite is irreducible to the successive infinite makes it meaningless to argue against the former by using the latter as *yardstick*.

But all these considerations still side-step the basic fact that most of the treated positions enter into a combat that takes place on the wrong battlefield! The real issue is not whether it is possible to develop a sound mathematical (or: set theoretical) analysis of the meaning of the successive infinite or of the at once infinite and then apply it to the numerical and spatial side of an actual physical movement. The issue is whether or not spatial continuity could be *arithmetized* fully[47] and whether or not it is the task of a mathematical theory of number (or space) to explain the core meaning of *motion* in the sense of *defining* it in quantitative terms or in terms of the static meaning of space.

The employment of the *at once infinite* in the mathematics of Weierstrass, in fact misguided him to an understanding devoid of phoronomic and physical connotations, i.e., stripped of the idea of *constancy* and *change*. Boyer remarks:

> In making the basis of the calculus more rigorously formal, Weierstrass also attacked the appeal to intuition of continuous motion which is implied in Cauchy's expression that a variable *approaches* a limit. Previous writers generally had defined a variable as a quantity or magnitude which is not constant; but since the time of Weierstrass it has been recognized that the ideas of variable and limit are not essentially phoronomic, but involve purely static considerations. Weierstrass interpreted a variable x as simply a letter designating any one of a collection of numerical values. A continuous variable was likewise defined in terms of static considerations: If for any value x_0 of the set and for any sequence of positive numbers d_1, d_2, ..., d_n' however small, there

[44] "We ought not to call space a *compositum*, but a *totum*, because in it its parts are possible only in the whole, and not the whole by its parts" (Kant, 1787:467).
[45] This idea was initiated by Weyl (see Weyl, 1966:61). See also Salmon, 2001:26 ff. and Thomson (in Salmon, 2001:89 ff.) and Benecerraf (in Salmon, 2001:103 ff.).
[46] Grünbaum distinguishes between *infinite divisibility* and an actual infinite *dividedness* (Grünbaum, 1952:300).
[47] This problem is related to Zeno's paradoxes of plurality.

> are in the intervals $x_0 - d_i$, $x_0 + d_i$ others of the set, this is called continuous (Boyer, 1959:286).[48]

This position even caused Russell to settle for the idea that Zeno's arrow is truly at rest at every moment of its flight!

> After two thousand years of continual refutation, these sophisms were reinstated, and made the foundation of a mathematical renaissance, by a German professor, who probably never dreamed of any connection between himself and Zeno. Weierstrass, by strictly banishing all infinitesimals,[49] has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest (Russell, 1956:347).

The alternative approach advanced in this article holds that only when the *uniqueness* and *mutual coherence* of number, space and movement are observed is it possible to avoid the threat of antinomies inherent in Zeno's arguments against plurality and motion.

The spatial metaphysics of Parmenides, for that matter, inspired Zeno to defend a view of *unitary wholeness* that *excludes* plurality. In other words, Zeno wants to *deny* the 'part'-element of the spatial whole-parts relationship while at the same time holding on to the 'wholeness' which entails it.[50] His position is that reality is both *one* and *indivisible*. Yet, in order to *argue* for his position, he explored the whole-parts relation in his argument that is aimed at the *denial* of plurality! The reason why Zeno considers plurality to be self-contradictory is that plurality requires a *number* of (indivisible) units and because it also implies that reality is *divisible* (see Guthrie, 1980:88). But divisibility threatens the wholeness of a *unit*, since anything divisible has to be a magnitude which must be infinitely divisible. The supposed *indivisibility* of a *unit* clashes with its *infinite divisibility*. "Hence, since plurality is a plurality of units, there can be no plurality either" (Guthrie, 1980:89).

---

48 However, in the course of their development during the 20th century both logic and mathematics realized that it is impossible to side-step the idea of *constants* and *variables* – thus showing that an analysis even of the meaning of number cannot be separated from the interconnections between various *modes of explanation*.

49 Of course the new introduction of *infinitesimals* in the non-standard analysis of Abraham Robinson outdates this remark of Russell. Robinson developed his new theory on the basis of a fertile use of Cantor's theory of *actually infinite sets* (transfinite cardinalities). A number *a* is called *infinitesimal* (or *infinitely small*) if its absolute value is less than *m* for all positive numbers *m* in □□(□ being the set of real numbers). According to this definition 0 is *infinitesimal*. The fact that the infinitesimal is merely the correlate of Cantor's transfinite numbers, is apparent in that *r* (*not equal to* 0) is infinitesimal if and only if *r to the power of minus 1* (r^{-1}) is infinite (cf. Robinson, 1966:55ff).

50 This is a quasi-Wittgensteinean position. Whereas Wittgenstein had to throw away the ladder after climbing up it (*Tractatus*, 6.54), Zeno started on top with wholeness and then discarded the ladder of infinite divisibility supporting it. The reverse took place in intuitionistic mathematics, which started with the original spatial whole-parts relation but then distorted it by accentuating the part-element (with its implied infinite divisibility) at the cost of the whole-element (with its givenness all at once). The intuitionistic theory of the real numbers and the continuum followed a similar kind of Wittgensteinean approach – it used the "spatial ladder of wholeness" but immediately afterwards discarded it while holding on to the infinite divisibility implied by it.

Therefore, by denying the *foundational meaning* of multiplicity, Zeno not only distorts the meaning of *number* (plurality) but also misrepresents the meaning of *space*. The infinite divisibility of a spatial whole analogically reflects the original and primitive numerical meaning of the successive infinite. Through spatial continuity the endlessness of the numerical infinite is "turned inwards." But divorced from its connections with number the meaning of space collapses.[51] The original numerical meaning of the unitary *one* is non-original within space, for within space the magnitude of an extended spatial figure (as a whole) provides a different context for unity – a unity (totality) that is *infinitely divisible*. The speculative (metaphysical) notion of a *unitary whole* excluding plurality robs both number and space of their meaning and mutual connections.[52]

From this subsection it is clear that the issues involved – regarding the uniqueness and mutual coherence of number, space and motion – transcend the confines of logic as such, although they certainly also cohere with the meaning of logical analysis. This consideration prompts us to look at the self-insufficiency of logic.

11. The limitations of logic

As long as one merely considers the logical principles of identity and non-contradiction (whether or not amended by the principle of the excluded middle), no *material* criterion of truth is available, for in terms of these principles one can at most affirm that two contradictory statements cannot both be true at the same time and within the same context.[53] What refers thought irrevocably *beyond* logic is first of all the *principium rationis sufficientis* (also known as *principium rationis determinantis* and *principium reddendae rationis*) – in English formulated as the "principle of sufficient reason." Since one can at most affirm that two contradictory statements cannot both be true at the same time and within the same context" it is clear that logic alone cannot resolve the contradiction. What is needed is to ask for *non-logical* (extra-logical) grounds, that is to say, a reference to states of affairs "in reality" ("outside" logic) is required.

This principle, originally formulated by Leibniz, was subjected to an extensive investigation by A. Schopenhauer in 1813. He called it the principle of *sufficient ground* of knowledge (*principium rationis sufficientis cognoscendi*) (Schopenhauer, 1974:156).

[51] Mathematical dimension theory explores the notion of dimension as an order of extension – captured by the natural numbers 0, 1, 2, 3, … – thus analogically reflecting the foundational meaning of number. In metrical spaces magnitude (such as length – one-dimensional extension; surface – two dimensional extension; and volume – three dimensional extension) in a correlated way analogically reflects the foundational meaning of number. Exploring the suggestions of Poincaré, Brouwer in 1913 introduced a precise (topologically invariant) definition of *dimension*, which was independently recreated and improved in 1922 by Menger and Urysohn. Menger's formulation (still adopted by Hurewicz and Wallman) simply reads: "a) the empty set has dimension -1, b) the dimension of a space is the least integer *n* for which every point has arbitrarily small neighborhoods whose boundaries have dimension less than *n*" (Hurewicz, 1959:4, cf. 24; cf. also Alexandroff, 1956:165, 167 note 12 regarding the *intuitive meaning* of dimension as it is present in the principle of invariance of Brouwer).

[52] Just recall our earlier remarks about *speed* and the length of a line where it was argued that within the domains of space, movement and the physical we can discern numerical analogies.

[53] This was already clearly understood by Kant (see Kant, 1787:84-85).

The general legacy of Leibniz is captured in the phrase: *there is nothing without a sufficient ground* (*nihil est sine ratione sufficiente*). Of course already Plato affirmed that assertions require a foundation (*Timaeus* 28a), whereas Aristotle distinguished four causes: *material*, *formal*, *effective* and *final* ones.

In his *Monadology* Leibniz formulates his view as follows:

> … and the second the *principle of sufficient reason*, by virtue of which we observe that there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases (Leibniz, 1976:646 – see Sections 44 and 196).

Ultimately the combined perspective of the principle of sufficient reason and the ontic requirement to avoid antinomies by acknowledging what is unique and irreducible opens up the plea for a *non-reductionist ontology*. In terms of this remark one can view each and every *monistic ism* as an argument against the position defended in this article.[54] Our conjecture is that the untenability of such a truly monistic orientation is shown by the antinomies that it entails. An interesting feature of an antinomic position is that it always reaches the opposite for what it aims for.

In addition to what has been said about the antinomies entailed in the space metaphysics of the school of Parmenides, we briefly consider another example by looking at the intentions of *historicism*. Under the heading: *Change and Permanence: On the Possibility of Understanding History* Hans Jonas examines the impasse of historicism. He argues that radical historical skepticism is self-defeating (Jonas, 1974:241). The problem is that *change* can only be detected on the basis of an *enduring* or *persistent* element, of something *constant*.[55] For example, if *law* is intrinsically *historical* it is supposed to have 'happened' somewhere in the past – which is not at all the case, for jurisprudence knows much about *legal history*. If the jural itself *was* history, it could not have *had* a history. The *irony* of radical historicism is therefore that the opposite of what is aimed for is achieved – if everything *is* history, there is nothing left that can *have* a history. Jonas refers to the said element of *constancy* as something *transhistoric* in his assessment that runs parallel with the irony just mentioned: "Actually, there is no paradox in this. For history itself no less than historiography is possible only in conjunction with a transhistoric element. To deny the transhistorical is to deny the historical as well" (Jonas, 1974:242).

Every scientific methodology (and epistemology) is founded in an ontology. This insight is captured in the title of an article of Neemann: "Das Primat der Ontologie vor dem der Metholodologie" ("The Primacy of Ontology before that of Methodology" – see Neemann, 1986). The idea of a non-reductionist ontology carries it to its ultimate epistemological consequences by making a plea to side-step the logical contradictions entailed in underlying antinomies.

12. The foundational role of the *principium exclusae antinomiae*

If the *principium rationis sufficientis* refers thinking beyond the limits of pure logicality, the logical principle of non-contradiction is further enriched by an underlying ontical principle,

[54] Monistic isms are for example arithmeticism, holism, physicalism, vitalism, psychologism, logicism, historicism, and so on.

[55] The term 'constancy' is preferable to 'permanence'.

namely the principle forbidding inter-modal reductions which invariably result in *antinomies*. This principle is *ontical* in nature and should be called the ontical principle of the *excluded antinomy* (*principium exclusae antinomiae*).[56]

The perennial philosophical problem of explaining the coherence of what is unique and irreducible (the "coherence of irreducibles") therefore opens the way to an acknowledgment of the foundational position of the *principium exclusae antinomiae* in respect of the logical principle of *non-contradiction* – and at once it explains why the distinction between *antinomy* and *contradiction* is not a purely *logical* distinction. The *principium exclusae antinomiae* not only depicts the *limits* of logic but at once also underscores the significance of a commitment to transcend the one-sidedness of reductionnsitic isms within philosophy and the various disciplines. For this reason we understand our argument as being supportive of a non-reductionistic ontology. Laying bare theoretical antinomies in an 'ismic' stance serves as a strengthened form of immanent criticism – no thinker can turn away when it has been shown that her position is internally antinomic. In fact, once an antinomy has been articulated, the challenge is reverted, for then an alternative must be presented not subject to the same immanent criticism. If it is successful, i.e., if the alternative view put forward does not harbour a similar antinomy, then progress has been made in the critical intellectual encounter.

13. Concluding remark

The logical principle of sufficient reason refers us to those (extra-logical) grounds on the basis of which a valid argument can be pursued. However, if something truly basic (primitive) and indefinable becomes the victim of an attempt to reduce what actually is *irreducible*, theoretical thought gets entangled in genuine antinomies. Since the latter bring to light an attempt to reduce different functional modes of reality to each other (such as the Eleatic attempt to reduce motion to space), antinomies demonstrate that they are *inter-modal* in nature – thus differing from the *intra-modal reference* of a mere logical contradiction – such as the confusion of two *spatial* figures in the (illogical) concept of a "square circle." In order to explain some of the intricacies of this distinction between contradiction and antinomy – a distinction that exceeds the confines of pure logic – we had to take into account related problems, such as the meaning of unity and diversity, the problem of reductionism and the issue regarding the urge of monistic isms to find one all-embracing mode of explanation. In the final analysis it turned out that the need to distinguish between contradiction and antinomy amounts to a plea for the acknowledgement of a non-reductionist ontology. The aim of such an ontology is to avoid the dead alleys accompanying all instances of a metaphysical reductionism because the latter always terminate in the antinomic position of one or another monistic orientation.

14. References

Agazzi, E. 1991. The Problem of Reductionism in Science. In: Episteme, A Series in the Foundational, Methodological, Philosophical, Psychological, Sociological, and Political Aspects of the Sciences, Pure and Applied, Volume 18, *Editor*: Mario Bunge,

[56] See Dooyeweerd, 1997-II:36 ff.

Foundations and philosophy of Science Unit, McGill University. Boston: Kluwer Academic Publishers.

Alexandroff, P.S. 1956. *Einführung in die Mengenlehre und die Theorie der reellen Funktionen*. Berlin: Deutscher Verlag der Wissenschaften.

Beth, E.W. 1965. *Mathematical Thought*. Dordrecht-Holland: D. Reidel Publishing Company.

Boyer, C.B. 1959. *The history of the calculus and its conceptual development*. New York: Dover.

Cantor, G. 1962. *Gesammelte Abhandlungen* (1932), Hildesheim: Georg Olms Verlag.

Cassirer, E. 1953. *Substance and Function*, New York (first edition of the English translation of *Substanzbegriff und Funktionsbegriff*: 1923. First German edition 1910).

Cassirer, E. 1957. *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*, Stuttgart: Kohlhammer Verlag.

Clark, M. 2002. *Paradoxes from A to Z*. London: Routledge.

Copi, I.M. 1994. *Introduction to Logic*, Ninth Edition. New York: Macmillan Publishing Company.

Cornford, F.M. 1966. *The Republic of Plato*. Translated with Introduction and Notes. Oxford: Clarendon Press.

Dennett, D.C. 1995. *Darwin's Dangerous Idea. Evolution and the Meanings of Life*. New York: Simon & Schuster.

Derrida, J. 1993. *Aporias*. Translated by Thomas Dutoit. Stanford: Stanford University Press.

Diels, H. & Kranz, W. 1959/60. *Die Fragmente der Vorsokratiker.* Band I-III. Berlin: Weidmannsche Verlagsbuchhandlung.

Dobzhansky, T. and Ayala, F.J. 1974. *Studies in the Philosophy of Biology. Reduction and Related Problems*. Berkeley: University of California Press.

Dooyeweerd, H. 1997. *A New Critique of Theoretical Thought*, Collected Works of Herman Dooyeweerd, The Edwin Mellen Press, A-Series Vols. 1-4 (General Editor D.F.M. Strauss.)

Einstein, A. 1959. *Autobiography*, in: Schilpp, 1959 (pp.2-95).

Fraenkel, A., Bar-Hillel, Y., Levy, A. & Van Dalen, D. 1973. *Foundations of Set Theory*, 2nd revised edition, Amsterdam: North-Holland Publishing Company.

Fränkel, H. 1968. Zeno von Elea im Kampf gegen die Idee der Vielheit. In: *Um die Begriffswelt der Vorsokratiker*, *Wege der Forschung*, Band IX, Editor Hans-Gerog Gadamer, Darmstadt: Wissenschaftliche Buchgesellschaft (pp.425 ff.).

Gardner, M. 1968. *Mathematical Puzzles and Diversions*. Harmondsworth: Penguin Books.

Goodfield, J. 1974. Changing Strategies: A Comparison of Reductionist Attitudes in Biological and Medical Research in the Nineteenth and Twentieth Centuries. In: Dobzhansky and Ayala (pp.65-86).

Greene, B. 2003. *The Elegant Universe*. New York: W.W. Norton & Company Inc.

Guthrie, W.K.C. 1980. *A History of Greek Philosophy*. Volume II. *The Presocratic Tradition from Parmenides to Democritus*. Cambridge: Cambridge University Press.

Grünbaum, A. 1952. A consistent conception of the extended linear contimuum as an aggregate of unextended elements, in: *Philosophy of Science*, Vol.19, nr.2, April 1952 (pp.288-306).

Grünbaum, A. 1967. *Modern Science and Zeno's Paradoxes*. Middletown: Wesleyan University Press.

Grünbaum, A. 1974. *Philosophical Problems of Space and Time*. Dordrecht (Holland): D. Reidel Publishing Company (second, enlarged edition).

Heine, E. 1872. Die Elemente der Functionenlehre. In: *Journal für reine und angewandte Mathematik*. Band 74, Berlin (pp.172-188).

Hörz, H. 1967. Article on *Physics* in: *Naturforschung und Weltbild*, Berlin.

Hurewicz, W. and Wallman, H. 1959. *Dimension Theory*, 5th edition, Princeton: Princeton University Press.

Janich, P. 1975. Tragheitsgesetz und Inertialsysteem. In: *Frege und die moderne Grundlagenforschung*, (Editor Chr. Thiel). Meisenheim am Glan.

Jonas, H. 1974. *Philosophical Essays*: *From Ancient Creed to Technological Man*. Prentice-Hall, INC.: Englewood Cliffs, New Jersey.

Kant, I. 1783. *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können*, (Felix Meiner Edition, Hamburg 1969).

Kant, I. 1787. *Kritik der reinen Vernunft* (1781); 1787 (Felix Meiner Edition, Hamburg 1956).

Koestler. A. and Smythies, J.R. (Editors) 1972. *Beyond Reductionism*. London.

Lakoff, G. & Johnson, M. 1999. *Philosophy in the Flesh. The Embodied Mind and Its Challenge to Western Thought*. New York: Basic Books.

Lakoff, G. and Núñez, R.E. 2000. *Where Mathematics Comes From. How the Embodied Mind brings Mathematics into Being*. New York: Basic Books.

Lorenzen, P. 1972. Das Aktual-Unendliche in der Mathematik, in his work: *Methodisches Denken*, Frankfurt am Main, Reprinted in Meschkowski, 1972 (pp.157-165).

Maier, A. 1964. *Ausgehendes Mittelalter*. Vol.I, Rome: Edizioni di Storia e Letteratura.

Meschkowski, H. (Editor) 1972. *Grundlagen der Mathematik*. Darmstadt: Wissenschaftliche Buchgesellschaft.

Neemann, U. 1986. Das Primat der Ontologie vor dem der Metholodologie. In: *Philosophia Naturalis*, 23(1):70-81.

Northrop, E.P. 1964. *Riddles in Mathematics*. Harmondsworth: Penguin Books.

Popper, K.R. Scientific Reduction and the Essential Incompleteness of All Science. In: Dobzhansky, 1974 (pp. 259-284).

Putnam, H. 1982. *Reason, Truth and History*. Cambridge: Cambridge University Press.

Quine, W.V.O. 1953. *From a Logical Point of View*. Cambridge Massachusetts: Harvard University Press.

Ritter, J. 1971 (Editor). *Historisches Wörterbuch der Philosophie*. Volume I, Stuttgart: Schwabe & Co Verlag.

Rombach, H. 1965-66. *Substanz, System, Struktur; die Ontologie des Funktionalismus und der philosophische Hintergrund der modernen Wissenschaft*. Freiburg: Alber.

Ryle, G. 1977. *Dilemmas, met 'n voorwoord deur René Meyer*. Pretoria: J.L. Van Schaik. (Achilles en die Skilpad – pp.50-69).

Salmon, W. (Ed.). 2001. *Zeno's Paradoxes.* New York: Bobs-Merrill. [Contributors: Resolution of the paradox, by A. Shimony. – Introduction, by W. C. Salmon. – The problem of infinity considered historically, by B. Russell. – The cinematographic view of becoming, by H. Bergson. – Achilles and the tortoise, by M. Black. – Achilles on a physical racecourse, by J. O. Wisdom. – Tasks and super-tasks, by J. Thomson. – Tasks, super-tasks, and the modern Eleatics, by P. Benacerraf. – Comments on Professor Benacerraf's paper, by J. Thomson. – Zeno and the mathematicians, by G. E. L. Owen. – Modern science and refutation of the paradoxes of Zeno. Zeno's metrical paradox of extension. Modern science and Zeno's paradoxes of motion. By A. Grünbaum. – Appendix: Sets and infinity, by W. C. Salmon. First published in 1970 by New York: Bobs-Merrill.]

Schilpp, P.A. 1951 (Editor). *Albert Einstein, Philosopher-*Scientist. Vol. I. London: Harper & Row Publishers.

Schopenhauer, A. 1974: *On the Fourfold Root of the Principle of Sufficient Reason*, translation by E.F.J. Payne, La Salle, Ill.: Open Court.

Smith, G.L., 1994. *On Reductionism*. Sewanee, Tennessee – available on the WEB at: http://smith2.sewanee.edu/texts/Ecology/OnReductionism.html (accessed on 22-01-2005).

Spivak, G.C. 1999. *A Critique of Postcolonial Reason*. Toward a History of the Vanishing Present. Cambridge: Harvard University Press.

Stafleu, M.D. 1980. *Time and Again*, *A Systematic Analysis of the Foundations of Physics*. Toronto: Wedge.

Stafleu, M.D. 1987. *Theories at Work*: *On the Structure and Functioning of Theories in Science*, *in Particular during the Copernican Revolution*, Lanham: University Press of America.

Stegmüller, W. 1970. *Main Currents in Contemporary German, British and American Philosophy*. Dordrecht-Holland: D. Reidel Publishing Company.

Strauss, D.F.M. 2007. Die Grenzen der Logik Übersteigen: Zum Unterschied zwischen Widerspruch und Antinomie. In: *Die Suid-Afrikaanse Tydskrif vir Natuurwetenskap en Tegnologie*, 26(1):37-61.

Strauss, D.F.M. 2009. *Philosophy*: *Discipline of the Disciplines*. Grand Rapids: Paideia Press

Teensma, E. 1969. *The Paradoxes*. Assen: Van Gorcum.

Vaihinger, H. 1949. *The Philosophy of "As If"*. London: Routledge & Kegan Paul (translated by C.K. Ogden).

Wang, H. 1988. *Reflections on Gödel*. Cambridge, Massachusetts: MIT Press.

Weyl, H. 1921. Ueber die neue Grundlagenkrise der Mathematik, *Mathematische Zeitschrift*. Band 10, 1921 (pp.39-79).

Weyl, H. 1932. *Das Kontinuum*, 2nd impression, Berlin.

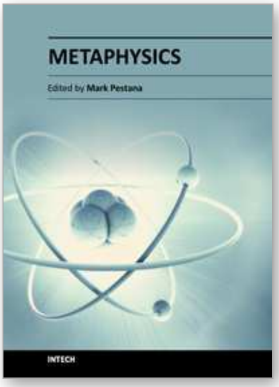
Weyl, H. 1946. Mathematics and Logic. In: *American Mathematical Monthly*, Vol. 53, 1946 (pp.2-13).

Weyl, H. 1966. *Philosophie der Mathematik und Naturwissenschaft*, 3rd revised and expanded edition, Wenen 1966.

Waldenfels, B. 1971. *Aporie*, *Aporetik*. In: Ritter 1971 (pp.447-448).

Weingartner, P. 1991. *Reductionism and Reduction in Logic and in Mathematics*. In: Agazzi, 1991 (pp.119-148).

Wolff, K. 1971. Zur Problematik der absoluten Überabzählbarkeit. In: *Philosophia Naturalis*, Band 13 (pp.399-404).



Metaphysics
Edited by Dr. Mark Pestana

ISBN 978-953-51-0646-3
Hard cover, 148 pages
Publisher InTech
Published online 13, June, 2012
Published in print edition June, 2012

It is our hope that this collection will give readers a sense of the type of metaphysical investigations that are now being carried out by thinkers in the Western nations. We also hope that the reader's curiosity will be peaked so that further inquiry will follow.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

D.F.M. Strauss (2012). Metaphysics Between Reductionism and a Non-Reductionist Ontology, Metaphysics, Dr. Mark Pestana (Ed.), ISBN: 978-953-51-0646-3, InTech, Available from: http://www.intechopen.com/books/metaphysics/the-significance-of-a-non-reductionist-ontology-for-metaphysics

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821