An Approach-Based on Response Surfaces Method and Ant Colony System for Multi-Objective Optimization: A Case Study

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1. Introduction

Numerical optimization is the search approach we adopted in order to determine the best mechanical design. Several algorithms related to the problems at hand were developed, most of them single-objective. However, given the complexity of the products involved and the multiple objectives of the design considered, the researchers focused on the optimization algorithms for such problems. In short, the optimization problems have multiple objectives, and in many cases, there are multiple constraints. Design processes often require expensive evaluations of objective functions. That is particularly the case when such performance indexes and their constraints are obtained through intermediate simulations by finite elements involving fine meshes, many freedom degrees and nonlinear geometrical behaviours. To overcome these difficulties, the response surface method (RSM) is employed (Myers & Montgomery, 2002; Roux et al., 1998; Stander, 2001; Zhang et al., 2002) to replace a complex model by an approximation based on results calculated on certain points in search space.

When an adequate model is obtained with the RSM approach, it then becomes necessary to consider the optimization step. The method used to find the best solution assesses several objectives simultaneously; since some such objectives are fundamentally conflicting vis-à-vis of another, we therefore need to establish a compromise. Existing literature shows that desirability or metaheuristic functions are normally used, the most common being the genetic algorithm (GA). Sun & Lee (Sun & Lee, 2005), present an approach which associates the RSM and GA with the optimal aerodynamic design of a helicopter rotor blade. The ACO is a metaheuristic, which has been successfully used to solve several combinatorial optimization problems. We however see that very little exists in terms of documentation for optimization using ACO, as far as multiobjective problems are concerned. Some works lead us to believe that ant colonies can produce an optimum situation faster than the GA (Nagesh, 2006; Liang, 2004). In the literature, ACOs are used almost exclusively for “Travelling Salesman Problem” (TSP), quadratic assignment problem allocation (QAS),
constraint satisfaction problems (CSP), design manufacturing systems (DMS), and for discrete and combinatorial optimization problems. Our contribution consists in an extension of the ACO in the multiobjective optimization of mechanical system design in a continuous field. This paper starts with the modelling process with RSM, and then goes on to describe the ACO and the Hybrid method developed for a problem regarding multiobjective optimization with constraints. An application of the suggested method for optimizing the mechanical process design is presented.

2. Modeling with RSM

RSM is a collection of statistical and mathematical techniques used to develop, improve and optimize processes (Myers & Montgomery, 2002). Furthermore, it has important applications in the design and formulation of new products, as well as in the improvement of existing products.

The objective of RSM is to evaluate a response, i.e., the objective physical quantities, which are influenced by several design variables. When we use RSM, we seek to connect a continuous answer \( Y \) with continuous and controlled factors \( X_1, X_2, \ldots, X_k \), using a linear regression model which can be written (Myers & Montgomery, 2002) as:

\[
y = f_\beta(X_1, X_2, \ldots, X_p) + \varepsilon
\]

Since the response surface is described by a polynomial representation, it is possible to reduce the optimization resolution process time by assessing the objectives with their models rather than using more complex empirical models such as those obtained through the FEM analysis. Although the specific form of response factor \( f_\beta \) is unknown, experience shows that it can be significantly approximated using a polynomial.

In the case of two factors, the linear regression model is one of the simplest available, and corresponds to a first-degree model with interaction, and which has the following form:

\[
y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \varepsilon
\]

Whenever this model is unable to describe the experimental reality effectively, it is common practice to use a second-degree model, which includes the quadratic effects of the factors involved:

\[
y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \varepsilon
\]

Where \( y \) is the response (study objective, for example, the total manufacturing cost); \( \varepsilon \) is the estimate of the error; \( X_1 \) and \( X_2 \) are influential factors of the coded response (e.g., design variables).

The unknown parameters of this mathematical model, \( \beta_i \) values, are estimated through the least-squares technique, and the adjustment quality of the model is assessed using traditional multiple linear regression tools.

Ideally, the number of experiments carried out, either with the finite element model (FEM) or using other approaches, during the application of RSM, should be as small as possible, in order to reduce data-processing requirements. Properly selecting the points to be used for
the simulation will allow a reduction of the variance of the coefficients of the mathematical model, which will in turn ensure that the response surfaces obtained are more reliable. To that end, we need to determine the experimental design to be adopted in order to obtain the most interesting simulation for this problem. The central composite design (CCD) was employed in the case of the second-order response surface, but other types of plans, such as the complete factorial design and the fractional factorial design, are also available for use.

Once the mathematical models are obtained, we need to verify that they produce an adequate approximation of the actual study system. The statistic selection criterion is the coefficient of determination $R^2$, which must be as close as possible to 1 ($0 < R^2 < 1$).

Once this stage is completed, we will have all the equations which make up our multiobjective optimization problem. Generally, such problems are as the following form:

Find $x = [x_1, x_2, x_3, ..., x_n]^T$ 
Which minimize $f(x) = \{f_1(x), f_2(x), f_3(x), ..., f_n(x)\}$ 
Subject to $g_j(x) \leq 0$ for $j = 1, m$ 
$x_i^L \leq x_i \leq x_i^U$ for $i = 1, n$ 

(4)

To optimize this problem, we explored the ant colony algorithm (ACO). Some options are offered for this kind of problem, such as the desirability function and the genetic GA used by some authors, such as Sun & Lee (Sun & Lee, 2005) or Abdul-Wahad & Abdo (Abdul-Wahad & Abdo, 2007). The literature shows that for many problems, the ant colony approach produces better results in terms of quality solutions and resolution speed, as compared to the GA. This allowed us to begin this research with the resolution of the multiobjective continuous optimization problem in mechanical design.

3. Ant colony algorithm approach

The ACO metaheuristic, called the ant system (Dorigo, 1992), was inspired by studies of the behaviour of ants (Deneubourg et al., 1983; Deneubourg & Goss, 1989; Goss et al., 1990), as a multi-agent approach for resolving combinative optimization problems such as the TSP.

Ants communicate among themselves through the "pheromone", a substance they deposit on the ground in variable amounts as they move about. It has been observed that the more ants use a particular path, the more pheromone is deposited on that path, and the more attractive it becomes to other ants seeking food. If an obstacle is suddenly placed on an established path leading to a food source, ants will initially go randomly right or left, but those choosing the side that is in fact shorter will reach the food more quickly, and will make the return journey more often. The pheromone concentration on the shorter path will therefore be more strongly reinforced, and it will eventually become the new preferred route for the stream of ants; however, it must also be borne in mind that the pheromone deposited along the way does evaporate. Works by Colorni et al. (Colorni et al., 1992), Dorigo et al. (Dorigo et al., 1996; Dorigo et al., 1999; Dorigo et al., 2000) provide detailed information on the operation of the algorithm and on the determination of the values of the various parameters (see Fig. 1).

In our field, the ACO has been used very sparingly, and has been focused primarily on single-objective problems (Chegury, 2006). For multiobjective problems, the ACO has hardly
been used at all (Zhao, 2007), and when used, has been mainly on combinatorial optimization problems. The importance of this work therefore lies in its attempt to adapt continuous ant colonies to multiobjective problems.

Fig. 1. Experimental setup and drawings of the selection of short branches by a colony of "Linephitema humile", 4 and 8 min after the bridge was placed (Dorigo et al., 2000)

4. Proposed design optimization approach

The objective of this chapter is to determine the best design for a mechanical system such as a plane wing, an engine, etc., or for an unspecified mechanical process that sometimes simultaneously optimizes several conflicting objectives. The ACO, like the GA, requires an objective function which can be quickly assessed. We use RSM modeling to determine such objective functions, and the ACO as the research method. Reducing the resolution time in the optimization process requires a reduction of the preciseness of the assessment of objective functions, since we use an approximate modeling of our objectives instead of their exact representations.

Each objective $f_i$ is expressed according to the variables of real design $x_i$ which influence its value. The multiobjective optimization model obtained with RSM is:

$$
\text{Minimize} \quad f(x) = \{f_1(x), f_2(x), f_3(x), \ldots, f_n(x)\}
$$

subject to \quad $x_i^L \leq x_i \leq x_i^U$ \quad for $i = 1, n$

(5)

After obtaining a mathematical model for that problem, the optimization phase must be able to determine the best compromise solution for the various objectives.

The steps for the general ACO metaheuristic for compromise solutions for combinatorial problems presented by Gagné et al. (Gagné et al., 2004) constitute an interesting approach to be considered in our resolution process for developing the fitness function.

4.1 Continuous ant colonies

There are several ant colony algorithms available for continuous optimization, the first of which was developed by Bilchev & Parmee (Bilchev & Parmee, 1995), and named CACO
(Continuous Ant Colony Optimization), using ant colonies for local searches and calling upon revolutionary algorithms for global searches. Ling et al. (Ling et al., 2002) present an unspecified hybrid algorithm whose main premise is to consider the differences between two individuals on each dimension as many parts of a path on which the pheromones are deposited. The evolution of the individuals dealt with mutation and crossing-over operators. This method thus tries to reproduce the construction mechanism of the solution component by component.

Monmarché et al. (Monmarché et al., 2000) developed the API algorithm which takes the primitive ant behaviour of the species *Pachycondyla Apicalis*, and which does not use indirect communication by tracks of pheromone. In this method, it is necessary to start by positioning a nest randomly on the research space, after which ants are distributed randomly over it. These ants explore their "hunting site" locally by evaluating several points within a given perimeter. Socha (Socha, 2004) presents the ACO algorithm for continuous optimization which tries to maintain the iterative construction solutions for continuous variables. He considers that the components of all solutions are formed by the various optimized variables. Moreover, before considering the algorithm from the ant’s point of view, he opts to operate at the colony level, with the ants being simply points to be evaluated. Pourtakdoust & Nobahari (Pourtakdoust & Nobahari, 2004) developed the CACS (Continuous Ant Colony System) algorithm, which is very similar to that of Socha. Indeed, in CACS, as is the case with ACO, for continuous optimization, the core of the algorithm consists in evolving a probability distribution which for CACS is normal.

4.2 Proposed algorithm

Once the steps used in making a choice regarding the elements to be included in the resolution process are explained. We present the new proposed algorithm for our approach (see Fig. 2a and Fig. 2b).

**Step 1: System configuration**

Determine the objectives of this study, the constraints and the variables which can influence these objectives. Evaluate the field of application of these variables.

**Step 2: RSM**

Set up an experimental design, carry out tests, and model the various objectives according to influential parameters.

**Step 3: Seek ideal point**

Using RSM, determine distinct optimum for each study objective.

**Step 4: Optimization function formulation**

a. State user preferences (weighting of the objectives).

b. The various objectives are expressed in a single function: the fitness function. It acts as an equation which for each objective, expresses the standard and balanced distance at the ideal point $F^*$ of an unspecified solution $k$, whose various objectives are given by $F_k$. This function makes it possible to standardize objectives in order to reduce the adverse effects obtained from the various measuring units, as well as the extent of the field of the variables, in order to not skew the fitness function (Gagné et al., 2004):
Determine system configuration
(Step 1/Fig. 2a)

Experimental design to determine the
tests
(Step 2/Fig. 2a)

Simulation to find the values of each
objective $Y_i$ for each test
(Step 2/Fig. 2a)

Obtain the determinative model of
each objective $Y_i$ by RSM
(Step 3 & 4/Fig. 2a)

Apply ACO to search the best
compromise solution of multiobjective
problem with the RSM model
(Step 5/Fig. 2a ➔ see Fig. 2b)

Evaluate optimal solution by simulation
(Step 6/Fig. 2a)

Narrow the domain of each design
variables
(Step 7/Fig. 2a)

At optimal design:
\[
\frac{RSM - SIMUL}{SIMUL} < e
\]

Yes
No

End

Figure 2a. Flow chart of the optimization approach
• Seek ideal points of the various objectives and statements of user’s preferences
• Determination of fitness function of compromise
  (Step 5a/Fig. 2b)

• Generate, randomly, R initial ants
• Distributed these ants in the design space
• According to the value of the fitness function, create 2 groups of ants: global and local groups
  (Step 5b/Fig. 2b)

Global search
  (Step 5c/Fig. 2b)

Local search
  (Step 5d/Fig. 2b)

Sort solutions according to fitness value
  (Step 5e/Fig. 2b)

Print best fitness value
  (Step 5e/Fig. 2b)

Iteration number equal to a given number of iterations?

Yes

No

End

Figure 2b. Flow chart of the ACO process
where $\mathbf{F}^*$ is a solution vector corresponding to the ideal point of each separate objective, and probably expressed by:

$$
\mathbf{F}^* = \{F_1^*, F_2^*, ..., F_k^*\}
$$

(7)

where $F_i^* = \min_{x \in S} f_i(x)$.

$\mathbf{F}^*$ generally corresponds to an unrealizable solution. $S$ is the space of acceptable search, and $\mathbf{F}^{nad}$ is the Nadir point, which represents the maximum values for each objective in the set of optimal Pareto solutions:

$$
\mathbf{F}^{nad} = \{F_1^{nad}, F_2^{nad}, ..., F_k^{nad}\}
$$

(8)

where $F_i^{nad} = \max_{x \in S} f_i(x)$.

**Step 5:** Determine compromise solution

a. Randomly generate R initial ants corresponding to feasible solutions.

To apply the ACO methodology for continuous optimization function problems, the field must be subdivided into a specific area, R, distributed by chance. Next, we need to generate feasible solutions representing the initial ants, each forming a part of the research area to be explored.

b. The fitness function of these solutions is assessed, and the values obtained are lines in descending order.

We obtain our initial ants "R" and the proportion of the higher values of R will be taken to constitute the global ants "G".

c. Apply a global search to a percentage of the initial ants, with "G" constituting the "worst" solutions available.

The percentage of global ants is an important parameter of CACO, which can be changed depending on the problem at hand. A global search creates new solutions for "G" by replacing the weaker parts of the existing field. This process is composed primarily of two genetic operators. In the terminology of CACO, these are called random walk and trail diffusion. In the random search process, the ants move in new directions in search of more recent and richer sources of food.

In the CACO simulation, a global search is conducted in all fields through a process that is equivalent to a GA crossover and mutation.

- **Crossover or random walk:** The crossover operation is conducted to replace inferior solutions with superior ones, with the crossover probability (CP).
- **Mutation**: The replaced solutions are further improved by mutation. The mutation step is completed in CACO by making an addition or proportional subtraction to the mutation probability. The mutation step size is reduced or increased as per Eq. 9. (Mathur et al., 2000)

\[
\Delta = R(1 - r^{(1-T)^b})
\]

where \( r \) is a random number from 0 to 1, \( R \) is the maximum step size, \( T \) is the ratio of the current iteration number and that of the total number of iterations, and \( b \) is a positive parameter controlling the degree of nonlinearity.

- **Trail diffusion**: In this step, the field of the global search is gradually reduced, as the search progresses. This reduction makes it possible to increase the probability of locating the optimum through more concentric search procedures. Trail diffusion is similar to the arithmetic GA crossover. In this step, two parents are randomly selected from the parent population space. The elements of the child’s vector can be any one of the following:

1. The child corresponds to an element from the first parent
2. The child corresponds to an element from the second parent
3. The child is a combination of the parents (Eq.10) (Mathur et al., 2000)

\[
X_{child} = (\alpha)X_{i(parent1)} + (1-\alpha)X_{i(parent2)}
\]

where \( \alpha \) is a uniform random number ranging from [0 to 1].

The probability of selecting one of the three options depends on the mutation probability. Thus, if the mutation probability is 0.5, option 3 can be selected with a probability of 50%, whereas the probability of selecting option 1 or 2 is 25%.

d. **Send local ants** \( L \) in the various \( R \) areas

Once the global search is completed, the zones to which you send the local ants are defined and a local search can begin.

In a local search, the local ants choose the area to be explored among the areas of the matrix \( R \), according to the current quantity of pheromones in the areas. The probability of choosing a solution "i" is given by: (Mathur et al., 2000)

\[
P_i(t) = \frac{\tau_i(t)}{\sum_k \tau_k(t)}
\]

where "i" is the solution index and \( \tau_i(t) \) is the pheromone trail on the solution "i" at time "t".

After choosing its destination, the ant proceeds across a short distance. The search direction remains the same from one local solution to the next as long as there is improvement in the fitness function. If there is no improvement, the ant reorients itself randomly to another direction. If an improvement in the fitness function is obtained in the preceding procedure, the position vector of the area is updated. The quantity of pheromone deposited is proportional to the improvement of the fitness function. If, in the search process, a higher fitness function value is obtained, the age of the area is increased. This age of the area is
another major parameter in the CACO algorithm. The size of the ant displacement in a local search depends on the current age. The search ray is maximum for age zero, and minimal for the maximum age, with a linear variation.

e. Evaluate the fitness function for each ant obtained, and continue the iterative process, beginning with a global search until stop conditions are observed.

**Step 6:** Evaluate the best solutions (quasi optimal) by simulation or experimentation for the experimental design (Example, FEM)

**Step 7:** Evaluate the stop criterion \( \frac{RSM - SIMUL}{SIMUL} < \epsilon \) with SIMUL being the simulation result.

In the optimization design problem for a mechanical system, the number of design variables is very often equal to or higher than 3, and each one of them has a broad field of variation. Consequently, in our resolution process, it is possible for the search field for each design variable to be gradually narrowed for as long as the stop criterion has not been encountered. The search process ends when \( \frac{RSM - SIMUL}{SIMUL} < \epsilon \) with \( \epsilon \) being a margin of error defined beforehand.

### 5. Application: Numerical example of two-objective problem

In order to illustrate the performances of the recommended resolution approach used in this paper, we carried out the optimization of a multistage flash desalination process. The problem was taken from Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007), and was solved using the experimental designs, and optimized using desirability functions.

#### 5.1 Problem definition

Multistage flash (MSF) desalination is an evaporation and condensation process, which involves boiling seawater and condensing the vapour to produce distilled water. A more extensive description of the multistage flash desalination MSF considered in this work can be found in Hamed et al. (Hamed et al., 2001).

In this study, two performance objectives are considered: the maximization of the distillate produced rate (DF) and the minimization of the blow down flow rate (BDF). The operation variables which influence these objectives are presented in Table 1 **(Step 1)**. They include:

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Nomenclature</th>
<th>Low level</th>
<th>High level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seawater inlet temperature (ºC)</td>
<td>SWIT (A)</td>
<td>24</td>
<td>35</td>
</tr>
<tr>
<td>Temperature difference (ºC)</td>
<td>TD (B)</td>
<td>5.2</td>
<td>8.0</td>
</tr>
<tr>
<td>Last-stage brine level (mm)</td>
<td>LSLBL (C)</td>
<td>50</td>
<td>850</td>
</tr>
<tr>
<td>First-stage brine level (mm)</td>
<td>FSBL (D)</td>
<td>40</td>
<td>320</td>
</tr>
<tr>
<td>Brine recycle pump flow (m3/h)</td>
<td>BRPF (E)</td>
<td>8200</td>
<td>11 500</td>
</tr>
</tbody>
</table>

Table 1. Design parameters
5.2 Modeling with RSM

To express our objectives according to decision variables, we need to use modeling with RSM (steps 2 and step 3). We considered the experiments carried out by Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007), which helped us to design our model.

Abdul-Wahab & Abdo resorted to a two-level factorial design, carried out 64 experiments and five central-point tests with design variables coded on two levels: low (-1) and high (+1). The experimental design provides us with a linear regression model coded for each response in this study (see Fig. 3 & Fig. 4).

![Fig. 3. Modeling with RSM – part I](image1)

![Fig. 4. Modeling with RSM – part II](image2)
Finally, the equations representing our objectives are:

\[
\begin{align*}
DF &= 1041.61 + 20.45(B) + 18.65(C) + 120.29(E) \\
&\quad + 26.46(AC) + 30.08(CD) - 25.06(ABE) \\
BDF &= 1419.22 + 414.54(C) + 34.77(D) \\
&\quad + 28.71(AC) - 25.63(ABD)
\end{align*}
\]

(12)

We also note that in our experimental design, the variables C and E are the most influential on our objectives (see Fig. 5).

![Impact of the decision variables on study objectives](image)

Fig. 5. Impact of the decision variables on study objectives

Additional information on optimization, as well as the goals of the study, is summarized in the following table:

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Goal</th>
<th>Lower limit</th>
<th>Upper limit</th>
<th>Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>BDF</td>
<td>To minimize</td>
<td>830.53</td>
<td>2001.87</td>
<td>3</td>
</tr>
<tr>
<td>DF</td>
<td>To maximize</td>
<td>789.99</td>
<td>1284.34</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2. Constraints on objectives of study

5.3 Multiobjective optimization

Let us optimize the following problem with our CACO multiobjective approach:

Find \( x = [A,B,C,D,E]^T \)

which minimize \( f(x) = \{-DF(x), BDF(x)\} \)

subject to
\[
\begin{align*}
DF(x) &\leq 1284.34 \\
DF(x) &\geq 789.99 \\
BDF(x) &\leq 2001.87
\end{align*}
\]

(13)
The fitness function used, obtained by the CP (compromise programming) (Gagné et al., 2004) method, allows the search for solutions approaching the ideal point for each objective (Step 4):

$$fitness = \left[ \frac{5}{8} \left( \frac{DF_{max} - DF_i}{DF_{max} - DF_i^{Nad}} \right)^2 \right]^\frac{1}{2} + \left[ \frac{3}{8} \left( \frac{BDF_i - BDF_{min}}{BDF_i^{Nad} - BDF_{min}} \right)^2 \right]^\frac{1}{2} \quad (14)$$

The minimization of the fitness function enables us to reach our "BDF" minimization and "DF" maximization goals.

The result is a set of optimal Pareto solutions. We present more than one solution to the user in order to provide him with a margin of makeover. Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007), in their paper, present their 10 best solutions. We will do the same in order to make some comparisons.

Using the MatLab software (step 5), Figure 6 allows us to say that we get the best solution after 310 iterations. The staircase shape of the curve (Fig. 6) is explained by the memory effect that we used in the program code. Thus, when iteration produces a worse solution than the last one, this last solution (previous iteration) is retained.

The Table 3 presents the results obtained by the optimization process. The best solution is the number 1, while the 9 other solutions offer alternatives to user. These solutions meet problem constraints and, gives results which minimize "BDF" and maximize "DF" while remaining in the field of each decisional variable.

![Fitness function evaluation according to the iterations](Fig. 6. Values of the fitness function according to the iteration count)
Table 3. Optimal solutions

The above Table (see Table 3) present the 10 best results of our study. These solutions meet the constraints of the problem and give excellent results which minimize "BDF" and maximize "DF" while remaining within the confines of each decision variable. It’s interesting to observe the values of the decision variables in their respective fields. We can see that these best solutions are obtained under the following conditions (see Fig.7):

- Low temperature for seawater (SWIT) entering into the system
- The temperature difference (TD), which is high and similar for each of the solutions
- A final level of low salinity (LSBL)
- A first level of low salinity (FSBL)
- A high flow rate of the pump recycling salt (BRPF)

![Fig. 7. Value margins of variables for optimal solutions](image)

5.4 Comparison with authors’ results

A comparison between the results obtained with the desirability function ("DF") and the hybrid approach developed ("DF/CACO multiobjective") shows that the second gives better
quality results. Recall that this comparison is made between the results obtained by the proposed approach and those of Abdul-Wahab & Abdo.

<table>
<thead>
<tr>
<th># of solution</th>
<th>BDF desirability</th>
<th>BDF-CACO multiobjective</th>
<th>% BDF improvement</th>
<th>DF desirability</th>
<th>DF CACO multiobjective</th>
<th>% DF improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>991.725</td>
<td>979.903</td>
<td>1.19%</td>
<td>1224.52</td>
<td>1249.698</td>
<td>2.01%</td>
</tr>
<tr>
<td>2</td>
<td>1038.83</td>
<td>990.740</td>
<td>4.63%</td>
<td>1222.91</td>
<td>1245.603</td>
<td>1.82%</td>
</tr>
<tr>
<td>3</td>
<td>1033.34</td>
<td>988.099</td>
<td>4.386%</td>
<td>1213.11</td>
<td>1247.492</td>
<td>2.76%</td>
</tr>
<tr>
<td>4</td>
<td>1035.78</td>
<td>981.335</td>
<td>5.26%</td>
<td>1210.98</td>
<td>1245.577</td>
<td>2.78%</td>
</tr>
<tr>
<td>5</td>
<td>975.718</td>
<td>990.740</td>
<td>-1.54%</td>
<td>1181.75</td>
<td>1245.603</td>
<td>5.13%</td>
</tr>
<tr>
<td>6</td>
<td>965.137</td>
<td>985.796</td>
<td>-2.14%</td>
<td>1173.29</td>
<td>1245.552</td>
<td>5.80%</td>
</tr>
<tr>
<td>7</td>
<td>996.446</td>
<td>979.456</td>
<td>1.71%</td>
<td>1176.02</td>
<td>1244.994</td>
<td>5.54%</td>
</tr>
<tr>
<td>8</td>
<td>1035.8</td>
<td>987.596</td>
<td>4.65%</td>
<td>1166.87</td>
<td>1240.785</td>
<td>5.96%</td>
</tr>
<tr>
<td>9</td>
<td>973.657</td>
<td>997.412</td>
<td>-2.44%</td>
<td>1151.36</td>
<td>1236.892</td>
<td>6.92%</td>
</tr>
<tr>
<td>10</td>
<td>1005.05</td>
<td>982.990</td>
<td>2.19%</td>
<td>1141.94</td>
<td>1239.208</td>
<td>7.85%</td>
</tr>
</tbody>
</table>

Table 4. Results of multiobjective CACO versus the desirability function

Firstly, by observing the change in the response values we obtain for the various solutions (see Table 4), we can see that the solutions achieved with the hybrid approach vary much less than those obtained with the desirability function of Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007). It seems that our solutions are closer to each other. The reason is that the hybrid approach causes small displacements during the ant's research process. Thus when the fitness function decreases, the ants move over a short distance before re-test the function, if and only if, the obtained value is better than the previous one. Otherwise, the process reorients itself in case of declining performance.

Secondly and always in Table 4, by comparing our results with those of Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007) for the desirability function, the CACO-multiobjective approach shows that the second objective gives better quality results, with a 4.66% average improvement for the main goal (BDF & CACO-multiobjective), and 1.79% for the secondary one (DF & CACO-multiobjective).

Moreover, the observed variations in the answers values of the various solutions are visualized on figures 8a and 8b. (see below). These variations from the point of view of the BDF desirability function are shown on Fig 8a while those related to DF function are illustrated on Fig 8b. By observing these Figures, we observe that the solutions obtained with the CACO-multiobjective approach are smaller than those of classical approaches. As previously stated, these variations are explained by a small displacement of local ants, and when the fitness function decreases, ants move on a short distance before re-test the fitness function to obtain a new solution. These mechanisms and process and mechanisms are the same for the second desirability function visualized on Fig. 8b.

Following this application, and having obtained appreciable results, we can conclude that our algorithm functions correctly, while leading to coherent solutions, and that it has proven its effectiveness by obtaining better solutions than those of the authors, Abdul-Wahab & Abdo (Abdul-Wahab & Abdo, 2007).
6. Conclusion

This book chapter presents a new multiobjective optimization approach for mechanical system design. Various techniques have traditionally been employed to resolve this kind of problem, including an approach combining RSM, GA and a simulation tool such as FEM. We have the ACO, which allows the exploration of a combination which includes another optimization algorithm. The ACO captured our interest because we were able to note in various works that in multiobjective optimization, it does produce better results than the quadratic programming technique and the GA. The ACO thus appears to be an innovative and leading solution for design optimization, because it is completely generalized and independent of problem type, which allows it to be modified in order to optimize the design of a complex mechanical system, subject to various economical and mechanical criteria, and respecting many
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constraints. However, it must be recalled that the ACO was developed to resolve discrete problems, and that its use on continuous problems is constantly under development; our study contributes to the development of the continuous ACO for multiobjective problems.

The approach we present makes it possible to effectively optimize a mechanical design problem. The approach performs much better when compared to using the desirability function. The results of the application allow it to validate the suggested design optimization method.

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8. References


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This book attempts to bring together selected recent advances, tools, application and new ideas in manufacturing systems. Manufacturing system comprise of equipment, products, people, information, control and support functions for the competitive development to satisfy market needs. It provides a comprehensive collection of papers on the latest fundamental and applied industrial research. The book will be of great interest to those involved in manufacturing engineering, systems and management and those involved in manufacturing research.

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