1. Introduction

Spin is a very complex movement of an aircraft. It is, in fact, a curvilinear unsteady flight regime, where the rotation of the aircraft is followed by simultaneous rotation of linear movements in the direction of all three axes, i.e. it is a movement with six degrees of freedom. As a result, there are no fully developed and accurate analytical methods for this type of problem.

2. Types of spin

Unwanted complex movements of aircraft are shown in Fig.1. In the study of these regimes, one should pay attention to the conditions that lead to their occurrence. Attention should be made to the behavior of aircraft and to determination of the most optimal way of recovering the aircraft from these regimes. Depending on the position of the pilot during a spin, the spin can be divided into upright spin and inverted spin. During a upright spin, the pilot is in position head up, whilst in an inverted spin his position is head down.

The upright spin is carried out at positive supercritical attack angles, and the inverted spin at negative supercritical attack angles. According to the slope angle of the aircraft longitudinal axis against the horizon, spin can be steep, oblique and flat spin (Fig.2). During a steep spin,

Fig. 1. Unwanted rotations of aircraft
the absolute value of the aircraft slope angle is greater than 50 degrees, i.e. angle $|\upsilon| > 50^\circ$, during an oblique spin $30^\circ \leq |\upsilon| \leq 50^\circ$, and during a flat spin $|\upsilon| < 30^\circ$. According to direction of aircraft rotation, spin can be divided into a left and right spin. In a left spin (upright and inverted), the aircraft is rotating leftward, and during a right spin (upright and inverted) rightward. If the aircraft is observed from above, in a right upright and in a left inverted spin, the center of gravity of the aircraft will move in a clockwise direction, and vice versa while at a left upright and a right inverted spin.

The axis of spin is a an axis of a spiral by which the center of gravity is moving during a spin, and the spin radius is the radius of the horizontal projection of that spiral.

There exists inward and outward sideslip of the aircraft during a spin. The inward sideslip is when the air stream encounters the aircraft from the side of the inner wing, the wing in which direction the aircraft is rotating during a spin. The outward sideslip occurs when the air stream encounters the aircraft from the side of the outer wing.

Modern supersonic aircrafts (unlike older supersonic and even more the subsonic aircrafts) are characterized by a greater diversity of spin. This can be explained by the influence of constructive-aerodynamic properties of such aircrafts. Even for the same supersonic aircraft, the characteristics of upright and inverted spin can substantially differ depending on the initial angles of initiation (height, centering, etc.), regime length, position of rudder and ailerons during spin, etc. As a rule, these aircrafts have a distinct unevenness of motion and great fluctuations during spin.

The pilot has to study and reliably recognize the characteristic features of every spin type. Thus, he can quickly and accurately determine the type of spin and properly recover the aircraft from such a dangerous and complex motion. In modern aircrafts, spin can be divided into several types similar by characteristic features. The magnitude and character of changes in the angular velocities and load during a spin are taken into account as characteristic features. These determine the conditions for recovery from this regime, i.e. the value and sequence of rudder deflection. The given type includes all regimes of spin where in order
to recover the aircraft the same control has to be applied. According to this principle, spin
regime classifications are given on Figs. 3 and 4. In compliance with Figures 3 and 4, upright
spin has four, and the inverted spin has three regimes for recovery. On these images, letters
"N" and "L", alongside numerated ways of recovery, denote the upright and inverted spin,
respectively.

Fig. 3. Types of upright spin in modern aircrafts

Four basic methods can be applied for recovering from a upright spin (letter "N" denotes that
the method refers to a upright spin):

• Method N1N - recovery from spin with simultaneous positioning of elevator and rudder
in neutral position at neutral position of ailerons;
• Method N2N - recovery from spin by full rudder deflection opposite to spin with a delayed
setting (2 - 4 sec) of the elevator in neutral position at neutral position of ailerons;
• Method N3N - recovery from spin by rudder deflection, and a delayed (3 to 6 sec.) full
elevator deflection opposite to spin at neutral aileron position.
• Method N4N - recovery from spin as by method N3N but with simultaneous rudder and
aileron deflection, possibly by full for recovery (for supersonic aircrafts aileron deflection
is the appropriate deflection in the side of spin)

Recovery of modern aircrafts from inverted spin is carried out by three basic methods (letter
"L" denotes that the method applies to recovery from inverted spin):

• Method N1L - recovery from spin by simultaneous elevator and rudder positioning in
neutral with neutral aileron position;
• Method N2L - recovery from spin by rudder deflection totally opposite to spin with
delayed (2 to 4 sec.) elevator deflection into neutral position at neutral aileron position;
• Method N3L - recovery from spin by rudder deflection and an elevator deflection (delayed
2 to 4 sec.) totally opposite to spin at neutral aileron position.
3. Steady spin

A steady spin is a spin in which the aircraft does not change direction of rotation neither in roll or yaw (parameters p and r are unchanged sign). Rotation is very intense, and the average values of angular velocities do not change. Upright and inverted steady spins can be oscillating and uniform, and the upright can also be intensive.

The oscillating steady spin is characterized by very large changes in the amplitude of the rolling angular velocity and angular velocity slight changes yaw and pitch. It is usually a steep spin with average angles of attack of 30° or 40°. Changes of basic parameters during a left upright steady oscillating spin are shown on figure 5. Figure reflects an aircraft induced into spin at an altitude of 12.5 km at a velocity of 250 km/h. After a time interval \( t = 5 \) s a deflection of commands is carried out according to spin. Pilot has completely pulled the yoke to himself which has caused a negative rudder deflection (\( \delta_{hk} = -20° \)).

A positive deflection of the rudder is carried out \( \delta_{vk} = +20° \), whilst ailerons are in neutral position. After a time interval \( t = 6 \) s parameters become stable so the average angular velocity of yawing is \( r = -5 \frac{rad}{s} \) and of rolling \( p = -0.75 \frac{rad}{s} \), the average normal load coefficient is \( n_z = 1.25 \). However, after a time interval \( t = 8 \) s, the aircraft starts to oscillate and the following values are obtained: \( n_z = 1.4 \), \( p = 3.6 \frac{rad}{s} \) and \( r = 0.6 \frac{rad}{s} \). During that period the velocity on path V has oscillated between stabilized angular velocities of yawing and rolling, and an increase in velocity, which caused an increase in the normal load coefficient \( n_z \), so that during the interval \( t = (20 \div 25) \) s the normal load coefficient was \( n_z = 2 \div 3 \).

Unlike the oscillatory spin, the steady uniform spin is characterized by small amplitude oscillations of the aircraft, as well as intensive rotation of invariant direction. Figure 6 shows an example of a right-hand steady uniform spin. It can be seen that the aircraft was induced into spin at an altitude of \( H = 10.500 \) m and a velocity of \( V = 320 \) km/h. After an interval \( t = 6 \) s, commanding surfaces were deflected. First, a negative aileron deflection (\( \delta_k = -10° \)) can be observed resulting in a negative rolling moment, i.e. to lowering the right and lifting the left wing. Then the direction control was deflected in a negative direction (\( \delta_{hk} = -20° \)) which lead to a yaw to right. In addition, the yoke was drawn onto the pilot and thus a negative deflection (\( \delta_{hk} = -20° \)) was carried out on the elevator. At all time during spin, i.e. during
interval $t = (5 \div 40) \, s$, parameters (angular velocity of rolling $p$, angular velocity of yaw $r$, and velocity on path $V$) oscillated very little around the following values: $p_{sr} = -0.5 \, \text{rad}$, $r_{sr} = -0.5 \, \text{rad}$, and $V_{sr} = 250 \, \text{km/h}$. In addition, there was a noteworthy smaller loss of altitude of $\Delta H = 2.500 \, \text{m}$, than in the oscillating spin.

4. Unsteady spin

In some aircrafts, spin can periodically change the direction of rotation relative to normal and longitudinal axis. In some aircrafts, spin might even stop to oscillate, i.e. it will continue with certain oscillations. Spin at which the aircraft periodically changes direction of rotation regarding the normal and longitudinal axis, or even stops oscillating, is called unsteady.

During an unsteady spin, there is a non-uniform rotation with large amplitudes of aircraft parameter change. During the regime, usually there is a tendency towards an arbitrary transition of the aircraft from a spin of one direction into a spin of another direction, or from upright into inverted spin.

Unsteady spin in modern aircrafts has three forms: spin continuing with outbreak oscillations (diagram shown in Figure 7), spin continuing as a shape of a falling leaf on a spiral path (diagram shown in Figure 8), and spin during which oscillations of the aircraft are ascending (diagram shown in Figure 9).
4.1 Spin continuing with outbreak oscillations

Diagram of parameter change for this shape of spin is shown in Figure 7. From this diagram, it is obvious that this is a left upright spin. In addition, it can be noticed that there is a periodic change of parameters, especially the angular velocity of roll $p$, angular velocity of yaw $r$, and the coefficient of normal load $n_z$. Every cycle of outbreak (change of angular velocities in roll and yaw) stands for about 15 s. Between these cycles the aircraft has stopped rotating around it’s normal axis $r = 0$. Period of oscillations in every cycle was $t = 2.5$ s. Velocity on path ($V$), during spin was approximately constant $V = 280 \text{ km/h}$, whilst altitude loss during an interval of $t = 90$ s was $\Delta H = 8.000 \text{ m}$.

4.2 Spin continuing in the shape of a falling leaf on a spiral path - flat spin

The parameter change in this shape of spin is shown in Fig. 8. During this spin there were periodic changes in magnitude and direction of angular velocities ($p, r$), and with harsh changes of the aircraft position in space. The aircraft was leaning from one wing to the other, with a nose deflection to the left and right, so that it resembles the movement of falling leaves. During this regime, the aircraft’s center of gravity shifted according to a spiral path. The diagram shows that the normal load coefficient $n_z$ has a significant oscillation. During a period of $t = 40$ s loss in altitude was $\Delta H = 4.000 \text{ m}$.
4.3 Spin continuing with ascending (intensifying) oscillations

In this form of spin there is a significant increase of oscillations in pitch that lead to an increase in oscillations of the normal load coefficient \( n_z \), that began in \( t = 10 \) s. At time \( t = 55 \) s there is a rise in the normal load coefficient \( n_z = 6 \). Also, present are oscillations with a high frequency in angular velocity of rolling \( p \), and in the oscillation of the vertical rudder in the interval \( t = (40 \div 60) \) s. Figure 9 shows the diagram of parameter change for this shape of spin. It is important to note that on these diagrams the velocity on the path \( V \) and altitude \( H \) are not trustworthy because of the inaccuracy of the measuring device (Pitot tube). This came as a result of the flight regime at high angles of attack where, because of the separated flow, the flow parameters were significantly altered.

5. Phases of spin

A typical spin can be divided into two phases shown in Fig.10, and they are: spin entry, incipient spin, steady spin and recovery from spin.

5.1 The spin entry phase

The spin entry phase begins with the aircraft being at attack angles higher than the critical angle of attack. This is the condition known as wing stall. Influenced by many factors (geometrical or aircraft aerodynamic asymmetry, rudder or aileron deflection, etc.) or disturbances (vertical wind stroke), the flow around the aircraft is asymmetrical. Due to this asymmetrical flow, there are aerodynamic rolling and/or yawing moments and angular rolling and/or yawing velocities. This means that the aircraft holds an uncontrolled rotation with respect to all three abiding axes, i.e., an autorotation arises. During this rotation, the angle of attack can periodically be less than the critical angle of attack, i.e. the nose of the aircraft can periodically ascend and/or descend. At this stage, it is not possible to determine what type of spin will develop.
5.2 Incipient spin phase

The aircraft motion during the incipient spin phase is unsteady. Forces and moments acting on the aircraft are not in equilibrium. There are also, linear and angular accelerations. The aircraft rotates around the inclined axis, which is changing direction from horizontal to vertical. At this stage, the spin type can be determined.

5.3 Steady spin phase

During this phase, the aerodynamic and inertial forces and moments come into equilibrium. The aircraft is rotating downward around the vertical axis. The motion is steady, i.e. all motion parameters (attack angle, angular velocity, altitude loss per turn, time interval per turn, etc.) are constant.

5.4 Spin recovery phase

This stage begins by deflecting aircraft commanding surfaces into position for spin recovery. Autorotation stops and the aircraft enters dive as to increase flight speed. When the aircraft "accumulates" sufficient speed reserve, the pilot starts to pull out the aircraft from dive. This phase ends with the aircraft transitioning into horizontal flight.

6. Methods for upright spin recovery

As stated before, spin can only occur at overcritical attack angles. Due to this, for the aircraft to recover from spin it is necessary to decrease the attack angle, convert the aircraft to below...
Fig. 9. Spin with ascending oscillations

critical attack angles, at which autorotation stops. This comprises the basic task for spin recovery.

For some time after the discovery of the physical image of spin, propositions were made to decrease the attack angle with an appropriate deflection of the elevator. This method proved to be efficient only for some cases, when aerodynamic pitching moments, produced by elevator deflection, were greater than the inertial pitching moments by absolute value.

However, when using such a control method, in most cases the aircraft did not recover from spin, even at full deflection of elevators. The spin became only steeper, but it did not stop, especially for aircrafts with rear centering.

In development of aircrafts their mass increased, which meant an increase in inertial moments, specially the inertial pitching moment. Therefore, in order to recover from spin it was necessary to increase the aerodynamic pitching moments, as well. All the same, it became
clear that merely elevators could not achieve sufficient aerodynamic pitching moments for the purpose. Investigations have shown that in order to facilitate (sometimes, even to secure) spin recovery, previously the angular rate of rotation should be reduced (inertial moments decreased) for which an inward sideslip should be introduced, i.e. an aerodynamic yawing moment. This was achieved by deflecting the rudder opposite to spin.

As a result, the first scientifically based method for spin recovery was developed, by which a deflection was made opposite to spin, first by the rudder, then, after a delay (required because the inward sideslip, created by rudder deflection, could reduce the angular rate of autorotation), by the elevator. This was a, so-called standard method for spin recovery. However, it turned out that one standard method was not sufficient for modern aircrafts, characterized by vast diversity in spin regime.

Creating aerodynamic yawing moments, which in turn cause inward sideslip, is a powerful means for stopping, or at least greatly reducing the autorotation. Characteristics of aircraft recovery depend on the capability to achieve the best ratio between aerodynamic pitching and yawing moments, and between aerodynamic and appropriate inertial moments.

Four basic methods of spin recovery for modern aircrafts are as follows (letter "N" denotes method related to upright spin):

- **Method N1N** - spin recovery by simultaneous rudder and elevator positioning in neutral, with ailerons in neutral position;
- **Method N2N** - spin recovery by a full rudder deflection opposite to spin and a sequential elevator positioning in neutral (delayed by 2 - 4 sec), with ailerons in neutral position;
- **Method N3N** - spin recovery by rudder deflection, and sequential elevator deflection after 3 - 6 sec, completely opposite to spin, with ailerons in neutral position; and
- **Method N4N** - spin recovery by N3N method, but with simultaneous deflection of rudder and ailerons possibly at full for recovery (for supersonic aircrafts aileron deflection for recovery corresponds to a deflection to the side of spin.)

Deflection of controls and commanding surfaces during recovery from a left upright spin by the four basic methods is shown in Fig.11. Conditional labeling of yoke and pedal deflection that were adopted are shown in Fig.12.
The stated methods of spin recovery are structured by their efficiency or "force" increase, i.e. by the increase in aerodynamic moments created with rudder and elevator deflections for spin recovery. Therefore, the "weakest" method will be N1N and the "strongest" N4N.

Method N1N is recommended for aircraft recovery from upright unstable spin, method N2N for recovery from upright stable wavering spin, method N3N for recovery from upright stable uniform spin, and method N4N for recovery from upright stable intensive spin. These methods, as a rule, allow faster (with minimum time and altitude loss during recovery) and safer recovery of modern aircrafts from all possible upright spin regimes.

Fig. 11. Spin recovery methods for modern aircrafts from left upright spin (conditional notation of pedal and yoke position is shown in Fig. 12)

When the aircraft falls into spin, usually it is necessary to position rudder completely on the side of spin so that their efficiency will be best while deflecting for recovery. In this case, first, a maximum rudder deflection (greatest stroke) is obtained, and second, a dynamic ("shock") effect is used during an abrupt deflection of the rudder from one end position to the other. This kind of method applies only when recovering aircraft from sufficiently stable spin regimes. However, since the pilot cannot know beforehand the type of spin that will be created (stable, unstable,...), as a rule, he has to completely position the rudders onto the side of spin.
The discussed methods for modern aircrafts’ upright spin recovery (as well as method for inverted spin recovery), were developed by special investigations during flight. The possibility of applying four methods instead of a standard one (method N3N) substantially increases the assurance of spin recovery, which means a safe flight. On the other hand, this requires additional attention of the pilot, because now he has to choose among the four required methods to recover from spin and to remember the sequence of rudder activities for every method.

![Diagram](https://via.placeholder.com/150)

Fig. 12. Conditional annotation of deflections for yoke and pedals

However, in opinion of many pilots of high and medium proficiency, this problem does not cause extra caution, as it may seem at first glance. First, these spin recovery methods differ from one another only by "force", i.e. by the magnitude of required deflections and the interval of delay between them. Second, same rudders are used for spin recovery (only spin recovery by method N4N requires additional deflection of ailerons), and the direction of controls deflections is the same.

For subsonic aircrafts, just the use of "strong" method was generally appropriate, since there was danger of a "lack" of rudder for spin recovery. Hence, the required efficiency of subsonic aircraft rudders was conditioned, mostly by the need to ensure spin recovery, i.e. the ability to create sufficient aerodynamic moments to stop autorotation.

On the other hand, supersonic airplanes have the risk of excessive rudder "delivery" during recovery, i.e. creating excessively large aerodynamic moments. The possibility of exhibiting this danger has to do with that the required efficiency of the supersonic airplane rudder is chosen starting from the requirement to provide maneuverability in flight at high Mach numbers, contrary to subsonic aircrafts. Hence, as a rule, the efficiency is more than sufficient for a spin recovery. Excessive rudder deflection can significantly impair characteristics of recovery (for example, unnecessarily increase the steepness of dive after spin, and consequently loose altitude for spin recovery, etc.), or lead to impossible spin recovery (the plane changes from upright to inverted spin, from left to right spin, etc.). This explains the necessity to apply the "weak" methods (N1N and N2N) for spin recovery.

However, only "strong" methods should be used for recovery of supersonic airplanes from stable intensive and uniform spins.

Therefore, for recovery of modern supersonic aircrafts from upright spin it is necessary to use "weak", as well as "strong" methods. It should be in mind at all times that, "strong" methods
in any case are not above nor can they replace the "weak" methods. Each method has its area of application.

When choosing a recovery method the pilot must be guided only by the character of spin at the time he decided to recover from spin. Other data (for example, flight altitude) can be used only as supplementary for refining and hastening the determination of regime characteristics.

In all cases, it is necessary to deflect the rudders, if possible, more abrupt. Slow, sluggish deflecting will worsen the characteristics of spin recovery, and, sometimes, make it impossible to recover.

If recovery of aircraft is particularly difficult, the pilot must carefully choose the moment to start recovering, i.e. moment when to deflect the first rudder for recovery. The best moment for the recovery start is considered to be the moment when the plane stops yawing, the nose starts declining (in a inverted spin - nose starts inclining) and similar.

It is better for the pilot to count seconds instead of turns when deflecting rudder and elevator. Practice has shown that even small changes in the regime (accelerating or decelerating the aircraft) make it difficult and often impossible to properly count turns, especially at the notable positions of the aircraft in space (during periodic switching to inverted, etc.). It is not advantageous to count turns of the aircraft during spin, not only for recovery but during the regime, as well. There are several reasons for this. First, in an unstable spin regime (for example, in a flat spin where the aircraft’s motion acts like a "falling leaf") the term "turn" is meaningless. Second, for the pilot, in order to evaluate the situation, it is of more importance that he knows the time of the acting regime and the altitude loss during that time. In addition, it is always easier to count seconds of duration (also much safer), and often, sometimes, it could be the only possible way to determine the mentioned interval and delay of aircraft recovery from spin. The interval and delay are usually determined count seconds aloud.

The control of altitude change during a spin is one of the most important conditions of a safe flight, especially entering spin at low altitudes.

Therefore, the pilot must follow the altimeter reading during a spin. The altimeter shows absolute values of altitude with great errors. This is due to a greater change in the flow over the Pitot-tube at higher attack angles and yaw angles, and at high rotation rates during spin. Even so, this instrument allows a proper determination of the altitude loss (altitude difference).

Flight practice shows that the most often applied methods for spin recovery for supersonic aircraft are N1N and N2N. If the first attempt to recover from a spin fails (for example, with the method N1N), i.e. the self-rotation did not stop, the pilot has to reset flight commands into position for spin, and, after 2 - 4 sec repeat recovery but with a "stronger" method (N2N). Obviously, in the first attempt the pilot applied a "weak" method due to an improper determination of the character of the spin.

The choice of the required method for spin recovery greatly depends on the weight composition of the aircraft - mass distribution and centering.

A known fact is that subsonic aircrafts, with rectangular wings, characteristically have a large weight distribution along the wingspan, which in turn adds up to generating significant inertial rolling and yawing moments during spin. On the other hand, modern supersonic aircrafts have a characteristically large mass distribution along the fuselage axis, which contributes to generation of significant inertial pitching and yawing moments.
The sequence of actions with the rudders for spin recovery depends on the nature of the interaction of inertial rolling, yawing, and pitching moments, with the inertial moments created by rudder deflection during autorotation.

7. Methods for recovery from inverted spin

To recover modern aircrafts from an inverted spin there are three basic methods (letter "L" denotes the method for recovery from an inverted spin):

- Method N1L - spin recovery by simultaneous positioning elevator and rudder into neutral, with ailerons in neutral position;
- Method N2L - spin recovery by deflecting the rudder fully opposite, followed by a delayed (2 - 4 sec) elevator positioning into neutral, with ailerons in neutral position;
- Method N3L - spin recovery by fully deflecting the both rudder and elevator (delayed for 2 - 4 sec) opposite to spin direction, with ailerons in neutral position.

The N1L method is recommended for recovery from an unstable inverted spin, the N2L method from a stable wavering spin, and method N3L from an inverted stable uniform spin.

Deflection of controls and commanding surfaces during inverted spin recovery according to the three basic methods is shown in Fig.13, with the adopted notation shown in Fig.12.

For supersonic aircrafts, most often in use is the method N2L, because these aircrafts are most characterized by an unstable wavering inverted spin. In general, supersonic airplanes rarely fall into inverted spin; more often, it is a upright spin.

As a rule, airplanes of usual geometry more easily recover from an inverted spin than from a upright spin. This is explained by the fact that during this regime the autorotation is weaker, the rudder is more efficient (practically it is out of the wing and stabilizer flow), the efficient arrow angle of the vertical surfaces is decreased, and the average absolute values of attack angles are reduced.

However, despite of everything already said, for the pilot the inverted spin is always more difficult than the upright spin. This is conditioned by the unusual position of the pilot: hanging on restraint harnesses, head down, and a negative load \( n_z < 0 \) tends to detach him from the seat. In such conditions, the pilot could drop the yoke and release the pedals (he could lose control of the aircraft, especially if he is not firmly seated).

Sometimes, during aircraft wavering the pilot has great difficulty to visually determine type of spin - upright or inverted. This is expressed if the aircrafts longitudinal axis is close to the vertical axis - the plane will "swirl" at low, by absolute value, negative overcritical attack angles, which is typical for a stable inverted spin. In this case, in order to stop autorotation the rudders have to be put in neutral position.

In the absence of or inability to use visual landmarks (also, to have control over a classical spin), the pilot can easily determine the type of spin by sensation: if the seat is pressuring the pilot - it is a upright spin, if the pilot is detaching from his seat, i.e. hanging on restraint harnesses - it is an inverted spin). However, if the aircraft exits at high negative attack angles that randomly change during the inverted spin regime, determining spin type by sensation is inapplicable.

During an inverted spin, it is more difficult to determine the direction of rotation (whether the plane is rotating to the right or to the left). In a classical spin, when the position of the
nose of the aircraft to the horizon is practically unchanged, the direction of rotation can easily be determined according to the angular speed of rotation. However, when in inverted spin at high angular velocities of rolling, uneven motion of roll and pitch, it is impossible to determine the aircraft’s direction of rotation with the mentioned method. The situation becomes more complex, because during inverted spin rolling motion is opposite to rotation. For a pilot seated in front cockpit look forward this means that, for example, in a right-hand spin the direction of rotation will into the left.

In a upright spin, the situation is opposite: directions of roll and yaw are the same. As so, for example, in a right upright spin the pilot can see the nose of the aircraft turning to the right and plane leaning to the same side. A more experienced pilot in this matter can distinguish a upright spin from an inverted.

Pilots lacking of sufficient flights for spin recovery training often determine spin direction according to rolling direction, but not yawing direction, because the rolling angular velocity
is usually higher than the yawing angular velocity (except for a flat spin). Determination of spin direction in this manner is applied more often (rolling angular velocity increasing). Use of this manner of determining spin during an inverted spin will only disorient an insufficiently trained pilot.

Therefore, when recovering from this regime, it is necessary to have safe means of control to facilitate easy conservation of spatial orientation and assure possibilities for effective and proper actions with wings. Such means could be the yaw indicator (it hand always turns in yaw direction regardless on spin type) and attack angle indicator, and if it is not present - a normal load indicator. Attack angle indicator allows the pilot to reliably determine the spin type (upright or inverted), and the yaw indicator - its direction (left or right).

8. Spin modeling

Modeling is the most accurate graph-analytical method for determining aircraft characteristics prior to flight tests. Modeling of flight conditions, with initial data correction, quite properly reflects timely development of aircraft motion and enables more complete conclusions about flight test results.

The differential equations of aircraft motion relative to its center of gravity is obtained from the Law of conversation of momentum. This is the moment equation. Projecting these equations to axes of the attached coordinate system \((X_1, Y_1, Z_1)\), shown in Fig.14, whose axes we denote as \((X, Y, Z)\) for simplicity, the following system of differential equations is derived:

\[
m \left( \frac{dV_x}{dt} + q V_z - r V_y \right) = R_x + G_x
\]

\[
m \left( \frac{dV_y}{dt} + r V_x - p V_z \right) = R_y + G_y
\]

\[
m \left( \frac{dV_z}{dt} + p V_y - q V_x \right) = R_z + G_z
\]

\[
I_x \frac{dp}{dt} + (I_z - I_y) q r + I_{xy} \left( p r - \frac{dp}{dt} \right) = M_x
\]

\[
I_y \frac{dq}{dt} + (I_x - I_z) p r + I_{xy} \left( q r - \frac{dp}{dt} \right) = M_y
\]

\[
I_z \frac{dr}{dt} + (I_y - I_x) p q + I_{xy} \left( p^2 - q^2 \right) = M_z
\]
where:

- $V_x, V_y, V_z$ correspond to projections of the velocity of aircraft center of gravity with respect to the axes of the adopted coordinate system;
- $p, q, r$ represent projections of the aircraft angular velocities to the axes of the adopted coordinate system;
- $I_x, I_y, I_z$ - aircraft axial inertia moments with respect to axes of the adopted coordinate system;
- $I_{xy}$ - aircraft centrifugal inertia moment;
- $R_x, R_y, R_z$ - projections of aerodynamic forces acting on the aircraft with respect to axes of the adopted coordinate system;
- $G_x, G_y, G_z$ - projection of aircraft weight with respect to axes of the adopted coordinate system;
- $M_x, M_y, M_z$ - projection of resulting moments from external forces acting on aircraft, with respect to axes of the adopted coordinate system.

8.1 Motion equations for modeling

In order to simplify the task during tests, following assumptions are made: the angle of sideslip is small so that $\sin \beta \approx \beta$ and $\cos \beta \approx 1$, effects of Mach and Reynolds number are ignored, and motion is investigated with engines turn off. This study uses known kinematic relations that are, with the adopted simplifications, equal to:

$$
\dot{\theta} = r \sin \gamma + q \cos \gamma \\
\dot{\gamma} = p + (q \sin \gamma - r \cos \gamma) \tan \theta \\
\dot{H} = V \cos \alpha \sin \theta - V \sin \alpha \cos \gamma \cos \theta - V \beta \sin \gamma \cos \theta
$$

where:
- $\theta$ - pitch angle, angle between X axis and horizontal plane,
- $\gamma$ - angle of transverse inclination, angle between Y axis and vertical plane.

If it is assumed that velocity and altitude are unchanged, i.e. $V = \text{const}$ and $H = \text{const}$, and if the right-hand sides of Eq.(3) are approximated by a Taylor polynomial, the following simplified system of equations is obtained, a so-called System I:

$$
\dot{\alpha} = a_{11} \frac{1}{\cos \alpha} + a_{12} \frac{\beta p}{\cos \alpha} + a_{13} q + a_{14} \frac{\cos \theta \cos \gamma}{\cos \alpha} + a_{15} \\
\dot{\beta} = a_{21} \beta + a_{22} r \cos \alpha + a_{23} p \sin \alpha + a_{24} \cos \theta \sin \gamma + a_{25} \\
\dot{p} = a_{31} r q + a_{32} \beta + a_{33} p + a_{34} r + a_{35} + a_{36} \\
\dot{r} = a_{41} p q + a_{42} \beta + a_{43} p + a_{44} r + a_{45} q + a_{46} + a_{47} \\
\dot{q} = a_{51} p r + a_{52} \beta | + a_{53} r + a_{54} q + a_{55} + a_{56} \dot{\alpha} + a_{57} \\
\dot{\theta} = a_{61} r \sin \gamma + a_{62} q \cos \gamma \\
\dot{\gamma} = a_{71} p + a_{72} r \cos \gamma \tan \theta + a_{73} q \sin \gamma \tan \theta
$$
The coefficients involved in the system of equations (4) are defined by expressions:

\[
\begin{align*}
a_{11} &= -\frac{S \rho V}{2m} C_z(\alpha) & a_{12} &= -1 & a_{13} &= 1 & a_{14} &= \frac{g}{V} \\
a_{15} &= -\frac{S \rho V}{2m} C_{z_{jk}}(\alpha) \Delta \delta_{jk}(t) & a_{21} &= \frac{S \rho V}{2m} C_{y_k}(\alpha) \\
a_{22} &= 1 & a_{23} &= 1 & a_{24} &= \frac{g}{V} & a_{25} &= \frac{S \rho V}{2m} C_{y_{jk}}(\alpha) \Delta \delta_{jk}(t) \\
a_{31} &= \frac{I_y - I_z}{I_x} & a_{32} &= \frac{S I \rho V^2}{2 I_x} C_{l_p}(\alpha) & a_{33} &= \frac{S I \rho V^2}{2 I_x} C_{l_p}(\alpha) \\
a_{34} &= \frac{S I \rho V^2}{2 I_x} C_{l_p}(\alpha) & a_{35} &= \frac{S I \rho V^2}{2 I_x} C_{l_{jk}}(\alpha) \Delta \delta_{jk}(t) \\
a_{36} &= \frac{S I \rho V^2}{2 I_x} C_{l_k}(\alpha) \Delta \delta_{jk}(t) & a_{41} &= \frac{I_z - I_x}{I_y} \\
a_{42} &= \frac{S I \rho V^2}{2 I_y} C_{n_p}(\alpha) & a_{43} &= \frac{S I \rho V^2}{2 I_y} C_{n_p}(\alpha) \\
a_{44} &= \frac{S I \rho V^2}{2 I_y} C_{n_p}(\alpha) & a_{45} &= -\frac{I_p \omega_p}{I_y} \\
a_{46} &= \frac{S I \rho V^2}{2 I_y} C_{n_{jk}}(\alpha) \Delta \delta_{jk}(t) & a_{47} &= \frac{S I \rho V^2}{2 I_y} C_{n_{jk}}(\alpha) \Delta \delta_{jk}(t) \\
a_{51} &= \frac{I_y - I_z}{I_y} & a_{52} &= \frac{S b \rho V^2}{2 I_z} C_{m_p}(\alpha) & a_{53} &= \frac{I_p \omega_p}{I_z} \\
a_{54} &= \frac{S b \rho V^2}{2 I_z} C_{m_p}(\alpha) & a_{55} &= \frac{S b \rho V^2}{2 I_z} C_m(\alpha) \\
a_{56} &= \frac{S b \rho V^2}{2 I_z} C_{m_p}(\alpha) & a_{57} &= \frac{S b \rho V^2}{2 I_z} C_{m_{jk}}(\alpha) \Delta \delta_{jk}(t) \\
a_{61} &= 1 & a_{62} &= 1 & a_{71} &= 1 & a_{72} &= -1 & a_{73} &= 1 & (5)
\end{align*}
\]

If assumed that velocity and altitude are changeable over time, i.e. \( V = f(t) \) and \( H = f(t) \), and if the right-hand side of Eq.(1) and (3) (projections of aerodynamic forces, weights and moments) is approximated by Taylor polynomial, the following simplified system of equations can be obtained, a so-called 

**System II**:

\[
\begin{align*}
\dot{\alpha} &= b_{11} \frac{\rho V}{\cos \alpha} + b_{12} \frac{p \beta}{\cos \alpha} + b_{13} q + b_{14} \frac{\cos \theta \cos \gamma}{V \cos \alpha} + b_{15} \frac{\dot{V}}{V} \tan \alpha + b_{16} \frac{\rho V}{\cos \alpha} \\
\dot{\beta} &= b_{21} \rho V \beta + b_{22} \rho V \cos \alpha + b_{23} p \sin \alpha + b_{24} \frac{\cos \theta \cos \gamma}{V} + b_{25} \frac{\dot{V} \beta}{V} + b_{26} \rho V \\
\dot{\phi} &= b_{31} \rho V^2 \beta - b_{33} \rho V^2 p + b_{34} \rho V^2 r + b_{35} \rho V^2 + b_{36} \rho V^2
\end{align*}
\]
\[ \dot{r} = b_{41} p q + b_{42} \rho V^2 \beta + b_{43} \rho V^2 p + b_{44} \rho V^2 r + b_{45} q + b_{46} \rho V^2 + b_{47} \rho V^2 \]
\[ \dot{q} = b_{51} p r + b_{52} \rho V^2 |\beta| + b_{53} r + b_{54} \rho V^2 q + b_{55} \rho V^2 + b_{56} \rho V^2 \dot{\alpha} + b_{57} \rho V^2 \]
\[ \dot{\theta} = b_{61} r \sin \gamma + b_{62} q \cos \gamma \]
\[ \dot{\gamma} = b_{71} p + b_{72} r \cos \gamma \tan \theta + b_{73} q \sin \gamma \tan \theta \]
\[ \dot{V} = b_{81} \frac{\rho V^2}{\cos \alpha} + b_{82} \dot{V} \tan \alpha + b_{83} V q \tan \alpha + b_{84} \frac{V r \beta}{\cos \alpha} + b_{85} \frac{\sin \theta}{\cos \alpha} + b_{86} \frac{\rho V^2}{\cos \alpha} \]
\[ \dot{H} = b_{91} V \cos \alpha \sin \theta + b_{92} V \sin \alpha \cos \theta \cos \gamma + b_{93} \beta \cos \theta \sin \gamma \]

The coefficients involved in the system of equations (6) are defined by expressions:

\[
\begin{align*}
b_{11} &= -\frac{S}{2m} C_z(\alpha) \\
b_{12} &= -1 \\
b_{13} &= 1 \\
b_{14} &= g \\
b_{15} &= -1 \\
b_{16} &= -\frac{S}{2m} C_{z_hh}(\alpha) \Delta \delta_{hk}(t) \\
b_{21} &= \frac{S}{2m} C_{y_h}(\alpha) \\
b_{22} &= 1 \\
b_{23} &= 1 \\
b_{24} &= g \\
b_{25} &= -1 \\
b_{26} &= \frac{S}{2m} C_{y_h}(\alpha) \Delta \delta_{vk}(t) \\
b_{31} &= \frac{I_y - I_z}{I_x} \\
b_{32} &= \frac{S}{2I_x} C_{l_p}(\alpha) \\
b_{33} &= \frac{S}{2I_x} C_{l_p}(\alpha) \\
b_{34} &= \frac{S}{2I_x} C_{l_h}(\alpha) \\
b_{35} &= \frac{S}{2I_x} C_{l_h}(\alpha) \Delta \delta_{vk}(t) \\
b_{36} &= \frac{S}{2I_x} C_{l_h}(\alpha) \Delta \delta_{vk}(t) \\
b_{41} &= \frac{I_z - I_x}{I_y} \\
b_{42} &= \frac{S}{2I_y} C_{n_p}(\alpha) \\
b_{43} &= \frac{S}{2I_y} C_{n_p}(\alpha) \\
b_{44} &= \frac{S}{2I_y} C_{n_r}(\alpha) \\
b_{45} &= -\frac{I_p \omega_p}{I_y} \\
b_{46} &= \frac{S}{2I_y} C_{n_h}(\alpha) \Delta \delta_{vk}(t) \\
b_{47} &= \frac{S}{2I_y} C_{n_h}(\alpha) \Delta \delta_{vk}(t) \\
b_{51} &= \frac{I_x - I_y}{I_z} \\
b_{52} &= \frac{S}{2I_z} C_{m_h}(\alpha) \\
b_{53} &= \frac{I_p \omega_p}{I_z} \\
b_{54} &= \frac{S}{2I_z} C_{m_h}(\alpha) \\
b_{55} &= \frac{S}{2I_z} C_{m_h}(\alpha) \\
b_{56} &= \frac{S}{2I_z} C_{m_h}(\alpha) \\
b_{57} &= \frac{S}{2I_z} C_{m_h}(\alpha) \Delta \delta_{hk}(t) \\
b_{61} &= 1 \\
b_{62} &= 1 \\
b_{71} &= 1 \\
b_{72} &= -1 \\
b_{73} &= 1 \\
b_{81} &= -\frac{S}{2m} C_x(\alpha) \\
b_{82} &= 1 \\
b_{83} &= -1 \\
b_{84} &= -1 \\
b_{85} &= -g \\
b_{86} &= -\frac{S}{2m} C_{x_hh}(\alpha) \Delta \delta_{hk}(t) \\
b_{91} &= 1 \\
b_{92} &= -1 \\
b_{93} &= -1
\end{align*}
\]

Notation in previous equations are:
- $C_z$ - lift coefficient;
- $C_x$ - drag coefficient;
• $C_m$ - pitching moment coefficient;
• $C_{z_{\delta_hk}}$ - derivative of lift coefficient with respect to angle of deflection of the elevator;
• $C_{y_{\beta}}$ - derivative of sideslip force with respect to sideslip angle;
• $C_{y_{\delta_vk}}$ - derivative of sideslip force with respect to angle of deflection of the rudder;
• $C_{l_{\beta}}$ - derivative of rolling moment coefficient with respect to angle of sideslip;
• $C_{l_p}$ - derivative of rolling moment coefficient with respect to rolling angular velocity;
• $C_{l_{\delta_v}}$ - derivative of rolling moment coefficient with respect to rudder angle of deflection;
• $C_{l_{\delta_k}}$ - derivative rolling moment coefficient with respect to aileron angle of deflection;
• $C_{n_{\beta}}$ - derivative of yawing moment coefficient with respect to angle of sideslip;
• $C_{n_p}$ - derivative of yawing moment coefficient with respect to rolling angular velocity;
• $C_{n_{\delta_v}}$ - derivative of yawing moment coefficient with respect to rudder angle of deflection;
• $C_{n_{\delta_k}}$ - derivative of yawing moment coefficient with respect to aileron angle of deflection;
• $C_{m_{\beta}}$ - derivative of pitching moment coefficient with respect to angle of sideslip;
• $C_{m_q}$ - derivative of pitching moment coefficient with respect to pitching angular velocity;
• $C_{m_\dot{\alpha}}$ - derivative of pitching moment coefficient with respect to derivative of angle of attack over time;
• $C_{m_{\delta_hk}}$ - derivative of pitching moment coefficient with respect to elevator angle of deflection;
• $C_{x_{\delta_hk}}$ - derivative of drag coefficient with respect to elevator angle of deflection;
• $I_p$ - polar moment of inertia of engine rotor;
• $\omega_p$ - angular velocity of engine rotor;
• $\Delta \delta_{h_k}$ - change in elevator angle of deflection;
• $\Delta \delta_{v_k}$ - change in rudder angle of deflection;
• $\Delta \delta_k$ - change in aileron angle of deflection.

These coefficients are entirely determined with the aid of DATCOM\textsuperscript{1} reference, and in some additional literature\textsuperscript{2} their values are defined for the category of light aircrafts.

\textsuperscript{1} D. E. Hoak: \textit{USAF Stability and Control DATCOM}, (N76-73204), Flight Control Devision, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, 1975.

8.2 Methods of modeling

It is obvious that most of the coefficients in Exp.(6) are not constant, but vary with time, i.e. angle of attack.

If assumptions that speed and altitude are constant are rejected, a complete system of equations is obtained. In order to perform modeling a computer is used with its memory loaded with the simplified or complete system of equations, which depends on the regime that has to be studied or the accuracy of results. The computer loaded with data necessary to calculate the coefficients and other values in Eq.(6), for a given time \(t_1\), or the given angle of attack \(\alpha_1\). Data is obtained by testing models in wind tunnel or in flight. Then, the data is entered for moment \(t_2\) or attack angle \(\alpha_2\), and so on. Data is entered until the end of the observed time interval, and it is entered point-by-point. The accuracy of obtained results depends on the magnitude of the time change (time difference between two points), i.e. the angle of attack. The more points are entered, the more accurate the results will be. The computer will integrate and associate values for angle of attack, angle of sideslip, pitching angle and angular velocities for the observed time interval, for which data is entered, and results are obtained as shown in Fig.15, i.e. following functions are defined: \(\alpha = f(t), \theta = f(t), \beta = f(t), p = f(t), q = f(t) i r = f(t)\). For System II (Eq.7), following the same analogy, the

![Diagram for a timely development of a left-hand spin (System I)](image)

Fig. 15. Diagram for a timely development of a left-hand spin (System I)
8.3 Analysis of results from modeled flat spin (spin ongoing as a falling leaf on a spiral path)

An analysis of relations is made if effects from individual terms in equations of motion need to be determined (Eq.6). To do this, each member will be designated by the letter "A" with appropriate numerical indexes $i, j$ ($A_{ij}$), with "i" being the ordinal number of the equation $i = (1, n)$, and "j" being the ordinal number of the term in the equation $j = (1, m)$. For example, the first equation of Eq.6 will look:

$$\dot{\alpha} = A_{11} + A_{12} + A_{13} + A_{14} + A_{15},$$

where:

$$A_{11} = a_{11} \frac{1}{\cos \alpha}, \quad A_{12} = a_{12} \frac{\beta p}{\cos \alpha}, \quad A_{13} = a_{13} q, \quad A_{14} = a_{14} \frac{\cos \theta \cos \gamma}{\cos \alpha}, \quad A_{15} = a_{15}$$

Ratios of absolute values of terms in Eq.6 are assessed, as of these depend the magnitudes of effects of terms $\alpha, \beta, \theta, p, q$ and $r$, and therefore are shown as fractions:

$$\bar{A}_{ij} = \frac{|A_{ij}|}{\sum_{j=1}^{m} |A_{ij}|} \times 100 \quad [\%]$$
In case of modeling spin with the complete system of equations, terms $A_{ij}$ and $\bar{A}_{ij}$ are introduced in the same manner.

Fig. 17. Diagram of development for a flat spin (spin ongoing as a falling leaf)

Figure 17 shows results obtained for a numerically modeled spin. From diagrams (b), (c), (d), (e), and (f), it can be seen which terms have the most effect on results given in diagram (a). Diagrams (b), (c), (d), (e), and (f), were constituted by showing time on the abscissa, while values of $\bar{A}_{ij}$ ($i = \text{const}$) are "stacked" on the ordinate, one over the other. For example, value $\bar{A}_{11}$ is imposed from 0, value $\bar{A}_{12}$ is imposed from point where $\bar{A}_{11}$ ends, value $\bar{A}_{13}$ is imposed from point where $\bar{A}_{12}$, ... ends, value $\bar{A}_{im}$ is imposed from point where $\bar{A}_{i,m-1}$ ends, and ends at 100%. By doing so, these diagrams show which term $\bar{A}_{ij}$ has the most impact on values of $\alpha$, $\beta$, $\theta$, $p$, $q$, and $r$. For example, in diagram (b) it is shown that at moment $t = 23$ s, value of $\bar{A}_{13}$ is higher than of $\bar{A}_{11}$, $\bar{A}_{12}$, $\bar{A}_{14}$ and $\bar{A}_{15}$. This means that $\bar{A}_{13}$ has the most effect on values of function $\alpha = f(t)$ at the given time. When observed what $A_{13}$ is equal to, it
can be noticed that the value of angle of attack, at this moment, mostly depends on $q$, since $a_{13} = 1$. In this way, every term $A_{ij}$ on the right-hand side of Eq.(6) can be analyzed on its impact on the left-hand side of same equations. Analysis of every term $A_{ij}$ brings about a deeper understanding of the physical image of the studied regime. In addition, it enables the determination of terms that have the most effect on such a state, in cases when the aircraft does not fulfill necessary requirements for spin, and by appropriate modifications obtain an aircraft with necessary technical characteristics for spin.

9. References

The book substantially offers the latest progresses about the important topics of the "Mechanical Engineering" to readers. It includes twenty-eight excellent studies prepared using state-of-art methodologies by professional researchers from different countries. The sections in the book comprise of the following titles: power transmission system, manufacturing processes and system analysis, thermo-fluid systems, simulations and computer applications, and new approaches in mechanical engineering education and organization systems.

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