Application of Wavelet Analysis in Power Systems

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1. Introduction

When you capture and plot a signal, you get only a graph of amplitude versus time. Sometimes, you need frequency and phase information, too. However, you need to know whenever in a waveform the certain characteristics occur. Signal processing could help, but you need to know which type of processing to apply to solve your data-analysis problem.

Many books and papers have been written that explain WT of signals and can be read for further understanding of the basics of wavelet theory. The first recorded mention of what we now call a "wavelet" seems to be in 1909, in a thesis by A. Haar. The concept of wavelets in its present theoretical form was first proposed by J. Morlet, a Geophisicist, and the team at the Marseille Theoretical Physics Center working under A. Grossmann, a theoretical phisicist, in France. They provided a way of thinking for wavelets based on physical intuition. They also proved that with nearly any wave shape they could recover the signal exactly from its transform (Graps, 1995). In other words, the transform of a signal does not change the information content presented in the signal.

The wavelet functions are created from a single charasteristic shape, known as the mother wavelet function, by dialating and shifting the window. Wavelets are oscillating transforms of short duration amplitude decaying to zero at both ends. Like the sine wave in Fourier transform (FT), the mother wavelet $\psi(t)$ is the basic block to represent a signal in WT. However, unlike the FT whose applications are fixed as either sine or cosine functions, the mother wavelet, $\psi(t)$, has many possible functions. Fig. 1 shows some of the popular wavelets including Daubechies, Harr, Coiflet, and Symlet. Dilation involves the stretching and compressing the mother wavelet in time. The wavelet can be expanded to a coarse scale to analyze low frequency, long duration features in the signal. On the other hand, it can be shrunk to a fine scale to analyze high frequency, short duration features of a signal. It is this ability of wavelets to change the scale of observation to study different scale features is its hallmark.

The WT of a signal is generated by finding linear combinations of wavelet functions to represent a signal. The weights of these linear combinations are termed as wavelet coefficients. Reconstruction of a signal from these wavelet coefficients arises from a much older theory known as Calderon's reproducing activity (Grossmann & Morlet, 1984).

The attention of the signal processing community was caught when Mallat (Mallat, 1989) and Daubechies (Daubechies, 1988) established WT connections to discrete signal processing. To date various theories have been developed on various aspects of wavelets and it has been successfully applied in the areas of signal processing, medical imaging, data compression, image compression, sub-band coding, computer vision, and sound synthesis.

There is a plan in this chapter to study the WT applications in power systems. The content of the chapter is organized as follows:

The first part explains a brief definition of wavelet analysis, benefits and difficulties. The second part discusses wavelet applications in power systems. This section is consisted of the modeling guidelines of each application whose the goal is to introduce how to implement the wavelet analysis for different applications in power systems. Also there will be a literature review and one example for each application, separately. In the last part, the detailed analysis for two important applications of wavelet analysis, i.e. detection of the islanding state and fault location, will be illustrated by the authors.

Although, there have been a great effort in references to prove that one wavelet is more suitable than another, there have not been a comprehensive analysis involving a number of wavelets to prove the point of view suggested. Also, the method of comparison among them is not unified, such that a general conclusion is reached. In this chapter, algorithms are also presented to choose a suitable mother wavelet for power system studies.

In general, the properties of orthogonality, compactness support, and number of vanishing moments are required when analyzing electric power system waveforms for computing the power components. All these properties are well described in (Ibrahim, 2009).
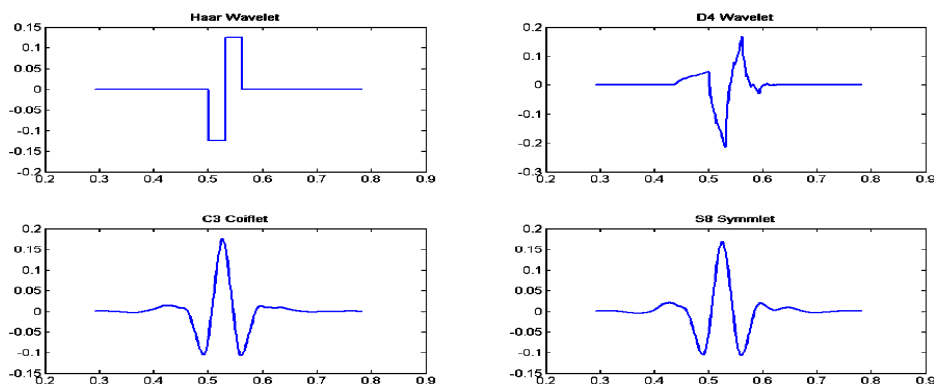

Fig. 1. Some of the popular wavelets used for analysis

2. Wavelet transform

There are several types of WTs and depending on the application, one method is preferred over the others. For a continuous input signal, the time and scale parameters are usually continuous, and hence the obvious choice is continuous wavelet transform (CWT). On the other hand, the discrete WT can be defined for discrete-time signals, leading to discrete wavelet transform (DWT).

2.1 Continuous wavelet transform (CWT)

The CWT is defined as:

$$CWT(a,b) = \int_{-\infty}^{+\infty} x(t)\psi^*_{a,b}(t)\,dt \qquad a > 0 \tag{1}$$

where $x(t)$ is the signal to be analyzed, $\psi_{a,b}(t)$ is the mother wavelet shifted by a factor (b), scaled by a factor (a), large and low scales are respectively correspondence with low and high frequencies, and * stands for complex conjugation.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad a > 0 \quad and \quad -\infty < b < +\infty \tag{2}$$

2.2 Discrete wavelet transform (DWT)

CWT generates a huge amount of data in the form of wavelet coefficients with respect to change in scale and position. This leads to large computational burden. To overcome this limitation, DWT is used. In other words, in practice, application of the WT is achieved in digital computers by applying DWT on discretized samples. The DWT uses scale and position values based on powers of two, called dyadic dilations and translations. To do this, the scaling and translation parameters are discreted as $a = a_0^m$ and $b = nb_0 a_0^m$, where $a_0 > 1$, $b_0 > 0$, and m, n are integers, then the DWT is defined as:

$$DWT(m,n) = \int_{-\infty}^{+\infty} x(t)\psi^*_{m,n}(t)\,dt \tag{3}$$

where $\psi_{m,n}(t) = a_0^{-m/2}\psi\left((t - na_0^m b_0)/a_0^m\right)$ is the discretized mother wavelet. The DWT, based only on subsamples of the CWT, makes the analysis much more efficient, easy to implement and has fast computation time, at the same time, with the DWT, the original signal can be recovered fully from its DWT with no loss of data. Note a continuous-time signal can be represented in a discrete form as long as the sampling frequency is chosen properly. This is done by using the sampling theorem, termed the Nyquist theorem: the sampling frequency used to turn the continuous signal into a discrete signal must be twice as large as the highest frequency present in the signal (Oppenheim & Schafer, 1989).

To implement the DWT, (Mallat, 1989) developed an approach called the Mallat algorithm or Mallat's Multi-Resolution Analysis (MRA). In this approach the signal to be analyzed (decomposed) is passed through finite impulse response (FIR) high-pass filters (HPF) and low-pass filters (LPF) with different cutoff frequencies at different levels. In wavelet analysis the low frequency content is called the approximation (A) and the high frequency content is called the details (D). This procedure can be repeated to decompose the approximation obtained at each level until the desired level is reached as shown in Fig. 2.

2.3 Wavelet Packet Transform (WPT)

The WPT is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis. In WPT, the details as well as the approximation can be split as shown in Fig. 3.
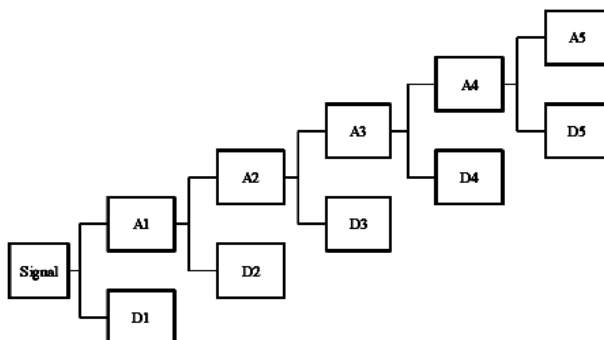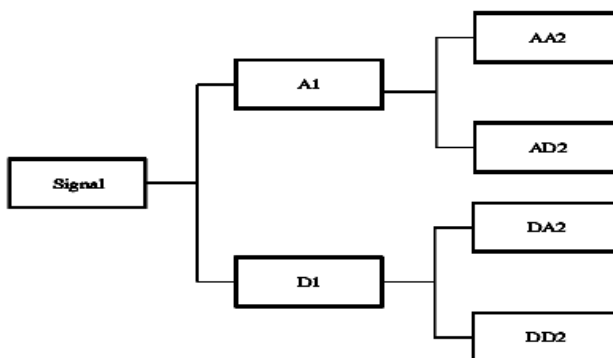
Fig. 2. Decomposition tree for DWT

Fig. 3. Decomposition tree for WPT

In view of the fact that WPT generates large number of nodes it increases the computational burden. In DWT only approximations are further decomposed thus reducing the level of decomposition and thereby computational attempts.

3. WT applications in power systems

In the main stream literature, wavelets were first applied to power system in 1994 by Robertson (Robertson et al., 1994) and Ribeiro (Ribeiro, 1994). From this year, the number of publications in this area has increased. The most popular wavelet analysis applications in power systems are as following:

- Power quality
- Partial discharges
- Forecasting in power systems
- Power system measurement
- Power system protection

- Power system transients

Fig. 4 shows the percentage of 196 IEEE papers based in each area (Source: search on IEEE Explore). One can conclude that most research are carried in the field of power quality and power system protection.
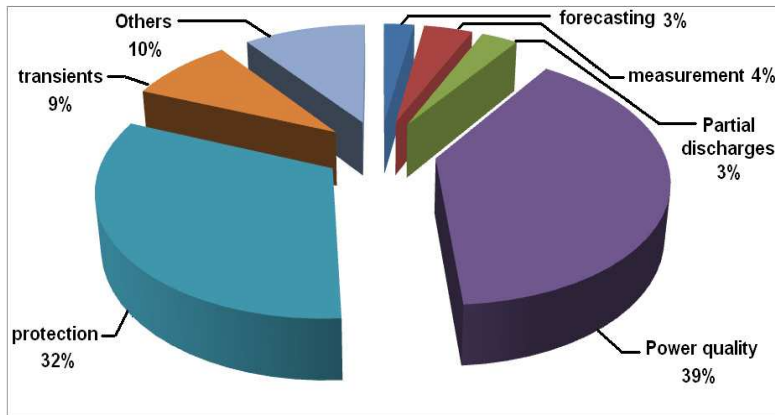

Fig. 4. Percentage of wavelet articles in different areas of power system

Next sections present a general description of wavelet applications in the selected areas of power systems.

3.1 Power quality (PQ)

In the area of PQ, several studies have been carried out to detect and locate disturbances using the WT as a useful tool to analyze sag, swell, interruption, etc. of non-stationary signals. These disturbances are "slow changing" disturbances. Therefore, it contains only the spectral contents in the low frequency range. Therefore, examining WT coefficients (WTCs) in very high decomposition levels would help to determine the occurrence of the disturbance events as well as their occurring time. Counting on this, the DWT techniques have been widely used to analyze the disturbance events in power systems.

Theoretically, all scales of the WTCs may include all the features of the original signal. However, if all levels of the WTCs were taken as features, it would be difficult to classify diverse PQ events accurately within reasonable time, since it has the drawbacks of taking a longer time and too much memory for the recognition system to reach a proper recognition rate. Moreover, if only the first level of the WTCs were used, some significant features in the other levels of the WTCs may be ignored. Beside, with the advancement of PQ monitoring equipment, the amount of data over the past decade gathered by such monitoring systems has become huge in size. The large amount of data imposes practical problems in storage and communication from local monitors to the central processing computers. Data compression has hence become an essential and important issue in PQ area. A compression technique involves a transform to extract the feature contained in the data and a logic for removal of redundancy present in extracted features. For example, in (Liao, 2010) to effectively reduce the number of features representing PQ events, spectrum energies of the

WTCs in different levels calculated by the Parseval's Theorem are proposed. It is well known, in the digital signal processing community, that wavelets revolutionized data compression applications by offering compression rates which other methods could not achieve (Donoho, 1995).

The choice of the mother wavelet is crucial in wavelet analysis of PQ events and it can affect the analysis results. For recognizing the PQ events, maximum number of vanishing moments is the main required property. Beside, per IEEE standards, Daubechies wavelet family is very accurate for analyzing PQ disturbances among all the wavelet families. Nothing that going higher than db43 may lead to instability in the algorithm used to compute the db$N$ scaling filter which affects the filter's frequency response. This is due to the fact that computing the scaling filter requires the extraction of the roots of a polynomial of order $4N$ (Misiti et al., 2007). Moreover, higher Daubechies means that more filter coefficients will be processed which could influence the required memory size and the computational effort.

Although the WT exhibits its great power in the detection and localization of the PQ events, its ability is often degraded, in actual applications, due to the noises, particularly the white noise with a flat spectrum, riding on the signal. Therefore, to overcome the difficulties of capturing the disturbances out of the background noises in a low-SNR environment, a noise-suppression algorithm should be integrated with the WT. The noise-suppression methods for the noising-riding disturbances have been paid much attention in recent years, with different performances exhibited. However, the threshold is difficult to give in detecting the existence of PQ events.

To do a case study the technique proposed in (Liao, 2010) is studied. First the noise-suppression algorithm based on Brownian bridge stochastic process is applied to signals. After the noise-suppression procedure, the pure WTCs are employed in the feature extraction of the PQ event recognition system. The energy spectrum E_H of each level of the WTCs can be obtained with a distorted signal described by Parseval's theorem and the WTCs. The formula is shown as follows (Oppenheim et al., 1999):

$$E_H = \sum_{l=-\infty}^{\infty} \left| c_0(l) \right|^2 + \sum_{j=0}^{\infty} \sum_{i=-\infty}^{\infty} \left| d_j(i) \right|^2 \qquad (4)$$

To enhance the features of PQ events, the energy of the baseband is subtracted from the energy of the distorted signals caused by PQ events, which will derive the energy difference ΔE. Hence, using the differences of energy ΔE as the features of power distorted signals can easily distinguish different PQ events.

Further, in there a genetic k-means algorithm (GKA) based radial basis function (RBF) classification system is used for PQ event recognition. The detailed description of this system can be found in (Liao, 2010).

Four wavelets from Daubechies family db4, db8, db10, and db40 as mother wavelets were used to train and test the proposed PQ recognition system. The corresponding identification rates of PQ events reached 93%, 98%, 99%, and 99% on the average, respectively, for the testing cases. Since fewer coefficients of the mother wavelet can reduce calculation time and

make classifying PQ events faster, db8 as the mother wavelet has been good enough to acquire reasonable accuracy and efficient calculation in the recognition system. Hence, db8 was chosen as the mother wavelet in there. Also, the sampling rate of the input signals was set at 61440points/s, with 1024 sampling points for each cycle on the average. The level of noises, included random process of the stationary white Gaussian distribution, with the SNR value was set to be between 25 and 40dB.

Before establishing the recognition system, the features of training samples after the noise-suppression algorithm had to be obtained. Analyzing the energy spectrum of various signals, the dominant features of voltage sag and swell events were obtained as ΔEd_7 to ΔEd_9 mapped to the 7th to 9th energy spectrums of the WTCs.

Then, the normal signal and signals included voltage sag and swell together with their energy spectrum, with and without noise suppression, are obtained and shown in Fig. 5.
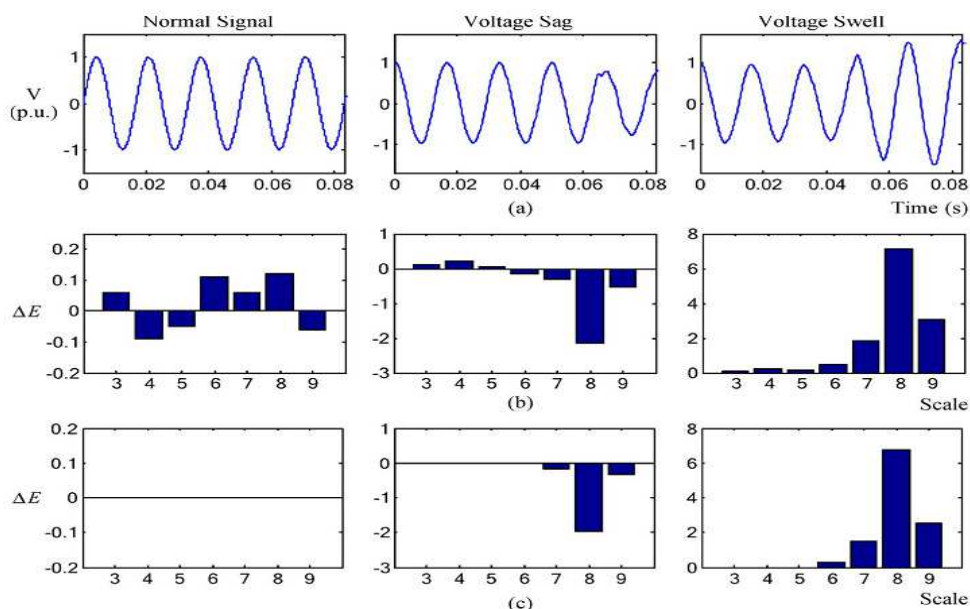

Fig. 5. (a) Actual field data of normal signal, voltage sag, and voltage swell (b) Energy spectrum of the actual signals without noise suppression (c) Energy spectrum of the actual signals with noise suppression

3.2 Partial discharges (PDs)

The PDs are difficult to detect due to their short duration, high frequency and low amplitude signals, but the capacity of the WT to zoom in time the signals with discontinuities unlike the FT, allows identifying local variations of the signal. Almost, DWT technique, among all the WT techniques, is almost proposed and used for detection, measurement and location of PDs.

Beside, PD monitoring has the major problem of electromagnetic interference (EMI). This noise often subsumes completely the very low level PD signals picked up by the sensors. This makes PD detection difficult, particularly for monitoring low level PDs. In addition, there are additional radio frequencies related to mobile phone traffic and so on.

For on-line PD measurement, these excessive interferences cause very low SNR, which means most of the WTCs have to be discarded, as they are noise associated. However, using modified coefficients, especially for on-line PDs measurement where most of the coefficients have to be modified, the IDWT is no longer a perfect reconstruction.

Under such circumstance a wavelet family having the linear phase characteristic is recommended. Linear phase characteristic is necessary as the phase delay and group delay of linear phase FIR filters are equal and constant over the frequency band. This characteristic ensures adjacent signals will not be overlapped together after reconstruction. But, filters with nonlinear phase characteristics will cause signal distortion in the time domain when the signal is filtered.

Also, to some extent, the WT is a measure of similarity. The more similar between the original signal and mother wavelet, the higher the coefficients produce (Zhang et al., 2004). For de-noising and reconstruction considerations, the optimal wavelet suitable for a given signal is the one that is capable of generating as many coefficients with maximal values as possible throughout the time scale domain (Misiti et al, 1996). Based on the above analysis, Bior wavelet family is almost obtained as the most suitable wavelet family to on-line analysis of the PDs.

Also, the number H of decomposition levels, dependent on sampling frequency, can be selected based on trial and error until PD-associated coefficients can be distinguished from noise at a certain WT level (Zhang et al., 2004). Then, coefficients associated with PDs are retained and coefficients corresponding to noise are discarded.

3.3 Forecasting in power systems

Demand forecasting is a key to the efficient management of electrical power systems. The works have been developed for short term electrical load forecasting by combining the WT and neural networks (NNs). As electrical load at any particular time is usually assumed to be a linear combination of different components, from the signal analysis point of view, load can be also considered as a linear combination of different frequencies. Every component of load can be represented by one or several frequencies. The process decomposes the historical load into an approximation part associated with low frequencies and several details parts associated with high frequencies through the WT. Then, the forecast of the part of future load is develop by means of a neural network (Yao et al., 2000) or adjusting the load by a regression method (Yu et al., 2000).

Beside, the increased integration of wind power into the electric grid, as it today occurs in some countries, poses new challenges due to its intermittency and volatility. Wind power forecasting plays a key role in tackling these challenges. (Wang et al., 2009) brings WT into the time series of wind power and verifies that the decomposed series all have chaotic characteristic, so a method of wind power prediction in short-term with WT-based NN model is presented. The obtained results show that the new model is a more effective method in the short-term prediction of wind power than the no WT NN model and ARMA model.

Moreover, price forecasting in a real electricity market is essential for risk management in deregulated electricity markets. Price series is highly volatile and non-stationary in nature. In (Aggarwal et al., 2008) a WT-based price forecasting is proposed. In this work, initially complete price series has been decomposed and then these series have been categorized into different segments for price forecasting. For some segments, WT based multiple linear regression (MLR) has been applied and for the other segments, simple MLR model has been applied.

Daubechies wavelets are most appropriate for treating a non-stationary series (Reis & Silva, 2005). For these families of wavelets, the regularity increases as the order of the functions does. The regularity is useful in getting smoothness of the reconstructed signal. However, with the increase in the order, the support intervals also increase, which leads to poor local analysis and hence may cause the prediction to deteriorate. Therefore, low order wavelet functions are generally advisable. The Daubechies wavelet of order 1 (db1) is the Haar wavelet and is the only wavelet in this family that is discontinuous in nature, and therefore may not be suitable for load, wind or price signal analysis. However, in order to find out the appropriate order of the Daubechies wavelets, effect of the order of Daubechies wavelets, from order 2 to next, should be evaluated on the performance of prediction during the test period. It is necessary to say that according to the authors' research, Db4 mother wavelet have been the most of the applications in forecasting of power systems. Beside, the more levels the original signal is decomposed, the better stationary the decomposed signals are, but great errors will be brought about at the same time (Wang et al., 2009). So the number of decomposition levels should be determined as low as possible.

In (Saha et al., 2006) for forecasting of hourly load demand, Autoregressive (AR) model of coefficients obtained from WT is used. The forecast made by the model of the transformed data appears to be quite satisfactory. Hourly load demand data of past 51 weeks has been utilized for forecasting the demand of 52nd week.

Wavelet coefficients for each of the past 51 weeks demand data are calculated and modeling of time series is done.

The transfer function of AR process in order to transform the non-stationary time series into a stationary series is given by,

$$y_k = \frac{1}{1 - u_1 Z^{-1} - u_2 Z^{-2}} a_k \tag{5}$$

In other words the AR process of order 2 or AR(2) process is represented by

$$y_k = u_1 y_{k-1} + u_2 y_{k-2} + a_k \ \text{ for } k=3, 4, 5,\ldots, N.$$

where N is number of data points in the series, y_k is the kth observation or data point, u_1 and u_2 are the AR(2) model parameters and a_k is the error term (assumed zero mean random variables or white noise). Therefore, the error term is as: $a_k = y_k - u_1 y_{k-1} + u_2 y_{k-2}$. The error is minimized using least square algorithm and an obtained result by db2 mother wavelet is shown in Fig. 6. The accuracy of forecast is found to be within a satisfactory range.
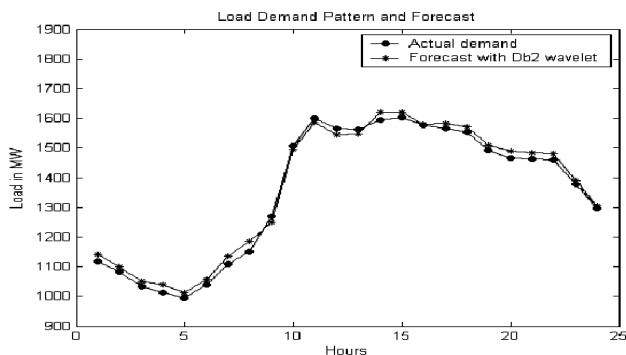
Fig. 6. Load demand pattern and forecast

3.4 Power system measurements

The advantage of using the WT for the application of power/energy and RMS measurements is that it provides the distribution of the power and energy with respect to the individual frequency bands associated with each level of the wavelet analysis.

There are two main approaches to the harmonics field. The first one, carries out an MRA using wavelet filter banks in a first step and usually the application of the CWT to the sub-bands in a second step (Pham & Wong, 1999); the second one, uses a complex wavelet transform analysis or continuous wavelet (Zhen et al., 2000).

There has not been much work on applying DWT for power and RMS measurements. It is important to say that in MRA implemented by DWT filter banks, a signal is decomposed into time-domain non-uniform frequency sub-band components to extract detailed information. For harmonic identification purposes however, it is more useful if the signal is decomposed into uniform frequency sub-bands. This can be achieved using WPT filter banks. The use of the WPT permits decomposing a power system waveform into uniform frequency bands. With an adequate selection of the sampling frequency and the wavelet decomposition tree, the harmonic frequencies can be selected to be in the center of each band in order to avoid the spectral leakage associated with the imperfect frequency response of the filter bank employed. In (Morsi & El-Hawary, 2009) a WPT application is developed for calculating PQ indices in balanced and unbalanced three-phase systems under stationary or non-stationary operating conditions. In order to handle the unbalanced three-phase case, the concept of equivalent voltage and current is used to calculate those indices.

In general, wavelet functions with a large number of coefficients have less distortion and smaller levels of spectral leakage in each output band than wavelets with fewer coefficients. Daubechies wavelet function with 20 coefficients (db20) (Parameswariah & Cox, 2002), and Vaidyanathan wavelet function with 24 coefficients (v24) (Hamid & Kawasaki, 2001; 2002) are proposed as the best solutions for harmonic analysis.

While the WPT provides uniform frequency bands, the main disadvantage is that the computational effort and required memory size increase much more in comparison with the

DWT as the number of levels increase. In (Morsi & El-Hawary, 2007) definitions of power components contained in the (IEEE Std. 1459–2000) are represented by DWT for unbalanced three-phase systems. Also in order to study system unbalance, the concept of symmetrical components is defined in the wavelet domain. The main disadvantage of DWT is the issue of spectral leakage (Barros & Diego, 2006). The errors due to spectral leakage depend on the choice of the wavelet family and the mother wavelet involved in the analysis. In (Morsi & El-Hawary, 2008) a wavelet energy evaluation-based approach is proposed to select the most suitable mother wavelet that can be achieved by evaluating the percentage energy of the wavelet coefficients at each level H

$$\%E_H = \frac{E_H}{E} \times 100 \tag{6}$$

where E is the energy of the original signal and E_H is the energy of the coefficients at each level

$$E_H = \int_R c_H^2(t)\,dt \ \ or \ \ E_H = \sum_{n \in Z} c_H^2(n) \tag{7}$$

Hence, the most suitable mother wavelet is that which satisfies minimum energy deviations for all decomposition levels.

In (Barros & Diego; 2008) a WPT-based algorithm is proposed to calculating harmonics. By selecting a sampling frequency of 1.6 kHz and using a three-level decomposition tree, the frequency range of the output is divided into eight bands with a uniform 100-Hz interval. The selected sampling window width is ten cycles of the fundamental frequency (200 *ms* in a 50 *Hz* system) as in the IEC Standard 61000-4-7. In each output band, the odd-harmonic frequencies are in the center of the band, this way avoiding the edges of the band where the spectral leakage is higher. Using the decomposition tree of WPT, the fundamental component and the odd-harmonic components from the third to the 15th order, from coefficients d_1 to d_8, can be investigated in the input signal. The RMS value of each harmonic component is exactly considered equal to the RMS value of each of the coefficients of the eight output levels. Also, based on the text above v24 and db20 were selected as the wavelet functions to implement the filter bank.

The distorted signal with 1% white Gaussian noise has been considered in order to study the performance of the algorithm proposed and to compare the results with the IEC approach.

To reduce the spectral leakage caused by the filtering characteristics of the method proposed, a double-stage process is used: First, the fundamental component of the input signal is estimated, and then, this component is filtered out; second, the proposed algorithm is applied to the resultant signal to compute the rest of the harmonic components without the interference of the spectral leakage due to the fundamental component.

Table I shows the results obtained, in the estimation of harmonic distortion of the waveform, using the proposed and IEC methods.

As can be seen, the effect of noise is not the same in the measurement of the different harmonic groups; the algorithm with the v24 wavelet function shows a better performance than using db20 and a similar noise immunity as in the IEC method.

Input signal		IEC method magnitude (%)	db20 magnitude (%)	v24 magnitude (%)
Harmonic order	Magnitude (%)			
1	100	100.01	100.01	100.01
3	1	0.98	1.12	1.03
5	2.5	2.53	2.46	2.50
7	1.1	1.08	0.97	1.05
9	0.2	0.19	0.52	0.31
11	0.3	0.30	0.31	0.31
13	0.1	0.12	0.15	0.16
15	0.1	0.24	0.16	0.15

Table 1. Noise immunity of the IEC and wavelet-packet methods

3.5 Power system protection

The potential benefits of applying WT for improving the performance of protection relays have been also recognized. In (Chaari et al., 1996) wavelets are introduced for the power distribution relaying domain to analyze transient earth faults signals in a 20 kV resonant grounded network as generated by EMTP.

There are two main criteria for the selection of the mother wavelet in power system relay protection. At first, the shape and the mathematical expression of the wavelet must be set such that the physical interpretation of wavelet coefficients is easy. Secondly, the chosen wavelet must allow a fast computation of wavelet coefficients.

In (Osman-Ahmed, 2003) the selection procedure of more suitable mother wavelet is shown for fault location. The most suitable mother wavelet is that which satisfies maximum energy for details coefficients at the first decomposition level based on (7), when fault is occurred. Also, if this energy value of each phase exceeds a predetermined threshold value, the disturbance is identified as a fault in that phase.

In (Bhalja & Maheshwari, 2008) another method is proposed to select an optimal mother wavelet for fault location. In this method, the ratio of the norm of details coefficients to approximation coefficients (RDA) is calculated. Then the mother wavelet having the highest RDA value is selected as the optimal mother wavelet.

The WT is also applied for islanding detection (Hsieh et al., 2008), the bars (Mohammed, 2005), motors (Aktas & Turkmenoglu, 2010), generators and transformers (Saleh & Rahman, 2010) protection. For these cases, entropy and minimum description data length (MDL) criteria are used to determine the optimal mother wavelet and the optimal number of levels of decomposition.

The MDL criterion selects the best wavelet filter and the optimal number of wavelet coefficients to be retained for signal reconstruction. The MDL criterion for indexes K (number of coefficients to be retained) and g (number of wavelet filters) is defined as:

$$MDL\left(K,g\right)=\min\left\{\frac{3}{2}K\log N+\frac{N}{2}\log\left\|\tilde{\alpha}_g-\alpha_g^{(K)}\right\|^2\right\},\ \ 0\le K<N;\ \ 1\le g\le M \tag{8}$$

where $\tilde{\alpha}_g=W_g*x$ denotes a vector of the wavelet-transformed coefficients of the signal x using wavelet filters (g) and $\tilde{\alpha}_g^{(K)}=\Theta^K\tilde{\alpha}_g=\Theta^K\left(W_g*x\right)$ denotes a vector that contains K nonzero elements. The thresholding parameter Θ^K keeps a K number of largest elements of the vector $\tilde{\alpha}_g$ constant and sets all other elements to zero. Letters N and M denote the length of the signal and the total number of wavelet filters, respectively. Number of coefficients K, for which the MDL function reaches its minimum value, is considered as the optimal one. With this criterion, the wavelet filters can also be optimized as well.

The entropy $En(x)$ of a signal $x(n)$ of length N is defined as:

$$En\left(x\right)=-\sum_{n=0}^{N-1}\left|x\left(n\right)\right|^2\log\left|x\left(n\right)\right|^2 \tag{9}$$

To determine the optimal levels of decomposition, the entropy is evaluated at each level. If there is a new level H such that

$$En\left(x\right)_H\ge En\left(x\right)_{H-1} \tag{10}$$

Then level H is redundant and can be omitted.

3.6 Power system transients

Voltage disturbances shorter than sags or swells are classified as transients and are caused by the sudden changes in the power system. On the basis of the duration, transient over voltages can be divided into switching surge (duration in the range of milliseconds) and impulse spike (duration in the range of microseconds). Surges are high energy pulses arising from power system switching disturbances either directly or as a result of resonating circuits associated with switching devices, particularly capacitor switching. Impulses on the other hand result from direct or indirect lightning strokes, arcing, insulation breakdown, etc.

The selection of appropriate mother wavelet without knowing the types of transients is a challenging task. For short and fast transient disturbances in power systems, the wavelet must be localized in time and oscillate rapidly within a very short period of time. This means short length of LPF and HPF filters. However, a very short filter length leads to a blockness problem (Akansu & Haddad, 2001).

For dyadic MRA, the minimum filter order is equal to two coefficients. However, for more freedom and to eliminate the blockness problem, the filter length must be greater than or equal to 4 coefficients (Akansu & Haddad, 2001). A literature survey by authors and past experience show that for short and fast transient disturbances, db4 and db6 wavelets are better while for slow transient disturbances db8 and db10 are more suitable.

At the lowest scale i.e. level 1, the mother wavelet is most localized in time and oscillates rapidly within very short period of time. As the wavelet goes to higher scale the analyzing

wavelets become more localized and oscillate less due to the dilation nature of the WT analysis. Hence, fast and short transient disturbances will be detected at lower scales whereas slow and long transient disturbances will be detected at higher scales. Thus, both fast and slow transients can be detected with a single type of analyzing wavelets.

Apart from the application of wavelets to introduce new identification, classification and analysis methods such as those presented previously, at the moment is also studied the application of wavelets to develop new components models; for example (Abur et al., 2001) extends the results of previous works (Magnago & Abur, 2000) and describes a transmission line model which is based on WT taking into account frequency dependence of modal transformation matrices into the transients simulation. This allows the use of accurate modal transformation matrices that vary with frequency and yet still remain in the time domain during the simulations.

Although wavelet analysis usually combined with a large number of neural networks provides efficient classification of PQ events, the time-domain featured disturbances, such as sags, swells, etc. may not easily be classified. In addition, some important disturbance frequency component may not be precisely extracted by WT. Therefore, (Reddy & Mohanta, 2004) presents a new transform by incorporating phase correction to WT and is known as S–transform. The S-transform separates the localizing-in-time aspect of the real valued Gaussian window with modulation (selection of frequency) so that the window translates, but does not get modulated. (Reddy & Mohanta, 2010) extends the use of S-transform for detection, localization and classification of impulsive transients. The results obtained from S-transform are compared with those obtained from WT to validate the superiority of S-transform for PQ and transients analysis of complex disturbances. To do a case study the technique proposed in (Liao, 2010) is again studied, but for capacitor switching leading to the oscillatory transient. The results obtained are shown in Fig. 7. The dominant features of capacitor switching event was obtained as ΔEd_3 to ΔEd_5 mapped to the 3th to 5th energy spectrums of the WTCs.
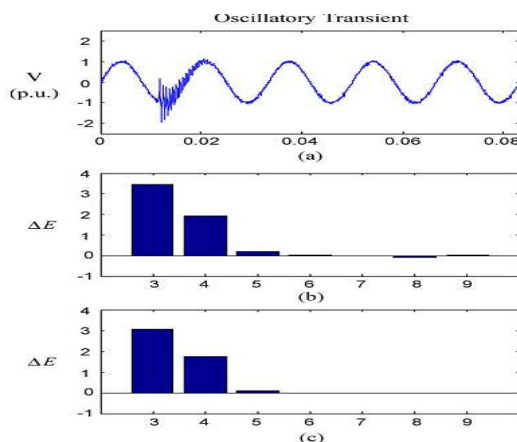

Fig. 7. (a) Simulated data of oscillatory transient with noise. (b) Energy spectrum of the simulated signals without noise suppression. (c) Energy spectrum of the simulated signals with noise suppression.

4. Investigation of WT application on islanding state and fault location

In this section, the detailed analysis of two important application of wavelet analysis, carried on detection of the islanding state and fault location by the authors, will be illustrated.

4.1 Islanding detection

4.1.1 Methodology

The proposed algorithm is based on the study of disturbances existed in the waveform of terminal current of DGs. It should be noted that once the islanding event is occurred, a transient component continues only for a very short time after the switching operation and then it is removed. But in non-islanding events this transient component continues for longer time, so it should be distinguished. In the proposed method, after studies done by the authors, it was found out that third decomposition level with 20 samples as the length of data window and 17 samples as the moving size of data window is accurate leading to detect the islanding state within maximum 54 samples i.e. 5.4 ms. In the first step, the ratio of maximum current magnitude in rth window to the previous window is calculated as follows:

$$Ratio - I_t(r) = \frac{min \ and \ max \ I_t(r)}{max \ I_t(r\text{ - }1)} \tag{11}$$

where, the threshold values are:

$$0.98 \leq Ratio - I_t(r) \leq 1.02 \tag{12}$$

These threshold values are selected according to simulate the different events. If the calculated ratio satisfies (12) then there is no problem and this means that islanding has not been occurred. For values out of range of (12), the following criteria could be used to check whether the islanding event is taken place or not:

$$Ratio - D_3(r) = \frac{maxD_3(r)}{maxD_3(r\text{ - }2)} \tag{13}$$

Considering different studies done, threshold value chosen for (13) is 0.02. This condition can be expressed by:

$$Ratio - D_3(r) \leq 0.02 \tag{14}$$

This threshold value is also adopted according to simulate the different events. If value of (14) is less than 0.02, then the islanding event is occurred and trip command should be issued for islanded DGs. The algorithm diagram is shown in (Shariatinasab & Akbari, 2010).

It is worth to point out that moving size of the data window in the proposed algorithm is an important parameter. As decreasing the moving size reduces total time of detection, so the moving size should be decreased as possible.

4.1.2 Case study

The study system consists of two synchronous DGs (DG1 and DG2) operating on PQ mode, and is a part of Iranian distribution network located in Tehran (Fig. 8). The data of the network are available in (Shariatinasab & Akbari, 2010).
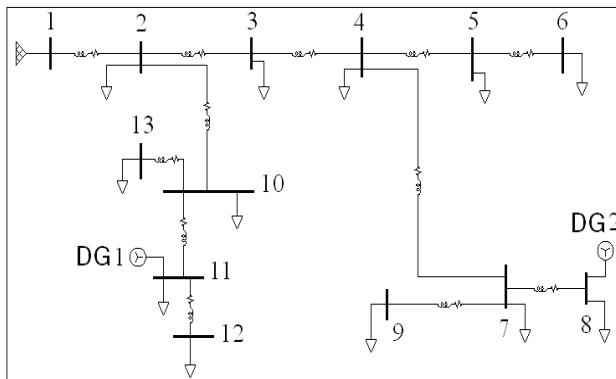

Fig. 8. Test system for islanding detection study

4.1.3 Simulation results

To be ensured of accuracy of the proposed algorithm, all the cases affecting the terminal current of DGs are analyzed. Figs. 9 and 10, show RMS current form and related three decomposition levels using 'Haar' mother wavelet for non-islanded DG1 and islanded DG2, respectively.
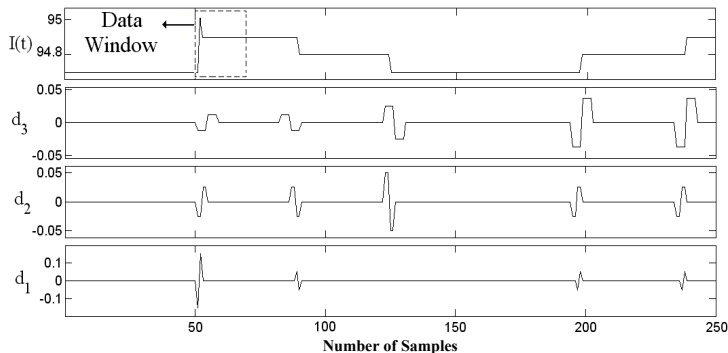
Fig. 9. The waveform of terminal current of DG1, due to breaker opening on line 7-8, d_1-d_3 are detail components of main signal
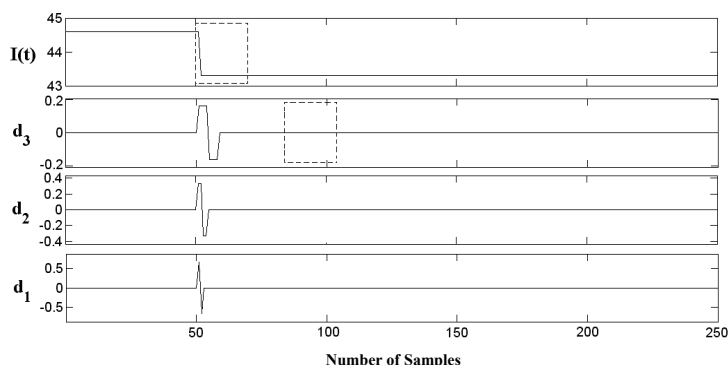
Fig. 10. The waveform of terminal current of DG2, due to breaker opening on line 7-8, d_1-d_3 are detail components of main signal

20 samples length data window is considered. In this window, ratio of the changed current is 1.003 for DG1 which satisfies (12). So as it is expected the algorithm would not issued a trip command for DG1. It is important to point out that in order to get a conservative result; it was assumed that the generated power of DG2 is equal to the customer load at the connected bus. Therefore, DG2 is islanded, the difference between the generated and consumed power in bus 8 will be zero, while the ratio of the changed current in related data window for DG2 is 0.0971 that is less than 0.98 and therefore (12) is not satisfied. Then data window is twice shifted to the right, either one up to 17 samples. In this new window, the obtained value of (13) is nearly zero, in which this value satisfies (14). So the proposed algorithm detects the islanding event in maximum time within 54 samples of 10 *kHz* sampling frequency, i.e. 5.4 *ms,* and issues a trip command for DG2.

The more research is done for various combinations and conditions of islanding for both DG1 and DG2 available in (Shariatinasab & Akbari, 2010).

In order to perform a comprehensive study to check the accuracy of the proposed method, motor starting and capacitor switching are also investigated; as they may cause a similar situation to islanding state and hence should be distinguished correctly. To perform the motor starting study, a 15 *kVA* induction motor starting is studied, and results are shown in Figs. 11-12. For DG1 the value of (11) obtained under this condition is 1.353 that is more than 1.02 and the value of (13) is 0.054 that is more than 0.02. Also, for DG2 the obtained value of (11) is 2.392 and the value of (13) is 0.079, in which both values are more than criteria adopted in the proposed algorithm. Hence, the proposed method distinguishes this situation correctly, i.e. an islanding state is not detected for DGs under motor starting condition.
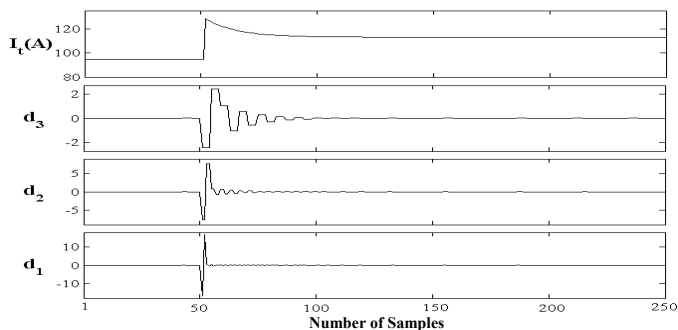
Fig. 11. The waveform of terminal current of DG1, due to motor starting at bus 3, d_1-d_3 are detail components of main signal
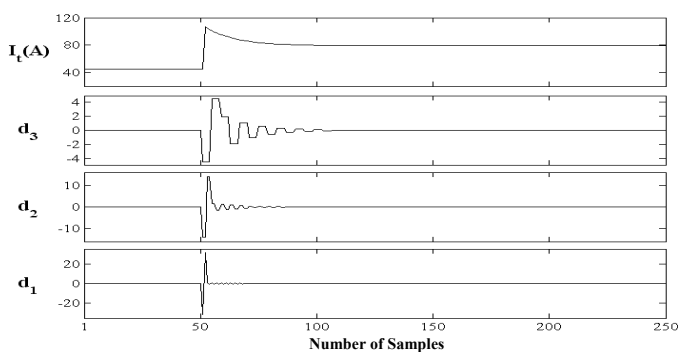

Fig. 12. The waveform of terminal current of DG2, due to motor starting at bus 3, d_1-d_3 are detail components of main signal

Results of the capacitor bank switching are also available in (Shariatinasab & Akbari, 2010).

4.2 Fault location

4.2.1 Methodology and study system

In this section, the fault location by DWT and a trained NN will be discussed. The case study is IEEE 9-bus test system as shown in Fig. 13. This system is a 400 kV transmission system included 3 generators and 6 lines. Each line is divided to 20 points and then a fault is separately applied in each point. Totally 120 faults is applied in 120 points. As the most of faults occurred in transmission systems have low fault impedance, so fault impedance was considered equal to zero in this study. Then the terminal current signal of G1, G2 and G3 during the fault is obtained with sampling rate 10 kHz. The fault signals collected in ETAP

software is then transformed to MATLAB software in order to apply the wavelet analysis. Only 46 samples/10 *kHz* sampling rate (equal to 4.6 *ms*) of data are considered after fault time. According to the analyses done, db4 mother wavelet was selected as a suitable solution.

After DWT analysis, it is necessary to extract the characteristics of this transform to provide inputs of NN. To this, 2nd norm (norm2) of signal details was considered as NN inputs. Also, the details of 5 levels were obtained as the optimal solution to train the NN.

To describe the work, norm2 of 3rd level details versus fault distance from a generator (G3) is illustrated in Fig. 14. As shown in Fig. 14, the more fault distance, the lower value of norm2 is reached. In this study, norm2 of details of five levels were used. The NN used in this study was consisted of 3 hidden layers either with 20 neurons. The optimal number of neurons was determined based on the trial and error approach. The transfer functions applied in input, hidden and output layers were considered **tansig**, **tansig** and **purelin**, respectively, and training algorithm was also considered as **trainlm**.
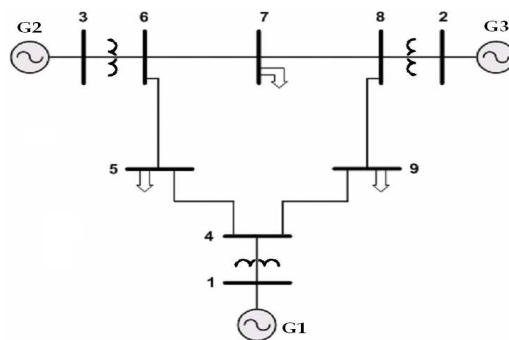

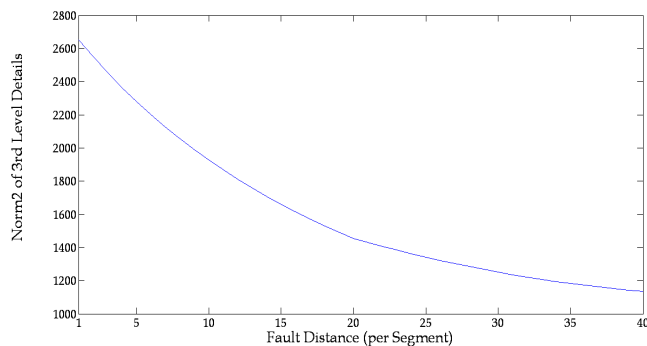Fig. 13. Schematic diagram of test system for fault location study

Fig. 14. Norm2 of 3rd level details (d3) for G3

4.2.2 Simulation results

For study system, fault was applied in 120 points which 85 points was considered as training patterns of NN and 35 points was considered for testing.

According to the definition in (IEEE Std. PC37.114, 2004), error percentage of fault location estimation is determined as follows:

$$error \% = \frac{error \; value}{line \; length} \tag{15}$$

Some results obtained from the proposed DWT-NN technique are shown in Table 2. As seen in results, the error values are reasonable values and satisfactory. According to 4.2.1, the time of the fault detection and location is 4.6 *ms* equal to 46 samples per 10 *kHz* sampling rate. Therefore, this technique can be well used to estimate the fault detection and location in a specific transmission system.

Real segment number	Calculated value	Error value	Error %
4	3.8086	-0.1914	-0.96
14	4.0467	0.0467	0.23
29	29.0903	0.0903	0.45
37	37.1833	0.1833	0.92
51	51.1034	0.1034	0.52
66	65.7872	-0.2128	-1.06
74	74.0679	0.0679	0.34
86	86.1994	0.1994	1.00
95	94.8874	-0.1126	-0.56
103	103.2897	0.2897	1.45
112	111.7134	-0.2866	-1.43

Table 2. The results of fault location under db4 mother wavelet and 5 decomposition levels

5. Conclusion

Wavelet transform is a powerful signal processing tool used in power systems analysis. The most of applications of wavelet analysis in power systems include analysis and study of power quality, partial discharges, forecasting, measurement, protection and transients. It transforms a time-domain waveform into time-frequency domain and estimates the signal in the time and frequency domains simultaneously.

The most popular applications of WT are related to CWT, DWT and WPT techniques. CWT generates a huge amount of data in the form of wavelet coefficients with respect to change in scale and position. This leads to large computational burden. To overcome this limitation, DWT is used, as do in digital computers by applying DWT on discretized samples.

According to the done research, DWT is also extensively used to analyze the most of phenomena of power systems. However, an extensive study should be carried on applying DWT for power and RMS measurements. Because in MRA implemented by DWT filter banks, a signal is decomposed into non-uniform frequency sub-bands. However, for harmonic identification purposes, it is more useful if the signal is decomposed into uniform frequency sub-bands. This can be achieved using WPT filter banks.

Further, Although there have been a great effort in references to prove that one wavelet is more suitable than another, there have not been a comprehensive analysis involving a number of wavelets to prove the point of view suggested. Also, the method of comparison among them is not unified, such that a general conclusion is reached.

Therefore, in this chapter for each application in power systems, it was tried to introduce principles and algorithms in order to determine the optimal mother wavelet. According to the literature review, Daubechies family has been the most of applications in power systems analysis. Further, often db4 have been the satisfactory results than the other mother wavelets of Daubechies family. However, it is should be noted that the type of mother wavelet, the number of decomposition levels and etc, may be changed from one application and/or condition to another and therefore not be generalized to all the cases.

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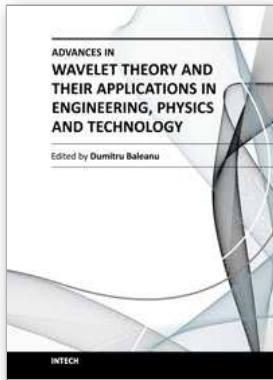
The use of the wavelet transform to analyze the behaviour of the complex systems from various fields started to be widely recognized and applied successfully during the last few decades. In this book some advances in wavelet theory and their applications in engineering, physics and technology are presented. The applications were carefully selected and grouped in five main sections - Signal Processing, Electrical Systems, Fault Diagnosis and Monitoring, Image Processing and Applications in Engineering. One of the key features of this book is that the wavelet concepts have been described from a point of view that is familiar to researchers from various branches of science and engineering. The content of the book is accessible to a large number of readers.

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