The Optical Transmission of One-Dimensional Photonic Crystals Containing Double-Negative Materials

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1. Introduction

A crystal is a periodic arrangement of atoms or molecules. The pattern with which the atoms or molecules are repeated in space is the crystal lattice. The crystal presents a periodic potential to an electron propagating through it, and both the constituents of the crystal and the geometry of the lattice dictate the conduction properties of the crystal.

Importantly, however, the lattice can also prohibit the propagation of certain waves. There may be gaps in the energy band structure of the crystal, meaning that electrons are forbidden to propagate with certain energies in certain directions. If the lattice potential is strong enough, the gap can extend to cover all possible propagation directions, resulting in a complete band gap. For example, a semiconductor has a complete band gap between the valence and conduction energy bands.

The optical analogue is the photonic crystal, in which the atoms or molecules are replaced by macroscopic media with differing dielectric constants, and the periodic potential is replaced by a periodic dielectric function (or, equivalently, a periodic index of refraction). If the dielectric constants of the materials in the crystal are sufficiently different, and the absorption of light by the materials is minimal, then the refractions and reflections of light from all of the various interfaces can produce many of the same phenomena for photons (light modes) that the atomic potential produces for electrons. One solution to the problem of optical control and manipulation is thus a photonic crystal, a low-loss periodic dielectric medium. (Joannnopoulos et al, 2008).

There has been growing interest in the development of easily fabricated photonic band gap materials operating at the optical frequencies. The reason for the interest in photonic band gap materials arises from the possible applications of those materials in several scientific and technical areas such as filters, waveguides, optical switches, cavities, design of more efficient lasers, etc. (Li et al., 2008; Wang et al., 2008).

The simplest possible photonic crystal consists of alternating layers of material with different dielectric constants: a one-dimensional photonic crystal or a multilayer film. This arrangement is not a new idea. Lord Rayleigh (1887) published one of the first analyses of

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the optical properties of multilayer films. This type of photonic crystal can act as a mirror for light with a frequency within a specified range, and it can localize light modes if there are any defects in its structure. These concepts are commonly used in dielectric mirrors and optical filters. (Joannopoulos et al., 2008)

Recently, photonic crystals containing metamaterials have received special attention for their peculiar properties (Deng & Liu, 2008). One kind of metamaterials is double-negative materials (DNG) whose electric permittivity \( \varepsilon \) and magnetic permeability \( \mu \) are simultaneously negative (Veselago, 1968), which can be used to overcome optical diffraction limit, realize super-prism focusing and make a perfect lens (Pendry, 2000). Another kind of metamaterials is single-negative materials (SNG), which include the mu-negative media (MNG) (the permeability is negative but the permittivity is positive) and the epsilon-negative media (ENG) (the permittivity is negative but the permeability is positive). These metamaterials possess zero-effective-phase gap and can be used to realize easily multiple-channeled optical filters (Zhang et al., 2007).

In this chapter are analyzed one-dimensional photonic crystals composed of two layers: A=dielectric material (TiO\(_2\)) or A=epsilon-negative material (ENG) and B=double negative material (DNG), from the point of view of their optical transmission. In the case in which A is a dielectric material are used the following materials properties: the magnetic permeability \( \mu_A=1 \) and the electric permittivity \( \varepsilon_A=7.0225 \). To describe the epsilon-negative material (ENG) it is used a transmission-line model (Eleftheriads et al., 2002): the magnetic permeability \( \mu_A=3 \) and the electric permittivity \( \varepsilon_A=1-100/\omega^2 \). For the double-negative material (DNG) are used the following material properties: the magnetic permeability \( \mu_B=1-100/\omega^2 \) and the electric permittivity \( \varepsilon_B=1.21-100/\omega^2 \). \( \Omega \) is the angular frequency measured in GHz.

An algorithm based on the transfer matrix method (TMM) was created in MATLAB and used to determine the optical transmission of the considered photonic structures. In the simulations the angular frequency \( \omega \) takes values from 0 to 9 GHz. In this chapter is analyzed the influence of various defects upon the optical transmission of the photonic crystal: the type of the material used in the defect layer, thickness and position of the defect layer upon the optical transmission.

2. Calculus method

The transfer matrix method (TMM) is widely used for the description of the properties of stacked layers and is extensively presented in (Born & Wolf, 1999). The transfer matrix method (TMM) provides an analytical means for calculation of wave propagation in multilayer media. This method permits exact and efficient evaluation of electromagnetic fields in layered media through multiplication of 2 x 2 matrices. The TMM is usually used as an efficient tool to analyse uniform and non-uniform gratings, distributed feedback lasers and even one-dimensional photonic crystals. The solution of the coupled mode (coupled modes TE and TM) equations is represented by a 2 x 2 transfer matrix which relates the forward and backward propagating field amplitudes. The grating structure is divided into a number of uniform grating sections which each have an analytic transfer matrix. The transfer matrix for the entire structure is obtained by multiplying the individual transfer matrices together.

Such an algorithm was implemented in MATLAB and used to determine the optical transmission of the considered photonic structures. In this approach were considered isotropic layers, nonmagnetic and a normal incidence of the incident light.
Let us consider for investigation a stack of \( m \) layers perpendicular on the OZ axis as it can be seen in figure 1.

![Fig. 1. Schematic representation of a multilayered structure](image)

Fig. 1. Schematic representation of a multilayered structure

Using the notations given in figure 1, it was considered known the refractive index of the medium from where the beam of light emerges \( n_0=1 \) (air), the refractive index of the medium in which the beam of light exits \( n_s=1.52 \) (glass), the intensities of the electric and magnetic fields \( E_m \) and \( H_m \) in the support layer of glass.

To determine the electric and magnetic fields in the air, \( E_0 \) and \( H_0 \), the following system was solved:

\[
\begin{bmatrix}
E_0 \\
H_0
\end{bmatrix} = M_1 M_2 \ldots M_m \begin{bmatrix}
E_m \\
H_m
\end{bmatrix}
\]

(1)

where

\[
M_j = \begin{bmatrix}
\cos \phi_j & \frac{\sqrt{\mu_j}}{\sqrt{\varepsilon_j}} \cdot \sin \phi_j \\
\frac{1}{\sqrt{\varepsilon_j}} \cdot \sin \phi_j & \cos \phi_j
\end{bmatrix}, j = 1..m
\]

(2)

and \( \phi_j = \omega \frac{1}{v} \cdot \sqrt{\varepsilon_j \mu_j} \cdot d_j \) the phase variation of the wave passing the layer \( j \). \( \varepsilon_j, \mu_j \) and \( d_j \) are the electric permittivity, the magnetic permeability, respective the thickness of the layer. \( v \) is the phase speed and \( \omega \) is the angular frequency. The relationship between the wavelength \( \lambda \) and the angular frequency \( \omega \) is \( \lambda = v f = \frac{2\pi f}{\omega} \).

Be multiplying the \( M_1, M_2, \ldots, M_m \) matrices we obtain a final matrix of the following shape:

\[
M = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\]

(3)
Using the matrix obtained above we calculate the optical transmission with the following relationship:

\[ T = \left( \frac{2}{M_{11} + M_{22} + i(M_{12} - M_{21})} \right)^2 \] (4)

The MATLAB code is based on these equations for optical transmission calculus.

3. Structures design

The studied structures were generated starting from the one-dimensional Thue-Morse sequence (Allouche & Shallit, 2003). The one-dimensional Thue-Morse sequence of \( N \) order, \( TM_N \), is a binary sequence of two symbols ‘A’ and ‘B’. \( TM_{N+1} \) is generated from \( TM_N \) in which we substitute ‘A’ with ‘AB’ and ‘B’ with ‘BA’. Thus \( TM_0 = \{A\} \), \( TM_1 = \{AB\} \), \( TM_2 = \{ABBA\} \), \( TM_3 = \{ABBABAAB\} \) etc.

In the simulations were used one-dimensional photonic crystals of the following types: \( (AB)_16 \), \( (ABBA)_8 \) and \( (ABBABAAB)_4 \), where A and B are two isotropic media with the refractive indexes \( n_A \) and \( n_B \).

The following particular cases of one-dimensional photonic crystal composed of two types of layers were considered:

a. The first case: A=dielectric material (\( \text{TiO}_2 \)) and B=double negative material with the permeability \( \mu_A=1 \), respectively \( \mu_B=1-100/\omega^2 \) and the permittivity \( \varepsilon_A=7.0225 \), respectively \( \varepsilon_B=1.21-100/\omega^2 \).

b. The second case: A=epsilon negative material and B=double negative material with the permeability \( \mu_A=3 \), respectively \( \mu_B=1-100/\omega^2 \) and the permittivity \( \varepsilon_A=1-100/\omega^2 \), respectively \( \varepsilon_B=1.21-100/\omega^2 \).

![Fig. 2. The one-dimensional analyzed photonic crystal's structure](image-url)
the optical transmission of the photonic crystal. The new structure of the crystal is presented in figure 3.

![Figure 3. The structure of the photonic crystal with a defect layer]

4. MATLAB simulation results

4.1 Optical transmission of 1D PC without defects

In these experiments the thickness of the layers were kept constant - \(dA = 24\,\text{mm}\) and \(dB = 6\,\text{mm}\) - and the frequency varied \(\omega \in (0, 9)\,\text{GHz}\). As it is specified earlier there will be two cases: case 1 in which A is a dielectric material and case 2 in which A is an epsilon negative material. In both cases B is a double negative material (Ramakrishna, 2005).

4.1.1 Case 1: A is a dielectric material

Figure 4 presents the optical transmission of the three types of the studied photonic crystals versus of the frequency of the light beam so that we can observe easily the band-gaps.

One easily observes that the \((AB)^{16}\) type photonic crystals have two band gaps, the \((ABBA)^{8}\) type crystals have three band-gaps and the \((ABBABAAB)^{4}\) type crystals have four band-gaps. The values are given in table 1.

<table>
<thead>
<tr>
<th>Crystal’s type</th>
<th>Band-gaps (GHz)</th>
<th>The mid-gap frequency of the gap (GHz)</th>
<th>The width of the band-gaps (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((AB)^{16})</td>
<td>(\omega \in (1.2, 2.4))</td>
<td>1.8</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (6.5, 7.5))</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>((ABBA)^{8})</td>
<td>(\omega = 0.8)</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (1.2, 1.5))</td>
<td>1.35</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (7, 7.5))</td>
<td>7.25</td>
<td>0.5</td>
</tr>
<tr>
<td>((ABBABAAB)^{4})</td>
<td>(\omega = 0.58)</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (1.25, 1.55))</td>
<td>1.4</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (1.9, 5))</td>
<td>3.45</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>(\omega \in (7, 7.5))</td>
<td>7.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1. The photonic band-gaps for \((AB)^{16}\), \((ABBA)^{8}\) and \((ABBABAAB)^{4}\) one-dimensional photonic crystal
Fig. 4. The optical transmission function of the frequency of the incident radiation for 1D PC of different types: a) \((AB)_6\); b) \((ABBA)_6\); c) \((ABBABAAB)_4\)
It is also observed that the \((AB)_{16}\) type crystals have wider band-gaps. The \((ABBA)_8\) type crystals have the smallest band-gaps. The maximum band-gap is obtained for a \((ABBABABAAB)_{4}\) type structure.

### 4.1.2 Case 2: A is an epsilon negative material

Figure 5 presents the optical transmission of the three types of the studied photonic crystals versus the frequency of the light beam so that to observe easily the band-gaps and the localized states.

![Graphs showing optical transmission for different types of photonic crystals](image)

Fig. 5. The optical transmission function of the frequency of the incident radiation for 1D PC of different types: a) \((AB)_{16}\); b) \((ABBA)_8\); c) \((ABBABABAAB)_{4}\)
From the graphics above it is observed that for the (AB)$_{16}$ type photonic crystal it is obtained a localized state with an optical transmission of 96% at the frequency of 0.99 GHz and a group of localized states with high optical transmission between the frequencies of 2.5 GHz and 3.5 GHz. One of these localized states has an optical transmission of 100%. This localized state is obtained at 3.07 GHz. For the (ABBA)$_8$ type structure it is obtained a localized state at 4.2 GHz with an optical transmission of 100%. The results obtained for the (ABBABAAB)$_4$ type crystal are not important because the optical transmissions of the localized states are very small (under 15%).

4.2 The optical transmission of 1D PC with defects

In this case it was introduced, in the structure of the one-dimensional photonic crystal, first an A type defect layer and then a B type defect layer. As before it is considered first that A is a dielectric material and then A is an epsilon negative material. B remains a double negative material in both cases.

In these experiments the thicknesses of the layers were kept constant - $dA=24\text{mm}$ and $dB=6\text{mm}$ - and the frequency was varied $\omega\in(0,9)\text{GHz}$.

4.2.1 Case 1: A a dielectric material

![Image](a)

![Image](b)
The Optical Transmission of One-Dimensional Photonic Crystals Containing Double-Negative Materials

Fig. 6. The optical transmission function of the frequency for 1D PC of (AB)$_{16}$ type: a) without defect; b) with A type defect; c) with B type defect.

Fig. 7. The optical transmission function of the frequency for 1D PC of (ABBA)$_{8}$ type: a) without defect; b) with A type defect; c) with B type defect.
Fig. 8. The optical transmission function of the frequency for 1D PC of (ABBABAAB)₄ type: a) without defect; b) with A type defect; c) with B type defect.

From the figures above it can be seen that only for the (AB)₁₆ type photonic crystal significant results were obtained.
4.2.2 Case 2: A an epsilon negative material

Fig. 9. The optical transmission function of the frequency for 1D PC of (AB)_{16} type:
a) without defect; b) with A type defect; c) with B type defect.
Inserting a A type defect layer lowers the optical transmission in the localized state obtained at 0.99 GHz while the insertion of a B type defect layer doesn’t have a big influence on the optical transmission of the (AB)_16 type photonic crystal.

Fig. 10. The optical transmission function of the frequency for 1D PC of (ABBA)_8 type: a) without defect; b) with A type defect; c) with B type defect.
From the figure above it is observed that like in the case of a \( (AB)_{16} \) type crystal in the case of a \( (ABBA)_8 \) type crystal the insertion of a \( A \) type defect layer lowers the optical transmission (from 100 % to 4 %) in the localized state obtained at 4.2 GHz while the insertion of a \( B \) type defect layer has little influence upon the optical transmission of the photonic crystal.

Fig. 11. The optical transmission function of the frequency for 1D PC of \( (ABABAAB)_4 \) type: a) without defect; b) with \( A \) type defect; c) with \( B \) type defect.
Analyzing the graphics above it was observed that by inserting a defect layer of A type or B type the optical transmission of the (ABBABAAB)$_4$ type photonic crystal in the localized state becomes even smaller (under 0.001 %).

Considering the results obtained till now the following types of photonic crystals will be studied farther more:

a. The (AB)$_{16}$ type crystal with a A type defect layer (A a dielectric material)
b. The (AB)$_{16}$ type crystal with a B type defect layer (A a dielectric material)
c. The (AB)$_{16}$ type crystal with a B type defect layer (A a epsilon negative material)

4.3 The optical transmission of 1D PC of (AB)$_{16}$ type with a defect layer

The influence of the thickness and the position in which is inserted the defect layer in the structure of the photonic crystal will be analyzed. Only the three cases enumerated above will be considered.

4.3.1 The (AB)$_{16}$ type crystal with a A type defect layer (A is a dielectric material)

![Graphs showing optical transmission](image)

**Fig. 12.** The variation of the position of the defect layer: a) (AB)$_{8}$A(AB)$_{8}$, b) (AB)$_{9}$A(AB)$_{7}$, c) (AB)$_{10}$A(AB)$_{6}$, d) (AB)$_{11}$A(AB)$_{5}$
From the simulations it is observed that the best results are reached if the defect layer is inserted after 8 (AB) groups. A localized state with high optical transmission (95%) is obtained at a frequency of 6.65 GHz.

Keeping the defect layer on position 8 (after 8 (AB) groups) we will now vary its thickness to see its influence upon the optical transmission or the frequency of the localized state.

![Graphs showing the variation of the thickness of the A type defect layer and the frequency of the localized state vs. the thickness of the defect layer](www.intechopen.com)
Analyzing the graphics above it is reached the conclusion that by increasing the thickness of the defect layer the localized state moves to right and by decreasing the thickness of the defect layer the localized state moves to left. The optical transmission of the localized state is not significantly influenced.

Father on the thickness of all the A type layers in the structure of the crystal is varied to see its influence upon the optical transmission of the photonic crystal.
It is observed that by varying the thickness of all A type layers we influence both the optical transmission and the frequency of the localized state that was initially obtained at 6.65 GHz. By increasing the thickness of the layers the optical transmission of the localized state decreases and the localized state moves to left – the localized state is obtained at smaller values of the frequency (as it can be seen in figure 15 a) and b)).

Fig. 15. a) the optical transmission of the localized state vs. the thickness of the A type layers in the structure of the (AB)_8A(AB)_8 type photonic crystal; b) the frequency at which is obtained the localized state vs. the thickness of the A type layers in the structure of the (AB)_8A(AB)_8 type photonic crystal.

4.3.2 The (AB)_16 type crystal with a B type defect layer (A is a dielectric material)

In figure 16 are presented some of the results obtained by changing the position of the defect layer. It is observed that the best position to insert the B type defect layer is after 8 (AB) groups.
Keeping the defect layer on position 8 (after 8 (AB) groups) its thickness will be varied to see its influence upon the optical transmission of the (AB)$_8$B(AB)$_8$ type photonic crystal.

It is observed that by increasing the thickness of the B type defect layer a localized state is obtained. The optical transmission and the frequency at which is obtained this state depend on the thickness of the defect layer as it can be seen in the graphics below. By increasing the thickness of the defect layer the localized state moves to right – the state is obtained at higher values of the frequency (figure 18 b)). The values of the localized state’s optical transmission have mainly an ascending trajectory (figure 18 a)).
Fig. 17. The variation of the thickness of the B type defect layer: a) dB=3 mm; b) dB=6 mm; c) dB=9 mm; d) dB=12 mm; e) dB=14 mm; f) dB=16 mm;

Fig. 18. a) the optical transmission of the localized state vs. the thickness of the B type defect layer in the structure of the (AB)₈A(AB)₈ type photonic crystal; b) the frequency at which is obtained the localized state vs. the thickness of the B type defect layer in the structure of the (AB)₈A(AB)₈ type photonic crystal.

Father on the thickness of all the B type layers in the structure of the crystal is varied to see its influence upon the optical transmission of the photonic crystal.

By increasing the thickness of all the B type layers in the structure of the (AB)₈A(AB)₈ type photonic crystal there are no longer obtained localized states but a series of band-gaps.
Fig. 19. The variation of the thickness of the B type layer: a) dB=3 mm; b) dB=6 mm; c) dB=9 mm; d) dB=15 mm; e) dB=21 mm; f) dB=24 mm;
### Layer thickness (mm) | Band-gaps (GHz) | The width of the band-gaps (GHz)
---|---|---
6 | $\omega \in (1.2, 2.7)$ | 1.5
 | $\omega \in (5.5, 7.6)$ | 2.1
9 | $\omega \in (0.85, 0.93)$ | 0.08
 | $\omega \in (1.8, 7.5)$ | 5.7
15 | $\omega \in (0.7, 0.72)$ | 0.02
 | $\omega \in (0.935, 0.995)$ | 0.06
 | $\omega \in (1.41, 1.76)$ | 0.35
 | $\omega \in (2.9, 8.1)$ | 5.2
21 | $\omega \in (0.78, 0.8)$ | 0.02
 | $\omega \in (0.98, 1.02)$ | 0.04
 | $\omega \in (1.3, 1.45)$ | 0.15
 | $\omega \in (1.95, 2.6)$ | 0.65
 | $\omega \in (4, 8)$ | 4
24 | $\omega \in (0.745, 0.795)$ | 0.01
 | $\omega \in (0.89, 0.92)$ | 0.03
 | $\omega \in (1.15, 1.2)$ | 0.05
 | $\omega \in (1.5, 1.7)$ | 0.2
 | $\omega \in (2.2, 3)$ | 0.8
 | $\omega \in (4.5, 8)$ | 3.5

Table 2.

#### 4.3.3 The $(AB)_{16}$ type crystal with a B type defect layer (A an epsilon negative material)

![Graphs](image1.png)

**Fig. 20.** The variation of the position of the defect: a) $(AB)_4B(AB)_{12}$; b) $(AB)_5B(AB)_{11}$; c) $(AB)_6B(AB)_{10}$; d) $(AB)_7B(AB)_9$
In figure 20 it is showed the influence of the position of the defect layer upon the transmission of the 1D PC. If the defect layer is inserted after an even number of (AB) groups the transmission in the localized states is near 100%. If the defect layer is inserted after an odd number of (AB) groups the transmission is low (under 30%).

![Graphs showing transmission](image)

Fig. 21. The variation of the thickness of the B type layer a) dB=3 mm; b) dB=6 mm; c) dB=9 mm; d) the frequency of the localized state vs. the position of the defect

Now it is analyzed the influence of the thickness of the B type defect layer upon the optical transmission of the 1D PC. The defect layer is inserted after 8 (AB) groups. The variation of the thickness of the defect layer doesn’t influence the optical transmission of the crystal but if it is varied the thickness of all B type layers in the crystal structure are obtained some interesting results as it can be seen in the following graphics.
From the graphics above it is observed that the thickness of the defect layer influences the value of the frequency at which we obtain the localized states (figure 21 d).

5. Conclusions

The insertion of a double negative material defect layer in the structure of the one-dimensional photonic crystal structure generates localized states. The optical transmission in these localized states and the value of the frequency at which the localized states are obtained depend on the thickness and position in which we have inserted the defect layer.

Analyzing the simulations results it is reached the conclusion that the best photonic crystal configurations are the following three:

a. The (AB)$_{16}$ type crystal with a A type defect layer (A a dielectric material)
b. The (AB)$_{16}$ type crystal with a B type defect layer (A a dielectric material)
c. The (AB)$_{16}$ type crystal with a B type defect layer (A a epsilon negative material)

From the results obtained it can be said that the studied structures can be used as optical filters.

6. References


The second volume of the book concerns the characterization approach of photonic crystals, photonic crystal lasers, photonic crystal waveguides and plasmonics including the introduction of innovative systems and materials. Photonic crystal materials promises to enable all-optical computer circuits and could also be used to make ultra low-power light sources. Researchers have studied lasers from microscopic cavities in photonic crystals that act as reflectors to intensify the collisions between photons and atoms that lead to lasing, but these lasers have been optically-pumped, meaning they are driven by other lasers. Moreover, the physical principles behind the phenomenon of slow light in photonic crystal waveguides, as well as their practical limitations, are discussed. This includes the nature of slow light propagation, its bandwidth limitation, coupling of modes and particular kind terminating photonic crystals with metal surfaces allowing to propagate in surface plasmon-polariton waves. The goal of the second volume is to provide an overview about the listed issues.

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