1. Introduction

A ballasted railway track system comprises several components among which steel rails, rail fasteners, timber, steel or concrete sleepers, granular ballast, sub-ballast, and subgrade materials are the main parts. Railway track systems are constructed to provide a smooth and safe running surface for passenger or freight trains. They are designed to sustain the force of lateral, longitudinal and vertical loads imposed on the track structure.

Due to the wide range of mechanical characteristics of the track complements, as well as the components' complex interaction, there is a lack of a comprehensive and precise understanding of track mechanical behaviour, particularly when the nonlinear and dynamic properties of the track are considered [1]. Several important criteria have been defined in the conventional track design methods to ensure that the weight of the load is safely transferred to the ground. These criteria include limits on rail and rail fastener stresses, rail deflections, sleeper stresses, contact pressure between sleepers and the ballast, and the pressure transferred to the supporting layers underneath the track [1].

The recent extensive increases in axle loads, speed and traffic volume, along with the need to improve passenger comfort and reduce track life cycle costs, have caused the subject of track design optimization to arise. Furthermore, complementary decision support systems require a more precise analytical and mechanistic approach to meet the design needs of modern railway track systems. These aspects highlight the necessity of a thorough review and revision of the current railway track designs.

A thorough review of conventional track design method was made by this author elsewhere [1]. This chapter (in section 2) presents a summery of the track design review adapted directly from the author work in [1]. The recent researches in the field of railway track analysis and design are also discussed. Addressing the limitations of the current codes of practice, some methods for improvement of the current track design methods are presented. These methods consider new research findings in the following areas;

- application of dynamic impact factors,
- accurate loading pattern for the sleepers,
- incorporation of nonlinear characteristics of the rail support system, and
- incorporation of plastic deflection of the track support system.
2. Conventional methods in analysis and design of railway tracks

A wide range of railway design codes including AREMA Manual, Euro Codes, UIC Leaflets, Australian Standards (AS), South African Railway Codes, and Indian Standards are available worldwide. The design methods in all of the available codes follow the same approach although they might have some minor differences. Due to great variety of structural elements used in the track system, the railway standards consider each track component as a single structural unit and suggest each complement be designed independently. Such an approach, subsequently, includes the interaction between track components by defining suitable boundary conditions and load transfer patterns [1]. Furthermore, since the dynamic response characteristics of the track are not sufficiently well understood to form the basis of a rational design method, current practices greatly rely on relating the observed dynamic response to an equivalent static response; this is carried out by making use of various load factors [1]. In the conventional methods, railway tracks are designed utilizing load bearing approach to ensure that the concentrated loads of the wheels are transferred to the track formation while the strength of the components are not exceeded [1]. Several important criteria are defined to secure this objective. They are detailed under.

2.1 Rail

Rail, as the most important track element, should be able to securely sustain wheel loads in the vertical, lateral, and longitudinal directions and subsequently transfer them to the underlying supports. Rail is a track element which is in direct contact with the rolling stock and therefore, it is very important to ensure its proper functioning [1].

The most important criteria used in the conventional design procedure are presented in Figure 1 [1]. As it is illustrated in this figure, rail design criteria are mainly divided into two categories: structural strength and serviceability. Structural strength criteria include wheel-rail contact stresses and rail bending stresses. Having satisfied the structural strength criteria, the serviceability requirements should be met to ensure the rail proper structural and operational performance. Moreover, it is important that track design engineers should have a deep understanding of the track operating conditions in order to make appropriate assumptions in the design process [1].

![Fig. 1. Recommended rail design criteria [1]](www.intechopen.com)
Current practices in the calculation of rail bending moments and vertical deflections are mainly based on the theory of “beam on elastic foundation model” [1]. This model was proposed for the first time by Winkler in 1867 and thereafter developed by Zimmerman in 1888 [2]. The basic assumption in the Winkler model is that the deflection of the rail at any point is proportional to the supporting pressure under the rail (Figure 2) [1].

![Beam on elastic foundation model](image)

Fig. 2. Beam on elastic foundation model [1]

The corresponding equations for the calculation of rail bending moment and rail deflection are as follows [1].

\[ y(x) = \frac{P\beta e^{-\beta x}}{2u} (\cos \beta x + \sin \beta x) \quad (1) \]

\[ M(x) = \frac{P}{4\beta} e^{-\beta x} (\cos \beta x - \sin \beta x) \quad (2) \]

where, \( y(x) \) and \( M(x) \) are the vertical deflection and the bending moment of the rail at the distance “\( x \)” from the load point, respectively. Parameter \( \beta \) is defined by the following equation:

\[ \beta = \left( \frac{u}{4EI} \right)^{0.25} \quad (3) \]

Winkler model is basically developed for a continuously supported beam on an elastic foundation. This approach neglects some conditions of railway tracks [1]. First, the assumption of continuous support under the rail does not reflect the effects of actual discrete support provided by cross sleepers. Second, this model does not include various track supporting layers (i.e. ballast, sub-ballast, and subgrade) and simply uses the Bernoulli-Euler beam theory to calculate rail deflections and bending moments. Third, it is assumed that the sleepers do not resist against rail bending despite their rotational stiffness [1]. Due to the above limitations, some researchers have questioned the reliability of the Winkler model [1].

Design procedure for a specific rail section always starts with the calculation of the design wheel load. This load is defined as the product of static wheel load and a corrective factor known as dynamic impact factor (DIF) to compensate for dynamic impact of wheel loads irregularities [1]. Taking into account various parameters which affect the magnitude of dynamic impact factor, several relationships for the estimation of this parameter have been proposed [1]. The main DIF suggestions are presented in Table 1.
<table>
<thead>
<tr>
<th>Developer</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AREMA</td>
<td>$\varphi = 1 + 5.21 \frac{V}{D}$</td>
</tr>
<tr>
<td>Eisenmann</td>
<td>$\varphi = 1 + \delta \eta . t$</td>
</tr>
<tr>
<td>ORE</td>
<td>$\varphi = 1 + \alpha' + \beta' + \gamma'$</td>
</tr>
<tr>
<td>BR</td>
<td>$\varphi = \frac{8.784(\alpha_1 + \alpha_2) V}{P_s} \left[ \frac{D_j P_u}{g} \right]^{1/2}$</td>
</tr>
<tr>
<td>India</td>
<td>$\varphi = 1 + \frac{V}{58.14 u^{0.5}}$</td>
</tr>
<tr>
<td>South Africa</td>
<td>$\varphi = 1 + 4.92 \frac{V}{D}$</td>
</tr>
<tr>
<td>Clarke</td>
<td>$\varphi = 1 + \frac{19.65 V}{D u^{1/2}}$</td>
</tr>
<tr>
<td>WMMTA</td>
<td>$\varphi = (1 + 3086 \times 10^{-5} V^2)^{0.67}$</td>
</tr>
<tr>
<td>Sadeghi</td>
<td>$\varphi = 1.098 + 8 \times 10^{-4} V + 10^{-6} V^2$</td>
</tr>
</tbody>
</table>

Table 1. Dynamic impact factor [1-2]

The magnitude of vertical rail deflection calculated using Equation (1) is greatly dependent upon track modulus [1]. Track modulus is defined as the load which causes unit vertical deflection in unit length of the rail. For typical tracks with light to medium rails, AREMA [6] recommends a value of 13.8 MPa [1].

The rail bending stress is usually calculated at the center of the rail base assuming the pure bending conditions are applicable [1]. The bending stress at the lower edge of the rail head also may be critical if the vehicles impose high guiding forces between wheel flange and rail head. Having calculated the magnitude of the rail bending stress, comparisons should be made between this stress and the allowable limit. AREMA [6] has recommended a practical methodology for calculation of rail bending stress based upon fatigue consideration and through the determination of several safety factors. According to this method, the allowable bending stress is defined as [1,7]:

$$\sigma_{all} = \frac{\sigma_y - \sigma_l}{(1 + A)(1 + B)(1 + C)(1 + D)}$$  \hspace{1cm} (4)

where, $\sigma_y$ is the yield stress of rail steel and $\sigma_l$ is the longitudinal stress due to temperature changes. $\sigma_l$ can be calculated using the following equation [1]:

$$\sigma_l = E.\alpha.\Delta t$$  \hspace{1cm} (5)

The parameters A, B, C, and D in Equation (4) are safety factors to account for rail lateral bending, track condition, rail wear and corrosion, and unbalanced super-elevation of track, respectively [1]. Some of the recommended values for the safety factors are presented in table 2 [1].
Wheel-rail contact stresses mainly include rolling and shear stresses [1]. The magnitude of these stresses is greatly dependent upon the geometry of ellipsoidal wheel-rail contact patch. Many investigations have been carried out to develop reliable formulations for the calculation of these stresses. The most applicable formulas are those suggested by Eisenmann [8]. He conducted an analysis of rolling and shear stress levels in which a simplifying assumption of uniform distribution over the wheel-rail contact area was made. In this analysis wheel and rail profiles were also represented by a cylinder and a plane, respectively (Figure 3) [1].

<table>
<thead>
<tr>
<th>Safety Factor</th>
<th>Hay</th>
<th>Clarke</th>
<th>Magee</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15%</td>
<td>15%</td>
<td>20%</td>
</tr>
<tr>
<td>B</td>
<td>25%</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>C</td>
<td>10%</td>
<td>10%</td>
<td>35%</td>
</tr>
<tr>
<td>D</td>
<td>15%-20%</td>
<td>25%</td>
<td>15%</td>
</tr>
</tbody>
</table>

Table 2. Values of rail bending stress safety factors [7]

Fig. 3. Uniform distribution of wheel-rail contact stress [5]

Based on Hertz’s theory, Eisenmann [8] suggested the following formula for the calculation of the mean value of the rolling contact stress [1]:

$$\sigma_{\text{mean}} = \frac{P_s \times 10^3}{2a \times 2b}$$  \hspace{1cm} (6)

where, 2b (mm) is the width of wheel-rail contact area. Eisenmann [8] adopted the value of 12 (i.e., 2b=12 mm). The contact length (2a) is also calculated by the following formula [1]:

$$2a = 3.04 \times \left[ \frac{P_s R_w \times 10^3}{2b E} \right]^{0.5}$$  \hspace{1cm} (7)

The values of wheel loads transferred to the rail head through the contact area often exceed the yield limit of the contacting materials [1]. In this situation, the resulting surface plastic deformations jointed with wear processes acts to flatten out the contact area. Therefore, the contact surface can be approximated by a rectangle of the length of 2a and the breadth of w based upon the assumption of contact between a plane (rail) and a cylinder (wheel). For such a condition, Smith and Liu [9] suggested the following formula for the calculation of the contact length [1]:
Considering required fatigue strength for rail steel, Eisenmann [8] proposed the limit value for mean rolling contact stress as a percentage of the ultimate tensile strength of rail steel. Based on this assumption, subsequent criterion was suggested [1]:

\[ \sigma_{all(ult)} = 0.5 \sigma_{ult} \]  

(9)

Shear stress distribution is chiefly occurs in the rail head area and is in a close relationship with the magnitudes of normal principal stresses [1]. Eisenmann [8] observed that the values of major and minor stresses do not follow the same reduction patterns with increasing depth from the rail head surface. Such a discrepancy results in the appearance of a maximum value of shear stress at a depth corresponding to half of the contact length [2]. Maximum shear stress value is simply interrelated to mean rolling contact stress values and is given by the following equation [1]:

\[ \tau_{\text{max}} = 0.3 \sigma_{\text{mean}} \rightarrow \tau_{\text{max}} = 410 \left( \frac{P_s}{R_w} \right)^{0.5} \]  

(10)

As indicated earlier, the magnitudes of shear and rolling contact stresses are interrelated [1]. Using the theory of shear strain energy applied for the condition in which the two principal stresses are compressive, the subsequent criterion for the shear stress limit can be obtained [1]:

\[ \tau_{\text{all}} = \frac{1}{\sqrt{3}} \sigma_{\text{mean}} \rightarrow \tau_{\text{all}} = 0.3 \sigma_{\text{ult}} \]  

(11)

Criteria related to the performance of the rails under operating conditions mainly include rail vertical deflection and rail wear life [2]. These criteria are presented hereunder.

AREMA [6] has proposed a limiting range for the magnitudes of vertical rail deflections. According to this recommendation, vertical rail deflections should be kept within the range of 3.175 to 6.35 millimetres [1]. Lundgren and his colleagues [10] has incorporated this recommendation and proposed a diagram presented in Figure 4 which presents the limit values of vertical rail deflection. This diagram is based upon the capability of the track to carry out its design task [1].

Fig. 4. Track deflection criteria for serviceability [2-10]
Domains indicated in Figure 4 are described as follows [1,2].

A: Deflection range for track which will last indefinitely.
B: Normal maximum desirable deflection for heavy track to give requisite combination of flexibility and stiffness.
C: Limit of desirable deflection for track of light construction (with rails weigh < 50 kg/m)
D: Weak or poorly maintained track which will deteriorate quickly.

The other serviceability criterion is the rail wear life [1]. Although many investigations have been carried out to develop a rational method for estimation of this parameter, the results at the best are still empirical and have no theoretical support [1].

The University of Illinois has conducted a research on some U.S. railway tracks in order to investigate the rail wear rate and consequently develop a method for the estimation of rail wear life [1]. The following formula is suggested by the Illinois University for the estimation of annual rail head area wear (mm²/year) [11]:

\[ W_a = W_t (1 + K_w D_c) D_A \]  

Having estimated \( W_a \) and considering the maximum rail head limit \( \theta_A \), the rail wear life could be calculated from the following formula [1]:

\[ T_y = \frac{\theta_A}{W_a} \]  

Danzig and his colleagues [12] from the AREMA association also carried out extensive investigations to find a proper formulation for the estimation of rail wear life. Based on the results obtained, they suggested the following equation which represents the rail wear life in terms of MGT passed over a specific time period [1]:

\[ T = \frac{1.839 K_C K_G K_R R_{Wi} (1.102 D_A) 1.565}{\sum_{i}^{n} \left[ \frac{1.102 D_i}{K_{Wi_i} K_{Ri} K_{Si}} \right]} \]  

Each railway industry has established its own allowable rail wear limits [1]. For instance, the envelope of maximum rail wear values for different rail sections set by Canadian National Railway is presented in Figure 5. Using the diagrams outlined in this figure, maximum allowable rail head height and width losses can be determined [1].

The acceptable rail wears usually range from 20 to 50 percent of the rail head area [1]. The weight of unit length of the rail, the amount of MGT passed over the track during its service life, and the train speed are the most important parameters which determine the proper values of allowable rail wear limits to be chosen. The more weight of unit length of the rail, the greater amount of rail head area reduction would be allowed. On the other hand, the greater amount of MGT and higher values of train speed call for more limited rail head area reduction [1].
2.2 Sleeper

Sleepers play important roles in railway track system [1]. The primary function of the sleepers is to transfer the vertical, lateral and longitudinal rail seat loads to the ballast, sub-ballast and subgrade layers. They also serve to maintain track gauge and alignment by providing a stable support for the rail fasteners [1-2].

The vertical-loads cause bending moments in the sleeper which their amounts are dependent upon the degree and quality of ballast layer compaction underneath the sleeper [1]. The performance of a sleeper to withstand lateral and longitudinal loading is relied on the sleeper’s size, shape, surface geometry, weight, and spacing [2].

<table>
<thead>
<tr>
<th>Loss of Height (mm)</th>
<th>Side Head Wear (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>66 (kg/m)</td>
</tr>
<tr>
<td>1.57</td>
<td>7.92</td>
</tr>
<tr>
<td>3.18</td>
<td>6.35</td>
</tr>
<tr>
<td>4.75</td>
<td>4.75</td>
</tr>
</tbody>
</table>
Current practices regarding the analysis and design of sleepers comprise three steps. These are [1]: 1) estimation of vertical rail seat load, 2) assuming a stress distribution pattern under the sleeper, and 3) applying vertical static equilibrium to a structural model of the sleeper.

Vertical wheel load is transferred through the rail and distributed on certain numbers of sleepers due to rail continuity. This is usually referred to as vertical rail seat load [1]. The exact magnitude of the load applied to each rail seat depends upon several parameters including the rail weight, the sleeper spacing, the track modulus per rail, the amount of play between the rail and sleeper, and the amount of play between the sleeper and ballast [2]. Based on these considerations, various relations are proposed for the amount of rail-seat loads. They are summarized in Table 3 [1].

Most of the expressions are simplified by reducing the number of influencing factors [1]. For instance, AREMA [13] recommends diagrams in which the percentage of wheel load transferred to the sleeper is only dependent on the sleeper spacing, track modulus, and the type of sleepers (Figure 6) [1].

<table>
<thead>
<tr>
<th>Developer</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talbot [2]</td>
<td>$q_r = S_u.y_{\text{max}}F_1$</td>
</tr>
<tr>
<td>ORE [14]</td>
<td>$q_r = \bar{q}C_1.P^*$</td>
</tr>
<tr>
<td>UIC [15] (Concrete Sleepers)</td>
<td>$q_r = \frac{P_s}{2}(1 + \gamma_p \times \gamma_v) \times \gamma_d \times \gamma_r$</td>
</tr>
<tr>
<td>Austraila [16] (Concrete Sleepers)</td>
<td>$q_r = j \times P_s \times \frac{D.F.}{100}$</td>
</tr>
<tr>
<td>Austraila [17] (Steel Sleepers)</td>
<td>$q_r = 0.5 \times F_2 \times P \times S \times \beta$</td>
</tr>
<tr>
<td>Sadeghi [18]</td>
<td>$q_r = 0.474P \times (1.27S + 0.238)$</td>
</tr>
</tbody>
</table>

* $\bar{q}$ is defined as the ratio of $q_r / P_s$ in which $q_r$ and $P_s$ are mean values of rail seat load and static wheel load, respectively. $C_1$ is a coefficient usually equals to 1.35.

Table 3. Relations for the calculation of rail seat load [1].

The exact contact pressure distribution between the sleeper and the ballast and its variation with time are highly important in the structural design of sleepers [1]. When track is freshly tamped the contact area between the sleeper and the ballast occurs below each rail seat. After the tracks have been in service the contact pressure distribution between the sleeper and the ballast tends towards a uniform pressure distribution [1]. This condition is associated with a gap between the sleeper and the ballast surface below the rail seat [2]. The most accepted contact pressure distribution patterns between sleeper and ballast are presented in Tables 4 and 5 [1].
Fig. 6. Estimation of rail seat load [13]

As indicated in Table 6, it is usually presumed that the uniform pressure under the sleeper distribute in certain portions of the sleeper length (area) [1]. This length (area) is referred to as “Effective Length (Area)” and commonly shown with “L (Ae)” in the literature. This assumption is made to facilitate the procedure of design calculations. The static equilibrium in vertical direction is then applied to acquire the magnitude of contact pressure under the sleeper. A factor of safety is also included to account for pressure variations in the sleeper support. Therefore, the average contact pressure between the sleeper and the ballast \( P_a \) (kPa) can be obtained by [1]:

\[
P_a = \left( \frac{q_r}{B.L} \right) F_3
\]  

(15)

Having determined sleeper loading pattern, the sleeper can be analyzed. For this purpose several structural model have been proposed in the literature. Table 6 indicates the sleeper structural model used in the current practice [1].

<table>
<thead>
<tr>
<th>Pressure Distribution</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>Laboratory test</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>Principal bearing on rails</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td>Tamped either side of rail</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td>Maximum intensity in middle</td>
</tr>
<tr>
<td><img src="image5" alt="Diagram" /></td>
<td>Uniform pressure</td>
</tr>
</tbody>
</table>

Table 4. Some contact pressure distribution patterns [16]
Pressure distribution pattern beneath sleeper

<table>
<thead>
<tr>
<th></th>
<th>After Tamping</th>
<th>After Accumulative Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_1 = 1.267q_r / L$</td>
<td>$w_1 = 1.596q_r / L$</td>
</tr>
<tr>
<td></td>
<td>$w_2 = 2.957q_r / L$</td>
<td>$w_2 = 2.436q_r / L$</td>
</tr>
<tr>
<td></td>
<td>$w_3 = 1.967q_r / L$</td>
<td>$w_3 = 1.974q_r / L$</td>
</tr>
<tr>
<td></td>
<td>$w_4 = 1.447q_r / L$</td>
<td>$w_4 = 1.687q_r / L$</td>
</tr>
</tbody>
</table>

Table 5. In-track sleeper loading pattern [18]

It should be noted that there are some differences between the sleeper structural models (i.e. sleeper loading patterns) suggested for the calculation of bending moments for each type of sleepers. It is indicated in Table 7 [1].

Table 6. Effective length (area) of sleeper support at rail seat [1].

<table>
<thead>
<tr>
<th>Developer</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AREMA [13]</td>
<td>Ae: Two third of sleeper area at its bottom surface</td>
</tr>
<tr>
<td>UIC [14-15]</td>
<td>$A_e = 6000$ cm$^2$ for $l = 2.5$ m</td>
</tr>
<tr>
<td></td>
<td>$A_e = 7000$ cm$^2$ for $l = 2.6$ m</td>
</tr>
<tr>
<td>Australia [16-17]</td>
<td>$L = (l - g)$ (1)</td>
</tr>
<tr>
<td></td>
<td>$L = 0.9 \times (l - g)$ (2)</td>
</tr>
<tr>
<td>Schramm [2]</td>
<td>$L = \frac{l - g}{2}$</td>
</tr>
<tr>
<td>Clarke [2]</td>
<td>$L = (l - g) \left(1 - \frac{(l - g)}{125t^{0.75}}\right)$ (3)</td>
</tr>
<tr>
<td>Clarke (simplified) [2]</td>
<td>$L = \frac{l}{3}$</td>
</tr>
</tbody>
</table>

1. For bending moment calculation at rail seat
2. For bending moment calculation at sleeper center
3. the parameter “$t$” is the sleeper height
<table>
<thead>
<tr>
<th>Sleeper Type</th>
<th>Developer</th>
<th>Rail Seat Moment</th>
<th>Center Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M_{r+}$ (kN.m)</td>
<td>$M_{c+}$ (kN.m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_{r-}$ (kN.m)</td>
<td>$M_{c-}$ (kN.m)</td>
</tr>
<tr>
<td>Timber</td>
<td>Battelle [2]</td>
<td>$q_r \left( \frac{l-g}{2} \right)$ (1)</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>Schramm [2]</td>
<td>$q_r \left( \frac{l-g-n}{8} \right)$</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>Raymond [2]</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>Steel</td>
<td>Australian Standard [16]</td>
<td>$q_r \left( \frac{l-g}{8} \right)$</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>UIC [15]</td>
<td>$\gamma_i q_r \frac{\lambda}{2}$ (2)</td>
<td>$0.5M_{r+}$</td>
</tr>
<tr>
<td>Concrete</td>
<td>Australian Standard [16-17]</td>
<td>$q_r \left( \frac{l-g}{8} \right)$</td>
<td>Max ${0.67M_{r+}, \frac{14}{14} }$</td>
</tr>
</tbody>
</table>

1. Less conservative and more realistic formula is also suggested by Battelle as $q_r \left( \frac{l-g}{8} \right)$

2. The effective lever arm can be obtained from $\lambda = \frac{L_p - e}{2}$

Table 7. Comparison of methods for calculation of sleepers bending moment [1]

From analyses of the sleeper, the bending moments are calculated [1]. The obtained bending moments are then compared with the sleeper ultimate bending capacities. AREMA [13] has developed a practical method for the estimation of ultimate bending capacities of sleepers. In this method, based on sleeper length, and type of bending moment (i.e. positive or negative), limit values are determined [1].
2.3 Rail fastener

Rail fasteners also known as fastening systems are used in railway track structure to fasten the rails to the sleepers and to protect the rail from inadmissible vertical, lateral, and longitudinal movements [1]. Moreover, these components serve as tools for gauge restraining, wheel load impact attenuation, increasing track elasticity, etc. There are various types of rail fasteners which mainly are classified from two points of view as presented in Figure 8 [1].

![Classification of rail fasteners](image)

Fig. 8. Classification of rail fasteners [1]

<table>
<thead>
<tr>
<th>Qualification Test</th>
<th>Design Code</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AREMA</td>
</tr>
<tr>
<td>Uplift restraint</td>
<td>✓</td>
</tr>
<tr>
<td>Longitudinal restraint</td>
<td>✓</td>
</tr>
<tr>
<td>Repeated load</td>
<td>✓</td>
</tr>
<tr>
<td>Torsional restraint</td>
<td>✓</td>
</tr>
<tr>
<td>Lateral restraint</td>
<td>×</td>
</tr>
<tr>
<td>Clip spring rate</td>
<td>×</td>
</tr>
<tr>
<td>Fatigue strength</td>
<td>×</td>
</tr>
<tr>
<td>Impact attenuation</td>
<td>×</td>
</tr>
</tbody>
</table>

Table 8. Comparison of design codes for rail fastener qualification tests [1]

Despite the important roles that rail fasteners play in track system, there is no practical and sufficient approaches available in the literature related to the analysis and design of rail fasteners [1]. Design criteria available in all railway standards related to fasteners design are limited to those dealt with laboratory qualification tests. AREMA manual [20-21], Australian Standard [22], and European Standards [23-27] are the most important design codes which include such criteria. A comparison of these standards is presented in Table 8 [1].

2.4 Ballast and sub-ballast layers

Ballast and sub-ballast layers are composed of granular materials and used in track structure mainly to sustain the loads transferred from the sleeper [1]. Other important functions of these layers include [1]: 1) to reduce the stress intensity transferred to the subgrade layer to the allowable level, 2) to absorb impact, noise and vibration induced by the wheels, 3) to restrict the track settlement, 4) to facilitate track maintenance operations, particularly those
related to the correction of track geometry defects, and 5) to provide adequate drainage for
the track structure. The sub-ballast layer is used as a separation layer between ballast and
subgrade layers [1].

The sizes of aggregates in the ballast and sub-ballast layers are proposed in the majority of
the railway standards or codes of practice. Theoretical, semi-empirical, and empirical
methods are used in order to determine the depth of these layers [1].

Theoretical determination of minimum required depth based on Boussinesq elastic theory
(applied to a uniform rectangular loaded area) (Figure 9) is performed using the numerical
solution of the following equation [1]:

\[
x \sigma_z = \frac{3 \sigma x}{2 \pi} \int_{-A}^{A} \int_{-B}^{B} \frac{d \varepsilon \, d \eta}{\left( (x - \varepsilon)^2 + (y - \eta)^2 + z^2 \right)^{3/2}}.
\]  

(16)

It should be noted that, ballast and sub-ballast layers are assumed as a single homogenous
and isotropic layer in Boussinesq elastic theory [1]. Although such an assumption seems to
be insufficiently accurate, ORE [28-29] have indicated the validity of Boussinesq elastic
theory based on the available field tests results (Figure 10). As it is apparent from Figure 10,
the results obtained from Boussinesq method reasonably remain within the envelope
obtained from experimental investigations [1].

Fig. 9. Application of Boussinesq’s elementary single vertical concentrated load over a
uniformly loaded rectangular bearing area [2]

Simplified semi-empirical methods are also employed in the analysis of the ballast [1]. It is
assumed that the load is distributed vertically with a load spread with a slope of 1 vertical to
1 horizontal or a slope of 2 vertical to 1 horizontal. It is also assumed that the stress
distribution is uniform at any given depth below the surface [1]. Only the average vertical
pressure at any depth can be calculated by this method while the Boussinesq method leads
to the maximum vertical pressure at a depth below the loaded area [1]. A comparison of the
vertical stress distribution calculated for both 1:1 and 2:1 load spread assumptions with the
theoretical Boussinesq solution is presented in Figure 11 [2]. It is clearly apparent that the
assumed 2:1 load spread distribution of vertical pressure more closely approximates the Boussinesq pressure distribution than that of 1:1 load spread distribution [2].

Considering the 2:1 load spread distribution as indicated in Figure 12, the minimum required ballast and sub-ballast depth can be calculated from the following formula [1]:

$$\sigma_z = P_d \frac{B.L}{(B + z)(L + z)}$$  \hspace{1cm} (17)

Having determined the allowable subgrade load carrying capacity and substituting it in the equations (16) or (17) the minimum required ballast and sub-ballast depth can be calculated [1].

Fig. 11. Comparison of vertical stress distribution under a uniformly loaded circular area based on Boussinesq equations and 1:1 and 2:1 distributions [2]
3. New developments in design of railway track system

As discussed in Section 2, the conventional track design methods are based on the Winkler model (a continuously supported beam on an elastic foundation). It includes several unrealistic assumptions, including:

- static point loads for the trains,
- continuous support under the rail,
- linear characteristics for the track support system, and
- linear uniform pressure distribution under the sleeper.

Although several factors have been taken into account to compensate for the errors caused by these assumptions, there are sometimes large discrepancies between the results obtained from conventional methods and those obtained in railway fields. This indicates that the effectiveness of the conventional track design is questionable from two aspects: cost efficiency and accuracy.

In the last decade, some improvements have been made in providing a better understanding of railway track systems. The new developments can be used to improve the current track design approach. They are detailed as follows.

3.1 Dynamic impact factor (DIF)

As indicated in the last section, various formulations have been suggested for the calculation of the track dynamic impact factor. Rail deflection and rail bending stresses have been the main criteria in the development of the majority of the available dynamic impact factor. It means that the current DIF can be only justifiable for the design of rails [29].

The mostly used suggestions for calculation of DIF for the design of rails are compared in Figure 13. It is apparent from this figure that train speed can considerably influence the values of dynamic impact factor. Sadeghi and his colleagues conducted a thorough field investigation into the effect of various track and rolling stock parameters on track dynamic impact factor [30]. They indicated that the suggestion made by AREMA and South African standards are rather accurate for track with lower speed while for the high speed transit, implementation of the suggestion made by ORE (UIC) is more appropriate [30].
Since AREMA recommendations are basically in accordance with heavy haul operations in which trains have low speeds, it would not be practical to use the AREMA formula for the railway high-speed operations. Therefore, the use of the AREMA recommendation (Table 1) for dynamic impact factor is suggested for heavy haul railway with the train speed less than 80 km/h. On the other hand, the use of mathematical expression proposed by ORE (indicated in Table 1) is more justifiable for the calculation of dynamic impact factor in high speed railway tracks [31].

![Fig. 13. Comparison of dynamic impact factor for design of rails [1]](image)

The effects of the dynamic characteristics of the load on the sleeper and the ballast (Rail-seat load and ballast-sleeper contact pressure) have been recently investigated by Sadeghi and his colleagues [31]. They indicated that the dynamic characteristics of the load have more impact on the amount of ballast-sleeper pressure at the points which experience less pressure. That is, the less pressure, the more influence due to the load dynamics. Since with an increase in load speed the pressure in the middle and cantilever parts of the sleeper considerably increases, we are led to believe that the load distribution under the sleeper will become increasingly uniform with increasing load speeds [31]. This means that the assumption of a uniform pressure distribution under the sleeper for train speeds of more than 120 km/h is justifiable [31].

The impacts of the dynamic properties of the wheel load on the increase of the loads between ballast and sleepers experimentally investigated by Sadeghi and his colleagues are presented in Figure 14. They have obtained this figure from dividing the results obtained from the load cells (installed under the sleepers) when trains were moving with the speed of 20 to 120 km/h by the static load cell values (i.e., speed is zero). The vertical axis in this figure is the dynamic coefficient factor or dynamic impact factor. Since the DIF calculated in figure 14 is based on the effects of dynamic load on sleeper loading pattern, implementation of this figure in the design of sleepers and ballast is more justifiable and lead to a better accuracy [31].
Fig. 14. DIF versus speed for calculation of pressure in various sections of sleepers [31]

### 3.2 Continuous support under the rail

Because of the limitations of the theory of beam on elastic foundation model, discussed in the last section, several attempts have been made to develop new modeling techniques to simulate railway track behavior. First, a model of beam on discrete support has been developed and more recently analyzed using a practical energy approach [3]. Second, Pasternak foundation and double beam models are also introduced [4] to take into account the interaction between track supporting layers and multi-layer nature of track support, respectively. Kerr [5] has reported the result of a research in which he showed that the effect of the rotational stiffness of the sleepers in calculation of rail deflection and bending moment are negligible.

Sadeghi and his colleagues investigated the importance of considering the discontinuity of the rail supports by comparing the results from the Winkler model (WM) and the discrete support model (DSM) [32]. They have shown that the maximum differences in the results from each method are at the most 2.5% and 11% for rail deflections and rail bending moments, respectively [32]. The difference between rail deflections decreases as the track stiffness increases but the difference in bending moments increases as the track stiffness increases [32]. A comparison of the results confirms that the assumption of continuous support under the rail (Winkler model) slightly increases rail bending moments and negligibly decreases rail deflections [32]. These negligible differences between the results obtained from the two methods along with the simplicity and practicality of the conventional method indicate that the assumption of continuous support under the rail is justifiable [32].

### 3.3 Non-linearity of track support system

The results obtained in the literature illustrate large differences between analyses of the track with and without consideration of nonlinearity of the track support system [2, 37],
indicating a need for incorporation of the nonlinear properties and plastic deformations of
the track support system. This is more apparent where the long term behavior of the track is
concerned. The major differences between results obtained from the conventional method
and the nonlinear track analyses come from the consideration of track plastic deformations
caused by accumulative loadings [33-34]. Note that the impact made by the consideration of
plastic deformation is influenced by the amount of wheel loads. This means that there are
two key influencing factors to be considered in the calculation of track design parameters:
the amount of accumulative loadings and the amount of wheel loads [33-34].

There are two ways to include the nonlinear properties of the rail support system in the
design procedure [32]: first, by using a new approach which includes nonlinear analysis of
the track; or second by applying appropriate correction factors to the conventional method.
Because the conventional track design approach is simple and user-friendly, the second
method seems more practical. For this purpose, two correction factors have been developed
in the literature for rail deflection and rail bending moment to compensate for the
assumption of linear properties for the track support system [32].

Sadeghi and his colleagues have obtained the ratios of rail deflections and rail bending
moments obtained from track nonlinear analysis to those from linear analysis for different
track conditions [32]. They have plotted these ratios against Mega Gross Tons passed
(MGTP) and against wheel loads. Using a curve-fitting method, they draw mathematical
correlations (expressions) between ratios of nonlinear to linear track responses and wheel
loads. Using the results obtained, they have proposed correction factors for the rail
deflection and rail bending moment as follows [32]:

$$f_{D-T} = (10^{-4}P^2 - 0.03P + 2.85) + (3 \times 10^{-5}P^2 - 8 \times 10^{-3}P + 0.62)T \quad P \geq 20 \text{ kN} \quad (18)$$

$$f_{D-C} = (10^{-4}P^2 - 0.02P + 2.45) + (10^{-5}P^2 - 3 \times 10^{-3}P + 0.28)T \quad P \geq 20 \text{ kN} \quad (19)$$

$$f_{M-T} = (-0.001P + 1.126) + (-3 \times 10^{-4}P + 0.08)T \quad P \geq 20 \text{ kN} \quad (20)$$

$$f_{M-C} = (-0.001P + 1.09) + (-3 \times 10^{-4}P + 0.04)T \quad P \geq 20 \text{ kN} \quad (21)$$

where, $f_{D-C}$ and $f_{M-C}$ are correction factors for rail deflection and rail bending moments in
concrete sleeper tracks respectively, $f_{D-T}$ and $f_{M-T}$ are correction factors for rail deflection
and rail bending moments in timber sleeper tracks, respectively, $P$ is the wheel loads (kN), and
$T$ is the amount of accumulative loadings in MGT. The rail bending moment and rail deflection
calculated by the conventional method should be multiplied by these correction factors to
compensate for the unrealistic assumption of linear properties for the track supports. The use of
these correction factors would improve the accuracy of the current track analysis method,
particularly when the track has undergone a large amount of accumulative loading [32].

3.4 Sleeper loading pattern

As indicated in the previous section, a uniform contact pressure distribution is assumed
between ballast and sleeper in the current practice. In other words, the current design
methods do not clearly include the effect of ballast material degradation on the ballast-sleeper pressure distribution pattern [35]. This means that the sleeper design approach awaits further improvements by the incorporation of long term effects of the loads and in turn, consideration of the ballast degradation.

In 2008, Sadeghi and his colleagues conducted a thorough field investigation into the relationship between rail seat loads, static wheel loads and train speeds [36]. Using the results obtained from the field, they proposed the following relationships for the calculation of rail seat load as a function of static wheel loads, train speeds, and sleeper spacing for the three types of sleepers. This reflects the concept of dynamic impact factor which can be used for the design of sleepers [36].

\[ q_r = (-3 \times 10^{-6} V^2 + 0.0018V + 0.4274)(1.27S + 0.238)P \text{ Timber sleepers} \] \hspace{1cm} (22)

\[ q_r = (-3 \times 10^{-6} V^2 + 0.0017V + 0.4659)(1.27S + 0.238)P \text{ Steel sleepers} \] \hspace{1cm} (23)

\[ q_r = (-3 \times 10^{-6} V^2 + 0.0018V + 0.4936)(1.27S + 0.238)P \text{ Concrete sleepers} \] \hspace{1cm} (24)

Comparisons of the experimental results obtained here and those currently used indicate that the American Code is slightly over-estimated for speeds higher than 80 km/h [36]. This can be due to the US railway trend towards higher axle loads and heavy haul operations which limits the reliability and applicability of American Code for high speed operations. On the other hand, the European suggestion is an under-estimation of the ratio of the rail seat load to the static wheel load for train speeds more than 60 km/h [36]. Results indicate that European suggestion is more reliable for timber sleeper in comparison with the steel or concrete sleepers. The results obtained in the field are more agreeable with that suggested by the Australian standard for steel and concrete sleepers, particularly for train speeds between 60 to 120 km/h [36].

Sadeghi and his colleagues have also experimentally investigated the pressure distribution between ballast and various sleepers [36]. They illustrated that the pressure between the ballast and the sleeper increases from the sleepers' edge and reaches its maximum value under the rail-seat position, then with a decreasing pattern, comes to the minimum value at the center of the sleeper. They showed that the contact pressure occurred under pre-stressed concrete sleepers is more uniform when compared with that under timber and steel sleepers [36]. The most non-uniform shape of the sleeper-ballast contact pressure occurs under the timber sleepers; this could be due to the higher bending rigidity of concrete sleepers in comparison with wood and steel sleepers [36].

In order to make the results practical, Sadeghi and his colleagues have brought the obtained stress distribution beneath the sleeper into a uniform shape [36]. It is indicated in Table 6 which presents uniform pressure distributions beneath the timber, steel and concrete sleepers with a reasonable accuracy. Most of the railway standards suggest the consideration of a uniform pressure distribution beneath all the area (or effective area) of the sleeper. Implementation of the new model (suggested in Table 9) can improve the accuracy of the current sleeper design approach [36]. It should also be mentioned that this new sleeper loading pattern is applicable to the curves as long as there is no cant deficiency or cant excess along the curve length (i.e., when the sleeper symmetric load distribution pattern is maintained [36]).
## 4. Summary and conclusions

In this chapter, the algorithm and limitations of the current track designed approach were discussed. In the current practice, several assumption are made: (1) each track complement (rail, sleeper, fastening system, and ballast) is design independently; (2) loads are considered static; (3) the theory of beam on elastic foundation is mainly used for the analyses of the track components; and (4) track support system is considered to have a linear mechanical behavior. Although several factors have been taken into account to compensate for the errors caused by these assumptions, there are sometimes large discrepancies between the results obtained from conventional methods and those obtained in railway fields. This indicates that the effectiveness of the conventional track design is questionable.

In the last decade, several experimental and theoretical investigations have been conducted in the field of railway engineering related to the track mechanical behaviors. Incorporation of new research findings into the track design method can lead to substantial improvements in the accuracy and reliability of the conventional methods.

1. The results recently obtained from field investigations indicate that the use of the AREMA suggestion for dynamic impact factor is appropriate for heavy haul railway with the train speed less than 80 km/h while the mathematical expression proposed by ORE (UIC) is more justifiable for the calculation of dynamic impact factor in high speed railway tracks. However, these expressions or formulations have been developed based on the rail design criteria and therefore, their accuracy for the design of sleeper or ballast is questionable. The use of newly obtained dynamic impact factor which has been developed based on the sleeper and ballast design criteria would improve the accuracy of the track design when sleepers and ballast layers are concerned.

### Table 9. Load distribution pattern under different sleepers [36]

<table>
<thead>
<tr>
<th>Sleeper Type</th>
<th>Parameter</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timber</td>
<td>a</td>
<td>0.753q_r / L</td>
<td>0.494q_r / L</td>
</tr>
<tr>
<td>Steel</td>
<td>a</td>
<td>0.719q_r / L</td>
<td>0.561q_r / L</td>
</tr>
<tr>
<td>Pre-stressed concrete</td>
<td>a</td>
<td>0.709q_r / L</td>
<td>0.581q_r / L</td>
</tr>
</tbody>
</table>
2. Results from field investigations and finite element analyses indicate that there are negligible differences between track models with a continuous support and those with discrete supports. Therefore, considering the practicality and simplicity of the conventional method, the assumption of continuous support under the rail seems to be justified.

3. It has been experimentally shown that there is a large impact on the track design parameters due to the nonlinearity of the track support system, indicating the necessity of improving current codes of practice. Using results from track fields, several models have been developed for the nonlinear response function of the track support system for different track loading and material conditions. Using these models and conducting a nonlinear analysis of the track, it was shown that there was a substantial increase in the maximum rail deflection and rail bending moment (the main track design parameters) when considering the nonlinearity of the track support system. In contrast with the current practice of assuming a linear relationship between wheel loads and rail deflection, a cubic relationship between wheel loads and rail deflections has been observed. It has been shown that the major differences between the conventional method and nonlinear track analysis come from the consideration of track plastic deformation caused by accumulative loadings. It has been also illustrated that the impact of the track nonlinearity on the analysis results is influenced by the magnitude of wheel loads. Therefore, plastic deformation and applied wheel loads are the key influencing factors in the nonlinear behavior of track support systems and they are in turn important factors to be considered in the track analysis and design. To compensate for the errors caused by the assumption of a linear response for the track support system, correction factors have been recently developed for rail deflection and rail bending moment, to be multiplied by the rail bending moment and rail deflection. The influences of the nonlinear properties of the track support system on the track design parameters have been the basis for the calculation of the correction factors. They are functions of the magnitude of wheel loads and the amount of accumulative loadings. It has been proved that the incorporation of these factors improves the accuracy and reliability of the conventional track design method.

4. It has been shown that there is a second order relationship between maximum rail seat loads and train speeds. From this point, formulae for the calculation of rail seat loads as a function of wheel loads, train speeds, and sleeper spacing for timber, steel and concrete sleepers have been recently proposed. It has experimentally shown that the pressure distribution pattern between the ballast and the sleeper is not a uniform contact pressure as proposed earlier by American codes (AREMA). The pressure between the ballast and sleeper for all three types of sleepers was found to decrease from the sleepers’ edge, reach its maximum value under the rail seat position, and then decrease to a minimum value at the center of the sleeper. This is in close agreement with the sleepers’ behaviour proposed by UIC. The pressure distribution beneath the concrete sleepers is more uniform than those of timber or steel sleepers. As the speed and axle loads increase, this pressure distribution becomes less uniform, indicating a need for improvement of the current track design approaches for heavy haul and high speed tracks. The assumption of a uniform pressure distribution between ballast and sleeper in the current sleeper design methods can be replaced with a more realistic uniform load distribution pattern beneath concrete, steel and timber sleepers newly proposed.
5. List of symbols

B  Sleeper breadth, m

c  height of the rail neutral axis from rail base, mm

D  Wheel diameter, mm

DA  Annual gross tonnage, MGT/year

c  Degree of curve, degrees

Di  Sub-tonnage, MGT/year

DJ  Track stiffness at the joint, kN/mm

D.F.  Load distribution factor

e  Width of the rail seat load distribution along sleeper thickness

E  Rail modulus of elasticity, N/mm²

F₁  Track support variation safety factor

F₂  Factor accounting for adjacent wheels interactions

F₃  Factor depending on the sleeper type and the standard of track maintenance

g  Gravitational constant, m/s²

I  Rail moment of inertia, mm⁴

Iₜ  Horizontal moment of inertia of the center of the sleeper cross section, mm⁴

Iₙ  Horizontal moment of inertia of the sleeper cross section at rail seat position, mm⁴

j  Load amplification factor

K₁  Wheel load class factor

KC  Track curvature and lubrication factor

KG  Track gradient factor

KR  Rail factor

Ks  Service type factor

KV  Speed class factor

Kw  Wear factor varying with the degree of curve

l  Sleeper length, m

Lₚ  Distance between rail seat axles and the end of the sleeper

n  Length of steel rail plate

P  Wheel load, kN

Ps  Static wheel load, kN

Pu  Unsprung weight of the wheel, kN

Rw  Wheel radius, mm

Rwt  Rail weight per unit length, kg/m

S  Sleeper spacing, mm

T  Rail wear life, MGT

Ty  Rail wear life, year

u  Track modulus, N/mm²

V  Train speed, km/h

Wt  Average rail head area wear term, mm²/MGT
Greek symbols

- $\alpha$: Coefficient of expansion
- $\alpha'$: Speed coefficient
- $\beta'$: Speed coefficient
- $\delta$: Factor related to track condition
- $\Delta t$: Temperature variation
- $\gamma'$: Speed coefficient
- $\gamma_d$: Load distribution factor
- $\gamma_i$: Dynamic increment of bending moment factor due to sleeper support irregularities
- $\gamma_p$: Impact attenuation factor of rail fasteners
- $\gamma_r$: Sleeper support condition factor
- $\gamma_o$: Speed related amplification factor
- $\eta$: Speed factor
- $\lambda$: Effective lever arm, m
- $\nu$: Poisson’s ratio

6. References


In railway applications, performance studies are fundamental to increase the lifetime of railway systems. One of their main goals is verifying whether their working conditions are reliable and safety. This task not only takes into account the analysis of the whole traction chain, but also requires ensuring that the railway infrastructure is properly working. Therefore, several tests for detecting any dysfunctions on their proper operation have been developed. This book covers this topic, introducing the reader to railway traction fundamentals, providing some ideas on safety and reliability issues, and experimental approaches to detect any of these dysfunctions. The objective of the book is to serve as a valuable reference for students, educators, scientists, faculty members, researchers, and engineers.

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