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Putting Einstein to Test – Astrometric Experiments from Space, Fundamental Physics and Local Cosmology

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1. Introduction

First experimental evidences of General Relativity (GR) came from Astrometry. Its measurements not only had put to evidence the existence of the effect of an excess of perihelion precession in the motion of the Solar System objects which couldn't be explained in terms of the Newtonian Mechanics (Le Verrier, 1859) but also provided a striking confirmation of Einstein's theory through the experiment conducted at Sobral and Principe by Dyson, Eddington & Davidson (1920) during the Solar Eclipse of 1919, which measured the light deflection foreseen by GR with an accuracy of $\sim$ 10%.

GR, however, has never been the only theory claiming to provide a detailed description of the Gravitational interaction beyond the Newtonian limit. Although GR is still the most favoured alternative, either because of the experimental results or of its simplicity, several other theories have been proposed. Examples of such theories are those where the field equations contain not only the metric tensor of GR, but also a scalar field coupled with the metric itself (Will, 2006), or the so-called fourth-order theories of gravity, where the scalar of curvature R in the field equations is replaced by a more complex function of this quantity, $f(R)$.

Tests of gravity theories within the Solar System are usually analysed in the framework of the so-called *Parametrized Post-Newtonian framework* which enables the comparison of several theories through the estimation of the value of a limited number of parameters. Among these parameters, γ and β are of particular importance for astrometry since they are connected with the classical astrometric phenomena of the light deflection and of the excess of perihelion precession in the orbits of massive objects. Besides its immediate implication for the fundamental physics problem of characterizing the best gravity theory, a precise estimation of these parameters has important consequences on the interpetation of observational evidences at different scales in space and time up to cosmological scales. This is the reason why these Solar System astrometric experiments can also be intended as a kind of "local cosmology" tests which will be addressed from space by planned missions like Gaia (Perryman et al., 2001) and, in the future, by other projects presently under study.

In the next section we will birefly recall the basic concepts standing at the foundations of the current gravity theories, and we will try to give a general overview of the "what" and the “why”, that is which are the alternatives available on the “theoretical market” and their motivations. In Section 3 we will show the “how”, i.e. the theoretical methods which have been developed to make the comparison among the various gravity theories easier, while in the following one we will concentrate on the problem of how it is possible to model a generic astrometric observable in a relativistic consistent way. Section 5 will adress the experimental side of this problem giving examples of data reduction techniques and of some experiments planned or under study, and finally the next and last section will come to the conclusions.

2. Theoretical background

The basic tenets of the so-called *calssical field theories* could be summarized, very shortly and roughly, as:

1. there exists something (the field source) producing a field;
2. this field tells to the particles how to move.

Mathematically, these two simple principles correspond to the *field equations* and to the *equations of motion* respectively.

The first and simplest theory of this kind is the newtonian theory of gravity, in which the Poisson equation

$$\nabla^2 U = 4\pi G\rho \tag{1}$$

is the field equation where the gravitational field U originates from a mass density ρ, while the Newtonian law of dynamics

$$\frac{\mathrm{d}^2 x^k}{\mathrm{d}t^2} = -\frac{\partial U}{\partial x^k} \tag{2}$$

are the equations of motion.

However, many other classical field theories aiminig at explaining how gravity works have been formulated. In GR the field equations write

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta} \tag{3}$$

where the metric $g_{\alpha\beta}$ plays the role of the potential U and the stress-energy tensor $T_{\alpha\beta}$ is the relativistic counterpart of the matter density ρ.[1] This statement can be better understood if one notices that:

1. under the assumption of weak gravitational fields and slow motion, the relativistic equations of motion approximate to

$$\frac{\mathrm{d}^2 x^k}{\mathrm{d}t^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial x^k}, \tag{4}$$

and the requirement that, within these hypothesis, they have to reproduce the newtonian ones implies that

$$g_{00} \simeq -1 + \frac{2U}{c^2}; \tag{5}$$

[1] The definition of the stress-energy tensor is more general and takes into account that in GR any form of energy can contribute to the curvature of the space-time.

2. the quantities $R_{\alpha\beta}$ and R basically depend on the derivatives of the metric, so that the Poisson equation can be derived from Eq. (3) under the same weak-field and slow-motion assumptions.

Another example of relativistic field theory of gravity is the Brans-Dicke theory. (Brans & Dicke, 1961) This theory is the simplest example of the class of the so-called *scalar-tensor* theories, in which the field equation is characterized by the presence of a scalar field ϕ coupled with gravity in addition to the metric tensor

$$\phi\left(R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R\right)+g_{\alpha\beta}\Box\phi-\phi_{,\alpha;\beta}=\frac{8\pi G}{c^4}T_{\alpha\beta}+\frac{\omega}{\phi}\left(\phi_{,\alpha}\phi_{,\beta}-\frac{1}{2}g_{\alpha\beta}\phi_{,\sigma}\phi^{,\sigma}\right). \tag{6}$$

The quantity ω is a coupling constant[2] which gives a measure of the deviations of this theory from GR. From this equation, in fact, it can be shown that the relation between ϕ and the trace T of the stress-energy tensor is

$$\Box\phi=\frac{8\pi G}{(2\omega+3)\,c^4}T \tag{7}$$

so that Eq. (6) reduces to the GR field equation (3) when $\omega\to\infty$. The maini motivation at the basis of this theory was to have a gravity theory which could fully incorporate the Mach principle, according to which the inertial phenomena were caused by the relative motion of the mass in the Universe. (Ciufolini & Wheeler, 1995)

The importance of scalar-tensor theories comes also from other more complex models introduced as a way to explain inflationary cosmological scenarios (like e.g. in Damour & Nordtvedt (1993)), and also in particle-physics-inspired models of unification such as string theory (Damour, Piazza & Veneziano, 2002).

We finally want to mention the so-called $f(R)$ theories. This class of theories stem from the idea that the lagrangian $\mathcal{L}$ from which the field equations can be derived can be more general with respect to that of GR. The latter, in fact, assumes that it is simply equal to the Ricci scalar, i.e. $\mathcal{L}=R$, while these theories relax this condition and take a generic dependence $\mathcal{L}=f(R)$. From a mathematical point of view, a consequence of this hypothesis is that the field equations are no more second-order differential equations, but fourth-order. From a physical point of view, the additional terms emerging into the field equation can be interpreted also as sources, rather than fields, and take the same role of $T_{\alpha\beta}$. From a phenomenological point of view, finally, these theories have been invoked to explain several observational phenomena like cosmological acceleration, galactic dynamics, gravitational lensing, etc. without the need to resort to non-barionic Dark Matter or to Dark Energy. A more detailed review of this kind of theories can be found in Capozziello & Faraoni (2011).

3. The theoretical tools of the gravity arena

As we have just pointed out in the previous section, several other theories alternative to GR has been formulated in order to describe how gravity works. This situation naturally calls for a framework in which it is possible to model the observations taking into account many viable theories at the same time. In this section we will briefly explain two of these frameworks: the

[2] In more general versions of scalar-tensor theories ω is not a constant but a function of ϕ.

Parametrized Post-Newtonian (PPN) and the Mansouri-Sexl. More details about these two subjects can be found in Will (2006) and in Mansouri & Sexl (1977a;b;c).

3.1 The PPN framework

As pointed out in Will (2006) a viable theory of gravity must be complete, self-consistent, relativistic, and with a correct Post-Newtonian limit. The Post-Newtonian limit of a gravity theory is the approximation of such theory under the assumptions of slow-motion and weak gravity field, i.e. when $v/c \sim U/c^2 \sim \epsilon \ll 1$.[3]

The PPN formalism is a powerful tool which simplifies the confrontation among the Post-Newtonian limits of the different theories of gravity. This is particularly useful also because these conditions hold in the Solar System, where local experiments are confined.

It can be shown that, in the slow motion and weak-field limit,

$$g_{\alpha\beta} \simeq \eta_{\alpha\beta} + h_{\alpha\beta}, \tag{8}$$

i.e. that the metric can be written as an expansion about the Minkowskian metric of Special Relativity where the perturbations $h_{\alpha\beta}$ are functions of the different smallness parameters $\sim \epsilon$ like U, v, etc.

The noticeable thing is that the expansions predicted by nearly every theory of gravity have the same structure apart from the coefficients multiplying each kind of perturbation.

In the PPN formalism these coefficients are replaced by appropriate parameters in such a way that it is possible to reproduce any specific theory simply by assigning specific values to them. Currently the number of parameters of the PPN formalism is set to 10 (Table 1), suitably chosen to give informations on general properties of the selected theory of gravity. On the other side, the formulation of the experimental measurements in the PPN formalism can be translated to the estimation of the values of these parameters, and thus in a direct selection of the viable theories.

The two most important parameters of the PPN formalism are γ and β, which are both 1 in GR. A precise estimation of these two parameters, has important *theoratical and observational implications*. It could help to *fulfill theoretical needs* because they are the phenomenological "trace" of a scalar field coupled with gravity of scalar-tensor theories which, as explained in the previous section, is related to:

- theories fully compatible with the Mach principle (Brans & Dicke, 1961);
- cosmological scenarios with inflationary stage (Damour & Nordtvedt, 1993);
- theories aiming to provide a formulation of a quantum theory of gravity (Damour, Piazza & Veneziano, 2002).

For example, there are formulations of scalar-tensor theories in which the scalar field evolves with time toward a theory close to GR leaving little relic deviations at present times (Damour & Esposito-Farèse, 1992). Such deviations from GR today range from 10^{-5} to a few times 10^{-7} for $|\gamma - 1|$, depending on the cosmological model (Damour & Nordtvedt, 1993).

[3] Actually it is also required that the internal energy Π and the pressure-over-density ratio p/ρ are $\sim \epsilon$.

Parameter	**Meaning**	**Value in**			
		GR	Brans-Dicke	Vector-Tensor	$f(R)$
γ	How much space-curvature is produced by unit rest mass?	1	$\frac{1+\omega}{2+\omega}$	γ'	γ'
β	How much nonlinearity in the superposition law for gravity?	1	1	β'	β'
ξ	Preferred-location effects?	0	0	0	0
α_1	Preferred-frame effects?	0	0	α_1'	0
α_2		0	0	α_2'	0
α_3		0	0	0	0
ζ_1	Violation of conservation of total momentum?	0	0	0	0
ζ_2		0	0	0	0
ζ_3		0	0	0	0
ζ_4		0	0	0	0

Table 1. List of the PPN parameters, and of their meanings and values in some theories of gravity.

Experiment	**Effect**	**Technique**	$\lvert\gamma-1\rvert$ **lower bound**
HIPPARCOS	Light deflection	Global Astrometry	$3 \cdot 10^{-3}$
VLBI	Light deflection	Radio Interferometry	$4.5 \cdot 10^{-4}$
Cassini	Shapiro time delay	Round-trip travel time of radar signals	$2.3 \cdot 10^{-5}$

Table 2. Current best estimations for the γ parameter.

On the front of the $f(R)$ theories, instead, it has been argued that the current estimations of the γ and β PPN parameters are not sufficient to provide serious constraints on such theories, which claim of being able to explain *observational evidences* about several astrophysical and cosmological problems without any need for Dark Matter (DM) or Dark Energy (DE) like, e.g. Capozziello, de Filippis & Salzano (2009):

- DE dynamics (acceleration of cosmological expansion);
- DM dynamics (galactic rotation curves, galaxy cluster masses);
- observational data from gravitational lensing;
- Tully-Fisher relation.

Again, it seems that the desired experimental accuracy can be set at level of 10^{-7} to 10^{-8} for $|\gamma - 1|$, and from 10^{-5} for $|\beta - 1|$ (Capozziello, Stabile & Troisi, 2006).

The most stringent experimental limit for γ available so far was set by the Cassini experiment (Bertotti, Iess & Tortora, 2003) which used the relativistic time delay effect on the propagation of radar signals (Table 2). This result will be overcome by future astrometric measurements

Experiment	Effect/Technique	Combination with γ	$\|\beta - 1\|$ lower bound
Radar observations of Mercury (1966-1990)	Perihelion shift excess of Mercury	$2\gamma - \beta$	$3 \cdot 10^{-3}$
Lunar Laser Ranging	Nordtvedt effect	$4\beta - \gamma$	$\sim 10^{-4}$
Radar observations of inner bodies (1963-2003)	"Grand fits" of Solar System equations of motion	$2\gamma - \beta$	$1 \div 2 \cdot 10^{-4}$

Table 3. Current best estimations for the β parameter.

from space (Vecchiato et al., 2003). The estimation of β depends on that of γ, since the former always appear in combination with the latter, and its present bounds are provided by different techniques, involving e.g. Lunar Laser Ranging measurements (Williams, Turyshev & Boggs, 2004) or "Grand fits" of the Solar System equations of motion obtained from the reduction of radar observations (Pitjeva, 2005; 2010) (Table 3). Once again, astrometric techniques could be used to improve on these limits (Vecchiato, Bernardi & Gai, 2010).

3.2 Beyond the PPN framework

Other interesting phenomena which can be used to test the gravity theories at a fundamental level are the higher-order quadrupole contribution to the light deflection, and the possible violations of the Local Lorentz Invariance (LLI). Both can be put to test with convenient astrometric experiments.

The first one is foreseen by GR and other theories of gravity when the light is deflected by perturbing bodies with non-spherically symmetric distributions of the mass. The coefficients of $g_{\alpha\beta}$, in fact, in this case does not depend only on the mass, but also on the higher order multipoles of the gravity field of the perturbing body. The deflection can then be described as a vectorial quantity with two components **n** and **m**

$$\Delta\psi = \Delta\psi_1 \mathbf{n} + \Delta\psi_2 \mathbf{m}$$

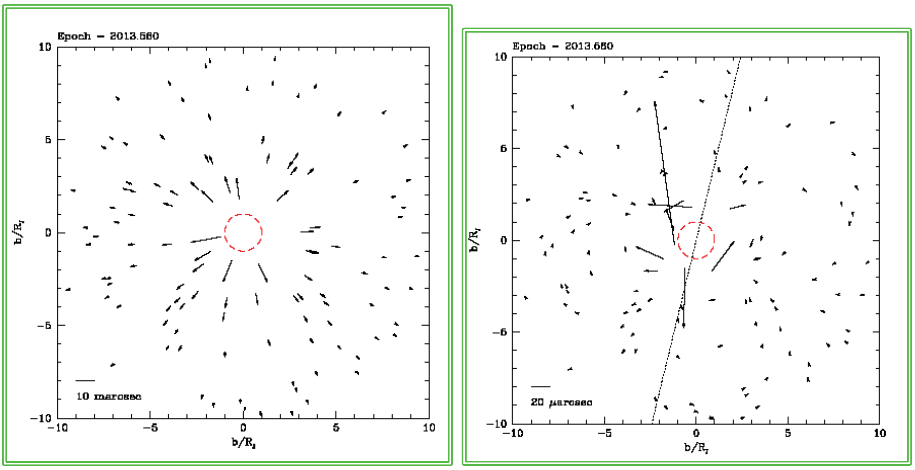
with respect to a reference triad $(\mathbf{n}, \mathbf{m}, \mathbf{z})$ where the **z**-axis is orthogonal to the celestial sphere.

The two components can be modelled as functions of a parameter ε whose value is 1 in GR like in the following formulae (Crosta & Mignard, 2006)

$$\Delta\psi_1 = \frac{2(1+\gamma)M}{b}\left[1 + \varepsilon J_2 \frac{R^2}{b^2}\left(1 - 2(\mathbf{n}\cdot\mathbf{z})^2 - (\mathbf{t}\cdot\mathbf{z})^2\right)\right] \tag{9}$$

$$\Delta\psi_2 = \frac{4(1+\gamma)M\varepsilon J_2 R^2 \gamma}{b^3}(\mathbf{m}\cdot\mathbf{z})(\mathbf{n}\cdot\mathbf{z}) \tag{10}$$

where M is the mass of the perturbing body, J_2 is the quadrupole component of its gravitational field, and b the impact parameter, i.e. the distance of maximum approach of the light path to the perturbing body.



(a) Spherically symmetic case. (b) Deflection pattern with $J_2 \neq 0$.

Fig. 1. A non-spherically symmetric distribution of the mass of a perturbing body produces specific patterns in the nearby light deflection.

This asymmetric perturbation on the light path induces specific patterns in the nearby light deflection (Fig. 1) which have never been mesuared up to now because of the smallness of this effect.

Tests of possible violations of the LLI are motivated by several theoretical models encompassing a large number of different subjects, from quantum gravity to varying speed of light cosmologies. A complete review of the tests and of the motivations linked to the LLI is out of scope here, and can be found in Mattingly (2005). Here we will limit ourselves to cite the Robertson-Mansouri-Sexl (RMS) formalism for its possible application to astrometry. A violation of the LLI, in fact, will show itself as a breaking of the Lorentz transformations. The RMS formalism is a way to describe this hypothetical breaking in a kinematical way.

In analogy to the PPN formalism, the RMS framework is developed under the assumption that $v \ll c$, and can be expressed as a generalization of the Lorentz transformations

$$
\begin{aligned}
T &= \frac{(t - \boldsymbol{\epsilon} \cdot \mathbf{x})}{a} \\
\mathbf{X} &= \frac{\mathbf{x}}{d} - \left(\frac{1}{d} - \frac{1}{b}\right)\frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{x})}{v^2} + \frac{\mathbf{v}}{a}t
\end{aligned}
\tag{11}
$$

depending on a set of arbitrary parameters $(a, b, d, \boldsymbol{\epsilon})$. The potential impact of this formalism on astrometric measurements comes from the fact that those LLI violations depending on $f = d/b$, show themselves as an aberration effect which therefore puts to test the same properties of the Michelson-Morley experiment. It is probable that violations depending on such aberration effect are out of reach for the present planned space experiments like Gaia (Klioner, 2008) but possible applications to new and more sensitive experiments has not been investigated yet.

4. Modelling the observations

4.1 Basics of the relativistic Theory of measurements

In order to understand why Astrometry can be used to set limits to the PPN parameters and therefore to put to test the different theories of gravity, it has to be shown how these parameters enter in the astrometric observable. The most basic measure in Astrometry, ideally, is the angle ψ_{12} between two observing directions, which in the usual Euclidean geometry reads

$$\cos\psi_{12} = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|}, \tag{12}$$

and can be represented as in Fig. 2 as the projection on a unit sphere of the two vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ connecting the observer and the objects P and Q.

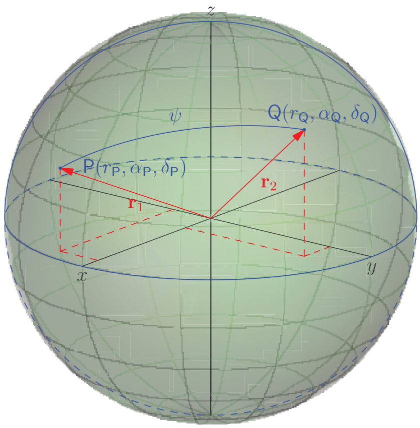


Fig. 2. Representation of the angular distance ψ on the unit sphere between two objects P and Q.

In order to give a correct interpretation of the experimental results, however, the measurements must be written in a proper relativistic way, i.e. following the prescription of the *theory of measurements*. The details of this formalism are out of scope in this article, and are fully developed in de Felice & Bini (2010). We therefore will give here only a brief overview of the main concepts needed to follow the present exposition.

The starting point is the definition of a physical observer, whose history, as shown in Fig. 3, is identified by his/her worldline $\Gamma(w)$ in the four-dimensional differential manifold representing the space-time. The four-vector tangent to this curve (i.e. the *four-velocity* of Γ) is $u^\alpha \equiv \dot{\Gamma}$, where the dot stands for the derivative with respect to the parameter w of the worldline, and if the four-velocity is unitary with respect to the metric $g_{\alpha\beta}$ of the space-time, that is if $g_{\alpha\beta}u^\alpha u^\beta = -1$, then u^α represents the observer at a given point of the worldline. The parameter τ of the curve Γ that satisfies the unitarity condition of u^α has the physical meaning of the *proper time* of the observer, so that u^α can be interpreted as the representation of the observer at each instant of his/her proper time.

With a certain degree of approximation, it can be said that at each point of Γ, u^α splits the space-time into the 1D and 3D subspaces parallel and orthogonal to itself, which are the *time*

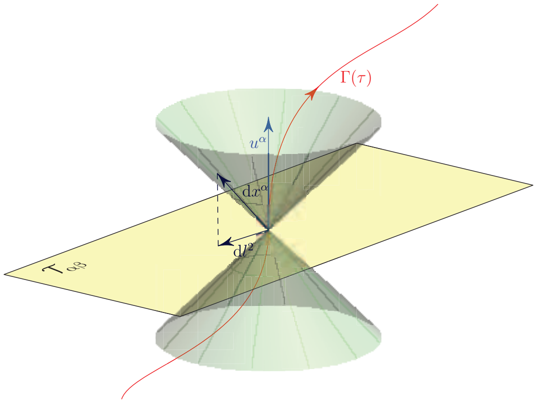


Fig. 3. Any time-like curve $\Gamma(\tau)$ in the space-time can represent the history of an observer, which is identified by its tangent vector u^α and whose proper time τ is the parameter of the worldline. Locally, u^α induces a $3+1$ splitting of the space-time where the two subspaces represent the space and the time associated to this observer.

direction and the *space* relative to the observer u^α respectively. More precisely, we can always define two operators $\mathcal{P}_{\alpha\beta} = -u_\alpha u_\beta$ and $\mathcal{T}_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$ which project any four-vector parallely and orthogonally to the observer. Quite obviously, these operators are called the *parallel* and *transverse* projector respectively, and any interval of space-time $\mathrm{d}s^2 = g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ can be written as

$$\mathrm{d}s^2 = -c^2\left(-\frac{1}{c^2}\mathcal{P}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta\right) + \left(\mathcal{T}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta\right), \tag{13}$$

where $\mathrm{d}\tau^2 = -c^{-2}\mathcal{P}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ has the physical meaning of the (square of the) infinitesimal time interval measured by the observer u^α, while similarly $\mathrm{d}l^2 = \mathcal{T}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ is the measured spatial distance associated to $\mathrm{d}s$.

4.2 Linking the observable with the unknown parameters

With the above indications in mind, and remembering the spatial nature of angular measurements, it is easy to understand why the relativistic counterpart of Eq. (12) can be written as

$$\cos\psi_{12} = \frac{\mathcal{T}_{\alpha\beta}k_1^\alpha k_2^\beta}{\sqrt{\mathcal{T}_{\alpha\beta}k_1^\alpha k_1^\beta}\sqrt{\mathcal{T}_{\alpha\beta}k_2^\alpha k_2^\beta}} \tag{14}$$

where, as shown in Fig. 4 k_1^α and k_2^α are the two observing directions (de Felice & Clarke, 1990). It has to be noticed, however, that the expression "observing direction" has not to be intended as the result of a measure. The observable is the angle at the left-hand-side of the equation, while the four-vectors $k_{1,2}^\alpha$ have the mathematical meaning of the tangents to the path of the incoming light rays connecting the positions of the observed objects to the observer, i.e. of the null geodesics of the observed photons. In other words, if s is the parameter of the null geodesic x^α, then

$$k^\alpha = \frac{\mathrm{d}x^\alpha}{\mathrm{d}s}$$

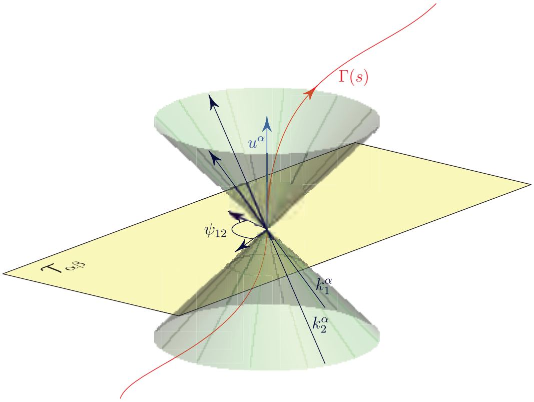


Fig. 4. The formula expressing the arc measured between two directions can be found considering that the relativistic viewing directions k_1^α and k_2^α – i.e. the tangents to the null geodesics connecting the observed objects to the observer and computed at the observation event – have to be projected onto the *spatial* hypersurface relative to u^α.

and this four-vector can be obtained by solving the geodesic equation (i.e. the relativistic equation of motion)

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{ds}\frac{dx^\gamma}{ds} = 0. \tag{15}$$

This formula has to be expressed as a function of the desired astrometric unknowns, namely the positions and proper motions of the observed objects. Moreover, as it is known from classical astrometry (see e.g. Smart, 1965), measurements of the same object change in time not only because of its intrinsic motion, but also because of the motion of the observer itself, with the aberration and parallax effects. In addition to this, a relativistic formulation of the astrometric measurements has to properly take into account the geometry of the space-time.

The most important thing to be noticed in Eq. (14) is that it is natural to choose a coordinate system of the space-time metric coincident with that of the desired catalog, and that this helps to express automatically the observer as a function of the needed parameters introducing immediately and naturally the observer-related effects of classical astrometry like the aberration and the parallactic displacement. If we imagine, just as an example, to use the Schwarzschild metric, its coordinates are already centered on the Sun, therefore the aberration is automatically taken into account in the observation equation since the transverse projector

$$\mathcal{T}_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$$

contains the (four-)velocity u^α expressed with respect to the origin of the coordinate system.

It's easy to understand that the dependence on the positions of the observed objects enters in the equation through the k_i^α, while the relativistic formulation of the observable, together with the correct solution of Eq. (15), can take into account of all the "relativistic effects" such as the light deflection, i.e. the geometry of the space-time. However, many ways to address the problem of the integration of the null geodesics have been developed so far, therefore the actual expression of Eq. (14) can vary according to the accuracy level of the model and/or to the specific technique used for the integration of the equation of motion. A sub-mas $(v/c)^2$ model is described in Vecchiato et al. (2003) and reference therein, while de Felice et al. (2004) is an example of a μas-level $(v/c)^2$ model which takes into account the gravitational influence of all the bodies of the Solar System. More accurate sub-μas models can be found in Klioner (2003) and references therein, Kopeikin & Schäfer (1999), and de Felice et al. (2006). Finally, Teyssandier & Le Poncin-Lafitte (2008) have applied the method of the Synge's World Function to obtain a relativistic expression of the basic astrometric observables without integrating directly the geodesic equations.

Whatever is the explicit method used for the integration of the null geodesic, the PPN parameters such as γ and β can be easily included in the formula of the observable by chosing an appropriate PPN expression of the metric tensor, however there exists an important difference between them. If we consider, e.g., the PPN approximation of the Schwarzschild metric[4] we can see that its expression in isotropic coordinates at the first Post-Newtonian order (1PN) can be written as

$$ds^2 = -\left(1 - 2U + 2\beta U^2\right) dt^2 + (1 + 2\gamma U)\,\delta_{ij} dx^i dx^j. \tag{16}$$

This shows that β enter the metric at the second order in the gravitational potential U, while γ is at the first order, and since the relativistic deflection of light is in general

$$\Delta\psi = f\left((g_{00})^{-1}, g_{\mu\nu}\right), \tag{17}$$

this explains why astrometric measurements exploiting the light deflection are particularly suited for the estimation of γ but not for that of β. The latter, actually, can also be addressed by astrometric measurements able to detect the well-known effect of the excess of perihelion shift

$$\Delta\omega = \frac{6\pi m}{a\,(1 - e^2)}\left[\frac{1}{3}\,(2 + 2\gamma - \beta) + f\,(\alpha_1, \alpha_2, \alpha_3, \zeta_2, J_2)\right] \tag{18}$$

in the orbit of massive objects. If we neglect the contribution from the other PPN parameters and from the quadrupole of the gravitational source, which in the case of the Sun is not important, we can see that this observation is affected by γ and β at the same order, and therefore β can be estimated given a prior knowledge of γ, usually known at a better order because of the above considerations.

4.3 Relativistic reference systems in astrometry

If the required measure is the observed direction of the object, instead of an angle between two directions, then one has to deal with a relativistic theory of reference systems. The Equivalence Principle states that for any observer it is always possible to find a locally inertial reference system where Special Relativity holds. This reference system is called *tetrad*, and it is represented by a set of 4 orthonormal four-vectors $\left(E^{\alpha}_{\hat{0}}, E^{\alpha}_{\hat{1}}, E^{\alpha}_{\hat{2}}, E^{\alpha}_{\hat{3}}\right)$ which can be built at

[4] The Schwarzschild metric considers the spherical non-rotating Sun as the only source of gravity.

any point P of the space-time by imposing the conditions

$$\begin{aligned} E^{\alpha}_{\hat{0}} &\equiv u^{\alpha} \\ \left(g_{\alpha\beta} E^{\alpha}_{\hat{a}} E^{\beta}_{\hat{b}}\right)_{\mathsf{P}} &= \eta_{\hat{a}\hat{b}} \quad \forall\, a, b = 1, 2, 3 \end{aligned} \tag{19}$$

where u^{α} is the four-velocity of the observer. In other words (see Fig. 5) the first axis is by definition the four-velocity of the observer and represents the direction of the proper time, while the other three axes are projected onto the spatial hypersurface of u^{α} and define a reference frame for the 3D space locally seen by the observer at each instant. The above conditions imply that the spatial axes of a tetrad are defined except for three arbitrary spatial rotations.

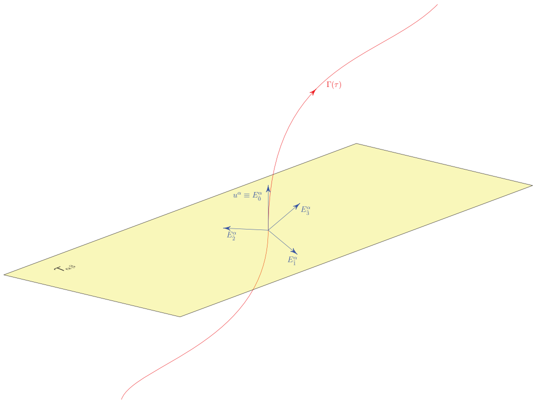


Fig. 5. A tetrad is a set of four orthonormal axes in the 4D space-time which locally represent a Minkowskian reference frame.

The formula which gives the direction cosine seen by an observer u^{α} on the a-th axis of the tetrad is

$$\cos\psi_{\hat{a},k} = \frac{\mathcal{T}_{\alpha\beta} E^{\alpha}_{\hat{a}} k^{\beta}}{\sqrt{\mathcal{T}^{\alpha}_{\alpha\beta} k^{\alpha} k^{\beta}}} \tag{20}$$

which can be easily obtained from Eq. (14) by substituting one observing direction k^{α} with the required tetrad axis and considering that, by construction, tetrad axes are such that $\mathcal{T}_{\alpha\beta} E^{\alpha}_{\hat{a}} E^{\beta}_{\hat{a}} = 1$ for $a = 1, 2, 3$.

5. Methods of data reduction and experimental overview

5.1 Estimation of the γ parameter

The position and motion of the objects in the sky can be determined by astrometric measurements using the two differents approaches of the global and relative astrometry.

5.1.1 The global astrometry approach

Global astrometric techniques have been used in the HIPPARCOS space mission to estimate the γ parameter at the 10^{-3} accuracy level Froeschle, Mignard & Arenou (1997). The same techniques, with a more advanced technology, will be applied by the Gaia space mission (Perryman et al., 2001) to push this measurement to a higher level of accuracy.

The main goal of the Gaia mission is the determination of a 6D map of $\sim 10^9$ objects of the Milky Way. To achieve this objective the observations can't be made in pointed mode. The satellite will rather scan continuously the celestial sphere, measuring the angular positions of the observed objects with respect to a local reference frame associated to the satellite's attitude, and accumulating several billions of observations on the entire sky. Each observation can be modelled as indicated in Section 4 and corresponds to an equation like the (20) which is highly non-linear. In general this equation can depend not only on the astrometric parameters and the PPN γ, but also on other kind of parameters, like those defining the satellite attitude or other instrumental ones needed for the calibration, which cannot always be determined *a priori* with sufficient accuracy[5]

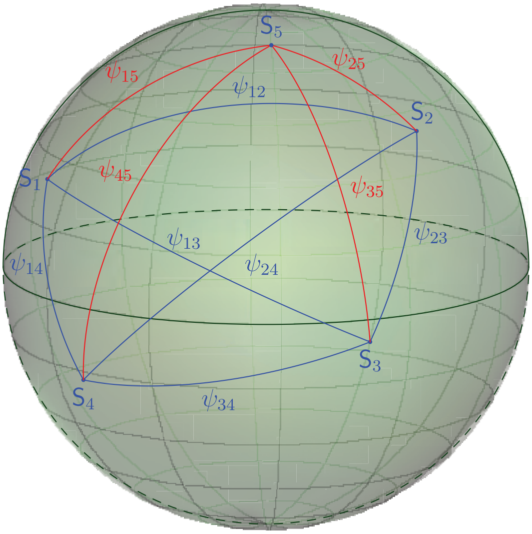
$$\cos\psi \equiv F\left(\underbrace{\alpha_*,\delta_*,\varpi_*,\mu_{\alpha*},\mu_{\delta*}}_{\text{Astrometric parameters}},\underbrace{a_1^{(j)},a_2^{(j)},\ldots}_{\text{Attitude}},\underbrace{c_1,c_2,\ldots}_{\text{Instrument}},\underbrace{\gamma,\ldots}_{\text{Global}}\right). \tag{21}$$

Several billions of observations are accumulated during the five years of the mission lifetime. The final result therefore will be a big and sparse system of equations (up to $\sim 10^{10} \times 10^8$ in the case of Gaia, see Lammers & Lindegren, 2011). Since the solution of such systems of non-linear equations is not practically feasible, the observation equations (21) are linearized about a convenient starting point represented by the current best estimation of the required unknowns. In this case the problem is converted into that of the solution of a big and sparse system of linear equations

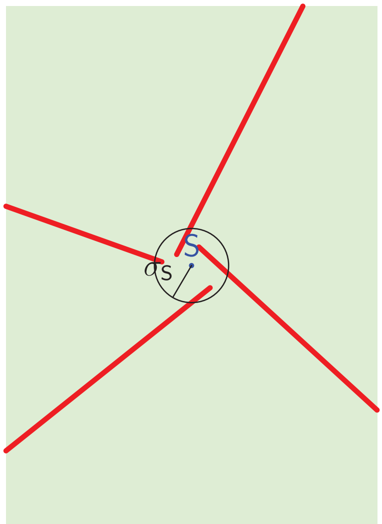
$$\begin{aligned}-\sin\psi\,\mathrm{d}\psi = &\underbrace{\left.\frac{\partial F}{\partial\alpha_*}\right|_{\bar{\alpha}_*}\delta\alpha_* + \left.\frac{\partial F}{\partial\delta_*}\right|_{\bar{\delta}_*}\delta\delta_* + \left.\frac{\partial F}{\partial\varpi_*}\right|_{\bar{\varpi}_*}\delta\varpi_* + \cdots}_{\text{Astrometric parameters}}\\ &\underbrace{\sum_{ij}\left.\frac{\partial F}{\partial a_i^{(j)}}\right|_{\bar{a}_i^{(j)}}\delta a_i^{(j)}}_{\text{Attitude}} + \underbrace{\sum_i\left.\frac{\partial F}{\partial c_i}\right|_{\bar{c}_i}\delta c_i}_{\text{Instrument}} + \underbrace{\left.\frac{\partial F}{\partial\gamma}\right|_{\bar{\gamma}}\delta\gamma + \cdots}_{\text{Global parameters}}\end{aligned} \tag{22}$$

whose unknowns are the *corrections* δx to the starting catalog values. Despite this simplification, the system is still too large to be solved with direct methods, and iterative methods are then used. Since the number of observations is much larger than the number of unknowns, the system is overdetermined and can be solved in the least-squares sense (Fig.6). The complexity of the observation equations and the dimensions of the problem are only two of the components contributing to the challenge of such global astrometry experiments. Another one comes from the correlations among some of the unknowns which has to be determined. As shown in Fig. 7, one of them involves the γ parameter, the angle between the two viewing directions (basic angle) and the parallax of the observed objects.

[5] The PPN γ, contrary to the largest part of the unknowns, appears on every equation of the system. Conventionally such kind of unknowns are called *global parameters*.



(a) Measured arcs accumulate faster than the number of unknowns in global sphere reconstruction.



(b) Conceptual representation of least-squares fit of measurement errors.

Fig. 6. Concept of sphere reconstruction with global astrometric measurements in the case of arc measurements. The pictures show how repeated observations of the same stars can accumulate a number of measurements greater than the number of unknowns (a) thus allowing to find them by solving an overdetermined system of equations in the least-squares sense (b).

When all of these complications are solved, the γ parameter can thus be obtained as a by-product of the system solution. The foreseen accuracy for the Gaia mission is at the $10^{-6} - 10^{-7}$ level (Vecchiato et al., 2003) i.e. one or two orders of magnitude better than the present estimations.

5.1.2 The relative astrometry approach

Other proposals have been made in order to attempt a high-precision estimation of the γ parameter with techiniques of relative astrometry.

Under many aspects this approach is the opposite of the global one examined in the previous section. Global astrometry is specially fitted to deduce positions and motions of stellar objects with respect to a common global inertial reference frame, and it resorts on the measurements of large angles on the sky. Relative astrometry, instead, is targeted to the high-precision estimation of the *relative* position and motion of celestial objects, i.e. with respect to some distant background objects, using small-field measurements.

The design of the proposed medium-class mission LATOR (Turyshev et al., 2004) is based on a three-spacecraft configuration involving two micro-satellites moving on a trailing-Earth orbit and the International Space Station (ISS). The two satellites are pushed away from the Earth until they reach the opposite side of the Sun. Each satellite is equipped with a laser

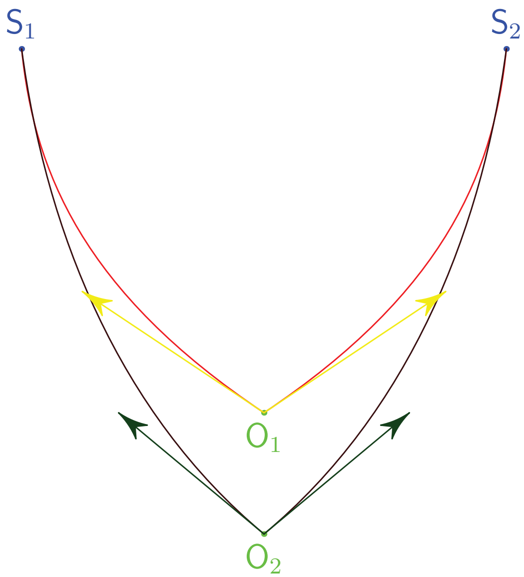


Fig. 7. The γ parameter is highly correlated with parallax, in the sense that any variation in the arc measured by two stars can be interpreted with almost no distinction as a variation of the distance of the stars or of the PPN parameter as well. In any Gaia-like global astrometric sphere reconstruction this parameter is also correlated with the variations of the basic angle, i.e. of the angle between the two viewing directions.

beam which can be detected by a long baseline interferometer housed in the ISS. In this way it is possible to measure the angular distance of these two "artificial stars". These measures adds to those done with an additional laser-ranging system onboard of each spacecraft which is able to measure the distances among them. Gravity should deform the shape of the large triangle formed by the two satellites and the ISS, making LATOR able to reach the 10^{-8} level of accuracy on the estimation of γ.

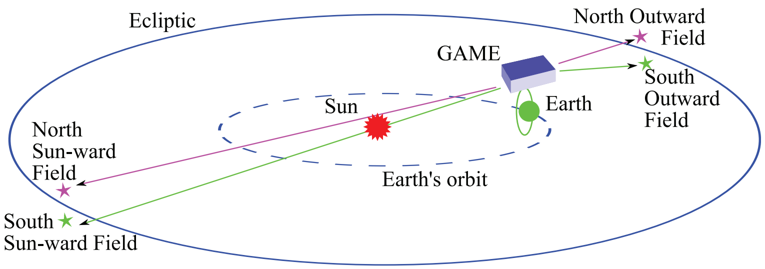


Fig. 8. Schematic depiction of the GAME mission concept.

The estimation of the same PPN parameters is the main scientific goal of GAME, another astrometric mission whose mission concept is sketched in Fig. 8.

The instrument concept is based on a multiple-field, multiple-aperture Fizeau interferometer, observing simultaneously several sky regions close to the solar limb. A beam combiner folds the telescope line of sight on different directions on the sky, separated by a base angle of a few degrees ($\simeq 4°$).

This satellite will measure repeatedly the arcs between selected star pairs, placed in the different fields of view of the telescope, when they are close to the Sun. It will observe again the same arcs after some months, when the Sun is at a large angular distance. In the first configuration the arcs are larger because of the gravitaional pull of the Sun, so that their comparison makes it possible to estimate the γ parameter.

The GAME mission and instrument concepts are highly scalable and the final performances vary according to the selected configuration. A small-mission version with two fields of view, capable to reach a final accuracy between 10^{-6} and 10^{-7} in its 2-yr lifetme, was proposed in Gai et al. (2009); Vecchiato et al. (2009). At present both smaller versions eligible to be hosted in high-altitude baloon flights, and larger medium-class implementations of the same concept are under study. The former have the capability of performing in few days quick and cheap tests at the same level of the Cassini experiment (i.e. 10^{-5}), while the latter is able to reach the 10^{-8} level of accuracy thanks to a longer duration (five years) and a more complex instrument configuration with up to 4 fields of view and simultaneous front and rear observations which enable a better control of the systematic errors.

5.2 Estimation of the β parameter

As explained in Section 4.2, the astrometric attempts for the β parameter estimation, are not based on the light deflection, but rather on high precision reconstruction of the orbits of massive bodies in the Solar System. Since the perihelion shift effect increases with the eccentricity of the orbit and decreases with the distance from the Sun, the most convenient targets are Mercury and the Near-Earth Objects.

Gaia will not be able to estimate the value of β from Mercury since this planet cannot be observed by the satellite because it is too close to the Sun. It will instead revert to the observation of NEOs, which are tipically in less favourable conditions, but can somewhat compensate because of their large ($\sim 10^5$) number. Numerical simulations, however, have shown that Gaia will not be able to improve on the accuracy of this parameter, and will rather reach the current best estimations at the 10^{-4} level using the satellite observations for some 1300 of these objects (Hestroffer et al., 2009).

On the other side, simple order-of-magnitude calculations suggest that with GAME, especially in its medium-class mission implementation, the β parameter can be estimated at an unprecedented level of accuracy.

The shift excess for Mercury is $\Delta\omega \simeq 0.104''/\mathrm{orbit}$. If we consider the simplified scenario in which Mercury is observed only close to superior conjunction[6] the "observed" displacement from the predicted newtonian position is

$$\Delta\alpha \simeq 0.27\Delta\omega \simeq 28\,\mathrm{mas}. \tag{23}$$

[6] This is the most favourable condition because the displacement due to $\Delta\omega$ is largest from the point of view of the observer, except for the inferior conjuction where, however, the planet has the faintest magnitude. See Fig. 9

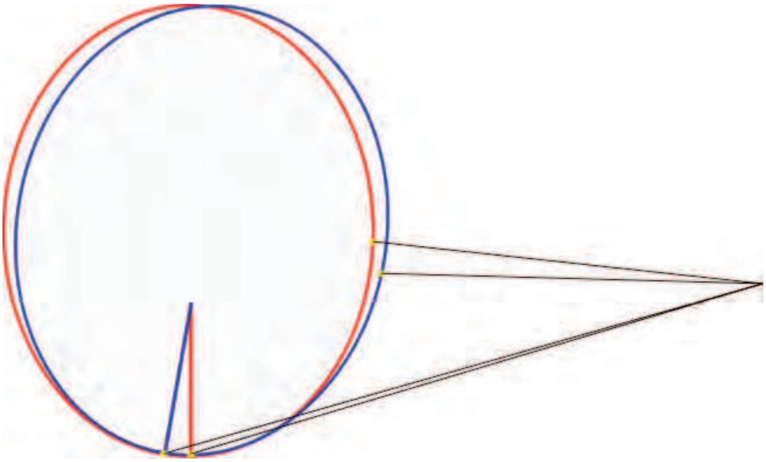


Fig. 9. In principle observations at quadrature are not good for estimating the β parameter.

The integrated magnitude of Mercury in this geometry is $V_{\rm merc} \gtrsim -1.5$, and thus its position can be estimated with a formal relative accuracy of $\sigma_{\Delta\alpha}/\Delta\alpha \simeq 0.5$ mas with a single exposure of 0.1 s by the medium-class GAME. Therefore, from Eqs. (18) and (23) it easy to deduce that

$$\frac{\sigma_\beta}{\beta} \simeq 3\frac{\sigma_\Gamma}{\Gamma} = 3\frac{\sigma_{\Delta\alpha}}{\Delta\alpha} \sim 5 \cdot 10^{-2}$$

with this single observation. If, for the sake of simplicity, we consider the case in which Mercury is observed for one day each orbit in these conditions, after 5 years it is possible to collect some $5 \cdot 10^8$ observations, which would bring the final accuracy to $\approx 2.5 \cdot 10^{-6}$ on β.

5.3 Estimation of the quadrupole effect

The quadrupole light deflection effect (q-effect) has its preferred targets on the stars which can be observed close to the largest and most oblate planets of the Solar System, like Jupiter or Saturn, where the effect is larger. The maximum amount of deflection which can be ascribed to the J_2 component of the gravitational field is in fact 240 μas and 95 μas for these two planets respectively, and this quantity has to be compared with the 10 μas for the next largest contribution coming from Neptune, or with the ~ 1 μas at the solar limb.

Similarly to the case of γ, the ε parameter of the quadrupole deflection can be estimated with a *global* approach, as the result of a sphere reduction in which this effect has been appropriately modeled, or with a *differential* approach, in which the configurations of suitable stellar fields are compared when Jupiter/Saturn is in between, and some time later, when the planet has moved away.

At the moment only the performances of Gaia in the differential case and for Jupiter has been investigated in some detail. The results depend obviously on the initial condition of the scanning law of the satellite, because they determine if a favourable configuration of bright stars around Jupiter can be detected. With the present scanning law, the most positive situation will happen in mid-2017, when there will be the possibility to determine the ε parameter at the 10σ-level.

It has to be added, however, that GAME, as a pointed mission is probably in a more convenient condition for this kind of measurements.

6. Conclusions

In this paper we have given a brief overview of the motivations at the basis of the quest for the determination of the most reliable gravity theory, and of the possible tests inspired by the theoretical scenario. It has also been shown how astrometry, probably the most ancient experimental discipline, can play an important role in these tests. What have eventually emerged, is that it is likely that in the next one or two decades some crucial experiment will be successfully conducted, the outcome of which will be able to revolutionize our view of the physics and of cosmology at a fundamental level.

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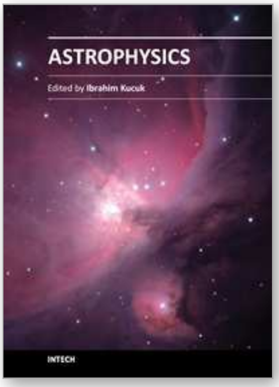

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