Analysis Precision Machining Process
Using Finite Element Method

Xuesong Han
School of Mechanical Engineering, Tianjin University,
P.R. China

1 Introduction

Machining is the process of removing the material in the form of chips by means of wedge shaped tool[1]. The need to manufacture high precision items and to machine difficult-to-cut materials led to the development of the newer machining processes. The dimensional tolerance achieved by precision machining technology is on the order of 0.01 μm and the surface roughness is on the order of 1 nm. The dimensions of the parts or elements of the parts produced may be as small as 1 μm, and the resolution and the repeatability of the machine used must be of the order of 0.01 μm (10 nm). The accuracy targets for ultra-precision component cannot be achieved by a simple extension of conventional machining processes and techniques. They are called precision machining processes, notwithstanding that the definition of conventional and traditional changes with time. Unlike conventional machining processes, precision machining processes are not based on the removing the metal in the form of chips using a wedge shaped tool. There are a variety of ways by which the material may be removed in precision machining processes. Some of them are abrasion by abrasive particles, impact of water, thermal action, chemical action and so on.

When metal is removed by machining there is substantial increase in the specific energy required with decrease in chip size. It is generally believed this is due to the fact that all metals contain defects (grain boundaries, missing and impurity atoms, etc.), and when the size of the material removed decreases, the probability of encountering a stress-reducing defect decreases. Since the shear stress and strain in metal cutting is unusually high, discontinuous microcracks usually form on the metal-cutting shear plane. If the material being cut is very brittle, or the compressive stress on the shear plane is relatively low, microcracks grow into gross cracks giving rise to discontinuous chip formation[2]. When discontinuous microcracks form on the shear plane they weld and reform as strain proceeds, thus joining the transport of dislocations in accounting for the total slip of the shear plane. In the presence of a contaminant, the rewelding of microcracks decreases, resulting in decrease in the cutting force required for chip formation. Owing to the complexity of elastic-plastic deformation at nanometer scale, the world wide convinced precision materials removal theory is not built up until now.

There are two basic approaches to the analysis of metal cutting process, namely, the analytical and the numerical method. As the complexity associate with the precision machining process, which involve high strains, strain rates, size effects and temperature, various simplifications and idealizations are necessary and therefore important machining features such as the strain
hardening, strain rate sensitivity, temperature dependence, chip formation and the chip-tool interface behaviors are not fully accounted for by the analytical methods. Experimental studies on precision machining are expensive and time consuming. Moreover, their results are valid only for the experimental conditions used and depend greatly on the accuracy of calibration of the experimental equipment and apparatus used. Advanced numerical techniques such as Finite Element Method is a potential alternative for solving precision machining problems.

Finite Element Method (FEM) which is originated from continuum mechanics, has already been justified as successful method in analyzing complicated engineering problem\cite{3-8}. There are many advantages of using FEM to investigate machining: multi-physical machining variables output can be acquired (cutting force, chip geometry, stress and temperature distributions), improving precision and the efficiency comparing with Try-Out-Method and so on. In the last three decades, FEM has been progressively applied to metal cutting simulations. Starting with two-dimension simulations of the orthogonal cutting more than two decades ago, researches progressed to three-dimensional FEM models of the oblique cutting, which capable of simulating metal cutting processes such as turning and milling. Increased computation power and the development of robust calculation algorithms (thus widely availability of FEM programs) are two major contributors to this progress. Unfortunately, this progress was not accompanied by new developments in precision machining theory so the age-old problems such as the chip formation mechanism and tribology of the contact surfaces are not modeled properly. Further, even at a moderate cutting speed, the strain rates are quite high, almost of the order of $10^4$ per second and the temperature rise is also quite large. As a result, the visco-plasticity and temperature-softening effects become more important compared to strain-hardening. Therefore, the material properties associated with these two effects should be known for a range of strain rates and temperatures occurring in typical machining processes. Additionally, to incorporate the temperature rise in the analysis, one needs to solve the heat transfer equation governing the temperature field in conjunction with the usual three equations governing the deformation field. For plastic deformation, these equations are coupled, and hence difficult to solve.

In material removal processes at the precision scale, the undeformed chip thickness can be on the order of a few microns or less, and can approach the nanoscale in some cases. At these length scales, the surface, subsurface, and edge condition of machined features and the fundamental mechanism for chip formation are much more intimately affected by the material properties and microstructure of the workpiece material, such as ductile/brittle behavior, crystallographic orientation of the material at the tool/chip interface, and micro-topographical features such as voids, secondary phases, and interstitial particulates. Characterizing the surface, subsurface, and edge condition of machined features at the precision scale in the FEM analysis are of increasing importance for understanding, and controlling the manufacturing process. There are still many challenges in the investigation of precision machining by means of FEM.

As mentioned above, this chapter will give some key factors on numerical modeling of precision machining and current advancements.

2. The flow stress characteristics of the workpiece materials

The flow stress characteristics are an important issue in the numerical analysis which is directly affects the loads and stresses in the precision machining. The flow stress is generally
considered as function of strain, strain rate and temperature. Many research works justify that the influence of strain rate on flow stress become more important when the temperature becomes higher. It is important to build the appropriate flow stress models fit for different working conditions.

Accuracy and reliability of the predictions heavily depend on the materials flow stress at cutting areas such as high deformation rates and temperatures and variable friction characteristics at tool-chip interface which are not completely understood and need to be determined. Materials property at local shear band is very complex in the precision machining which makes it difficult to build up real robust flow stress model fitting for manufacturing process. Most of the energy consumption limited to local cutting area and transformed into heat which complicated the distribution of temperature at the local deformation area. The temperature plays an important role in the unstable chip flow. Larger plastic deformation rate and the intense friction at the tool-chip interface increase the heat generation rate and lead to the material softening thus decreasing the strain hardening ability and instability of materials flow. Therefore, the instability of shear behavior is directly induced by materials flow. Presently, researchers can’t build up reasonable materials constitutive relationship which can characterize strain rate and the temperature and reflect the variation of materials property in the precision machining process.

Sound theoretical models based on atomic level material behavior are far from being accomplished. Semi-empirical constitutive models are widely utilized. Several material constitutive models are used in Finite Element (FE) simulation of metal cutting, including rigid-plastic, elasto-plastic, viscoplastic, elasto-viscoplastic and so on. These models take into account the high strains and temperatures reportedly found in metal cutting. Among others, the most widely used is the Johnson and Cook\textsuperscript{[7]} (JC) model which is a thermo-elasto-visco-plastic material constitutive model expressed as follows:

\[
\bar{\sigma} = \left[ A + B (\dot{\varepsilon})^n \right] \left[ 1 + C \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right] \left[ 1 - \left( \frac{T - T_0}{T_m - T_0} \right)^m \right]
\] (1)

where \( A \) is the initial yield stress of the material at the room temperature, strain rate 1/s and \( \bar{\varepsilon} \) represents the equivalent plastic strain. The equivalent plastic strain rate \( \dot{\varepsilon} \) is normalized with a reference strain rate \( \dot{\varepsilon}_0 \). Temperature term in JC model reduces the flow stress to zero at the melting temperature of the work materials, \( T_m \), leaving the constitutive model with no temperature effect. In general, the parameters \( A, B, C, n, \) and \( m \) of the model are fitted to the data obtained by several material test conducted at low strains and strain rates and at room temperature as well as Split Hopkinson Pressure Bar (SHPB) test at strain rates up to 1000/s and at temperatures up to 600 °C. JC model provides good fit for strain-hardening behavior of metals and it is numerically robust and can easily be used in FE simulation models.

Besides, there are two major problems with the use of the discussed model and its method of the determination of its constants. First, only few laboratories and specialist in the world can conduct SHPB testing properly, assuring the condition of dynamic equilibrium. None of the known tests in metal cutting was carried out in these laboratories. Second, the high strain rate in metal cutting is rather a myth than reality. Third, the temperature in the so-
called primarily deformation zone where the complete plastic deformation of the work materials takes place can hardly exceed 250 °C. It is understood that the mechanical properties of the work material obtained at room temperature are not affected by this temperature so metal cutting is a cold working process, although the chip appearance can be cherry-red. Fourth, it is completely unclear how to correlate the properties of the work materials obtained in SHPB uniaxial impact testing with those in metal cutting with a strong degree of stress triaxiality.

3. The chip separation criterion on different materials used in the FEM

Presently, two FE methods exist for analyzing the precision machining process. In the first method, it is assumed that the chip formation is continuous and the shape of the chip is known in advance. Thus, the process is analyzed as a steady-state process. This method is called Eulerian method. In this method, a chip separation criterion is not required. In the second method, the process is analyzed from the beginning to the steady state chip formation. This is called Updated Lagrangian Formulation. In this method, a chip separation criterion is required to predict the chip geometry. Early applications of finite element method to the machining process were mainly Eulerian method. The main objective of many of these studies was to predict the temperature distribution and therefore, the determination of deformation and stress fields was only an intermediate step. These studies considered the machined material as rigid-plastic. But, later applications of Eulerian formulation to machining process also included viscoplastic effects. All of these applications have considered only orthogonal machining. The first finite element study of the machining process using a modified Lagrangian Formulation was made by Strenkowski and Carroll[8]. A critical value of the equivalent plastic strain was used to model the separation of a chip. Later on, several researchers used the Updated Lagrangian Formulation for analyzing two- and three-dimensional machining processes. The criterion used for chip separation has been based on controlled crack propagation or some geometrical considerations. Remeshing technique has been used to simulate the chip formation.

As the size of the material removed decreases in the precision machining, the probability of encountering a stress-reducing defect decreases. There are some new disciplines dominate the chip separation process. The metal cutting process is different from general metal forming process as there are always accompanied with chip separation or materials removal phenomenon. The separation of chip is of utmost important about numerical simulation of precision machining. The simulation results can only be meaningful only if the reasonable chip separation criteria which can reflect materials mechanical and physical property (such as morphology of chip, force, temperature and the residual stress etc.) were applied in the simulation model. Besides, the criterion for chip separation should be invariant for definite materials but not change with the different working conditions. In the metal cutting process, some kinds of materials may generate continuous chip while others may generate saw-like chip thus different materials fracture criteria should be included in the finite element model.

Presently, there are two kinds of chip separation criteria, namely, the geometric criterion and the physical criterion. Materials removal (chip separation) using geometric criterion is realized through the variation of size of deformable body. On the other hand, the physical criterion is based on if some key physical parameters approached the critical value, these
physical criterion includes effective plastic strain criterion, strain energy density criterion and the fracture stress criterion and so on.

3.1 Fracture mechanics criterion

3.1.1 Stress intensity factor

In reality, chip separation process can be assumed as the formation and development of crack. Under what conditions and what manners can the materials be cut off is closely related with the fracture criterion\textsuperscript{[2]}. Consider plane crack extending through the thickness of flat plane. There are three independent kinematic movements of the upper and lower crack surfaces with respect to each other. These three basic modes of deformation are illustrated in figure 1, which presents the displacements of the crack surface of a local element containing the crack front. Any deformation of the crack surface can be viewed as a superposition of these basic deformation modes, which are defined as follows:

1. Opening mode, the crack surfaces separate symmetrically with respect to the planes xy and xz
2. Sliding mode, the crack surfaces slide relative to each other symmetrically with respect to the planes xy and skew-symmetrically with respect to plane xz
3. Tearing mode, the crack surfaces slide relative to each other skew-symmetrically with respect to both planes xy and xz.

![Fig. 1. Three basic modes of crack extension (i) Opening mode; (ii) Sliding mode; (iii) Tearing mode](image)

The stress and deformation fields associated with each of these three deformation modes will be determined in the sequel for the case of plane strain and generalized plane stress. Solid materials is defined to be in a state of plane strain parallel to the plane xy if

\[ u=u(x,y), v=v(x,y), w=0 \]  

where \( u, v, w \) denote the displacement components along the axes \( x, y \) and \( z \). Chip separation originated from crack while the static, stable or extension of the crack are all closely related with the distribution of stress field around the crack. The study of stress field near the crack tip is of great important as this field govern the fracture process that takes place at the crack tip.

a. Opening mode

Infinite plate with a crack of length 2a subjected to equal stresses \( \sigma \) at infinity is give by
If we place the origin of the coordinate system at the crack tip \( z = a \) through the transformation

\[ \zeta = z - a \]  

Then the equation (3) takes the form

\[ Z_1 = \frac{\sigma(\zeta + a)}{\sqrt{\zeta(\zeta + 2a)}} \]  

using polar coordinates, \( r \) and \( \theta \) we have

\[ \zeta = re^{i\theta} \]  

the stress near the crack tip can be derived as follows:

\[ \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left( \frac{1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}}{2} \right) \]  

\[ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left( \frac{1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}}{2} \right) \]  

\[ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \]  

\[ u = \frac{K_I}{4G\sqrt{2\pi}} \left[ \frac{r}{4} \left( 2\beta - 1 \right) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right] \]  

\[ v = \frac{K_I}{4G\sqrt{2\pi}} \left[ \frac{r}{4} \left( 2\beta + 1 \right) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right] \]  

\[ w = 0 \]  

here \( \sigma_x \), \( \sigma_y \) and \( \tau_{xy} \) are the stress component, \( u \), \( v \) and \( w \) are the displacement component, \( G \) is the shear modulus, \( \mu \) is the poisson ratio, \( \beta = 3 - 4\mu \). The \( K_I \) is the stress intensity factor and expresses the strength of the singular elastic stress field. As put forward by Irwin\(^9\), equation (7) ~ (9) applies to all crack tip stress fields independently of crack/body geometry and the loading conditions. The stress intensity factor depends linearly on the applied load and is a function of crack length and the geometrical configuration of the cracked body.

\[ K_I = \lim_{|\zeta| \to 0} \sqrt{2\pi |\zeta| Z_1} \]  

Equation (13) can be used to determine the \( K_I \) stress intensity factor when the \( Z_I \) is known.
b. Sliding mode

Following the same procedure in the previous case, and recognizing the general applicability of the singular solution for all sliding mode crack problems, the following equations for stresses and displacements are obtained:

\[
\begin{align*}
\sigma_x &= -\frac{K_{II}}{\sqrt{2\pi r}}\sin\theta \left(2 + \cos\frac{\theta}{2} \cos\frac{3\theta}{2}\right) \\
\sigma_y &= \frac{K_{II}}{\sqrt{2\pi r}}\sin\theta \cos\frac{\theta}{2} \cos\frac{3\theta}{2} \\
\tau_{xy} &= \frac{K_{II}}{\sqrt{2\pi r}}\cos\theta \left(1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right)
\end{align*}
\]

The \( K_{II} \) is the sliding mode stress intensity and can be obtained as following

\[
K_{II} = \lim_{|\gamma| \to 0} i\sqrt{2\pi} \gamma Z_{II}
\]

c. Tearing mode

\[
K_{III} = \lim_{|\gamma| \to 0} \sqrt{2\pi} \gamma Z_{III}
\]

The stress intensity factor is a fundamental quantity that governs the stress field near the crack tip. Several methods have been used for the determination of stress intensity factors as listed following:

a. Theoretical method (Westergaard semi-inverse method and method of complex potentials)

b. Numerical method (Green’s function, weight functions, boundary collocation, alternating method, integral transforms, continuous dislocations and finite element method)

c. Experimental method (photoelasticity, holography, caustics)

Theoretical method is generally restricted to plates of infinite extent with simple geometrical configurations of cracks and boundary conditions. For more complicated situations one must result to numerical or experimental methods.

The stress intensity factor is one of the key parameters for characterizing stress field around crack, which can be used as the criterion for crack extension.

1. Single mode criterion

The single mode criterion can be expressed as follows:

\[
K_i \geq K_{IC}, \quad K_{II} \geq K_{IIC}, \quad K_{III} \geq K_{IIIIC}
\]
2. Mixed mode criterion

The mixed mode criterion can be acquired using Ellipsoid Criterion:

\[
\left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IIC}} \right)^2 + \left( \frac{K_{III}}{K_{III}} \right)^2 \geq 1
\]  

(20)

3.1.2 J-integral theory

The stress intensity factor can only be applied to small yield around crack tip, other appropriate parameters should be developed to evaluated the large fracture strength. Rice\textsuperscript{[10]} introduced path independent line integral as the elastic-plastic parameter for characterizing the status of crack which also named as \( J \)-integral. Hutchinson\textsuperscript{[11]} and Rice and Rosengren\textsuperscript{[12]} showed that \( J \) uniquely characterizes crack tip stress and strains in nonlinear materials. Thus the \( J \) integral can be viewed as both an energy parameter and a stress intensity parameter. After that, many researchers investigate the \( J \)-integral which establish the theoretical foundation of the path independent \( J \)-integral and its use as a fracture criterion. Presently, the main efforts in the study of elastic-plastic fracture mechanics is building up the evaluating method on fracture strength using \( J \)-integral while the yield materials around crack tip can be considered as non-linear elastic materials.

As for crack in the nonlinear elastic continuum medium, Rice\textsuperscript{[10]} found that the integral around crack tip is path independent and is given by:

\[
J = \int_{\Gamma} \left( w \, dy - T_i \frac{\partial u_i}{\partial x} \, ds \right)
\]  

(21)

here \( w \) is the strain energy density, \( T_i \) is the component of the traction vector, \( u_i \) is the displacement vector component and \( ds \) is a length increment along the contour \( \Gamma \). The stress energy density is defined as:

\[
w = \int_{0}^{\varepsilon_{ij}} \sigma_{ij} \, d\varepsilon_{ij}
\]  

(22)

Fig. 2. Arbitrary contour around the tip of a crack

here \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the stress and strain tensors separately. The traction is a stress vector normal to the contour. That is, if we were to construct a free body diagram on the material
inside of the contour, \( T_i \) would define the normal stress acting at the boundaries. The components of the traction vector are given by:

\[
T_i = \sigma_{ij} n_j
\]  

(23)

here \( n_j \) is the component of the unit vector normal to \( \Gamma \).

As for linear elastic materials, there some relationship as follows:

\[
J = \frac{\kappa + 1}{8\mu} \left( K_i^2 + K_{ii}^2 \right) + \frac{1}{2\mu} K_{iii}^2 = G
\]  

(24)

As for nonlinear elastic materials, the system potential enclosed by curve \( \Gamma \) can be computed as follows:

\[
\Pi = \int_{\Gamma} W(\varepsilon) dA_i - \int_{\Gamma} p_j u_j d\Gamma
\]

(25)

Therefore

\[
J = \frac{\partial \Pi}{\partial a}
\]

(26)

here \( a \) is the crack length. The \( J \) integral is essentially variation rate of system potential energy which is mainly transform into irreversible plastic work. If the work needed to extend crack a unit length is a constant, then the \( J \) integral based elastic-plastic fracture criterion can be deduced. It is because the \( J \) integral can be used to characterize the elastic plastic stress field solved by deformation theory that the \( J \) integral is selected as elastic plastic fracture criterion.

In 1968, Hutchinson\[11\], Rice and Rosengren\[12\] investigated the elastic plastic stress field around crack using deformation theory and acquired singular solution as follows:

\[
\sigma_{ij} = \left( \frac{J}{a\varepsilon_i \sigma_{ij} I r} \right)^{m+1} \tilde{\sigma}_{ij}
\]

(27)

\[
\varepsilon_{ij} = a\varepsilon_i \left( \frac{J}{a\varepsilon_i \sigma_{ij} I r} \right)^{m+1} \tilde{\varepsilon}_{ij}(\theta)
\]

(28)

\[
\mathbf{u}_i = \left( \frac{J}{a\varepsilon_i \sigma_{ij} I r} \right)^{m+1} \mathbf{\tilde{u}}_i(\theta)
\]

(29)

here \( I \) is definite integral of \( \theta \), \( \mathbf{\tilde{u}}_i \) is a function of \( \theta \). In reality, it is difficult to solve the \( J \) integral using equation (27) ~ (29) because of the complex regular expression of \( \tilde{\sigma}_{ij}, \tilde{\varepsilon}_{ij} \) and \( \tilde{u}_i \). The numerical method and the energy method are the two practical solutions. The numerical method mainly makes use of elastic-plastic finite element method and integrates along several paths around crack tip and acquires the \( J \) integral. The final \( J \) integral can be computed as follows:
\[ J = \sum_{i} \frac{I_i}{n} \]  

(30)

here \( J_i \) is the \( J \) integral corresponding to path \( \Gamma_i \), \( n \) is the number of integrate path. The integrate path is generally continuous smooth curve which can reduce the error resulted by the discontinuous surface force.

3.2 Geometrical criterion

The geometrical criterion mainly takes effect through judging if the geometrical size of materials exceeding the criterion. Figure 3 shows the geometrical model in which a separation line is defined. The nodes at the chip side and the nodes at workpiece side are overlapped at the beginning. But the separation of two nodes occurs when the distance \( D \) between the tool cutting edge (point \( d \), in Figure 3) and the node immediately ahead (node \( a \)) becomes less than a predefined critical value thus the machined surface and the chip bottom are generated.

Fig. 3. Geometrical criterion model

Usui and Shirakashi\[13\] first put forward the geometrical criterion and found it is a stable criterion. Komvopoulos and Erpenbeck\[14\] pointed that there should be enough distance between tool tip and the overlap point to prevent the convergence problem resulted by the excessive distortion of finite element mesh. Zhang and Bagchi\[15\] brought forward that the geometrical distance should be less than 30 percent to 50 percent of element length. Furthermore, they also put up a new geometrical separation criterion which is based upon the ratio of geometrical distance to depth of cut which is equivalent to the microscopic fracture mechanics criterion.

The geometrical criterion is simple to be used in the FE computation. However, the distance \( (D) \) between tool tip and the separation point is closed to zero which result in the difference between the set value of \( D \) with the reality. The selection value of \( D \) will have a great influence upon the convergence of FE simulation and only the experienced researcher can deduce appropriate valuable critical value. In addition, the separation line which separates the mesh of chip and that of the workpiece should be built up in advance. Figure 4 shows the FE simulation of precision machining process based on geometrical separation criterion.
4. Materials deformation behavior in the precision machining

The depth of cut in the precision machining is very small, chips are formed at very narrow regions. The work material is subjected to extremely high plastic deformation and the strain rates can reach the values of about $10^5$ s\(^{-1}\). The large strain and high strain rate plastic deformation evolves out of hydrostatic pressure that travels ahead the tool as it pass over. The zone has, like all plastic deformations an elastic compression region that becomes the plastic compression region as the field boundary is crossed. The plastic compression generates dense dislocation tangles and networks which lead to the materials shear after the materials experience fully work hardened. The theory of micro-plasticity, which mathematically describes the stress and strain at small scale, is adopted to calculate the distributions of stress and strain in the distorted bodies.

![Separation line](image_url)

Fig. 4. FE simulation based on geometrical separation criterion

4.1 Plastic deformation and chip formation in the precision machining titanium alloy

The numerical analysis method applied to materials cutting process can be divided into two categories, namely, the elastic-plastic FEM and the rigid-plastic FEM. Furthermore, thermo-elastic FEM and the thermo-rigid FEM are introduced if the temperature and the velocity are considered in the materials processing technology. The simulation results are almost same whether the problem analysed by either elastic-plastic FEM or rigid-plastic FEM if the size of the workpiece and the amount of discreted element are same for these two methods. The elastic-plastic FEM mainly applied to solve the residual stress and the elastic recovery while the rigid-plastic FEM cannot solve this type of problems as it ignored elastic deformation and thus it has higher solution efficiency.

In this research work, the commercial finite element analysis package (Advantedge®) is utilized to gain good understanding of the materials deformation behavior underlying machining of titanium alloy. Among the different alloys of titanium, Ti-6Al-4V is by far the most popular with its widespread use in the chemical, surgical, ship building and aerospace industry. The primary reason for wide applications of this titanium alloy is due to its high strength-to-weight ratio that can be maintained at elevated temperatures and excellent corrosion and fracture resistance. On the other hand, Ti-6Al-4V is notorious for poor machinability due to its low thermal conductivity that causes high temperature on the tool face, strong chemical affinity with most tool materials, which leads to premature tool failure, and inhomogeneous deformation by catastrophic shear that makes the cutting force...
fluctuate and causes tool wear, thereby aggravating tool-wear and chatter. This poor machinability has limited cutting speed to less than 60 m/min in industrial practice. Numerical analysis of Ti-6Al-4V machining process using finite element method is of great importance on understanding the physical essence and optimizing the machining technique parameters.

4.2 Finite element formulation

The FEM mesh is constituted by elements that cover exactly the whole of the region of the body under analysis[3]. These elements are attached to the body and thus they follow its deformation. Metal cutting process is a large deformation and finite strain related elastic-plastic process. Therefore, both nonlinear material property and the nonlinear geometry property ought to be considered in the numerical analysis. Presently, typical finite element formulations used in metal cutting include Lagrangian or Eulerian method. Lagrangian formulation bases upon the original geometry which also termed as particle coordinates description. Eulerian formulation bases upon the deformed geometry which termed as floating coordinate description. These formulations are particularly convenient when unconstrained flow of material is involved, i.e., when its boundaries are in frequent mutation. In this case, the FE mesh covers the real contour of the body with sufficient accuracy. On the other hand, the Eulerian formulation is more suitable for fluid-flow problems involving a control volume. In this method, the mesh is constituted of elements that are fixed in the space and cover the control volume. The variables under analysis are calculated at fixed spatial location as the material flows through the mesh. This formulation is more suitable for applications where the boundaries of the region of the body under analysis are known a prior, such as in metal forming.

Although both of these formulations have been used in modelling metal cutting processes, the Lagrangian formulation is more attractive due to the ever-mutating of the model used. The Eulerian formulation can only be used to simulate steady state cutting. As a result, when the Lagrangian formulation is used, the chip is formed with thickness and shape determined by the cutting conditions. However, when one uses the Eulerian formulation, an initial assumption about the shaped of the chip is needed. This initial chip shape is used for a matter of convenience, because it considerably facilitates the calculations in an incipient stage, where frequent problems of divergence of algorithm are found.

The Lagrangian formulation, however, also has shortcomings. First, as metal cutting involves severe plastic deformation of the layer being removed, the elements are extremely distorted so the mesh regeneration is needed. Second, the node separation is not well defined, particularly when chamfered and/or negative rake or heavy-radiused cutting edge tools are involved in the simulation. Although the severity of these problems can be reduced to a certain extent by a denser mesh and by frequent re-meshing, frequent mesh regeneration causes other problems.

These problems do not exist in the Eulerian formulation as the mesh is spatially fixed. This eliminates the problems associated to high distortion of the elements, and consequently no re-meshing is required. The mesh density is determined by the expected gradients of stress and strain. Therefore, the Eulerian formulation is more computationally efficient and suitable for modelling the zone around the tool cutting edge, particularly for ductile work.
materials. The major drawback of this formulation, however, is that the chip thickness should be assumed and kept constant during the analysis, as well as the tool–chip contact length and contact conditions at the tool–chip and tool–workpiece interfaces. As the chip thickness is the major outcome of the cutting process that defines all other parameters of this process so it cannot be assumed physically. Consequently, the Eulerian formulation does not correspond to the real deformation process developed during a real metal cutting process.

The Lagrangian formulation\cite{16} under finite deformation is as follows:

\[
\left\{ p^\alpha \right\} = \iiint_{V_0} \left[ B^T \right] \dot{S}_ij dV_0 + \iiint_{V_0} \left[ B^T \right] \ddot{S}_ij dV \tag{31}
\]

where \( \left\{ p^\alpha \right\} \) denotes the column vector of external force exerted at the discrete element nodes, \( \left[ B^T \right] \) is the geometry matrix in the case of finite strain conditions and the \( \left[ B^T \right] \) is the additional item induced by the geometric nonlinear conditions.

### 4.3 Finite element model and simulation results

The corresponding mesh is refined in some region as severe plastic deformation may be induced under material surface which is shown in figure 5. The most fundamental and crucial characteristic of metal cutting process lies in the formation of chip. In reality, the chip is not exactly “cut” but “sheared” away from the work material which forms a clear distinction between machining plastic metal and other materials. Figure 6 shows the chip formation process during precision machining of titanium alloy. Chip formed with the tool approaching the material from the right side and the chip flow in curved fashion. When the original chip thickness or feed rate or depth of cut is compared with the chip thickness after cutting, the deformation can be clearly observed. This deformation is fundamental for the

<table>
<thead>
<tr>
<th>Material</th>
<th>Titanium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (mm)</td>
<td>0.4 × 0.01 × 0.1</td>
</tr>
<tr>
<td>Physical Property</td>
<td>Elastic-Plastic Solid</td>
</tr>
<tr>
<td>Depth of Cut(µm)</td>
<td>5</td>
</tr>
<tr>
<td>Speed (mm/s)</td>
<td>200</td>
</tr>
<tr>
<td>Temperature( ℃)</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1. FEM simulation parameters

Fig. 5. FE simulation model
metal cutting process and involves large deformations of materials with very large strains and very high strain rates. The produced chip is in contact with the tool face in a highly pressurized zone causing sticking friction which transforms to sliding friction further up on the tool face. A large amount of heat is generated in the cutting zone as a result of plastic work and friction causing temperature rise in the tool and chip.

There are three main plastic deformation areas in this precision machining process as shown in figure 6, namely, the first plastic deformation region, which dominates the kind and the morphology of the chip and generated large amount of heat, the degree of plastic deformation is closely related with materials stress-strain relationship; the second plastic deformation region where the intense tribology process is generated between bottom of chip
and rake face of cutting tool; the third plastic deformation region where the tribology behavior is generated between materials machined surface and the clear face of cutting tool. With the cutting in of tool, the elastic deformation is initially induced at the contact interface between cutting tool and materials. After that the titanium alloy becomes going into yield state with the further successively feeding of cutting tool and the plastic deformation region gradually comes into being ahead of cutting tool. The successive feeding of cutting tool results in the contraction of the elastic deformation and expansion of plastic deformation. The full contact between cutting tool and workpiece comes into being and the elastic-plastic deformation is generated. The simulation results show that fairly concentrated shear separates the nearly unstrained work materials from the fully strained chip. But no obvious region of secondary deformation is generated close to the rake face of tool. The contact length between rake face of cutting tool and the bottom is very small which also justifies most of the cutting process are accomplished by the local tool tip.

![Simulation results of cutting force](image1)

**Fig. 7. Simulation results of cutting force**

![Deformation area in the metal cutting](image2)

**Fig. 8. Deformation area in the metal cutting**

Metal cutting process at nanometer scale involves plastic deformation in small localized regions where opposing surface contact or in the interior of workpiece materials. As for chip formation, the single-shear plane model and practically all its “basic mechanics” have been
known since nineteenth century and referred as the Merchant (or Ernst-Merchant model) model\cite{1}. This model has been the basis for most of the present metal cutting analysis. The first orthogonal model was brought forward in 1937 by Piispanen\cite{1} and termed as card model. In this model, the material cut is assumed as a deck of cards inclined to the cutting direction which is shown in figure 9. Merchant assumed the chip to be formed over an infinite thin plane called shear plane. This shear plane starts from the cutting edge of the tool and crosses the chip on an angle with the cutting direction, which is termed as shear angle. When the chip passes the shear plane it is sheared away from the workpiece and increases in thickness. In this simulation, no single shear plane is observed in the whole precision machining process. On the other case, there some maximum stress band is continuously generated in front of cutting tool. This shear band possesses irregular geometry shape which extends from first deformation region to third deformation region.

Fig. 9. Card of cutting process

A zone of plastic deformation extends underneath the machined surface. This subsurface deformation will result in compressive stresses in the machined surface. Though the stress patterns are those with the load applied by the tool still present, elastic recovery caused by the unloading of the tool is not expected to significantly change the stress distribution close to the free surface. So the stress in the machined surface sufficiently far away from the tool can be taken to be the residual stress. The location of the nodes along the machined surface when compared with the location of tool cutting edge yields information about the elastic recovery of the machined surface after it passes under the tool. The elastic spring-back of the machined surface is found to be far less than the radius curvature of cutting edge which justify that most of the material in front of the rounded cutting edge is actually pushed ahead of the tool and not into the machined surface.

The simulation results also shows that the continuous internal curling chip is generated under current working conditions. At the beginning, part of chip adjacent to the tool tip begins to curl and form helix circle with small radius. After that, the larger helix circle surround the previous small one is gradually formed with the feeding of the cutting tool. The deformation coefficients ($\xi = \frac{t_c}{t_u}$) is gradually increased in this process which result in the increasing of cutting force (figure 7). The stress along the free surface (back) of chip is
tensile. It is also tensile along the surface of chip which has moved out of the contact with the tool rake face (front) while the $\sigma_{yy}$ in the middle of the chip is compressive. Such a distribution of stress is the critical factor to develop initial formation of chip.

Presently, the hypotheses propounded by various researchers to explain the curvature of the chip include (i) The cutting moment causes the chip to bend; (ii) The ‘crushing’ of chip in the secondary shear zone and the resultant acceleration of the work material in moving through the secondary shear zone causes the chip to lengthen along this side (the front side). This can also results in a curvature of the chip which is similar to the curvature of a bimetallic strip; (iii) The shear plane is curved in such a way that the shear plane angle is smaller near the exit of shear plane. Thus the chip velocity on the back side is smaller than the average chip velocity which causes the chip to curl.

The bending moment on the chip considered as a beam would result in compressive stress along the free surface (back) of the chip if hypothesis (i) was true. Crushing of the chip in the secondary shear zone will result in compressive $\sigma_{yy}$ in the front (underside) of the chip. Only a curved shear plane would result in a stress distribution similar to that given by the finite element analysis, while simultaneously accounting for curl of the chip. It should be noted that though the chip does accelerate (due to secondary shear) as it flows along the rake face of tool, this is just an accessory to chip curl and not the cause of chip curl. The reason for the curvature of the shear plane can be found from a detailed analysis of the stress distribution in the zone of plastic deformation. Work in this direction is in progress.

5. Conclusion

With the increasing of high quality and accuracy of modern automated machining technology, numerical simulation of machining technology such as FEM is starting to emerge. The FEM based virtual machining simulation has the capability of calculating the results of process variables about the precision machining process used for optimization the cutting process thus providing many benefits to the metal cutting application. Presently, FEM is mainly of use to mechanical and materials engineering, as a tool to support process understanding, materials machinability development and tool design. The research efforts show that the model used in FEM of precision metal cutting process should be adequate to the process. But the concept of FE model should be broadened in order to embrace important facets physics including uncertainty, which has been axiomatized out of modern cutting research. Breakthrough in these directions will have considerable impact by making metal cutting simulation useful for practical optimization of various metalworking operations including the cutting and machine tools, the metal working fluids and fixtures and so on.

6. References


[10] G. R. Irwin, Fracture, Encyclopedia of Physics, VI (Elasticity and plasticity), Springer-Verlag, 551–590


Continuum Mechanics is the foundation for Applied Mechanics. There are numerous books on Continuum Mechanics with the main focus on the macroscale mechanical behavior of materials. Unlike classical Continuum Mechanics books, this book summarizes the advances of Continuum Mechanics in several defined areas. Emphasis is placed on the application aspect. The applications described in the book cover energy materials and systems (fuel cell materials and electrodes), materials removal, and mechanical response/deformation of structural components including plates, pipelines etc. Researchers from different fields should be benefited from reading the mechanics approached to real engineering problems.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
