1. Introduction

In the next thirty years the annual global consumption of energy will rise by more than 50% (Hochbaum & Yang, 2010). Currently most of the energy production comes from the combustion of fossil fuels; nonetheless the deriving CO$_2$ emissions represent a real risk for the safety of the environment and for human health. Not secondarily, the prompt availability of fossil fuels is extremely influenced by geo-political factors. In the very last decades much attention has been devoted to the development of green renewable energetic sources as possible viable alternative. Owing to its almost ubiquitous availability, solar energy seems to be the most promising way to produce alternative energetic sources. The optimal choice of the materials for the device assembling, according to their prompt availability, in conjunction with recycling spent modules and thinning the semiconductor layers (these two latter are the key-points in reducing the material related sustainability deficits (Fthenakis, 2009)) will help passing from the gigawatt to the terawatt level, that required by the global consumption (Feltrin & Freundlich, 2008). The idea of getting energy from the sun is almost 150 years old. It derives from the discovery of the photovoltaics effect (1839) observed for the first time by the French physicist Edmond Becquerel in an experiment lead with an electrolytic cell made up of two metal electrodes: a weak electrical current was detected by exposing to sunlight a silver coated platinum electrode immersed in electrolyte. From the initial discovery many years have passed until the very first practical application, due to the celebrated Russian physicist Abram Ioffe, consisting in low efficient rectified thallium sulfide cells, has been realized.

The very basic components of a modern solar cell consists in a $p$-$n$ junction, an N-semiconductor, and a P-one, in conjunction with the two electrical contact metal layers that provide the current flow internally and externally to the cell. The main physical quantity that describes a solar cell is the energy conversion efficiency ($\eta$), defined as the quantity of sunlight converted into electric power. Materials like crystalline Si and GaAs can reach efficiency of about 25%, although efficiencies for most commercially available multicrystalline Si solar cells are below 20%. Solar cells constituted by only one semiconductor material are characterized by a maximum energy efficiency for a bandgap ($E_G$) between 1.4 and 1.6 eV (Green, 1982).
The other key performance characteristics are represented by the photocurrent density ($J_{sc}$), the fill factor (FF), and the open circuit voltage ($V_{oc}$), related (also with the incident light power density, $P_s$) by the general expression:

$$\eta = \frac{J_{sc} \cdot FF \cdot V_{oc}}{P_s}$$  (1)

Solar cells can operate in a wide range of current ($I$) and voltage ($V$); the maximum power point of the cell is defined as the point that maximizes the product between $I$ and $V$, in a process that involves the continuous increase of the resistive load on an irradiated cell from short circuit to open circuit regime. Modern cells can track the power by measuring continuously $V$ and $I$, equilibrating the load, in order to achieve always the maximum power, regardless of the lighting conditions.

The initial technology of the modern solar cells based on the well-assessed single and multicrystalline Si cells (the latter cheaper but with reduced quality due to the presence of grain boundaries) has been progressively replaced by the thin film based one, the so-called “Second-generation solar cells”, usually constituted by GaAs, CuInSe, CIGS (Copper Indium Gallium Selenide), or CdTe, which represent a cheap and simple alternative technology in photovoltaics (PV). A particular mention is deserved by the emerging technology based on the combination of dyes anchored on metal oxides (DSSC, dye sensitized solar cells, (O’Regan & Graetzel, 1991) consisting in porous layers of TiO$_2$ nanoparticles.

The usage of layered materials with different bandgaps is a desirable procedure for increasing the efficiency of the final PV device. Such assembling procedure results in the splitting of the solar spectrum in several parts. In this way, indeed, photons of different energies are absorbed depending on the used material: the stacking of higher bandgap material on the surface able to absorb high-energy photons with lower-energy photons absorbed by the lower bandgap material beneath is the key step for these Multi-Junction (MJ), “Third-generation” solar cells, characterized by the reduction of the transmission and that of the thermalization losses of hot carriers. According to the assembling procedure, MJs are monolithic or stacked. Owing to the requirement of electronic and lattice match of the constituents, the monolithic ones present additional intrinsic difficulties; they are epitaxially grown on Ge substrate and constituted by lattice matched (In,Ga)P and GaAs. The reader may understand the structure of similar devices observing Figures 1-2. In Figure 1, a sketch of (Ga,In)P/(Ga,In)As/Ge is reported, while Figure 2 shows its spectral irradiance of the solar AM1.5 spectrum in conjunction with the parts of the spectrum used. “AM”(Air-Mass) refers to the spectrum of the incident light and corresponds to the shortest, direct optical path length through the Earth atmosphere, that in conjunction with “1.5” represents the standard for the characterization of terrestrial power-generating panels. More precisely, AM is 1/cos($z$), where $z$ represents the zenith angle. The choice of AM1.5 as standard stems from the fact that it corresponds to $z \approx 43^\circ$, representing the yearly average at mid-latitudes. AM0 is the standard for space cells. In the mechanical stacked MJ the separate connections of top and bottom cells do not mandatorily require current matching, making the combination of bandgaps quite flexible. The only tight requirement is thus the transparent contacts. Anyway, the lattice mismatch may cause crystal dislocations introducing levels in the gap and thus mediating the Shockley-Read-Hall recombination.

Primary role in the rising success of monolithic tandem solar cells must be ascribed to the National Renewable Energy Laboratory (NREL) activity and in particular to the research
conducted by J. M. Olson group (Olson et al., 1985). In details, the initial tandem solar cells based on GaAs and GaInP were made via a vapour phase epitaxy growth process and revealed problems related to the purity of source materials. The initial drawbacks were progressively solved during the nineties and subsequently due to their high efficiency and power-to-mass ratio, systems based on GaInP/GaInAs/Ge have represented and still represent the most reliable choice for communication satellites (Dimroth, 2006). In Figure 3, the calculated conversion efficiency of a triple-junction solar cell as function of the bandgap of the single junctions forming the stacking is reported.

Fig. 1. General structure of a multi-junction solar cell.

Fig. 2. Spectral irradiance of the solar AM1.5 spectrum with the regions of the spectrum used by a GaInP/GaInAs/Ge 3-junction solar cell.

MJ systems are usually constituted by three different possible substrate: Ge, GaAs, and InP. The introduction of chemical components in the different layers is beneficial (Yamaguchi et al., 2005): Al in the top cell has the property of increasing $E_G$ to values that cover a larger part of the solar spectrum, while few amounts of In reduce the mismatch between layers. In the context of the Japanese “New Sunshine Project”, InGaP/InGaAs/Ge monolithic integrated cells have reached an efficiency of 31.7%, while for the stacked InGaP/GaAs//InGaAs ones an efficiency of 33.3% has been reported (Yamaguchi, 2003), representing at the time of their production the efficiency World Guinness for such cells.
Metamorphic triple-junction Ga$_{0.44}$In$_{0.56}$P/Ga$_{0.92}$In$_{0.08}$As/Ge terrestrial concentrator solar cells (owing to the macroscopic differences between terrestrial spectrum and the conditions in space, the structure of the solar cell must be adjusted. Therefore, the optimum bandgap combination of materials is not the same) have been recently grown and with the usage of a buffer structure, able to minimize the dislocation formation (King et al., 2007), a record of 40.7% in efficiency has been established in 2007 at 240 suns and at AM1.5D. Even more recently (Guter et al., 2009) a new World Guinness has been established for the metamorphic triple-junction solar cell: an efficiency of 41.1% has been achieved under 454 suns and at same standard conditions: this latter cell combines Ga$_{0.35}$In$_{0.65}$P (top cell), Ga$_{0.83}$In$_{0.17}$As (middle cell) with a Ge bottom cell. An electrically inactive buffer is used in order to make inactive the formation of threading dislocations that in any case have densities below $10^6$ cm$^{-2}$. Such highly efficient cells are integrated by the usage of a Fresnel lens placed ~10 cm over the cells, ensuring a concentration of the sunlight increased by a factor ranging between 400 and 500.

Fig. 3. Detailed balance calculations for the efficiency of different triple-junction solar cell structures under the AM1.5d ASTM G173 – 03 spectrum at 500 kW/m$^2$ and 298 K. The black dots represent structures with efficiencies from 60.5% to 61.0% and hence mark the optimal band gap combinations. The gray dots represent structures with an efficiency of 59.0%– 60.5% (see contours). Five specific triple junction solar cell structures are also shown. The lattice-matched Ga$_{0.5}$In$_{0.5}$P/Ga$_{0.99}$In$_{0.01}$As/Ge (LM), two metamorphic GaInP/GaInAs/Ge [(1.8, 1.29, 0.66 eV for MM1) and (1.67, 1.18, 0.66 eV for MM2)], as well as two inverted metamorphic GaInP/GaInAs/GaInAs [(1.83, 1.40, 1.00 eV for Inv1) and (1.83, 1.34, 0.89 eV for Inv2)] devices. [Reprinted figure 3 with permission from: Guter, W.; Schöne, J.; Philipps, S. P.; Steiner, M.; Siefer, G.; Wekkel, A.; Welser, E.; Oliva, E.; Bett, A. W. & Dimroth, F., Applied Physics Letters, Vol. 94, No. 22, pp. 223504-3 ISSN 1077-3118. (2009). Copyright (2011) by the American Institute of Physics].

The continuous research of new materials in PV and the optimization of their performances motivated us to study, in recent years, the electronic and the structural properties of the
alloys formed by IV group elements and the III-V compounds. \((\text{GaAs})_{1-x}(\text{Ge}_2)_x\), one of the \((\text{A}^{\text{III}}\text{B}^{	ext{IV}})_{1-x}(\text{C}^{\text{IV}})_x\), nonisovalent alloys, has direct gap ranging between 0.5 and 1.4 eV, ideal for visible-IR light absorption, and lattice constants matching with that of GaAs (5.66 Å (Wang & Ye, 2002)), representing the best candidate in some technologically oriented applications (extra junction of two junction Ga_{0.52}In_{0.48}P/GaAs solar cell (Norman et al., 1999)).

In this chapter we review recent results from modelling the structural and electronic properties of such non-isovalent alloys widely utilized in MJ solar cells. The relevance of Density Functional Theory (DFT) and GW (G: Green function, W: screened Coulomb interaction) calculations in this field is discussed and the importance of large modellization in accounting the clusterization phenomena is shown. We also review the basic concepts associated with the stabilizing self-compensation mechanism, both in defective supercells and in alloys.

2. Semiconductor alloys for photovoltaics: The case of \((\text{GaAs})_{1-x}(\text{Ge}_2)_x\):

A brief overview

The isovalent class of semiconductor alloys represents the most common one (Wei & Zunger, 1989; Shan et al., 1999; Wei et al., 2000; Janotti et al., 2002; Deng et al., 2010) (i.e., IV/IV, (III−V)/(III−V), and (II−VI)/(II−VI) ). The alloys belonging to this class are characterized by a reduced band offsets between the constituents (< 1 eV), by a small and composition-independent bowing coefficient, and by a lattice mismatch that is usually below 5% (Zunger, 1999). Differently, the introduction of a low-valent and high-valent element into a III-V compound generates a so called “nonisovalent” alloy (Yim, 1969; Bloom, 1970; Funato et al., 1999; Wang & Zunger, 2003; Osorio et al., 1999; Greene, 1983) (i.e., (III−V)/IV, (III−V)/(II−VI)). High carrier mobilities, an enhanced presence of free electrons and holes according to the growth conditions, and the reduced donor−acceptor charge compensation are the attractive characteristics that make this class matter of deep investigation. In the case of ZnSe–GaAs, Wang et al. report only small change in the final alloy bandgap, \(E_G\) as result of the introduction of larger bandgap II−VI dopants into the III−V host, while the opposite (GaAs in ZnSe) causes sharp drops in the bandgap of the alloy (Wang & Zunger, 2003).

In recent years, many other papers have been focused on the class of (III-V)_{1-x}(IV_2)_x nonisovalent alloys both at theoretical (Holloway, 2002; Newman et al., 1989; Osorio & Froyen, 1993; Ito & Ohno, 1992; Ito & Ohno, 1993; Newman & Jenkins, 1985; Bowen et al., 1983) and experimental (Green & Elthouky, 1981; Barnett et al., 1982; Alferov et al., 1982; Banerjee et al., 1985; Noreika & Francombe, 1974; Baker et al., 1993; Rodriguez et al., 2000; Rodriguez et al., 2001) level. The employment of nonequilibrium grown techniques that incorporate high dopant concentrations in semiconductor has boosted the attention towards this class of alloys. In particular, despite the mutual insolubility of the constituents, the homogeneous single crystal (GaAs)_{1-x}(Ge)_x can be synthesized as metastable alloy by sputter deposition technique (Barnett et al., 1982), metal-organic chemical vapor deposition (MOCVD)(Alferov et al., 1982), molecular beam epitaxy (MBE) (Banerjee et al., 1985), and rf magnetron sputtering (Rodriguez et al., 2000; Rodriguez et al., 2001). The direct gap “tailorability” for (GaAs)_{1-x}(Ge)_x alloys is observed in a final large, negative, and asymmetric, \(V\)-shaped, bowing of the bandgap. Barnett et al. (Barnett et al., 1982) report a minimum value of about 0.5 eV at Ge concentration of about 35%: the optical absorption of
homogeneous single-crystal metastable $(\text{GaAs})_{1-x} (\text{Ge}_2)_x$ alloys, grown using ultra-high-vacuum ion-beam sputter deposition is there investigated. Herbert Kroemer, Nobel prize in 2001 for his studies in semiconductor heterostructures used in electronics, clearly states “…If lattice matching were the only constraint, the Ge-GaAs system would be the ideal heterosystem, as was in fact believed by some of us — including myself — in the early 1960s”. (Kromer, 2001). Then he adds: “…Covalent bonds between Ge on the one hand and Ga or As on the other are readily formed, but they are what I would like to call valence mismatched, meaning that the number of electrons provided by the atoms is not equal to the canonical number of exactly two electrons per covalent bond” (Kromer, 2001). And indeed, at equilibrium condition, phase separation between GaAs-rich domains and Ge-rich domains occurs since GaAs and Ge are mutually insoluble due to the formation of what Kroemer calls valence mismatched, otherwise called “bad” bonds (octet-rule violating bonds, i.e., Ga−Ge, As−Ge) (Osorio et al., 1991a). In the ordered GaAs-rich phase, Ga and As preferentially form donor–acceptor pairs, whereas in the Ge-rich phase, they are randomly distributed in the alloy forming a mixture of n-type and p-type semiconductors. The origin of the large bowing is ascribed to this ordered → disordered transition (Newman et al., 1989).

Several other theoretical models have been developed in order to describe such zinc-blend-to-diamond phase transition. They are based on thermodynamic (Newman & Dow, 1983; Newman et al., 1989; Gu et al., 1987; Koiller et al., 1985), percolation (D’yakonov & Raikh, 1982), and stochastic growth approaches (Rodriguez et al., 2000; A Rodriguez et al., 2001; Kim & Stern, 1985; Davis & Holloway, 1987; Holloway & Davis, 1987; Preger et al., 1988; Capaz et al., 1989).

Owing to their intrinsic difficulty in taking into account possible different growth conditions, models based on the percolation method (D’yakonov & Raikh, 1982) predict a critical concentration for the transition at 0.57, quite far from the experimentally reported. In “growth models” the alloy configuration depends on the kinetics of the growth. In such methods no explicit functional minimization is performed (Osorio et al., 1991b) and once that atoms have satisfied the set of growth rules imposed by the model, they are considered as frozen, without any further possibility of including the influence of thermal effects. The prediction of the alloy configuration is based on Monte Carlo (MC) models and the atomic position of single layers depends on the epitaxial growth direction. Thus, Long Range Order (LRO) of the final structure depends on the growth direction. Imposed requirement in such models is the “wrong” bond formation (i.e., III−III and V−V) forbiddance. The absence of Sb-Sb bonds in $(\text{GaSb})_{1-x} (\text{Ge}_2)_x$ alloy detected via extended X-ray absorption fine structure (EXAFS) experiments (Stern et. al., 1985) confirms the appropriateness of such imposed condition. Kim and Stern have proposed a model specifying in the set of growth rules the equivalent probability for the $A^\text{III}$ and $B^\text{V}$ species in the site occupancy, obtaining a critical concentration for the phase transition ($x_c$) that is $0.26(+0.03/-0.02)$ on (100) substrate (Kim & Stern, 1985). Such model only considers Short Range Order (SRO), reporting the critical composition below which LRO is present. They also report the $x_c$ dependence by the growth morphology: a planar growth along [100], another, still planar along the [111] direction, and finally a spherical growth model are studied. In this last model the critical Ge concentration is calculated to be smaller than 0.18 (even if this last value is affected by computational limitations that lead to possible inaccuracy). Davis and Holloway have developed another model implemented by MC simulation and analyzed via an analytical approximation. In their model the formation of Ga-Ga and As-As nearest neighbour bonds is forbidden, and
additionally every gallium atom forms a bond with As atom present in excess during the growth process (Davis & Holloway, 1987; Holloway & Davis, 1987). They find a value for \( x_c \approx 0.3 \) on (100) oriented substrates (Davis & Holloway, 1987). No phase transition is at variance predicted along the <111> direction (Holloway & Davis, 1987), with remnant ZB phase present in all the range of composition. This prediction confirms the experimental findings for \((\text{GaAs})_{1-x} \text{(Ge)}_x\), obtained via High Resolution X-Ray Diffraction (HRXRD) on several substrate orientations (Rodriguez et al., 2000; Rodriguez et al., 2001).

Rodriguez et al. (Rodriguez et al., 2001) compared LRO behaviour with the results of Raman scattering, the experimental procedure for evaluating alloy SRO (Salazar-Hernández, et al. 1999; Olego & Cardona, 1981). In details, they find different LRO for each growth direction.

The long range order parameter \( S \) for \((\text{GaAs})_{1-x} \text{(Ge)}_x\) alloys and more in general for the \((\text{III-V})_{1-x} \text{(IV)}_x\) is expressed as (Shah et al., 1986):

\[
S(x) = 2r(x) - 1
\]

where \( r \) represents the probability that any Ga (As) occupies its site in the lattice. This corresponds to \( r=1 \) (0.5) in the case of perfect LRO (disordered crystal). The analysis of Rodriguez of the optical gap and Raman scattering, shows that near-neighbour correlations (SRO) extremely influence the optical properties while, at variance, there is no impact of the substrate orientation and the LRO on the optical properties. Figure 4 reports the mean cluster reciprocal length obtained by Rodriguez by the Monte Carlo simulation for different orientations of the alloy. It is evident from the plot the overall tendency of GaAs clusters in the alloys to reduce their size (fragmentation), increasing the Ge concentration, \( x \). That kinetic models better describe the phase transition than thermodynamic ones is well assessed. The reasons stem from the fact that thermodynamic models do not include the details of the critical composition \( x_c \) as a function of kinetic growth, the phase transition critical concentration must be explicitly added as input, and additionally there are no restrictions on the formation of Ga-Ga and As-As bonds. Also a determining role in the nature of the alloy is played by the growth temperature (Banerjee et al., 1985): \((\text{GaAs})_{1-x} \text{Ge}_x\) layers epitaxially grown on GaAs (100) substrates at different temperatures analysed by TEM revealed that at \( T_g = 550 \degree \text{C} \), Ge separated from GaAs into domains of \( \sim 100 \\text{Å} \). Differently, single-phase alloys are still detected at \( T = 430 \degree \text{C} \).

The research described in this chapter has been motivated by the fact that so wide potential applicability of this class of alloys in PV is astonishingly not supported by a deepen knowledge both at Density Functional Theory (DFT) and post-DFT level of their electronic, structural, and optical properties. We thus decided to examine the bowing in \((\text{GaAs})_{1-x} \text{Ge}_x\) alloys, searching the microscopic origin of this intriguing and not yet clarified phenomenon. In particular, at first we have theoretically analyzed the properties of four different intermediate structured compounds that range between “pure” GaAs and “pure” Ge \( x_{\text{Ge}} = 0.25, 0.50 \) (two samples), 0.75) (Giorgi et al., 2010). Enlarging our models to ones ranging between 8 and 64 atoms, we have investigated the impact of clustering effects and that of the cluster shape on the bandgap bowing (Kawai et al., 2011). For the alloy electronic properties calculations two of the methods which are reported to give extremely reliable results have been employed: the Quasiparticle Self-consistent GW (QSGW) approximation approach (van Schilfgaarde et al., 2006) developed by Mark Van Schilfgaarde and his group at Arizona State University and the frequency dependant GW
method implemented in the VASP code (VASP) by Shishkin and co-workers. (Shishkin & Kresse, 2006; Shishkin & Kresse, 2007; Fuchs et al., 2007; Shishkin et al., 2007).

Fig. 4. Left, mean cluster lengths reciprocal (1/<l>) from Monte Carlo simulations for (a) (111) and (b) (110) alloys. Full up (down) triangles show the behaviour with Ge concentration for the GaAs phase (antiphase) component. The full squares give the domain size for the Ge component. Right, simulated average cluster reciprocal lengths for (a) (112) and (b) (113) alloys. Full up (down) triangles show the behaviour with Ge concentration for the GaAs phase (antiphase) component. The full squares give the domain size for the Ge component. The lines joining the points are shown for visual aid only. [Reprinted figure 4 with permission from: Rodriguez, A. G.; Navarro-Contreras, H. & and Vidal, M. A., Physical Review B, Vol. 63, No. 11, pp. 115328-9, ISSN 1550-235X. (2001). Copyright (2011) by the American Physical Society].

The initial analysis of the 8-atoms cell revealed that the reduction of the bandgap for intermediate x values in (GaAs)_{1-x}(Ge_x) alloys takes place with a lattice constant increase and a symmetry reduction with the formation energies linearly related with the number of bad bonds in each model (Giorgi et al., 2010); the subsequent analysis on the shape and the clusterization effects present in these alloys have confirmed at first the main role that SRO plays on asymmetric bandgap behaviour (Rodriguez et al., 2001), further confirming experimental results, like the tendency of large clusters in the alloy to fragment (McGlinn et al., 1988). Large models have been also employed to refine the shape of the asymmetric bowing.

3. Computational details

Using Blöchl’s all-electron projector-augmented wave (PAW) method (Blöchl, 1994; Kresse & Joubert, 1999), we performed spin-polarized calculations by using density functional
theory (DFT), within both the local density approximation (LDA) (Perdew & Zunger, 1981; Ceperley & Alder, 1980) and the generalized gradient approximation (GGA) of Perdew and Wang (Perdew, 1991; Perdew et al., 1992) and of Perdew, Burke, and Ernzerhof (PBE, Perdew et al., 1996). In details, electrons in the semicore for both Ga and Ge have been considered. Cutoff energies of 287 and 581 eV were set as the expansion and augmentation charge of the plane wave basis. The force convergence criterion for these models was 0.01 eV/Å.

3.1 The initial case of eight atom unit cells

The initial (GaAs)$_{1-x}$Ge$_{2x}$ models consisting of 8 atoms were optimized with a 10 x 10 x 10 $\Gamma$-centered $k$-points sampling scheme. The reliability of our initial structures has been checked by re-calculating all the total energies with the generalized LMTO method scheme developed by Methfessel et al. (Methfessel et al., 1996). We found almost identical results for what regards structural properties and heats of reaction, indicating that the results are well converged. The thermodynamic stability of these alloys was calculated as

$$\Delta E_{\text{form}} = E_{\text{alloy}} - \{(1-x)E_{\text{GaAs}} + xE_{\text{Ge}}\} \quad (3)$$

derived from the product-reactant equation:

$$\text{GaAs} + 2x \text{ Ge} \rightarrow (\text{GaAs})_{1-x}\text{Ge}_{2x} + x \text{GaAs} \quad (4)$$

Owing to the methodological derived large cancellation of errors, both LDA and GGA are expected to predict reasonable heats of reaction like for that of Eq. (3). At variance with structural properties, optical ones are much less well described, with a well documented underestimation of semiconductor bandgaps. Also dispersion in the conduction band is affected by this DFT shortcoming: for Ge, the LDA gap is negative and $\Gamma_{1c}$ is lower than $L_{1c}$ in contradiction to experiment. Also the $\Gamma$–X dispersion is often strongly affected: in GaAs, $X_{1c}$–$\Gamma_{1c}$ is about twice the experimental value; underestimations that are generated by the self-interaction error (Perdew & Zunger, 1981) and the discontinuities in the derivatives of exchange-correlation energy (Perdew & Levy, 1983; Sham & Schlüter, 1983).

In the prediction of semiconductor optical properties one of the best method is based on the GW approximation of Hedin (Hedin, 1965). This approximation is a perturbation theory around some noninteracting Hamiltonian, $H_0$. In particular, the quality of this Hamiltonian highly impacts on the quality of the final GW result. In conjunction with a “safe” choice of the Hamiltonian, it must be stressed that for reliable results the use of an all-electron method is highly recommended (Gomez-Abal et al., 2008). To satisfy both the requirements, in this initial stage of our calculations, we have adopted here an all-electron method, where not only the eigenfunctions are expanded in an augmented wave scheme, but the screened coulomb interaction $W$ and the self-energy $\Sigma = iGW$ are represented in a mixed plane-wave and local-function basis (Kotani & van Schilfgaarde, 2002; Kotani et al., 2007). In addition, all core states are treated at the Hartree-Fock level. In the following we briefly describe the methodology key points.

Usually, in literature the initial Hamiltonian ($H_0$) for GW calculations is an LDA derived guess; thus usual GW method may be named CLDAW/LDA approximation. Many limitations characterize this CLDAW/LDA approaches as reported in previous literature (van Schilfgaarde et al., 2006b). The QuasiParticle Self-Consistent GW (QSGW) approximation (van
Semiconductor energy band structures are well described with uniform reliability. Discrepancies with experimental semiconductor bandgaps are small and highly systematic and the origin of the error can be explained in terms of ladder diagrams missing in the random phase approximation (RPA) to the polarizability (Shishkin et al., 2007). The RPA results in a systematic tendency for the dielectric constant, $\varepsilon_\infty$, to be underestimated. The error is very systematic: $\varepsilon_\infty$ is too small by a factor of approximately 0.8, for a wide range of semiconductors and insulators. This fact and also the fact that the static limit of $W$ mainly controls the quasiparticle (QP) excitations, provides a simple and approximate remedy to correct this error: $\Sigma - V_{xc}^{LDA}$ is scaled by 0.8.

3.2 The extension up to 64 atoms

The Short Range Order effects (i.e., those involving shape and the clusterization of GaAs/Ge regions) on asymmetric bandgap behaviour was confirmed in the alloys synthesized by ion-beam sputtering techniques (McGlinn et al., 1988) and rf sputtering techniques (Rodriguez et al., 2001). In order to model alloy structures reproducing the SRO effect and deeply understand their effect on the bandgaps, larger supercells (ranging from 8 to 64 atoms) have to be mandatorily investigated to make reliable the comparison with experiments. In the case of the optimization of larger supercells we made use of similar settings of those of the initial 8-atom cells, being the force convergence criterion still 0.01 eV/Å. At variance with the initial case, the number of $k$-points was still $10 \times 10 \times 10$ for models constituted by eight atoms, but in this case we used the Monkhorst-Pack (MP) scheme (Monkhorst & Pack, 1976), checking in this way the possible impact on the final results: our calculations revealed that MP scheme and $\Gamma$-centred sampling schemes gave identical results. For the $n_x \times n_y \times n_z$ multiplied supercell models derived from the eight atom unit cell we thus used a $8/n_x \times 8/n_y \times 8/n_z$ $k$-points sampling scheme. For such larger supercell $E_G$ calculation, we employed a GGA+$GW_0$ (Fuchs et al., 2007) scheme, using the eigenvalues and wave functions obtained at GGA level as initial guess for $GW_0$ calculations (eigenvalues only updated, screened potential kept fixed). For the GGA calculations, we used the Perdew-Burke-Ernzerhof (PBE) functional (Perdew et al., 1996). Cut-off energy for response function is 90 eV, and the number of frequency points for dielectric function is 48. The number of unoccupied bands was increased up to 200. A $6 \times 6 \times 6$ $\Gamma$-centred sampling scheme was used for eight atom models. For the $n_x \times n_y \times n_z$ supercell models of the initial eight atom unit cell we used a $\Gamma$-centered $4/n_x \times 4/n_y \times 4/n_z$ $k$-point sampling scheme.

4. Discussion

4.1 The initial case of eight atom unit cells

A common starting point for both approaches is represented by the calculations at the DFT level of the structural optimized parameters of the two most stable polymorphs of GaAs, zincblende (ZB, group 216, $F-43m$, $Z=4$) and wurtzite (WZ, group 186, $P63mc$, $Z=2$) and of Ge in its cubic form (group 227, $Fd-3m$, $Z=8$) Ge and GaAs, reported in Table 1. The choice of using LDA in all the subsequent calculations stems from the fact that in this context LDA reproduces structural properties closer to experiment than GGA. Both Ga–As and Ge–Ge bond lengths are 2.43 Å in their most stable polymorph.

ZB–GaAs is constituted by interpenetrating $fcc$ sublattices of cations (Ga) and anions (As). The diamond lattice of Ge may be through of as the ZB structure with Ge occupying both
cation and anion sites. In this section we consider 8-atom (GaAs)$_{1-x}$Ge$_{2x}$ alloy models that vary the Ge composition, including pure GaAs ($x=0$) to $x=0.25$ (Ge dimers), $x=0.50$ (4 Ge atoms), $x=0.75$ (6 Ge atoms) and finally pure Ge ($x=1$). Figure 5 reports the structures of the four intermediate alloys. At first we performed an analysis of the Ge dimer in bulk GaAs, at site positions $(0.25, 0.25, 0.5)$ and $(0., 0.25, 0.75)$, i.e. the alloy model I.

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<td>227, Fd-3m, Z=8</td>
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<tr>
<td><strong>Our analysis, PAW/PW91</strong></td>
<td><strong>ΔE</strong></td>
<td><strong>B</strong></td>
<td><strong>ΔE</strong></td>
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<tr>
<td></td>
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<td>79.01</td>
<td>+0.03</td>
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<td><strong>Lattice constant (Å)</strong></td>
<td>$a=5.739$</td>
<td>$a=4.040, c=6.668$</td>
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<td><strong>B</strong></td>
<td><strong>ΔE</strong></td>
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<td>65.94</td>
<td>+0.023</td>
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<td>$a=4.045, c=6.670$</td>
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<td><strong>Previous study (GGA)</strong></td>
<td><strong>B</strong></td>
<td><strong>Lattice constant (Å)</strong></td>
<td>$a=5.74^a, 5.722^b$</td>
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<td><strong>Previous study (LDA)</strong></td>
<td><strong>ΔE</strong></td>
<td><strong>B</strong></td>
<td><strong>ΔE</strong></td>
</tr>
<tr>
<td></td>
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<td>75.7$^e$, 77.1$^f$</td>
<td>+0.0120$^g$</td>
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<tr>
<td></td>
<td><strong>Lattice constant (Å)</strong></td>
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<td>$a=3.912, c=6.441^i$</td>
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<td>5.508$^h$, 5.644$^h$</td>
<td>5.391$^i$, c=6.407$^e$</td>
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<tr>
<td><strong>Experimentally</strong></td>
<td><strong>ΔE</strong></td>
<td><strong>B</strong></td>
<td><strong>ΔE</strong></td>
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<tr>
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<td>77.1$^i$</td>
<td>+0.0117$^h$</td>
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<tr>
<td></td>
<td><strong>Lattice constant (Å)</strong></td>
<td>$a=5.649^j, 5.65i$</td>
<td></td>
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Table 1. The energy difference ($\Delta E$ per unit eV) between ZB and WZ polymorphs of GaAs, lattice constant, $a$, and bulk moduli $B$ (GPa) of GaAs (ZB) and Ge (diamond). ($^a$Arabi et al., 2006; $^b$A. Wronka, 2006; $^c$Bautista-Hernandez et al., 2003; $^d$Wang & Ye, 2003; $^e$Wang & Ye, 2002; $^f$Kalvoda et al., 1997; $^g$Yeh et al., 1992; $^h$Murayama & Nakayama, 1994; $^i$Hellwege & Madelung, 1982; $^j$Singh, 1993; $^k$CRC, 1997-1998; $^l$Levinstein, 1999.)

This model can be considered a highly concentrated molecular substitutional Ge$_2$ defect in GaAs, for which we predict stability owing to the donor-acceptor self-passivation mechanism (Giorgi & Yamashita, 2011). For a better understanding of this last aspect concerning self-compensation mechanism, we invite the reader to take a look at Section 4.3,
where the stability of Ge substitutional defects (donor, acceptor, and donor-acceptor pairs) in GaAs matrix and its relationship with alloy self-compensation stabilizing mechanism is deeply discussed.

Fig. 5. The four (GaAs)$_{1-x}$Ge$_{2x}$ alloy models investigated. For visual aid, we here report the 2 x 2 x 2 enlarged cells [Ga, small purple; As, large pink; Ge, intermediate green].

The overall sequence of I is a repeated “sandwich-like” structure, \ldots/As/Ge–Ga/Ge–As/Ga/\ldots along the (001) direction. The bond lengths are 2.38 (Ga–Ge), 2.42 (Ge–Ge), 2.44 (Ga–As), and 2.47 Å (Ge–As) – only slightly differing from the calculated values in bulk Ge and GaAs (2.43 Å). The small electronegativity variation ($\Delta \chi$) is the explanation of this reduced difference in the bondlength, being Ga–Ge and Ge–As nearly covalent two-center bonds ($\chi_{Ga}$=1.81, $\chi_{Ge}$=2.01, and $\chi_{As}$=2.18).

In the alloy I the total number of III-IV and IV-V “bad bonds” (Osorio & Froyen, 1993; Kroemer, 2001; Rodriguez et al., 2001) is 12, or 37.5% of the total. According to the Bader analysis (Henkelmann et al., 2006; Tang et al., 2009; Sanville et al., 2007), in the pure host, the difference in electronegativity is responsible for charge transfer from cation to anion. In the alloy formation process, the introduction of Ge reduces the ionic character of the GaAs bond, while increasing the ionic character of the Ge–Ge bond. When a Ge dimer is inserted in GaAs, 0.32 electrons are transferred away from Ge$_{Ga}$ site, while Ge$_{As}$ gains 0.21 electrons. The charge deficit on Ga, is reduced from 0.6 electrons in bulk GaAs to 0.47e, while the charge excess on As is reduced from 0.6e to 0.5e. The heat of reaction according to Eq. (3) was 0.55 eV, and the optimized lattice parameter was $a$=5.621 Å. We have also considered Ge donors (Ge$_{Ga}$) and acceptors (Ge$_{As}$) in the pure 8-atom GaAs host cell, separately. The formation energy has been computed according to the Zhang-Northrup formalism (Zhang & Northrup, 1991). In particular, we calculate $\Delta E$ to be 1.03 eV for Ge$_{Ga}$ and 0.84 for Ge$_{As}$. The sum of the single contributions (1.87 eV) is larger than the heat of formation of the dimer, structure I (0.55 eV). This is ascribable to the fact that in the model alloy at least one correct bond III-V is formed while in the separate Ge$_{Ga}$ (IV-V) and Ge$_{As}$ (IV-III) cases only bad bonds are formed. The isolated Ge$_{Ga}$ is a donor; the isolated Ge$_{As}$ is an acceptor, thus both of them are unstable in their neutral charged state. We have tested it
in another work (Giorgi & Yamashita, 2011) where we calculated +1 and -1 as the most stable charged state for Ge\textsubscript{Ga} and Ge\textsubscript{As} (both isoelectronic with GaAs), for most of the range of the electronic chemical potential. That the stabilization energy 1.32 eV (i.e., 1.87-0.55) is only slightly smaller than the host GaAs bandgap establishes that the self-passivating donor-acceptor mechanism is the stabilizing mechanism of this Ge dimer. As previously stated, for a further and deeper analysis of Ge substitutions in GaAs the reader take a look on Section 4.3.

We considered two alternative structures for the \(x=0.50\) case. In the II\textsubscript{a} structure Ge atoms are substituted for host atoms at \((0.5, 0., 0.5), (0.5, 0.5, 0), (0.75, 0.75, 0.25),\) and \((0.75, 0.25, 0.75)\). The initial cubic symmetry lowers toward a simple tetragonal one: the optimized lattice parameters were found to be \(a = 5.590\) Å, \(b = c = 5.643\) Å. The 4 intralayer bond lengths were calculated to be Ga–Ge (2.39 Å), Ge–As (2.48 Å), Ge–Ge (2.42 Å), and Ga–As (2.44). Because of the increased amount of Ge, structure II\textsubscript{a} was less polarized than I, as confirmed by the slightly more uniform bond lengths. In II\textsubscript{a} alloy the number of “bad bonds” is 16 (i.e., 50%) and \(\Delta E\) rises to 0.72 eV. In the II\textsubscript{b} structure Ge atoms are substituted for host atoms at \((0.25, 0.25, 0.25), (0.25, 0.75, 0.75), (0.75, 0.75, 0.25),\) and \((0.75, 0.25, 0.75)\). This structure consists of a stack of pure atomic layers, \(\cdots/[Ga/Ge/As/Ge] \cdots\), and thus it contains only nearest neighbors of the (Ga-Ge) and (Ge-As) type: thus all bonds are “bad bonds” in this II\textsubscript{b} compound. Bond lengths were calculated to be 2.40 Å and 2.49 Å, respectively, and optimized lattice parameters were \(a = c = 5.682, b = 5.560\) Å. In this structure, \(\Delta E = 1.40\) eV, almost double that of II\textsubscript{a} with identical composition. According to phase transition theory, the symmetry lowering for the two intermediate systems is the fingerprint of an ordered–disordered phase transition. The calculated deviation from the ideal cubic case (\(c/a=1\)) is 0.94% and 2.15% for II\textsubscript{a} and II\textsubscript{b} models, respectively, confirming energetic instability for the II\textsubscript{b} alloy. The last model, III, \([Ge] = 0.75\), consists of pure Ge except that Ga at \((0., 0., 0.)\) and As at \((0.25, 0.25, 0.25)\). The calculated bond lengths were 2.39, 2.43, 2.45, and 2.48 Å for Ga–Ge, Ge–Ge, Ga–As, and Ge–As, respectively. Cubic symmetry is restored: the optimized lattice parameter \((a = 5.624\) Å) is nearly identical to structure I. The formation energy as from Eq.(3) is almost the same as I (~0.54 eV). Indeed, I and III are formally the same model with the same concentration (37.5\%) of Ge in GaAs (I) and GaAs in Ge (III) and the same number of wrong bonds, 12. For sake of comparison between these two cases, we have also calculated the formation energy of a single substituted Ge in the cell.

We have also made a preliminary calculation of the stability of isolated Ga acceptors (Ga\textsubscript{Ge}) and As donors (As\textsubscript{Ge}) vs that of the substitutional molecular Ga\textsubscript{AsGe2} in Ge pure host supercell consisting of 64 atoms; for such concentrations (0.0312=1Ge/32GaAs unit and 0.0156=1/64GaAs), the molecular substitutional Ga\textsubscript{AsGe2} is only stabilized by 0.057eV with respect to the separate couple acceptor-donor. This small stabilization for Ga\textsubscript{AsGe2} compared to isolated Ga\textsubscript{Ge} and As\textsubscript{Ge} confirms the expected similar probability of finding a mixture of \(n\)-type and \(p\)-type semiconductors in the “disordered” Ge-rich phase. We have used the most stable polymorph of the elemental compounds (orthorhombic Ga and rhombohedral As) for the chemical potential, \(\mu\), of both elements (Mattila & Nieminen, 1996). Ga-rich (\(\mu_{Ga}=\mu_{Ga}^{bulk}\)) and As-rich (\(\mu_{As}=\mu_{As}^{bulk}\)) conditions have been considered, respectively. In the case of the 8-atom cells, the formation energy for Ga\textsubscript{Ge} and As\textsubscript{Ge} are 0.26 eV and 0.58 eV, respectively. The model III stabilizes the isolated Ga and As substitutions by 0.30 eV, i.e. the \(\Delta E\) between the alloy and the isolated substitutionals.
At variance with alloy model I, the large energy difference (~0.4 eV) between the stabilization energy and the Ge host bandgap (0.67 eV at 300 K (Kittel, 2005)) reveals that other factors, and not only a self-compensating donor-acceptor mechanism, impact on the final stability of this III alloy model.

Our calculations reveal an almost exactly linear relationship between the formation energy and the number of bad bonds, as reported in Figure 6. Such relationship is verified at least for systems containing same number of Ge donors (Ge$_{Ga}$) and Ge acceptors (Ge$_{As}$).

Fig. 6. Heat of formation ($\Delta E$) of the alloy models vs. the number of “bad bonds”.

This striking result confirms that the electronic structure of these compounds is largely described in terms of independent two center bonds. For stoichiometric compounds, it suggests an elementary model Hamiltonian for the energetics of any alloy with equal numbers of Ge cations and anions. On all the optimized structures QSGW calculations have been performed and also for the pure GaAs and Ge 8-atom cells. From Figure 7, where the QSGW bandgaps as function of [Ge] are reported, one can see the good reproduction of the asymmetric bowing both at $\Gamma$ and R points. In particular, QSGW calculated bandgaps for pure GaAs and Ge are 1.66 and 1.04 eV on $\Gamma$. IIb model (100% of bad bonds), whose bandgap is not reported in Figure 7, has $E_G < 0$ at both the two points, confirming the tight relationship between high concentration of bad bonds and reduced values of the bandgap.

Fig. 7. QSGW calculated bowing of the bandgap at $\Gamma$ and R vs. different concentrations of Ge atoms.
4.2 Extended models: The quantitative description of the asymmetric bowing minimum

The subsequent analysis focused on extended alloy models. In particular, we have taken into account here models ranging between 8 and 64 atoms. Our choice has the two-fold target of confirming the initial results comparing different methodologies for bandgap calculations, and that of investigating the impact that cluster shape and size has in the bandgap itself. It is also supposed that enlarging the size of the models can extremely improve the reproducibility of the asymmetric V-shape of the bandgap bowing. The Special Quasi Random Structures (SQS) methodology (Zunger et al., 1990), developed to incorporate SRO and local lattice distortions in alloy systems, has been widely employed in literature for the description of alloy properties (see Fiorentini & Bernardini, 2001). We stress that the modellization we have here chosen does not lead mandatorily towards a global minimum for each concentration, stemming this choice from the metastable nature of (GaAs)$_{1-x}$(Ge)$_x$ alloys, grown only at non-equilibrium condition (Barnett et al., 1982; Rodriguez et al., 2001; Banerjee et al., 1985; Alferov et al., 1982). The appropriateness of our modellization is confirmed by the high reproducibility of the experimental results we have obtained. McGlinn. (McGlinn et al., 1988) have experimentally found that Ge regions start forming networks as the Ge concentration increased in the range between 0. and 0.3, and at [Ge]=0.3 such networks are connected with each other. This mechanism is accompanied by the GaAs region fragmentation towards size-reduced clusters. Such experimental finding is our driving force in the choice of the enlarged models: we based our study on models reproducing Ge-clusterized alloys at 0< x <0.3 and GaAs-clusterized alloys at 0.3< x <1. As stated, experimental results report only the formation of size reduced GaAs clusters for [Ge] > 0.3, thus we decided to compare and discuss local geometry effects and their influence on the bandgap of the two specular models at x = 0.375 (such Ge concentration represents the closest one in our models to the experimentally reported concentration, 0.3, where bandgap minimum is found (Barnett et al., 1982)): a Ge-clusterized (IIIa) and a GaAs-clusterized (IIIb) one. Going back to the discussion regarding our two models at x=0.375, IIIb and IIIa (whose structure is reported in Figure 8), the former alloy has larger bandgap than the latter, revealing that at low Ge concentration $E_G$ continues decreasing as Ge concentration increases, as long as the alloy geometry is characterized by the presence of Ge clusters embedded in GaAs host, i.e., in a quantum dot-like fashion. Differently, when GaAs turns to clusterize in Ge network, the bandgap stops decreasing. The relationship between the calculated direct gaps along all the range of x is reported in Figure 9, in conjunction with the experimental values. The close resemblance between our theoretical fitting and the experimental one reveals the extreme suitability of our models in order to reproduce the asymmetric bandgap bowing of (GaAs)$_{1-x}$(Ge)$_x$ alloys: a sharp $E_G$ decreasing at 0< x <0.3 accompanied by the subsequent smooth increasing at 0.3< x <1.

Up to now we were able to explain and demonstrate that the bandgap minimum detection is due to the switching of the embedded cluster in the host from a quantum-dot-like fashion (Ge in GaAs) to an anti-quantum-dot like fashion (GaAs in Ge). Let us go one step further and let us try to understand the chemical origin of this minimum in the bowing. As we mentioned, in (GaAs)$_{1-x}$(Ge)$_x$ alloys, the acceptors and donors, when nearest-neighbors, are subjected to stabilizing self-compensation mechanism, while between bad-bond pairs similar mechanism does not occur due to the presence of residual local positive and/or negative charges. However, if such charges can be delocalized, the compensation
can be effective even beyond nearest-neighbor atomic sites. Thus, since the charge distribution on the bad bonds describes the nearest-neighbor atomic chemical environment, it represents a highly valuable analysis to estimate the SRO. In order to investigate the effect of the cluster type switching on bad bonds, we analyzed the charge distribution of IIIa and IIIb models according to the Bader charge analysis scheme (Henkelmann et al., 2006; Sanville et al., 2007; Tang et al., 2009).

Fig. 8. (GaAs)\textsubscript{1-x}(Ge\textsubscript{x})\textsubscript{v} models at $x = 0.375$. Left: Top and lateral views of IIIa model. Right: Top and lateral views of IIIb model. [Pink, large: Ga atoms; purple, small: As atoms; green, intermediate: Ge atoms respectively].

Fig. 9. Our calculated (GGA+\textit{GW}_0, (VASP)) and experimental band gap behaviour (Rodriguez et al., 2001) vs. Ge concentration ($x$). Red solid line is fitted to experimental results by two line expressions. Blue points are the calculated values. Blue dotted line is fitted to blue points by two line expressions similar to the red solid one. For $x = 0.375$, the model IIIb bandgap is assumed.

In optimized GaAs, charge on Ga (As) atoms is $+0.55e$ ($-0.55e$). In model IIIa, containing Ge clusters, the charges range between $+0.40$ and $+0.52$ for Ga, $-0.53$ and $-0.48$ for As, and between $-0.29$ and $+0.32$ for Ge, respectively. The charge reduction for Ga and As with
respect to those of pure GaAs are easily explained in terms of electronegativity, \( \chi \). \( \Delta \chi \) are smaller on bad bonds than on correct Ga-As bonds. Charges on model IIIb are +0.30 on Ga, -0.41 on As, and +0.04/+0.08 on Ge, respectively, values extremely reduced if compared with those of IIIa. Such charge lowering is explained in terms of number of bad bonds that single Ga or As atom forms. In IIIa, each Ga and As atom forms 1 or 2 bad bonds. In IIIb, on the other hand, each Ga and As atom forms 3 bad bonds. The increase of bad bonds on single Ga and As atom causes partial delocalization of charges, as a consequence of the reduced \( \Delta \chi \) between their constituting atoms. This result also impacts the VBM charge density distribution on \( \Gamma \); indeed, in the IIIa model it is localized around the As and Ge, whose electronegativities are larger than Ga (strong charge localization on the formed bad bonds). Thus, similarly to the valence band forward shift ascribed to the charge density localization (non-bonding), the sharp bandgap decrease in the range \( 0 < [Ge] < 0.3 \) is ascribed to the enhancement of the non-bonding character of the VBM. Differently, in IIIb the VBM is highly delocalized in the whole crystal. Here, the strong bonding character of VBM stabilizes the system, causing the backward shift of VBM and also the smooth bandgap opening at \( 0.3 < x < 1 \), confirming that SRO effects play the main role in the asymmetric bandgap bowing of this class of alloys.

### 4.3 On the stability of Ge donor and acceptor defects

In previous sections we have widely taken advantage of the concept of self-passivation. We here focus on this very basic concept of semiconductor physics, showing that regardless the nature of the cell we are considering, alloys or supercells, the self-passivation stabilization mechanism between Ge donor and Ge acceptor pairs is the main stabilizing process in these IV-doped/III-V systems effective also for non nearest neighbor couples. We aim to demonstrate that alloys are super-concentrated defective cells (Giorgi & Yamashita, 2011).

The formation energy of the defect is defined as the contribution deriving from the formation energy of the defect in its state of charge, plus the contribution of Ga and As potentials in GaAs, and the potential of the substituting Ge. The thermodynamic stability of the charged substitutional Ge defects is calculated as (Zhang & Northrup, 1991):

\[
E_{\text{form}} = E_{\text{Dtot}} - (n_{\text{Ga}} + n_{\text{As}})\mu_{\text{GaAs(bulk)}}/2 - (n_{\text{Ga}} - n_{\text{As}})(\mu_{\text{Ga(bulk)}} - \\
\mu_{\text{As(bulk)}} + \Delta \mu)/2 + q(\mu_{\text{e}} + E_{\text{VBM}}) - n_{\text{Ge}}\mu_{\text{Ge}}
\]

where \( \Delta \mu \) is the chemical potential difference between bulk GaAs (\( \mu_{\text{GaAs(bulk)}} \)) and bulk Ga (\( \mu_{\text{Ga(bulk)}} \)) and As (\( \mu_{\text{As(bulk)}} \)), respectively, ranges between \( \pm \Delta H_{\text{form}} \). \( \Delta H_{\text{form}} \) is the heat of formation of bulk GaAs. Beyond the two extreme conditions of \( \Delta \mu = \pm \Delta H_{\text{form}} \) (Ga-rich and As-rich conditions, respectively) precipitation takes place. \( \mu_{\text{e}} \) is the electronic chemical potential and \( E_{\text{VBM}} \) the energy of the top of the valence band (VBM). \( n_{\text{Ga(As, Ge)}} \) is the number of atoms of Ga (As, Ge) in the supercell, while \( q \) is its total charge. It is straightforward that this equation is the extension (including the charged case and in the stoichiometric case) of Eq. (3) reported in Section 3.1. Similarly for the alloy case, \( \mu_{\text{Ga(As)}} \) is calculated from the orthorhombic (trigonal) polymorph for Ga (As) (Mattila & Nieminen, 1996; Giorgi & Yamashita, 2011). A method based on the combination of the Potential Alignment (PA) for the correction of supercells with a net charge and image charge correction (Lany & Zunger, 2004; Makov & Payne, 1995) has been applied.
To improve the description of the bandgap we employed an LDA + U scheme (Dudarev et al., 1998; Giorgi & Yamashita, 2011). VBM and CBM was also corrected and aligned with the LDA +U obtained bandgap. The Ge₂ quasi-molecule dimer defect best reproduces the high Ge-doping concentration because of the equally probable substitution of one Ga and one As atom, and represents the starting point for our analysis on the self-compensation mechanism in such systems. The neutral state is the most stable along the whole bandgap, as a consequence of the stabilization induced by the mentioned donor-acceptor self-passivating mechanism. The formation energy of the Ge molecular defect is reported in Figure 10.

![Fig. 10. Top: Formation Energy vs. electronic chemical potential of Ge₂ dimer defect. Bottom: Stabilization Energy vs. Ge concentration (red) and distance (green) in alloy and defective cells.](image)

We also considered the case of isolated Ge₅Ga and Ge₅As (Ge₅Ga···Ge₅As) pairs in the supercell. Comparison between the ΔE of the three structures (Ge₂Ga₅As₅, d=2.43 Å, Ge₅Ga···Ge₅As₅, d=4.65, 7.30 Å, respectively) reveals the net tendency for Ge to cluster; the first configuration is indeed 0.36 eV more stable than the second and 0.41 eV more stable than the third. This energy difference is due mainly to the formation of one III–IV (Ga–Ge) and one IV–V (Ge–As) bond and the breaking of one IV–IV (Ge–Ge) bond. To evaluate the impact of distance between pairs of substitutional s on the self-compensating mechanism, we have calculated the Ge–Ge pair correlation $J_{\text{Ge}_{i\alpha}\text{Ge}_{j\beta}}(\text{Ge}_{i\alpha}\text{Ge}_{j\beta})$ as:

$$J_{\text{nn}}(\text{Ge}_{i\alpha}\text{Ge}_{j\beta}) = [E_{\text{tot}}(\text{GaAs:Ge}_{i\alpha}\text{Ge}_{j\beta}) + E_{\text{tot}}(\text{GaAs})] - [E_{\text{tot}}(\text{GaAs:Ge}_{i\alpha}) + E_{\text{tot}}(\text{GaAs:Ge}_{j\beta})]$$

where $E_{\text{tot}}(\text{GaAs:Ge}_{i\alpha}\text{Ge}_{j\beta})$ is the energy for the double Ge-substituted GaAs and $E_{\text{tot}}(\text{GaAs:Ge}_{i\alpha})$ is that for the single substituted states ($h_{i\alpha}$ and $h'_{j\beta}$). The most correlated pair is the quasi-molecular Ge₂Ga₅As defect (~1.55 eV), while for the other two cases we calculate a
correlation energy of \(-1.18 \text{ eV}\) for \(\text{Ge}_{\text{Ga}} \cdots \text{Ge}_{\text{As}}\) at \(d = 4.65 \text{ Å}\) and \(-1.15 \text{ eV}\) for \(\text{Ge}_{\text{Ga}} \cdots \text{Ge}_{\text{As}}\) at \(d = 7.30 \text{ Å}\). The SRO included in the difference of the correlation energy between three distances account for electronic and steric effects. Indeed, the direct formation of Ge–Ge may partly release the stress resulting from substitution of Ga and As in the host. A comparison between the defective supercells and the alloys is here straightforward (see Figure 10, self-compensation mechanism for the two systems).

As in the case of defective supercells also in alloy models, “bad bond” formation and the self-compensation mechanism were considered destabilizing/stabilizing driving forces of the final alloy; we may indeed evaluate the SRO effects on both systems and find a unified trend for alloys and defective supercells. The stabilization energy for the quasi-molecular defect is 
\[
(E_{\text{Form}}(\text{Ge}_{\text{Ga}}^0) + E_{\text{Form}}(\text{Ge}_{\text{As}}^0)) - E_{\text{Form}}(\text{Ge}_{2}\text{GaAs}) = 1.60 \text{ eV} = -J_{\text{nn}},
\]
the correlation energy. Effects related to the Ge–Ge direct bond formation give a contribution of \(1.01 \text{ eV}\) to the total stability \((1.60 \text{ eV} - 0.59 \text{ eV})\). The same contribution for one Ge–Ge bond direct formation in an 8-atom cell alloy was found to be \(0.78 \text{ eV}\). Increasing the number of Ge in the alloy results in an increase in the stabilization energy, i.e., \(1.51 = (1.87 - E_{\text{Form}}/2) \text{ eV}\) for two \(\text{Ge}_2\) and \(1.69 \text{ eV} = (1.87 \text{ eV} - E_{\text{Form}}/3)\) for three \(\text{Ge}_2\). In the present case, we evaluate how the distance between pairs influences the stabilization energy, or in other words, how self-passivation increases the stability of the overall systems. The stabilization energy for the \(\text{Ge}_{\text{Ga}} \cdots \text{Ge}_{\text{As}}\) at \(d = 4.65 \text{ Å}\) is \(1.18 \text{ eV}\); for \(\text{Ge}_{\text{Ga}} \cdots \text{Ge}_{\text{As}}\) at \(d = 7.30 \text{ Å}\) the same energy is \(1.15 \text{ eV}\). The difference between formation energy and stabilization energy gives the self-passivation contribution, which is \(0.28 \text{ eV}\) for the former \((d = 4.65 \text{ Å})\), \(0.19 \text{ eV}\) for the latter \((d = 7.30 \text{ Å})\); in both cases self-passivation is reduced by the distance. A quantitative trend between the Ge atomic distance and the amount of stabilization due to the self-passivation mechanism is thus computed. Indeed, owing to the reduced difference in terms of atomic radius between the impurity and the host atoms, the stabilizing energy resulting from direct Ge–Ge formation is due purely to electronic factors. This confirms that defective supercells can be considered, regardless of the concentration of the substitutional atoms, as precursors of \((\text{GaAs})_{1-x}\text{Ge}_x\) alloys or, similarly, such alloys can be considered as extremely concentrated defective cells.

5. Conclusion

In this chapter we have reviewed the technology behind the Multi-Junction technology in solar cell assembling based on IV-doped III-V alloy showing the importance of Density Functional Theory as a tool for the prediction of the structural and electronic properties of these alloys. After an initial study focusing on eight atom cells, we have extended the analysis to systems constituted by up to 64 atoms. We detected a linear relationship between formation energy and number of bad bonds in the alloys. The relevance of this result stems by the fact that for stoichiometric compounds an elementary model Hamiltonian for the energetics of any alloy with equal numbers of Ge cations and anions as function of the number of bad bonds can be developed. The bandgap bowing for these alloys is confirmed by \(GW\) calculations. Increasing the cell size we were able to quantitatively reproduce the asymmetric bandgap bowing of \((\text{GaAs})_{1-x}\text{Ge}_x\) alloys. This finding stems from an extremely suitable model choice: moving from previous experimental results, we found that Ge-clusterized alloys at \(0 < x < 0.3\) and GaAs-clusterized ones at \(0.3 < x < 1\) are the best in reproducing the asymmetric V-shape of the bowing. Turning from Ge-cluster to GaAs-cluster embedding alloys at concentrations close to the experimental reported for the
bandgap minimum is the key-point for the interpretation of this controversial phenomenon. The last part of the Chapter has been dedicated to the discussion of the stability of the Ge donor-acceptor defects in the GaAs supercells. Regardless the distance between Ge pairs in both defective supercells and alloys, the self-passivation mechanism results the driving force of the stabilization of IV-doped III-V systems, being sensitively effective in the former case also for Ge pairs non nearest-neighbors.

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7. References


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VASP, Vienna Ab-initio Simulation Package.


Photovoltaics have started replacing fossil fuels as major energy generation roadmaps, targeting higher efficiencies and/or lower costs are aggressively pursued to bring PV to cost parity with grid electricity. Third generation PV technologies may overcome the fundamental limitations of photon to electron conversion in single-junction devices and, thus, improve both their efficiency and cost. This book presents notable advances in these technologies, namely organic cells and nanostructures, dye-sensitized cells and multijunction III/V cells. The following topics are addressed: Solar spectrum conversion for photovoltaics using nanoparticles; multiscale modeling of heterojunctions in organic PV; technologies and manufacturing of OPV; life cycle assessment of OPV; new materials and architectures for dye-sensitized solar cells; advances of concentrating PV; modeling doped III/V alloys; polymeric films for lowering the cost of PV, and field performance factors. A panel of acclaimed PV professionals contributed these topics, compiling the state of knowledge for advancing this new generation of PV.

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