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1. Introduction

The demand for wireless services has increased rapidly in the past years and this trend is expected to continue faster in the future. So far most of the available spectrum resources have already been licensed and there is not much bandwidth left to set up new services [Farhang-Boroujeni et al. 2008]. On the other hand, studies reveal that a large percent of licensed spectra is rarely used. The basic idea of Cognitive Radio (CR) is to allow unlicensed users to use the licensed bandwidth under certain conditions [Weiss & Jondral, 2004; Mitola & Maguire, 1999]. In other words, CR is a new method to satisfy ubiquitous demand for wireless services while there is not enough unlicensed spectrum for use in the developing systems.

In order to utilize licensed bandwidth by unlicensed or secondary users (SUs), certain conditions should be considered. One of the most important issues is that in a CR system, SUs are allowed to transmit and receive data over portions of the licensed spectra when licensed users or primary users (PUs) are inactive. Communication among SUs should not interfere with the primary system. Secondary users have to detect portions of free spectrum to avoid PUs. This can be done through spectrum sensing and then careful considerations are required to limit any transmission in the purposed spectrum [Haykin 2005].

The physical layer (PHY) of a CR system should be able to utilize unoccupied spectra for the desired communication. In fact, it often occurs that pieces of spectra that are not used by PUs have discontinuous nature. The PHY layer of a CR system should be flexible enough to utilize such spectra as efficiently as possible. In essence, the PHY of a CR network should be able to shape the output spectrum dynamically and simply. Therefore, Orthogonal Frequency Division Multiplexing (OFDM) is a main candidate to implement the PHY layer of a CR system due to its ability to bring the required flexibility to the system inherently by simple implementation [Weiss et al. 2004a, b].

OFDM is a Multi-Carrier Modulation (MCM) technique that has been used in many conventional systems such as Wireless Local Area Networks (WLANs), Long Term Evolution systems (LTE), Digital Video Broadcast (DVB), and Digital Audio Broadcast (DAB). This is a robust method for transmitting and receiving data over frequency selective channels. Because of its excellent performance in multipath fading channel and simple implementation using Fast Fourier Transfom (FFT) algorithm, it has been the most popular MCM method.
However, one of the major drawbacks of the OFDM is relatively large Out-of-Band (OOB) components spectrum that may be inconsistent with Power Spectral Density (PSD) specification of WLANs, LTE, DVB, and other OFDM based standards as well as CR networks requirements. In other words, the most important drawback of OFDM based CR systems is the large OOB radiation that originates from the high level side-lobes of Inverse FFT (IFFT) modulated subcarriers. These side-lobes cause unwanted interference among SUs and also between SUs and PUs. Therefore, the OOB radiation of OFDM has been a considerable issue either in conventional applications or in CR networks.

The importance of the OOB components of the OFDM spectrum is high enough for CR application to challenge the application of OFDM based CR systems. This has resulted in more attention towards other MCM methods such as Filter-Bank. These are alternative approaches that have been proposed and considered to be used for CR networks within recent years [Farhang-Boroujeni et al. 2008 & Amini et al. 2005]. Filter-Bank based MCM methods are able to shape the spectrum and reduce the OOB spectrum components of the output signal significantly by imposing large computational load to the system.

Consequently, shaping the spectrum and reduction of the OOB components are the elementary steps for OFDM based CR networks. In this chapter, the use of Partial Response Signaling (PRS), also known as correlative coding, will be investigated in order to shape the spectrum of the OFDM based PHYs. In section 2, we provide a review of the subject and cite the most important works in the field with emphasis on more recent literature. The main focus of this section is the problem of OOB reduction in OFDM PHYs. Section 3 is devoted to description of the PRS including history, definition, theoretical background, properties, and various types of the PRS. In section 4, the analytical PSD of the OFDM signal is calculated for the case where modulated symbols are correlated. Investigation of the spectral shaping of the OFDM signal using PRS is addressed in section 5. This section also discusses block-diagram of the required receiver in the proposed scenario. Simulation results are provided in section 6. The effect of the proposed method on the Peak-to-Average-Power-Ratio (PAPR), a significant issue for OFDM PHYs, and word error rate (WER) is also investigated. In addition this section contains application of the proposed method for spectral shaping in CR networks. Finally, summary and conclusions along with proposed topics for future research are presented in section 7.

2. State-of-the-art

It is inferred from the nature of CR systems that their PHY system should possess not only high flexibility in spectral shaping but also they need to provide a spectrum with very low OOB components. On the other hand, as it is stated in the previous section, existence of the relatively large OOB components in the OFDM spectrum is one the most serious shortcomings of this MCM technique especially for CR networks. In this section a brief review of existing methods for spectral shaping and reduction of OOB components in OFDM spectrum will be introduced.

The problem is illustrated in figure 1. This figure shows the available bandwidth and OOB components for an OFDM PHY with 3 modulated subcarriers. The OOB components are fairly large and cannot be tolerated in most applications.
In the following, an overview of the previously suggested methods for spectral shaping and OOB reduction in OFDM systems is given. In [Weiss et al. 2004] and several standard documents, the insertion of guard bands at the border of OFDM spectrum has been proposed [IEEE 802.16, 2004; IEEE 802.11, 1999]. The drawback of this approach is the less effective use of the bandwidth. Another common method is time windowing at the transmitter. In this method, the time domain signal is multiplied by a proper window to smooth transition between consecutive OFDM blocks [Nee & Prasad, 2000]. Various window functions such as raised-cosine (RC), hamming, hanning, and so on can be used for this purpose. In this approach the effective time duration of transmitted signal is extended and inter-block interference is introduced [Nee & Prasad, 2000; EN 301 958, 2001]. In [Xu et al. 2009], a new class of window functions has been derived for OOB reduction but there is no investigation on the effect of the proposed method on the system performance.

In [Yu et al. 2010], an alternative windowing method has been suggested for OFDM based CRs where time windowing is applied to the entire OFDM symbol before addition of the guard interval. Although this method introduces a controlled amount of Inter-carrier-interference in the system, it does not waste time-frequency resource as opposed to conventional windowing methods.

A different technique is dual to time windowing and uses pulse shaping filter. In fact, each subcarrier is multiplied by a pulse shaping function which is equal to the convolution of time domain transmitted signal with the impulse response of pulse shaping filter. This approach suffers from high complexity and lack of guard interval [Naghsh & Omidi, 2010]. It should be noted that guard interval plays an important role in avoiding inter-block interference between OFDM blocks. Besides, guard interval not only extends the time duration of transmitted signal and reduces side-lobe levels but also increases robustness of system against synchronization errors [Phillip, 2001].

In [Brandes, 2006], insertion of the cancellation carrier at the edge of available bandwidth has been proposed. Proper values for cancellation carriers in each OFDM block have been
obtained by solving a convex optimization problem. This method reduces OOB components considerably but has high complexity, and increases PAPR and WER. In [Cosovic, 2006], the modulated symbols in each OFDM block have been multiplied by proper sequence to reduce OOB components. Proper sequence can be found by solving a convex optimization problem. In addition to high complexity, this method results in WER loss.

In [Noreen & Azimi, 2010], a technique for OOB reduction in OFDM-based CRs has been suggested that maps groups of two input symbols onto extended constellation such that subcarriers in each group become 180 degrees out of phase. The mentioned technique does not require side information but degrades both PAPR and WER.

In the adaptive symbol transition method, the transition between consecutive OFDM blocks is smoothed adaptively. Multiple choice sequences method performs a mapping of each transmission sequence into a specific set of sequences. From this set, the sequence which offers the maximum reduction of out-of-band radiation is chosen for the actual transmission. These two approaches are discussed thoroughly in [Mahmoud & Arsalan, 2008a; Cosovic & Mazzoni, 2006] respectively. Note that in [Ghassemi et al. 2010], a generalization of the multiple choice sequences method has been proposed that reduces OOB and PAPR jointly.

Some references propose combination of the existing methods. For example, in [Mahmoud & Arsalan, 2008b] combination of the cancellation carrier insertion and RC windowing has been suggested. It has been shown that this method works well for small gaps in the spectrum in the cost of lower spectral efficiency and computational load. This reference also contains a review of the most important techniques for spectrum shaping in OFDM based CRs.

In [Yuan & Wyglinski, 2009, 2010], combination of the cancellation carrier insertion and frequency filtering has been proposed for side-lobe reduction of OFDM spectrum in CR PHYs. This method can reduce OOB components considerably in the cost of lower spectral efficiency, higher PAPR level, and higher WER.

In [Sokhandan & Safavi, 2010], a new method has been suggested that may be thought as a generalization of the combination of the cancellation carrier insertion and adaptive symbol transition. In essence, this method optimizes the value of cancellation carriers and time extension of the OFDM symbol jointly. The mentioned technique requires high computational load since it should solve an optimization problem for each OFDM symbol. Also, it does not affect the PAPR level and there is no report about its effect on the WER.

Finally we cite the work of [Zhou et al. 2011] that suggests mapping of the antipodal symbol pairs onto adjacent subcarriers to achieve a faster decay of OOB components in OFDM-based CR systems. It also proposes the use of power control scheme and channel coding with different rate for the balance between further reduction of the side-lobes level and system performance.

In this chapter, we propose a novel method to achieve side-lobe suppression. Our proposed method is to introduce proper carrier-by-carrier PRS for spectral shaping of the OFDM signal [Naghsh & Omidi, 2010]. In this approach a controlled amount of correlation is introduced among modulated symbols on each subcarrier in consecutive OFDM blocks. In this method the effective time duration and bandwidth of transmitted signal will remain unchanged. Also, guard interval can be used as before [Naghsh & Omidi, 2010].
It should be noted that the term “partial response OFDM” that has been used in [Vadde, 2001; Vadde & Gray, 2001, Kim & Km, 2005; Syed-yusof et al. 2006] refers to the introduced PRS between modulated symbols on the subcarriers in an OFDM block in order to decrease Inter-carrier-interference due to the receiver phase noise, PAPR reduction, etc. Therefore, stated methods are fundamentally different from our approach since our proposed method is developed for spectral shaping and OOB reduction by introducing correlation between consecutive OFDM blocks across the time (refer to section 5).

3. Partial response signalling

Partial response signalling (PRS), also known as correlative coding, was introduced for the first time in 1960s for high data rate communication [lender, 1960]. From a practical point of view, the background of this technique is related to the Nyquist criterion. Therefore, we present a brief review of this concept.

Assume a Pulse Amplitude Modulation (PAM), according to the Nyquist criterion, the highest possible transmission rate without Inter-symbol-interference (ISI) at the receiver over a channel with a bandwidth of W (Hz) is 2W symbols/sec. To achieve this rate, the transfer function of the overall system, i.e., $H(f)$, should satisfy $\sum_{m=-\infty}^{\infty} H(f - 2mW) = $ cte.

The only transfer function that satisfies the Nyquist criterion over a $W$ (Hz) bandwidth is

$$H(f) = \begin{cases} 1, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

$H(f)$ is an ideal Low Pass Filter (LPF) and is not practical for implementation. Therefore, in practice, other $H(f)$ functions, i.e., $\hat{H}(f)$, that satisfy Nyquist criterion over a wider bandwidth have to be used. It can be shown that a wider bandwidth for $\hat{H}(f)$ in comparison with $H(f)$ results in smoother transition of frequency response in cut off regions. For example, the required bandwidth for $\hat{H}(f)$ with RC shape\(^1\) is $(1 + \alpha)W$ where $\alpha$ is the roll-of-factor that controls the shape and bandwidth of the used transfer function. Note that we have $0 < \alpha \leq 1$ and larger $\alpha$ means more smoothed frequency shaping.

The underlying assumption here is that the modulated data over consecutive pulses are independent while PRS relaxes this assumption and allows for a controlled amount of correlation in the system. Although this controlled correlation leads to ISI, it can be easily compensated for at the receiver side as it has a known pattern. Figure 2 shows the block diagram of Doubinary system which is commonly used for PRS.

Assume that $\{d_k\}$ denotes the sequence of modulated symbols with rate of $2W$ symbols/sec. The controlled correlation is introduced to the system by a digital filter adding two independent successive symbols, i.e., $a_k = d_k + d_{k-1}$ (This scheme is usually denoted by

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\(^1\) Raised Cosine Shape.
1+D where D is the T second delay operator\(^2\). Then, the sequence of \(\{a_k\}\) is passed through an ideal Nyquist transfer function which guarantees no additional ISI at the receiver. Finally, at the receiver, removing of the controlled correlation and symbol detection is performed by means of a maximum likelihood sequence detector (MLSD) algorithm such as the Viterbi algorithm getting \(\{z_k\}\) as its input. In some special cases, symbol detection can be implemented very easily as will be shown in section 6.

\[
G(f) = 2H(f)\cos(\pi fT)\exp(j\pi fT)
\]

As it is observable in figure 3, G(f) has a gradual roll-off to the band edge in comparison with H(f) and hence is practical for implementation.

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\(^2\) Note that \(T \triangleq 1/2W\)
The effect of the PRS on the error rate of the system is investigated in section 6. It should be noted that nine patterns of more common PRS schemes are introduced in [Passupathy, 1977; Kabal & Passupathy 1975]. It has been shown that all of those schemes require 2M-1 or 4M-1 level slicer in the receiver for M-array PAM. In addition, it has been shown that using PRS brings out more synchronization robustness for the system.

4. Analytical PSD of the OFDM signal

In this section, the PSD of OFDM signal will be drawn analytically. In essence, the OOB components are considered as PSD components that are out of the permitted bandwidth as it is illustrated in figure 1. Basically the finite symbol duration of OFDM symbol results in these OOB components, however, other reasons such as phase transition between successive blocks may contribute to this phenomenon.

In the OFDM, the baseband signal can be represented as

\[ x(t) = \sum_{l=1}^{N} \sum_{k=0}^{\infty} d_{k,l} w(t-kT_s) \exp(j2\pi f_l(t-kT_s-T_{GI})) \]  

where \( l \) and \( k \) are the subcarrier and time indexes, complex number \( d_{k,l} \) is the modulated symbol on \( l \)-th subcarrier at \( k \)-th time interval, \( w(t) \) is the window function, e.g., rectangular, RC, hamming, etc., \( f_l \) is the \( l \)-th subcarrier frequency that equals to \( 1/T_{FFT} \), \( T_{FFT} \) is the pure OFDM block time duration, \( T_s \) is the total time duration of an OFDM block, \( T_{GI} \) is the guard interval (GI) duration, and finally \( N \) is the number of subcarriers. Clearly, we have \( T_s = T_{FFT} + T_{GI} \).

By defining

\[ g_l(t) = \sum_{k=0}^{\infty} d_{k,l} w(t-kT_s) \exp(j2\pi f_l(-T_{GI}-kT_s)) \]  

the baseband signal can be written as

\[ x(t) = \sum_{l=1}^{N} g_l(t) \exp(j2\pi l\Delta f t) \]  

where \( \Delta f = 1/T_{FFT} \).

Generally, a linear filter may be used to implement PRS for the system and make intentional correlation of length \( L \) among modulated symbols on each subcarrier as shown in Figure 4.

Indeed, each subcarrier is treated by such filters individually and a carrier-by-carrier PRS scheme is organized as shown in figure 5. Then we can write

\[ a_{k,l} = \sum_{n=0}^{L} \alpha_n d_{k-n,l} \]  

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Fig. 4. Implementation of PRS in OFDM Carriers using N digital filters [Naghsh & Omidi, 2010]

Fig. 5. PRS for $l$-th subcarrier in general case [Naghsh & Omidi, 2010]

where $\{a_n^l\}$ are the linear filter coefficients for $l$-th subcarrier. Then, the baseband signal becomes

$$x(t) = \sum_{l=1}^{N} b_l(t) \exp(j2\pi l\Delta f t) \quad (5)$$

where $b_l(t)$ is defined as

$$b_l(t) = \sum_{k=-\infty}^{\infty} a_{k,l} w(t - kT_s) \exp(j2\pi f_l (-T_{Cl} - kT_s)) \quad (6)$$
In [Proakis, 2007], it has been shown that the PSD of the random process $b_i(t)$ is given by

$$
B_i(f) = \frac{1}{T_s} |W(f)|^2 \left| \sum_{n=0}^{L} \alpha_n^l \exp(-j2\pi f n T_s) \right|^2 \left[ \sum_{m=-\infty}^{\infty} E\{d_{k,l}d_{k+m,l}^*\} \exp(-j2\pi f m T_s) \right]
$$

(7)

where $W(f)$ is the Fourier transform of the window function, i.e., $w(t)$ and $E$ denotes the expected value operator.

Under the assumption of uncorrelated data sequence we have

$$
E\{d_{k,l}d_{k+m,l}^*\} = \delta(m)
$$

and hence $B_i(f)$ can be simplified to

$$
B_i(f) = \frac{1}{T_s} |W(f)|^2 \left| \sum_{n=0}^{L} \alpha_n^l \exp(-j2\pi f n T_s) \right|^2
$$

(8)

In the following, we assume that data sequence is uncorrelated since it is the output of the source coder or scrambler. Also, in the rest of this chapter we assume that the effect of the GI can be neglected, i.e., $T_{FF}\gg T_{GI}$. Therefore, as various subcarriers carry uncorrelated modulated symbols at $k$-th block for any $k$, the PSD of OFDM signal becomes

$$
X(f) = \sum_{i=1}^{N} B_i(f - l\Delta f)
$$

(9)

It is realized from (8) and (9) that the PSD of OFDM signal depends mainly on the window function, number of subcarriers, and $\{\alpha_n^l\}$, the coefficients of linear filter used to introduce PRS on $l$-th carrier. In this chapter, we use these coefficients to shape the spectrum and reduce the OOB components.

If the common RC window is used, $w(t)$ can be written as

$$
w(t) = \begin{cases} 
\sin^2\left(\frac{\pi}{2} \left(5 + \frac{t}{T_R}\right)\right) & \text{for } -\frac{T_R}{2} \leq t \leq \frac{T_R}{2} \\
1 & \text{for } \frac{T_R}{2} \leq t \leq T_s - \frac{T_R}{2} \\
\sin^2\left(\frac{\pi}{2} \left(5 + \frac{t-T_s}{T_R}\right)\right) & \text{for } T_s - \frac{T_R}{2} \leq t \leq T_s + \frac{T_R}{2}
\end{cases}
$$

(10)

where $T_R$ is the transition time and $W(f)$ can be expressed as [Naghsh & Omidi, 2010]

\[^{3}\text{without loss of generality it is assumed that } E\{|d_{k,l}|^2\} = 1\]
by choosing $T_R$, various windows can be synthesized, e.g., choosing $T_R = 0$ results in rectangular window. The effect of the increasing of $T_R$ is shown in Figure 6. According to this figure, higher $T_R$ leads to lower side-lobes for $W(f)$. It should be noted that lower side-lobes level of $W(f)$ reduces OOB components of the OFDM spectrum according to (8) and (9). This also results in higher effective time duration of the window as it can be inferred from (10).

In the following, it is assumed that the rectangular window is used. Therefore, from (8), (9) and (11), $X(f)$ becomes

$$X(f) = T_s \sum_{l=1}^{N} \left| \text{sinc}(T_s[f - l\Delta f]) \right|^2 \left| \sum_{n=0}^{L} \alpha_n^l \exp(-j2\pi[f - l\Delta f]nT_s) \right|^2$$

To obtain conventional OFDM signal PSD from (12), assuming there is no PRS ($L = 0, \alpha_0^l = 1, \forall l$), yields

$$X(f) = T_s \sum_{l=1}^{N} \text{sinc}^2(T_s[f - l\Delta f]).$$

In the next section, we investigate how a proper choice of $\{\alpha_n^l\}$ can shape the spectrum and result in lower OOB in OFDM spectrum.
5. OFDM spectral shaping using PRS

In this section the advantage of introducing proper PRS between modulated symbols on each subcarrier, or proper carrier-by-carrier partial response signaling, is investigated. Considering (8) and (9), proper choice of this controlling term

\[
\alpha_l \exp(-j2\pi[f - l\Delta f]nT_s) \quad (14)
\]

in (12) can be used for shaping each subcarrier’s spectrum separately, and hence overall PSD shaping will be possible for OOB component reduction. As mentioned in the previous section, assume that the data sequence, \(d_{k,l}\), is uncorrelated. Otherwise, each subcarrier’s spectrum is expressed as (7) and our discussion still will be valid.

Now, we rewrite the PSD of OFDM signal as

\[
X(f) = T_s \sum_{l=1}^{N} \left| \text{sinc}(T_s[f - l\Delta f]) \right|^2 \sum_{n=0}^{L} \alpha_n \exp(-j2\pi[f - l\Delta f]nT_s) \right|^2 \quad (15)
\]

Figure 7 shows schematic of the carrier-by-carrier PRS method and intentionally introduced correlation between modulated symbols on each subcarrier across the time.

**Correlation (due to partial response signaling)**

**Fig. 7. Schematic of the carrier-by-carrier PRS method [Naghsh & Omidi, 2010]**

Considering (15), in general, each subcarrier may be processed based on its own PRS pattern depending on \(\alpha_n^1\). In this case, more degrees of freedom are available and better spectral
performance is achievable. But, it results in high complexity because each subcarrier must be treated individually for symbol modulation and detection. In the following, assume that all subcarriers have the same PRS pattern. It means

$$\alpha_n^1 = \alpha_n \quad 1 = 1, 2, \ldots, N$$  \hspace{1cm} (16)

Then, according to (15), PSD of OFDM signal becomes

$$X(f) = T_s \sum_{i=1}^{N} \left[ \text{sinc}(T_s[f - i\Delta f]) \right] \left[ \sum_{n=0}^{L} \alpha_n \exp(-j2\pi[f - i\Delta f]nT_s) \right]^2$$  \hspace{1cm} (17)

In (17), the PRS pattern should be selected for shaping the PSD and specially OOB components reduction. Clearly, large \(L\) gives more degrees of freedom, but leads to more states for receiver slicer. Hence \(L\) is dictated to the transmitter based on acceptable complexity for the receiver.

In general, in the receiver, the effect of PRS on each subcarrier symbol could be removed by means of an MLSD. In other words, this controlled correlation might be assumed as known channel impulse response and thus detection could be implemented by MLSD methods such as Viterbi algorithm.

It should be noted that not every PRS scheme can improve OOB radiation and thus in this context ‘proper PRS’ term has been used. In fact, the controlling term in (14) is the discrete Fourier transform of the PRS sequence \(\{\alpha_n^1\}\), regardless of \(i\Delta f\). Therefore, it is reasonable that sequences with low-pass frequency characteristic are selected to reduce OOB components; because the PSD of conventional OFDM system is multiplied by the Fourier transform of the chosen sequence according to (17). In the next section, we introduce two common examples of these proper schemes.

Although symbols with proper PRS pattern will result in lower OOB radiation compared to the case of no PRS, but it will increase the number of equivalent signal space components, which leads to increased WER because of more neighborhoods for each component. In addition, in the proposed method, error propagation may occur because of transmitting symbols by PRS (some controlled correlation); fortunately, these effects can be reduced by pre-coding schemes [Passupathy, 1977; Kabal & Passupathy 1975]. The general form of pre-coding for M-array modulation used in PRS with integer coefficient could be written as

$$\alpha_0 d_{k, 1} = c_{k, 1} - \sum_{i=1}^{L} \alpha_i d_{k-i, 1} \mod M$$  \hspace{1cm} (18)

where \(c_{k, 1}\) is the original uncorrelated sequence and \(d_{k, 1}\) is the pre-coded sequence. The necessary and sufficient condition for uniquely retrieving \(c_{k, 1}\) from \(d_{k, 1}\) is that \(\alpha_0\) and \(M\) be relatively prime [Kobayashi, 1971]. In this condition

$$c_{k, 1} = \sum_{i=0}^{L} \alpha_i d_{k-i, 1} \mod M$$  \hspace{1cm} (19)
For example in the Duobinary system, pre-coding can be implemented as

\[ d_{k,1} = c_{k,1} \oplus d_{k-1,1} \]  

(20)

where \( \oplus \) denotes XOR. It can be inferred from (18) and (19) that pre-coding scheme for non-binary cases with “\( \mod M \)” calculations imposes more complexity on transceiver. In the binary case pre-coding could be easily implemented using logical devices, even for large \( L \).

Based on the above discussion, the total block diagram of the transmitter and receiver can be illustrated as in figure 8 and figure 9. Note that S/P, P/S, D/A, and A/D denote serial-to-parallel, parallel-to-serial, digital-to-analog, and analog-to-digital converter, respectively.

Fig. 8. Transmitter block diagram of the carrier-by-carrier PRS OFDM [Naghsh & Omidi, 2010]

Fig. 9. Receiver block diagram of the carrier-by-carrier PRS OFDM [Naghsh & Omidi, 2010]
6. Numerical results

6.1 Basic results

In this section we assume an OFDM system with $N=64$ subcarriers, similar to IEEE 802.11 standard, which are modulated using BPSK modulation and we will use 1000 consecutive blocks for simulation. Also, it is assumed that permitted normalized bandwidth is in the $[0.2, 1.8]$ interval and 10% of the subcarriers are set to zero at the bordering of the spectrum. It should be noted that for fair comparison, PSDs are normalized such that the total transmitted power in the permitted bandwidth remains constant.

Figure 10 shows the resulting PSD of the proposed method in comparison with conventional OFDM where $L=1$. Note that all of the subcarriers have the same PRS pattern and values of $\alpha_0, \alpha_1$ have been selected based on well-known Duobinary system which has low-pass frequency characteristic [Passupathy, 1977; Kabal & Passupathy 1975; Kretzmer, 1966], $\alpha_0 = 1, \alpha_1 = 1$. In this case, the transmitted symbols are $\{-2, 0, 2\}$ and analytic PSD using (17) becomes

$$X_{\text{Duobinary}}(f) = T_1 \sum_{l=1}^{N} \left[ \text{sinc}(T_s[f - l\Delta f])^2 \left| 1 + \exp(-j2\pi T_s[f - l\Delta f]) \right|^2 \right]$$

If no pre-coding is used in the transmitter, the receiver should detect the modulated symbols on each subcarrier by means of MLSD after performing FFT. But, by using the pre-coding of (20), receiver can be implemented easily based on the absolute value of received noisy symbol after performing FFT [Naghsh & Omidi, 2010]

$$\begin{cases} c_{k,1} = 0 & \text{if } |y_{k,1}| = 2 \\ c_{k,1} = 1 & \text{if } |y_{k,1}| = 0 \\ c_{k,1} = 1 & \text{if } |y_{k,1}| \geq 1 \\ c_{k,1} = 0 & \text{if } |y_{k,1}| \leq 1 \end{cases}$$

Reduced OOB components in Duobinary carrier-by-carrier PRS OFDM are observable in figure 10. Explicit ripples in the available bandwidth in figure 10 are due to introduced inter-block-interference (because of PRS) similar to frequency selectivity at multipath channel [Naghsh & Omidi, 2010].

Now, let $L=2$ and choose a class-2 PRS which is another low-pass frequency characteristic sequence. In this case the coefficients are $\{\alpha_0 = 1, \alpha_1 = 2, \alpha_2 = 1\}$ and five levels will be produced as $\{-4, -2, 0, 2, 4\}$ [Passupathy, 1977; Kabal & Passupathy 1975; Kretzmer, 1966]. A typical receiver in this case will use MLSD after performing FFT. In this case, based on (17), analytic PSD expression can be written as

$$X_{\text{class-2}}(f) = T_1 \sum_{l=1}^{N} \left[ \text{sinc}(T_s[f - l\Delta f])^2 \left| 1 + 2\exp(-j2\pi T_s[f - l\Delta f]) + \exp(-j4\pi T_s[f - l\Delta f]) \right|^2 \right]$$
By using some trigonometric equality, the PSD becomes

\[
X_{\text{class-2}}(f) = T_s \sum_{l=1}^{N} |\text{sinc}(T_s[f - l\Delta f])|^2 \\
\times (6 + 8\cos(2\pi T_s[f - l\Delta f]) + 2\cos(4\pi T_s[f - l\Delta f]))
\]

In figure 11, comparison of the PSD of a subcarrier and overall modulated signal for conventional OFDM, Duobinary carrier-by-carrier PRS OFDM, and a class-2 carrier-by-carrier PRS OFDM are illustrated. It is clear from this figure that larger \(L\) results in lower OOB components while it imposes more complexity. Also, as we expect, more ripples exist for larger \(L\) because of the existence of more (controlled) correlation in the system.

Fig. 10. PSD comparison of conventional and Duobinary carrier-by-carrier PRS OFDM for 802.11a
Fig. 11. a) A subcarrier, b) Overall PSD; comparison of conventional, Duobinary, and a class-2 carrier-by-carrier PRS OFDM
6.2 Comparison with other methods

As mentioned in the introduction, there are several OOB reduction methods. Some of them such as cancellation carrier insertion have high complexity and so they are not suitable for practical applications. On the other hand, some methods such as time windowing, insertion of more guard band at the border of spectrum, and frequency filtering are very common in various applications and they could be implemented with acceptable complexity. Therefore, we compare our proposed method to the latter. The results are presented in figure 12 and Table 1. In time windowing, two RC windows are used with $T_R / T_s$ equal to 6% and 11% respectively. According to Figure 12 and Table 1, this method has failed to reduce the first peak of OOB components. In addition, larger $T_R / T_s$ will result in better OOB reduction while producing more inter-block interference and leads to less spectral efficiency. In the frequency filtering approach, the OOB components are filtered by an FIR filter with length 9, about 15% of an OFDM block, in the transmitter. Similar to time windowing, this method has failed to reduce the first peak of OOB components effectively. It should be noted that although better OOB reduction is possible by frequency filtering method, it requires long FIR filter that not only imposes more complexity to the system but also extends blocks in time domain resulting in more inter-block interference. In the last method, about 20% of subcarriers are set to zero at the border of the spectrum. Note that for all of the previous methods mentioned value was 10%. Therefore, the available bandwidth is not used efficiently\(^4\). Figure 12, and Table 1 show that our proposed method can reduce the main side-lobe peak more effectively in comparison with other methods while windowing and frequency filtering can reduce average OOB more effectively.

\[\text{Fig. 12. PSD comparison of conventional, windowed, filtered, and guard band inserted OFDM with the proposed method}\]

\[^4\] In this simulation, the total transmitted power of the Guard insertion method is less than that of other methods.
<table>
<thead>
<tr>
<th>Method</th>
<th>Peak OOB, dB</th>
<th>Mean OOB, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional OFDM</td>
<td>-9.98</td>
<td>-17.11</td>
</tr>
<tr>
<td>Duobinary OFDM</td>
<td>-13.75</td>
<td>-19.84</td>
</tr>
<tr>
<td>Class-2 OFDM</td>
<td>-13.78</td>
<td>-21.48</td>
</tr>
<tr>
<td>Time window OFDM (6%)</td>
<td>-11.92</td>
<td>-22.02</td>
</tr>
<tr>
<td>Time window OFDM(11%)</td>
<td>-12.46</td>
<td>-23.92</td>
</tr>
<tr>
<td>Frequency filtered OFDM</td>
<td>-13.04</td>
<td>-27.86</td>
</tr>
<tr>
<td>Guard insertion (20%)</td>
<td>-13.61</td>
<td>-18.51</td>
</tr>
</tbody>
</table>

Table 1. Comparison of peak and mean values of OOB components for investigated methods

### 6.3 PAPR and WER analysis

Now we examine our proposed method, carrier-by-carrier PRS OFDM, in the view of PAPR as a challenge for many OFDM systems and also WER. Assume that in each OFDM block the carrying symbols, $a_{k, \ell}$, are independent and identically distributed, hence

$$E\{a_{k, \ell}^* a_{k, \ell}^*\} = \delta(n) \quad (25)$$

Hence, after performing IFFT, the assumption of independency is still valid due to the orthonormality of IFFT basis [Vadde, 2001; Baxley, 2005]. Similar to conventional OFDM systems, for large $N$ the Complementary Cumulative Distribution Function (CCDF) of PAPR can be written as

$$\text{prob}\{\text{PAPR} \geq z\} = 1 - (1 - e^{z^2})^{N} \quad (26)$$

Therefore, we expect that carrier-by-carrier PRS OFDM does not lead to greater PAPR compared to conventional OFDM systems.

Figure 13 shows the CCDF of PAPR for conventional OFDM, Duobinary, and a class-2 carrier-by-carrier PRS OFDM system. For the MonteCarlo simulation $10^4$ OFDM blocks have been tested. This figure indicates that carrier-by-carrier PRS between consecutive OFDM blocks does not trade off higher PAPR for the better OOB reduction.

Now we investigate our proposed method in the view of WER. As mentioned in section 5, the introduced correlation between subsequent symbols on each subcarrier will cause error propagation and higher WER in the system. In [Passupathy, 1977; Kabal & Passupathy 1975], the lower band and upper band of the WER in AWGN channel for single carrier PRS systems have been calculated as follows

$$P_{e,\text{lower}} \leq P_{e} \leq \frac{M^{L-1} P_{e,\text{lower}}}{M - 1} \frac{P_{e,\text{lower}} (M^{L-1} - 1) + 1}{M^{L-1}} \quad (27)$$

where

$$P_{e,\text{lower}} = 2(1 - \frac{1}{M})Q(\alpha_0 / \sigma) \quad (28)$$
and $P_e$ is the WER, $M$ is the modulation alphabet size, $\sigma^2$ is the variance of the Gaussian noise, and $Q(x)$ is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-u^2/2) du$$  \hspace{1cm} (29)$$

Fig. 13. The CCDF of PAPR for conventional, Duobinary, and a class-2 carrier-by-carrier PRS OFDM

Although (27) and (28) have been drawn under the implicit assumption of decision feedback detector, they can be useful for overall WER behavior of the system.

In [Kobayashi, 1971], for single carrier system, the exact WER has been calculated for MLSD in Duobinary case for both non pre-coded data sequence and pre-coded data sequence according to (18). If a real $M$-array modulation is used, we will have

$$e_{MLSD} = 2M(M-1)Q(d)$$  \hspace{1cm} (30)$$

where $e_{MLSD}$ is WER in non pre-coded data sequence system, $d^2 = A^2 / 2\sigma^2$, and $A$ is the smallest distance between modulated symbols in the constellation. Similarly, the WER in the pre-coded data sequence, $e_{p_{MLSD}}$, becomes

$$e_{p_{MLSD}} = 4(M-1)Q(d)$$  \hspace{1cm} (31)$$

It could be realized from (30) and (31) that $e_{p_{MLSD}} \leq e_{MLSD}$, and the equality holds only for $M = 2$. It means that for $M > 2$, pre-coding prevents error propagation and hence it improves WER performance. Although in binary modulation, $M = 2$, pre-coding yields no WER gain, it results in very simple symbol detection according to (22).
In figure 14, the simulated WER for the same OFDM system of previous section is shown. It is reasonable that larger $L$ would result in higher WER because of more correlation between symbols. Table 2 shows the approximated SNR loss due to PRS in comparison with conventional OFDM system for the two investigated cases.

![Fig. 14. WER comparison for conventional, Duobinary, and a class-2 carrier-by-carrier PRS OFDM](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>Approximated SNR loss @ WER= 10^{-3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duobinary OFDM</td>
<td>0.9dB</td>
</tr>
<tr>
<td>Class-2 OFDM</td>
<td>1.7dB</td>
</tr>
</tbody>
</table>

Table 2. Approximated SNR loss @ WER=10^{-3} for two investigated systems compared to conventional OFDM

### 6.4 Application to CR PHY

In this subsection we examine our proposed method for a hypothetical CR scenario. Figure 15 shows the PSD of conventional OFDM, Duobinary, and a class-2 carrier-by-carrier PRS OFDM for the hypothetical cognitive radio physical layer with three available frequency notches.

The OOB reduction of carrier-by-carrier PRS OFDM and less interference by SUs in CR application can be realized clearly from this. As mentioned above, larger $L$ can make this situation more attractive. It is worth noting that carrier-by-carrier PRS OFDM method can be generally implemented with acceptable complexity by means of a simple digital filter in the transmitter and an MLSD in the receiver. Note that in Duobinary case with pre-coding and BPSK modulation, receiver can be implemented by a simple slicer rather than the MLSD. Furthermore, this method will not limit the system. In other words, in the proposed method, many other techniques of OOB radiation reduction can be used such as windowing, cancellation carrier insertion, frequency filtering, and so on in addition to the PAPR.
reduction techniques. Also, GI can be added to signal as before to improve inter-block-interference robustness and obtain better synchronization performance. Consequently, the carrier-by-carrier PRS OFDM method is a more appropriate candidate for PHY of CR networks than conventional OFDM.

Fig. 15. PSD comparison of conventional, Duobinary, and a class-2 carrier-by-carrier PRS OFDM for a hypothetical CR system

7. Conclusion

The reduction of OOB components of an OFDM spectrum is a challenge both in conventional applications and in CR networks. In this chapter using carrier-by-carrier PRS between consecutive OFDM blocks to reduce OOB radiation was introduced. This method neither increases effective time duration of signal nor decreases the bandwidth efficiency. Also, it is compatible with many other techniques and can be implemented in the existing systems by means of digital filtering in the transmitter and an MLSD, or in special case by a simple slicer, in the receiver. However, error propagation, and thus higher WER, may occur because of introduced controlled correlation which can be reduced by means of pre-coding. In addition, in the proposed system many other techniques of OOB radiation and PAPR level reduction can be used. Furthermore, GI can be added to signal as before. Simulation results show that this method can reduce OOB radiation effectively by an acceptable added complexity while it increases WER and PAPR remains unchanged. Also, it was showed that introducing correlation between more modulated symbols leads to better OOB radiation reduction, more complexity, higher WER, and the same PAPR level. Investigation of the combination of the proposed method with existing methods such as frequency filtering, cancellation carrier insertion, and etc. can be the topic of the future works. Also, derivation of the optimum correlation pattern, not necessarily PRS patterns, via an optimization problem considering OOB components and WER is an attractive subject.
8. Acknowledgment

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9. References


Kim, K. H. & Km, H. M. (2005). A Suppression Scheme of the ICI caused by Phase Noise based on Partial Response Signaling in OFDM systems. Fifth international conference on information, communication, and signal processing, pp. 253-257, Bangkok, 2005


The fast user growth in wireless communications has created significant demands for new wireless services in both the licensed and unlicensed frequency spectra. Since many spectra are not fully utilized most of the time, cognitive radio, as a form of spectrum reuse, can be an effective means to significantly boost communications resources. Since its introduction in late last century, cognitive radio has attracted wide attention from academics to industry. Despite the efforts from the research community, there are still many issues of applying it in practice. This book is an attempt to cover some of the open issues across the area and introduce some insight to many of the problems. It contains thirteen chapters written by experts across the globe covering topics including spectrum sensing fundamental, cooperative sensing, spectrum management, and interaction among users.

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