5

Fiber Bundles, Gauge Theories and Gravity

Rodrigo F. Sobreiro
UFF - Universidade Federal Fluminense, Instituto de Física, Campus da Praia Vermelha, Niterói, Brasil

1. Introduction

Motivated by the construction of a gravity theory independently of the metric structure of spacetime and on the stability of a quantum gravity theory many authors have developed schemes that allow a gauge theory to generate an effective metric, see for instance [1–9]. The models are constructed based on a gauge group G that possesses the Lorentz group $SO(1,3)$ as a stable subgroup. A symmetry breaking mechanism is imposed in order to G collapse to $SO(1,3)$. Mostly of these techniques are based on the de Sitter group and its variations. However, other groups are also considered such as the general linear and affine groups, see for instance [10–14], and also unitary groups [15]. The main motivation in the construction of a gauge theory of gravity that is metric independent is that the base space can be regarded as a flat one, and thus the standard quantization of gauge theories can be employed [16]. In fact, some of the cited works are in fact quantizable, at least perturbatively.

In the present work we consider the fiber bundle theory to describe gauge theories and gravity [17–20]. We then show that a gauge theory can be identified with a first order gravity if the principal bundle that describes the gauge theory can be identified with the principal bundle that describes gravity. We formally establish the conditions that the gauge theory must obey and the resulting gravity theory that emerges. The last is constructed from a mapping between the gauge principal bundle structures and the geometric setting of a gravity theory.

This work is organized as follows: In Sect. 2 we briefly review the fiber bundle description of gauge theories. Also in this section we enunciate some important results concerning reduction of principal bundles. The same approach to the first order gravity theories is displayed in Sect. 3. In Sect. 4 we discuss the emergent geometries that can be derived from a gauge theory in terms of formal theorems. In sect. 5 we collect our final remarks.

2. Gauge theories

2.1 Principal bundles for gauge theories

First we define two classes of principal bundles within gauge theories can be formally described. The first one is the principal bundle which localizes a gauge group [18] $G_R = (G, R)$ where G is a Lie group characterizing the fiber and structure group while R is the base space, a differential manifold with d_0 dimensions identified with spacetime. The total space G_R describes the localization of the Lie group G in the manifold R, assembling to each point $x \in R$ a different value for the elements of G. We shall refer to G_R as *gauge bundle*.

It is assumed that G_R is endowed with a connection 1-form Y. The connection 1-form is recognized as the gauge field, the fundamental field of gauge theories. The connection

1-form will be called gauge connection, or simply, connection. The gauge transformations are associated with coordinates changing of the total space with fixed base space coordinates, $(x, g) \to (x, g')$, which corresponds to a translation along the fiber, providing $Y(x, g) \longmapsto Y(x, g') = f^{-1}(x)\,(\mathrm{d} + Y(x, g))\, f(x)$, where $g' = gf$ and $\{g', g, f\} \subset G$. To every connection Y there is a curvature 2-form defined over G_R, namely $F = \nabla^2 = \mathrm{d}Y + YY$, where ∇ is the covariant derivative, $\nabla = \mathrm{d} + Y$, and d is the exterior derivative in R. The covariant derivative is defined from the parallel transport between fibers and the curvature is obviously recognized as the field strength in gauge theories.

The gauge connection does not belong to the former structure of the gauge bundle, it originates from a unique choice for the decomposition of the tangent space $T_q(G_R)$, in a point $q \in G_R$, into vertical and horizontal spaces. The mathematical structure that describes the dynamics of Y must contain all possible gauge connections that can be defined in G_R as well as the information of gauge transformations as the definition of equivalence classes for gauge connections. This task is achieved through the moduli bundle $\mathbb{Y} = (G_R, \mathcal{Y})$, see for instance [14, 18, 20–22]. In $\mathbb{Y}$, the fiber and structure group are the local Lie group G_R and the base space $\mathcal{Y}$ is the space of all independent connection 1-forms[1] Y, the so called moduli space. The typical fiber[2] $\pi^{-1}(Y)$ is a gauge orbit obtained from a configuration $Y(x) \in \mathcal{Y}$ and all of its possible gauge transformations $Y^g = g^{-1}(\mathrm{d} + Y)g$. Thus, the total space $\mathbb{Y}$ can be understood as the union of all gauge orbits which determine the equivalence classes in $\mathbb{Y}$.

The interpretation of the gauge and moduli principal bundles is as follows: The gauge bundle provides the localization of a Lie group and the existence of a gauge connection. To give dynamics for the connection one should consider all possible connections (together with a minimizing principle for a classical theory or a path integral measure for a quantum one [14, 22]). This dynamics is provided by the infinite dimensional moduli bundle.

2.2 Contraction of principal bundles

We now discuss some relevant results concerning gauge bundles:

Theorem 2.1. *Let $H_R = (H, R)$ be a reduced gauge bundle obtained from a former gauge bundle $G_R = (G, R)$, where $G = H \otimes K$ induces a Lie algebra decomposition $\widetilde{G} = \widetilde{H} \oplus \widetilde{K}$. If G_R is endowed with a connection form $Y = A + B$, where $A \in \widetilde{H}$ and $B \in \widetilde{K}$, then A defines a connection on H_R if, and only if, H is a stability group of G.*

Comment. This theorem[3] is a standard result [17, 23]. The formal proof can be found in [17]. It follows from the fact that a gauge transformation on a fiber $\pi^{-1}(x)$ will always keep A as a connection and B as an element of $\widetilde{K}$ as it can be seen from the decomposition of the gauge transformation in G_R. Obviously, this is a direct consequence of the stability of H. This result establishes that the original bundle imposes a connection on the reduced bundle, independently of the mechanism that led to H_R.

Corollary 2.2. *The space K defines an associated bundle $K_R = (H, R, K) \equiv H_R \times K$.*

[1] By independent we mean the set of gauge connections that cannot be related to each other through a gauge transformation, *i.e.*, they do not belong to the same equivalence class.
[2] We adopt the standard fiber bundle notation where $\pi : \mathbb{Y} \longmapsto \mathcal{Y}$ is the projection map.
[3] From now on the conditions for the validity of this theorem are assumed to hold.

Proof. The coset K is an invariant subspace with respect to the stability group H and thus a homogeneous space, which is the requirement for K to be the fiber of an associated bundle [20]. From Theorem 2.1 it is clear that a point $q \in G_R$ will split $q = (u, k)$ where $u \in H_R$ and $k \in \widetilde{K}$. Thus, we define the action of H on $H_R \times K$ by $(u, k) \longmapsto (uh, h^{-1}k)$ by taking the transitions functions to act on the fiber K while an element of the group suffers its own action from the right as allowed by the principal bundle nature of H_R. Ever since the point $x \in R$ is general, the proof holds for the entire bundle K_R. □

Comment. From Corollary 2.2 it is clear that the field B is a section over K_R [17, 18]. Thus, the component $B \in \widetilde{K}$ of the connection Y migrates to the sector of matter fields on H_R.

We consider now moduli bundles:

Theorem 2.3. *Let $\mathbb{Y} = (G_R, \mathcal{Y})$ be a moduli bundle constructed from $G_R = (G, R)$. Then the reduction $G_R \longrightarrow H_R$ induces a reduction on $\mathbb{Y}$ according to $\mathbb{Y} \longrightarrow \mathbb{A}$ where the reduced moduli bundle is $\mathbb{A} = (H_R, \mathcal{C})$. The base space $\mathcal{C} = \mathcal{A} \times \mathcal{B}$ is the decomposed moduli space of stable connections $A \in \widetilde{H}$ and independent sections $B \in \widetilde{K}$ on K_R while the fiber is the decomposed gauge orbit:*

$$A^h = h^{-1}(\mathrm{d} + A)h \,,$$
$$B^h = h^{-1}Bh \,. \tag{1}$$

Proof. Since G_R is the fiber of $\mathbb{Y}$ its reduction to H_R is equivalent to a split on the gauge orbit (1). Thus, the gauge orbit is reduced to the first of (1) where, $\mathcal{A} \subset \mathcal{Y}$, represented by independent elements $A \in \widetilde{H}$, define the reduced moduli space of connections. The space $\mathcal{B} = \mathcal{Y}/\mathcal{A}$, on the other hand, is the set of all fields B that cannot be related through a gauge transformation. Thus, a point in the base space can be defined as $C = (A, B)$ and the fiber is constructed by the action of H as $C \longmapsto C^h = (A^h, B^h)$. The reduced total space is the union of all reduced gauge orbits. The stable character of H ensures that there will be no mixing between the spaces $\mathbb{A}$ and $\mathbb{B} = \mathbb{Y}/\mathbb{A}$ along any gauge orbit. □

Comment. The infinite dimensional space $\mathcal{B}$ is equivalent to the set of all independent sections $B(x)$ that can be defined in K_R. Thus, the space $\mathbb{B}$ is the collection of all possible sections in K_R. The space $\mathbb{B}$ can be also understood as the fiber bundle $\mathbb{B} = (\Sigma(B), H_R, \mathcal{B})$ where the base space is $\mathcal{B}$ and a fiber $\Sigma(B)$ is the collection of all equivalent sections for a given $B \in \mathcal{B}$.

Corollary 2.4. *Define a composite field θ, which is an invariant representation of H, that can be constructed from the original set of connections. For each base space point C there is only one field $\theta(C)$. If an equivalence class C^h is defined then $\theta^h = \theta(C^h)$ is on the same equivalence class of $\theta(C)$ where $\theta^h = h\theta$.*

Proof. The field θ is, by construction, an invariant representation of H, thus, it transforms as $\theta \longmapsto \theta^h = h\theta$. The last expression defines the equivalence class for θ. Now, since $\theta = \theta(C)$, then $\theta(C')$, constructed in another point C', belongs to the same equivalence class of the original field if $\theta(C') = h\theta(C)$. However, the transformation of θ is induced by the action of the group on its dependence on C. Thus, $\theta(C') = \theta(C^g)$ for an element $g \in H$. Using again the definition of θ as an invariant representation, we have that $\theta(C^g) = g\theta(C)$. Thus, $g = h$. □

Comment. The field θ is a one to one map $\theta : C \longmapsto \theta(C)$ which establishes that for at each point C there will be only one θ such that if $C \sim C'$ then $\theta \sim \theta'$. In other words, in each fiber C^h there will be only one equivalence class for θ.

3. First order gravity

Gravity can be mathematically defined as a coframe bundle [12, 13, 20, 21], $C_M = (GL(d,\mathbb{R}), M)$, where M is a d-dimensional spacetime manifold. The structure group and fiber have a more deep meaning: In each point $X \in M$ one can define the cotangent space $T^*_X(M)$. The fiber is the collection of all coframes e that can be defined in $T^*_X(M)$ and which are related to each other through the action of the general linear group. As a consequence, the fiber is actually the group $GL(d,\mathbb{R})$. In terms of Sect. 2, the coframe bundle is also a gauge bundle for the general linear group with the addend that the gauge group is identified with geometric properties of M. The action of the group from the right are local gauge transformations while the action of the group from the left are general coordinate transformations.

Geometrically, the gauge connection Γ is related to the parallel transport, in M, between two near cotangent spaces. The curvature 2-form is obtained from the double action of the covariant derivative, $\Omega = \mathrm{d}\Gamma + \Gamma\Gamma$ while torsion, $T = \nabla e$, is the minimal coupling of coframes. It is evident that, besides Γ, which is the gauge field of gravity, e is just as relevant. Moreover, the metric tensor m in the tangent space $T(M)$ has to be introduced because the $GL(d,\mathbb{R})/SO(d-n,n)$ sector of the general linear group does not preserve a flat metric. In practice m enters as an extra independent field. Thus, in C_M, gravity possesses three fundamental fields, Γ, e and m, all relevant to determine spacetime geometry. A general theory of this type is a metric-affine gravity[4] [10–13].

Remarkably, the coframe bundle has a contractible piece $GL(d,\mathbb{R})/SO(d-n,n)$ where $SO(d-n,n)$ is obviously a stability group. This means that the coframe bundle can be naturally contracted down to $SO_M = (SO(d-n,n), M)$ [14, 17, 19]. The fact that the contraction is topologically favored has drastic consequences to the geometry, it means that every manifold M can assume a Riemannian metric, *i.e.*, the connection can always be chosen to be compatible with the metric. This means that the metric tensor can be set as a constant flat one, $m = \eta$, where the signature of η depends on n. As a consequence, a standard fiber at X is the set of all orthonormal coframes that can be obtained from an $SO(d-n,n)$ transformation acting on a fixed coframe. The group $SO(d-n,n)$ describes then the isometries in $T^*_X(M)$. From Theorem 2.1 the connection $\Gamma = \omega + w$ imposes an $SO(d-n,n)$ connection $\omega \in \widetilde{O}$, where $\widetilde{O}$ is the algebra of $SO(d-n,n)$ and $w \in \widetilde{GL}/\widetilde{O}$. A gravity theory constructed over SO_M is a standard Einstein-Cartan gravity. In this work we shall deal strictly with SO_M.

To construct a moduli bundle for gravity is not immediate as in pure gauge theories. If the coframe bundle is a gauge bundle then e is actually a matter field because it is a fundamental representation of the gauge group [24]. On the other hand, one can include the space of all independent e that can be defined in $T^*_X(M)$ as the coframe moduli space $\mathcal{E}$. Thus, defining the full moduli space as $\mathcal{G} = \mathcal{W} \times \mathcal{E}$, where $\mathcal{W}$ is the moduli space of spin-connections, the

[4] Metric-Affine gravities can be also generalized for the affine group $A(d,\mathbb{R}) = GL(d,\mathbb{R}) \ltimes \mathbb{R}^d$, however, the non-semi-simplicity of this group spoils the construction of an invariant action. We shall fix our attention to semi-simple groups.

gauge orbit is then

$$\omega^g = g^{-1}(\mathrm{d}+\omega)g \,,$$
$$e^g = ge \,, \tag{2}$$

with $g \in SO(d-n,n)$ and $W = (\omega, e) \in \mathcal{G}$. The moduli coframe bundle is $\mathbb{O} = (SO_M, \mathcal{G})$. This principal bundle is analogously equivalent to that described in Theorem 2.3. Thus, the space of all sections that can be defined over SO_M is actually the functional space of coframes. This space is equivalent to the fiber bundle $\mathbb{E} = (\Sigma(e), SO_M, \mathcal{E})$ where the fiber $\Sigma(e)$ is the set of all equivalent sections that can be obtain from an element $e \in \mathcal{E}$ through the action of SO_M.

4. Effective geometries

We now discuss the possibility of a gauge theory to be mapped into a gravity theory. We first discuss the map between gauge and coframe bundles and then we generalize the results for moduli bundles.

4.1 Gauge and coframe bundles

Theorem 4.1. *Let $H_R = (H, R)$ be a stable reduced bundle obtained from the gauge bundle $G_R = (G, R)$ which is endowed with a connection $Y = A + B$. Then G_R can define a geometry $SO_M = (SO(d-n,n), M)$ if and only if*

1. *The base spaces R and M are isomorphic;*
2. *The structure groups H and $SO(d-n,n)$ are related, at least, by a surjective homomorphism;*
3. *A composite field θ, which is an invariant representation of H, can be identified with an invariant fundamental representation of $SO(d-n,n)$.*

Proof. Condition *1* ensures that each point $x \in R$ can define a unique point in $X \in M$ while M will be entirely covered by the map with no overlapping points. Moreover, the algebraic structure defined in R will be preserved by the mapping. On the other hand, condition *2* ensures that the target group $SO(d-n,n)$ will be entire covered by the mapping. To construct the fiber at a cotangent space $T^*_X(M)$ in each point $X \in M$ we need two quantities: a coframe $e \in T^*_X(M)$ and the isometries of the cotangent space. The use of conditions *1* and *2* ensures the existence of the isometries. Since there is a fiber H in each point $x \in R$ and condition *2* ensures that H is at least homomorphic to $SO(d-n,n)$, this fiber defines the cotangent space $T^*_X(M)$ isometries. In addition, since there is one fiber for each point $x \in R$ there will be only one set of isometries for each $X \in M$, as it is evident from condition *1*. Condition *3* ensures that the field θ, in the fiber H_R, can be defined as the cotangent 1-form $e \in T^*_X(M)$, recognized as a coframe. Once more, the isomorphism of condition *1*, together with Corollary 2.4, ensures the uniqueness of e in X. Finally, a standard fiber in SO_R is obtained by the action of $SO(d-n,n)$ on e. A connection ω in SO_M emerges naturally from A. Again, condition *1* establishes that at a point X there will be only one ω while the action of H on A ensures that ω will transform correctly along the fiber $\pi^{-1}(X)$ under the action of the local isometries in $T^*_X(M)$. More explicitly, In each fiber $\pi^{-1}(x)$ a connection A can be defined. This definition ensures the existence of an equivalence class along the fiber. Thus, a section $s(x) : R \longmapsto H(x)$ is defined in such a way that $x \longmapsto q$, where $q = (x, g)$ and $g \in G$. In each point q the connection $A(q)$ can be identified with a connection $\omega(Q)$ in $SO(X)$ at a point $Q = (X, u) \in SO(X)$ and $u \in SO(d-n,n)$ is the $SO(d-n,n)$ equivalent of g such that $\pi(Q) = X$, $\pi(q) = x$ and $x \longmapsto X$. Condition *1* ensures that there will be only one connection $\omega(Q)$ for each $A(q)$.

Obviously, the reconstruction of the whole class of connections along a fiber is obtained from the action of the group on $\omega(Q)$. The final result is a mapping $G_R \longmapsto SO_R$ which is actually a set of mappings

$$\begin{aligned} R &\longmapsto M\ , \\ H &\longmapsto SO(d-n,n)\ , \\ \theta &\longmapsto e\ , \\ A &\longmapsto \omega\ . \end{aligned} \tag{3}$$

We remark that, in the mapping $G_R \longmapsto SO_R$, a contraction $G_R \longrightarrow H_R$ is assumed. □

Comment. If the map described in Theorem 4.1 is smooth and all fibers $\pi^{-1}(x)$ are mapped into fibers $\pi^{-1}(X)$ then this map is a bundle map. In that case, since each fiber $\pi^{-1}(x)$ is mapped into a fiber $\pi^{-1}(X)$ in a smooth way, a smooth map $R \longmapsto M$ is induced [20].

Comment. We remark that $\dim M = \dim T^*_X(M)$ and thus $\dim R = d_o$ is not necessary equal to $\dim M = d$. Notwithstanding, the bound $d \le d_o$ is a always valid. Furthermore, d is the dimension of the fundamental representation of $SO(d-n,n)$, as a consequence it coincides with the dimension of the invariant representation of H, namely θ. The case $d < d_o$ affects only a subsector of spacetime $R \supset R_{sub} \longmapsto M$, where $\dim R_{sub} = \dim M$. For instance, if $R = \mathbb{R}^{d_o}$, then the resulting full manifold is then $M \times \mathbb{R}^{d_o - d}$. The case $d = d_o$ deforms the entire spacetime. This case is more interesting because one can take the starting gauge theory as a description for quantum gravity. from now on, independently of the case, we shall call by M the full d_o-dimensional manifold formed by the deformed (d-dimensional subspace) and undeformed (($d_o - d$)-dimensional subspace) sectors.

Corollary 4.2. *If the space of p-forms in R are directly mapped into the space of p-forms in M, $\Pi^p_R \longmapsto \Pi^p_M$, then the map can be explicitly computed and depends exclusively on the metric tensors of R and M.*

Proof. By duality the map $\Pi^p_R \longmapsto \Pi^p_M$ induces a similar map for the Hodge dual space of $(d-p)$-forms, $*\Pi^p_R \longmapsto \star\Pi^p_M$, where $*$ is the Hodge operation in R while $\star$ is the Hodge operation in M. Thus, it is a straightforward exercise [9] to show that the map is given by

$$\frac{\partial X^\nu}{\partial x^\mu} = \left(\frac{\tilde{m}}{m}\right)^{1/2d} \tilde{m}^{\nu\alpha} m_{\alpha\mu}\ , \tag{4}$$

where $m_{\mu\nu}$ is the metric tensor in R, $\tilde{m}_{\mu\nu}$ is the metric tensor in M and m and $\tilde{m}$ are the respective determinants. The determinants are assumed to be non-vanishing. □

Comment. Since the mapping matrix (4) has an inverse, the geometry in M is unique.

4.2 Moduli bundles and gravity

Theorem 4.1 can be generalized for moduli bundles:

Theorem 4.3. *If the map $G_R \longmapsto SO_M$ exists then the map $\mathbb{Y} \longmapsto \mathbb{O} \oplus \tilde{\mathbb{B}}$ also exists. The space $\tilde{\mathbb{B}}$ is the target space associated with the space $\mathbb{B}$ or $\mathbb{B}/\Theta$ if $\Theta \subseteq \mathbb{B}$ where Θ is the functional space of all possible θ that can be defined in $\mathbb{Y}$.*

Proof. In a principal principal bundle G_R a connection Y can be defined. The collection of all possible connections Y defines the space $\mathbb{Y}$. The separation of $\mathbb{Y}$ into equivalence classes organizes this space as the set of all gauge orbits over the moduli space $\mathcal{C}$. According to Theorem 2.3, the gauge orbit splits as (1). Moreover, for each of these fibers one can associate a field θ, as allowed by Corollary 2.4. It is the pair (A, θ) that defines the geometric fields (ω, e) in SO_M. Thus, to construct an $\mathbb{O}$ structure for gravity is an easy task to collect all possible pairs $W = (\omega, e)$ emerging from Theorem 4.1. In fact, each pair (A, θ) and the associated orbit define a fiber $W^g \in \mathbb{O}$. That is ensured also by Condition *1* of Theorem 4.1. Thus, the fiber H_R over a point $(\omega, e) \in \mathcal{G}$ is obtained from $A \longmapsto \omega$ and $\theta(A, B) \longmapsto e$ and the respective action of $SO(d-n, n)$. The uniqueness of this mapping is ensured by Corollary 2.4.

The space $\mathbb{B} = (\Sigma(B), H_R, \mathcal{B})$ is a dynamical space and survives the mapping. For the case $\mathbb{B} \cap \Theta = \emptyset$ one can associate the moduli space with a set of independent fields $\mathcal{B} \longmapsto \tilde{\mathcal{B}}$ which are invariant representations of $SO(d-n, n)$. Theorem 4.1 ensures that the structure group H_R can be mapped into SO_M while the fibers $\Sigma(B)$ are identified with fibers $\Sigma(\tilde{B})$ over $\tilde{B}$. Thus, for each $B \in \mathbb{B}$ there will be a correspondent $\tilde{B} \in \tilde{\mathcal{B}}$ and the fiber $\Sigma(\tilde{B})$ is obtained from the action of $SO(d-n, n)$. Thus, $\tilde{\mathbb{B}} = (\Sigma(\tilde{B}), SO_M, \tilde{B})$. The proof for the case $\mathbb{B} \cap \Theta \neq \emptyset$ is totally equivalent. □

Comment. The final result is that of a gravity theory with an extra set of matter fields $\tilde{\mathbb{B}}$.

5. Final remarks

We have formally prove that a class of gauge theories can be deformed into a first order gravity theory and, possibly, with an extra set of matter fields. For that we have employed the theory of fiber bundles. The relevance (and motivation) of the present work is that it can be applied to quantum gravity models which are based on gauge theories that can generate an emergent gravity theory. The main problem in quantizing gravity is that the principles of general relativity are incompatible with those of quantum field theory. In fact, a quantum field theory can only be formulated in an Euclidean spacetime. For example, a quantum field is, by definition, an object that carries uncertainty fluctuations and is parametrized through spacetime coordinates, *i.e.*, a set of well defined real parameters. Now, if a coframe field is a quantum field[5], $\hat{e}(x)$, and from the fact that it defines a mapping from tangent coordinates x^a to world coordinates x^μ, then quantum fluctuations of $\hat{e}$ will induce spacetime to fluctuate as well, $\hat{x}^\mu = \hat{e}^\mu_a x^a$. Thus, a paradox is encountered because x must be a set of parameters instead of a fluctuating object.

On the other hand, if the starting gauge theory is constructed over an Euclidean manifold and it is renormalizable, then it can be an excellent candidate for a quantum gravity theory. All needed is that it emerges as a geometrodynamics at classical level. The class of theories that fits on this program are determined essentially by theorems 4.1 and 4.3.

A few practical examples are in order: In [15] a 4-dimensional $SU(2)$ gauge theory generates a deformation of the 3-dimensional space. Time is left untouched by he mapping. In this example, the resulting theory contains the Einstein-Hilbert action for the extrinsic curvature and the solution of the equations of motion predicts not only curvature but also torsion. Another example can be found in [9], where a deformed 4-dimensional spacetime emerges from a de Sitter type gauge theory over an Euclidean spacetime. In this case, a dynamical mass scale is responsible for the separation between the gauge and gravity phases. In general,

[5] The hat indicates the quantum nature of the field.

several emergent gravity theories fit to the results of this work, see for instance [1–8] where the Higgs mechanism is largely used to separate gauge and gravity phases.

We end this work by remarking that several issues are left for future investigation. Just to name a few: The role of matter fields living in the starting gauge theory; the generalization of the present results to include metric-affine gravities before the reduction of the coframe bundle; the role of the extra matter fields in the dark matter/energy problem; explicit computations in order to make reliable predictions that fit with actual data; and so on.

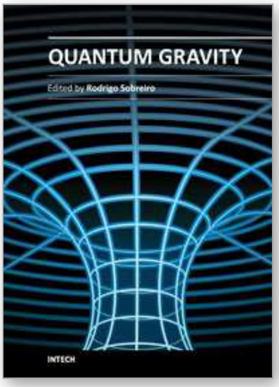
6. Acknowledgements

Conselho Nacional de Desenvolvimento Científico e Tecnológico[6] (CNPq-Brazil) is acknowledge for financial support. I also would like to express my gratitude to InTech for this opportunity.

7. References

[1] S. W. MacDowell, F. Mansouri, Phys. Rev. Lett. 38, 739 (1977).
[2] K. S. Stelle, P. C. West, J. Phys. A A12, L205-L210 (1979).
[3] K. S. Stelle, P. C. West, Phys. Rev. D21, 1466 (1980).
[4] A. A. Tseytlin, Phys. Rev. D26, 3327 (1982).
[5] H. R. Pagels, Phys. Rev. D29, 1690 (1984).
[6] P. Mahato, Phys. Rev. D70, 124024 (2004).
[7] R. Tresguerres, Int. J. Geom. Meth. Mod. Phys. 5, 171-183 (2008).
[8] E. W. Mielke, Phys. Lett. B688, 273-277 (2010).
[9] R. F. Sobreiro, A. A. Tomaz and V. J. V. Otoya, arXiv:1109.0016 [hep-th].
[10] F. W. Hehl, G. D. Kerlick and P. Von Der Heyde, Phys. Lett. B 63, 446 (1976).
[11] F. W. Hehl, J. D. McCrea, E. W. Mielke and Y. Ne'eman, Phys. Rept. 258, 1 (1995) [arXiv:gr-qc/9402012].
[12] A. Trautman, Czech. J. Phys. B 29, 107 (1979).
[13] A. Trautman, "Fiber Bundles, Gauge Fields, And Gravitation," *In *Held.A.(Ed.): General Relativity and Gravitation, Vol.1*, 287-308*
[14] R. F. Sobreiro, V. J. Vasquez Otoya, J. Geom. Phys. 61, 137-150 (2011).
[15] Y. .N. Obukhov, Theor. Math. Phys. 117, 1308-1318 (1998).
[16] C. Itzykson and J. B. Zuber, "Quantum Field Theory," New York, Usa: Mcgraw-hill (1980) 705 P.(International Series In Pure and Applied Physics)
[17] S. Kobayashi and K. Nomizu, "Foundations of Differential Geometry, Vol. 1" *New York, USA: John Wiley & Sons (1963).*
[18] M. Daniel and C. M. Viallet, Rev. Mod. Phys. 52, 175 (1980).
[19] C. Nash and S. Sen, "Topology And Geometry For Physicists," *London, Uk: Academic (1983) 311p.*
[20] M. Nakahara, "Geometry, topology and physics," Bristol, UK: Hilger (1990) 505 p. (Graduate student series in physics).
[21] R. A. Bertlmann, "Anomalies in quantum field theory," Oxford, UK: Clarendon (1996) 566 p. (International series of monographs on physics: 91).
[22] R. F. Sobreiro and V. J. V. Otoya, J. Phys. Conf. Ser. 283, 012032 (2011) [arXiv:1010.2946 [hep-th]].
[23] B. McInnes, Class. Quant. Grav. 1, 1 (1984).
[24] V. A. Rubakov, "Classical theory of gauge fields," Princeton, USA: Univ. Pr. (2002) 444 p

[6] RFS is a level PQ-2 researcher under the program *Produtividade em Pesquisa*, 304924/2009-1.



Quantum Gravity

Edited by Dr. Rodrigo Sobreiro

ISBN 978-953-51-0089-8
Hard cover, 100 pages
Publisher InTech
Published online 15, February, 2012
Published in print edition February, 2012

The unification between gravity and quantum field theory is one of the major problems in contemporary fundamental Physics. It exists for almost one century, but a final answer is yet to be found. Although string theory and loop quantum gravity have brought many answers to the quantum gravity problem, they also came with a large set of extra questions. In addition to these last two techniques, many other alternative theories have emerged along the decades. This book presents a series of selected chapters written by renowned authors. Each chapter treats gravity and its quantization through known and alternative techniques, aiming a deeper understanding on the quantum nature of gravity. Quantum Gravity is a book where the reader will find a fine collection of physical and mathematical concepts, an up to date research, about the challenging puzzle of quantum gravity.

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821