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GNSS in Practical Determination of Regional Heights

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1. Introduction

Describing the position of a point in space, basically relies on determining three coordinate components: the Cartesian coordinates (X, Y, Z) in rectangular coordinate system or latitude, longitude and ellipsoidal height (φ, λ and h) in ellipsoidal coordinate system, referred to any given reference ellipsoid. Today, of course, global navigation satellite systems (GNSS) is the best and most popular method for determining φ, λ and h, directly. The instantaneous determination of position and velocity on a continuous base, and the precise coordination of time are included in the objectives of GNSS, and positioning with GNSS base on ranging from known positions of satellites in space to the unknown positions on the earth or in space. Besides the geometrically described coordinates however, the natural coordinates, the astrogeodetic latitude, longitude and orthometric height (Φ, Λ, H), which directly refer to the gravity field of the earth, are preferable to take for many special purposes. In particular the orthometric heights above the geoid are required in many applications, not only in all earth sciences, but also in other disciplines such as; cartography, oceanography, civil engineering, hydraulics, high-precision surveys, and last but not least geographical information systems. Traditionally, these heights are determined by combining geometric levelling and gravity observations with millimetre precision in smaller regions. This technique, however, is very time consuming, expensive and makes providing vertical control difficult, especially in mountainous areas which are hard to access. Another disadvantage is the loss of precision over longer distances since each height system (regional vertical datum) usually refers to a benchmark point close to the sea level, which is connected to a tide gauge station representing the mean sea level (Hofmann-Wellenhof & Moritz, 2006).

In order to counteract these drawbacks of levelling, GNSS introduces a revolution also in the practical determination of the heights in regional vertical datum depending on the basic relation $H = h - N$ among the heights. This equation relates the orthometric height H (above the geoid), the ellipsoidal height h (above the ellipsoid), and the geoidal undulation N, as such, when the h is provided by GNSS and N exists from a reliable and precise digital geoid map, the orthometric height H can then be obtained immediately. This alternative technique for the practical determination of H is called GNSS levelling. In the recent decades the wide and increasing use of GNSS in all kinds of geodetic and surveying applications demands modernization of vertical control systems of countries. The current position is that, the most developed countries are concentrating efforts on establishing a dynamic geoid based vertical datum accessible via GNSS positioning (see e.g. Rangelova et al., 2010). Besides enabling the accurate determination of most up-to-date geoidal heights under the effects of secular dynamic changes of the earth for GNSS levelling purposes, it is envisioned that this new datum concept, will also provide a compatible vertical datum with global height system, which is crucial for studies related to large scale geodynamics and geo-hazards processes.

The accurate determination of orthometric heights via GNSS levelling requires a centimetre(s) accuracy of the geoid model. The level of achievable accuracy of the models varies depending on the computational methodology (assumptions used) and available data within the region of interest (Featherstone et al., 1998; Fotopoulos, 2003; Fotopoulos et al., 2001; Erol, 2007; Erol et al., 2008). The regional models provide better accuracies in comparison to global models. However, for many parts of the globe a high precision regional geoid model is not accessible usually due to lack of data. In these cases, depending on the required accuracy level of the derived heights, one may resort to applying global geopotential model values. An alternative way to determining discrete geoid undulation values is the geometric approach. The approach, which works well in relatively small areas, utilises the relationship between the GNSS ellipsoidal and regional orthometric heights at the known points to interpolate new values. In determining orthometric height with GNSS levelling, apart from consideration for the error budgets of each height data (h, H, N), it will also be necessary to take into account the systematic shifts and datum differences among these data sets, which also restrict the precision of determination. Since the regional vertical datum is not necessarily coincident with the geoid surface, the discrepancies between the regional vertical datum and the geoid surface are preferably accounted for using a special technique allowing for an improved computation of the regional heights with GNSS coordinates (Fotopoulos, 2003; Erol, 2007).

This chapter aims to review the geoid models for GNSS levelling purposes in Turkey and mapping the progress of the global and regional geoid models in Turkish territory. In this respect the study consists of two parts; the first part provides validation results of the recently released eight global geopotential models from satellite gravity missions namely; EGM96, EGM08 (of full expansion and up to 360 degree), EIGEN-51C, EIGEN-6C, EIGEN-6S, GGM03C, GGM03S and GOCO02S, as well as two Turkish regional geoid models TG03 and TG09, based at 28 homogeneously distributed reference benchmarks with known ITRF96 coordinates and regional orthometric heights. The validations consist of comparison of the geoid undulations between the used models and the observed height data (h, H). It should be noted that the results from the validations were evaluated against the reported precisions of the models by the responsible associations.

The second part of this chapter focuses on the determination and testing of the geoid models using geometrical approach in small areas and their assessments in GNSS levelling. In the numerical evaluation, two geodetic networks were used. Each network had 1205 and 109 reference benchmarks with known ITRF96 positions and regional vertical heights, established in these neighbour local areas. Since the topographical character, distribution and density of the reference benchmarks at each area were totally different, these networks provided an appropriate test bed for the local geoid evaluations. In the coverage of the second part, each network was evaluated independently. A group of modelling algorithms were run using the reference benchmarks and tested at the independent test benchmarks of each network. The applied modelling techniques for local geoids (ranging from simple to more comlex methods) including, the multivariable polynomial regression and artificial neuro-fuzzy inference systems (ANFIS). In the light of these conclusions, the roles of topography of the area of interest, the distribution and density of the reference benchmarks, and computation algorithm used in the precise determination of the geoid model and therefore the accuracy of regional heights from the GNSS levelling, were investigated. In addition to the investigation and review on local geoids, local improvement of the recent Turkish regional geoid using 31 reference benchmarks of Çankırı GNSS/levelling networks has also been included. The next section has been structured accordingly to report on these areas.

The outline for this chapter is as follows; the first section provides background information regarding the geoid models, the height data used to conduct the research are also presented and explained. As special emphasis has been given to the error sources affecting the used heights (h, H) and thus the accuracy of the geoid model (N), information relating to the global geoid models (EGM96, EGM08, EIGEN-51C, EIGEN-6C, EIGEN-6S, GGM03C, GGM03S and GOCO02S) and Turkish regional geoid models (TG03, TG09) has also been included in this section. In addition to an overview of the aforementioned models, a list of related references where they have been used in previous studies is provided for further reading purposes. Furthermore the numerical validations of the explained models are included within a sub-heading and validation results appropriately presented as graphics and tables.

The second section is focused on the methodology, and the theoretical background of the applied methods used in the calculation of the local geoid models. The local improvements of regional models are also summarized, and corresponding literature provided. The merits and limitations of each method are also referred to in this section. The outlined methodology was implemented using the three test network's data, in order to test the computation algorithms and demonstrate the role of the data and topographical patterns in geometrical modelling of the local geoids and local improvements of regional geoids. The findings are presented as graphics and tables.

The last section summarises the main conclusions of this research and some practical considerations for modernizing vertical control, as parallel to GNSS development are presented. This section therefore essentially focuses on how to evaluate the achievable accuracy of GNSS levelling. A brief discussion outlines some of the key concepts for providing users of GNSS with the proper information to transform ellipsoidal heights to heights associated with a regional vertical datum. To conclude the chapter, recommendations for future work in this area are also provided.

2. Global and regional geoid models: Methodology and data

GNSS ellipsoidal heights are purely geometric definitions and do not refer to an equipotential surface of the earth's gravity field, as such they cannot be used in the same way as conventional heights derived from levelling in many applications. In order for GNSS derived ellipsoidal heights to have any physical meaning in application, they must be transformed to orthometric heights referring to mean sea level (geoid). This transformation is applied using the geoidal heights (N) from a geoid model that must be known with sufficient accuracy (Fotopoulos et al., 2001; Fotopoulos, 2005). The computation methods of geoid models are many (Schwarz et al., 1987; Featherstone, 1998; Featherstone, 2001; Hirt and Seeber, 2007; Erol et al., 2008; Erol et al., 2009). The most commonly used methods for geoid surface construction are described in textbooks like Heiskanen & Moritz (1967), Vaniček & Krakiwsky (1986), Torge (2001). The so-called remove-restore (R-R) procedure is one of these methods; where a global geopotential model and residual topographic effects are subtracted (and later added back) (see Equations 1 and 2). The smooth resulting data set is then suitable for interpolation or extrapolation using for example least squares collocation with parameters (Sideris, 1994). According to R-R method, the reduced gravity anomaly is:

$$\Delta g = \Delta g_{FA} - \Delta g_{GM} - \Delta g_{H} \tag{1}$$

and the computed geoid height is:

$$N = N_{GM} - N_{\Delta g} - N_{ind} \tag{2}$$

where Δg_{GM} is the effect of the global geopotential model on gravity anomalies, Δg_{H} is the terrain effect on gravity, $N_{\Delta g}$ is the residual geoid height, which is calculated using Stokes integral (see Equation 3), N_{ind} is the indirect effect of the terrain on the geoid heights and N_{GM} is the contribution of the global geopotential model (expressed by the Equation 4), (Heiskanen & Moritz, 1967; Sideris, 1994). The residual geoid height, computed from Stokes's equation is;

$$N_{\Delta g} = \frac{R}{4\pi}\iint_{\sigma} \Delta g S(\Psi)\, d\sigma \tag{3}$$

where σ denotes the Earth's surface, Δg is the reduced gravity anomaly (Equation 1) and $S(\Psi)$ is the Stokes kernel function where Ψ is the spherical distance between the computation and running points (Haagmans et al., 1993; Sideris; 1994).

The global geopotential model derived geoid height using spherical harmonic coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, is;

$$N_{GM} \approx R \sum_{\ell=2}^{\ell \max} \sum_{m=0}^{\ell} \bar{P}_{\ell m}(\sin\theta)\left[\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right] \tag{4}$$

where R is the mean radius of the Earth, (θ, λ) are co-latitude and longitude of the computation point, $\bar{P}_{\ell m}$ are fully normalized Legendre functions for degree ℓ and order m, and ℓ_{max} is the maximum degree of the global geopotential model (Heiskanen & Moritz, 1967).

Following the Equation 2, it is obvious that the accuracy of the computed geoid heights depends on the accuracy of the three height components, namely N_{GM}, $N_{\Delta g}$ and N_{H} (Fotopoulos, 2003). The global geopotential model not only contributes to the long wavelength geoid information but also introduces long-wavelength errors that originate from insufficient satellite tracking data, lack of terrestrial gravity data and systematic errors in satellite altimetry. The two main types of errors can be categorized as either omission or commission errors. Omission errors occur from the truncation of the spherical harmonic series expansion (Equation 4), which is available in practice ($\ell_{max} < \infty$). The other major contributing error type is due to the noise in the coefficients themselves and termed as commission errors. As the maximum degree ℓ_{max}, of the spherical harmonic expansion increases, so does the commission error, while the omission error decreases. Therefore, it is important to strike a balance between the various errors. In general, formal error models should include both omission and commission error types in order to provide a realistic measure of the accuracy of the geoid heights computed from the global geopotential model. In the following section, recently released global geopotential models using the data from low earth orbiting missions such as CHAMP, GRACE and GOCE are exemplified and their performances in Turkish territory investigated. Parallel to the improvements in techniques, the new global geopotential models derived by incorporating the satellite data from these missions are quite promising (Tscherning et al., 2000; Fotopoulos, 2003).

The other errors in the budget contributing to the $N_{\Delta g}$ component stem from the insufficient data coverage, density and accuracy of the local gravity data. Obviously, higher accuracy is implied by accurate Δg values distributed evenly over the entire area with sufficient spacing, however there are some systematic errors such as datum inconsistencies, which influence the quality of the gravity anomalies too. The shorter wavelength errors in the geoid heights are introduced through the spacing and quality of the digital elevation model used in the computation of N_H. Improper modelling of the terrain is especially significant in mountainous regions, where terrain effects contribute significantly to the final geoid model. This is in addition to errors relating to the approximate values of the vertical gravity gradient (Forsberg, 1994). Improvements in geoid models according to the computation of N_H, will be seen through the use of higher resolution (and accuracy) digital elevation data, especially in mountainous regions.

2.1 Testing global geoid models

The global geopotential model used as a reference in the R-R technique has the most significant error contribution in the total error budget of the computed regional geoid models. Therefore employing an appropriate global model in R-R computations is of primary importance. Likewise, in areas where regional models exist, they should be used as they are more accurate compared to global models. However, many parts of the globe do not have access to a regional geoid model, usually due to lack of data. In these cases, one may resort to applying global geopotential model values (Equation 4) that best fit the gravity field of the region. Determining the optimal global model for either, using the base model in R-R construction of the regional geoid or estimating the geoid undulations in the region with a relatively low accuracy, it will be necessary to undertake a comparison and validation of the models with independent geoid and gravity information, such as GNSS/levelling heights and gravity anomalies (Gruber, 2004; Kiamehr & Sjöberg, 2005; Merry, 2007).

The global geopotential models are mainly divided into three groups based on the data used in their computation, namely satellite-only (derived from the tracking of artificial satellites), combined (derived from the combination of a satellite-only model with terrestrial and/or airborne gravimetry, satellite altimetry, topography/bathymetry) and tailored (derived by refining existing satellite-only or combined global geopotential models using regional gravity and topography data) models. Satellite-only models are typically weak at coefficients of degrees higher than 60 or 70 due to several factors, such as the power-decay of the gravitational field with altitude, modelling of atmospheric drag, incomplete tracking of satellite orbits from the ground stations etc. (Rummel et al., 2002). Although the effects of some of these limitations on the models decreased after the dedicated satellite gravity missions CHAMP, GRACE and GOCE (GGM02, 2004; GFZ, 2006; GOCE, 2009), the new satellite-only models still have full power until a certain degree, however rapidly increasing errors make their coefficients unreliable at high degrees (see e.g. Tapley et al., 2005; ICGEM, 2005). Whilst, the application of combined models reduce some of the aforementioned limitations, the errors in the terrestrial data effectively remain the same.

Theoretically, the observations, used in computation of the global models, should be scattered to the entire earth homogenously, but it is almost impossible to realise this exactly. As such, accuracy of quantities computed via global geopotential models, such as geoid undulation (Equation 4), is directly connected to the quality and global distribution of gravity data as well as to the signal power of satellite mission. The distribution and the availability of quality gravity data therefore plays a major role in the global model-derived values in different parts of the Earth. It may however be argued that, the various models may not be as good as they are reported to be, otherwise the differences between them should not be so great as they are (Lambeck & Coleman, 1983). As such, validating the models in local scale with in situ data before using them with geodetic and geophysical purposes is highly important (Gruber 2004). In this manner, Roland & Denker (2003) evaluated the fit of some of the global models to the gravity field in Europe using external data such as GPS/levelling and gravity anomalies. Furthermore, Amos & Featherstone (2003) included astrogeodetic vertical deflections at the Earth surface in the external data for validating the EGMs at that date in Australia. Similar evaluations were also undertaken by Kiamehr & Sjöberg (2005), Abd-Elmotaal (2006), Rodriguez-Caderot et al (2006), Merry (2007) and Sadiq & Ahmad (2009) in Iran, Egypt, Southern Spain, Southern Africa, Pakistan, respectively. Satellite altimeter data and orbit parameters were also used by Klokočník et al (2002) and Förste et al (2009) in comparative assessments of the EGMs. Erol et al (2009), Ustun & Abbak (2010) and Yılmaz & Karaali (2010) provided some specific results on spectral evaluation of global models and on their local validations using terrestrial data in territory of Turkey. Motivated research conducted by Lambeck&Coleman (1983) and Gruber (2004), we tested some of the recent global geopotential models having various orders of spherical harmonic expansion for Turkish territory, the results of which have been recorded later in this chapter. The listed global geopotential models in Table 1 were validated at 28 GNSS/levelling benchmarks, homogenously distributed over the country. The table provides the maximum degrees of the harmonic expansions, the data contributed for developing the models and also the principle references for further reading on these models. The reference data for validations are included by Yılmaz & Karaali (2010), hence the results from the models evaluated in both studies are comparable (see Figure 1 for the distribution of the benchmarks).

In evaluations, the geoid heights derived from the models (Equation 4) were compared with observations at the benchmarks, and the statistics of comparisons (see Table 2) were investigated. In the validation results, superiority of ultra-high resolution models EIGEN-6C (ℓ_{max} = 1420) and EGM08 (ℓ_{max} = 2190) in representing the gravity field in the region is naturally obvious given that these models comprise information relating to full content of gravity field spectrum. Considering the ±16.3 cm and ±17.9 cm accuracies of EIGEN-6C and

EGM08 in terms of root mean square errors of geoid heights, these models can be employed to obtain regional orthometric heights from GNSS heights for the applications that require a decimetre level accuracy in heights.

*Model	Degree	Data	Citation
EGM96	360	Satellite, gravity, altimetry	Lemoine et al., 1998
EGM08[a]	360	GRACE, gravity, altimetry	Pavlis et al., 2008
EGM08[b]	2190	GRACE, gravity, altimetry	Pavlis et al., 2008
EIGEN-6C	1420	GOCE, GRACE, LAGEOS, gravity, altimetry	Förste et al., 2011
EIGEN-6S	240	GOCE, GRACE, LAGEOS	Förste et al., 2011
EIGEN-51C	359	GRACE, CHAMP, gravity, altimetry	Bruinsma et al., 2010
GGM03C	360	GRACE, gravity, altimetry	Tapley et al., 2007
GGM03S	180	GRACE	Tapley et al., 2007
GOCO02S	250	GOCE, GRACE	Goiginger et al., 2011

* Related to the global geopotential models that were used in the study: *i*-) The adopted reference system is GRS80, *ii*-) The applied models are in tide free system, *iii*-) Zero degree terms were included in computations, *iv*-) The model coefficients are available from ICGEM (2011).

Table 1. Validated global geopotential models in the study

Some other conclusions drawn from the statistical inspection of the validation results that EGM08 provided improved results compared to its previous version EGM96 in the study region (compare the statistics of EGM96 and EGM08[a] in Table 2). Among the satellite only models EIGEN-6S fits best, and as such, can be recommended as a reference model for a future regional geoid of Turkey with R-R technique.

Model	ℓ_{max}	Type	min.	max.	mean	std. dev.
EGM96	360	Combined	-183.1	336.5	38.2	156.3
EGM08[a]	360	Combined	-105.0	47.6	-18.1	36.4
EGM08[b]	2190	Combined	-58.6	27.0	-4.5	17.3
EIGEN-6C	1420	Combined	-41.9	28.3	-4.1	15.8
EIGEN-6S	240	Satellite only	-77.5	85.0	-9.7	43.2
EIGEN-51C	359	Combined	-126.2	50.5	-21.8	38.9
GGM03C	360	Combined	-151.2	213.0	-2.4	76.3
GGM03S	180	Satellite only	-394.3	331.4	-18.5	198.1
GOCO02S	250	Satellite only	-87.2	90.9	-8.6	43.5

Table 2. Statistics of the geoid height differences between global models and observations (in centimetre)

The geoid height differences of EIGEN-6C and EIGEN-6S global models from the observed geoid heights at the reference benchmarks are illustrated in Figures 2 and 3, respectively. These differences can be compared and interpreted considering the topographic map of Turkey in Figure 1.

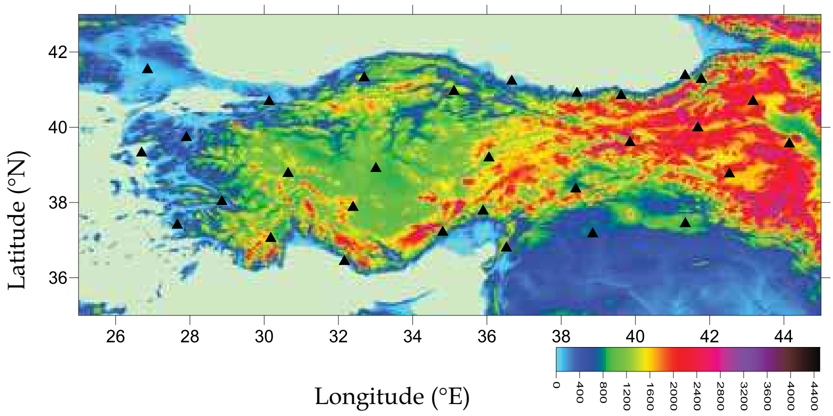


Fig. 1. Topographic map of Turkey and validation benchmarks (units metre) (using GTOPO30 data (USGS, 1997))

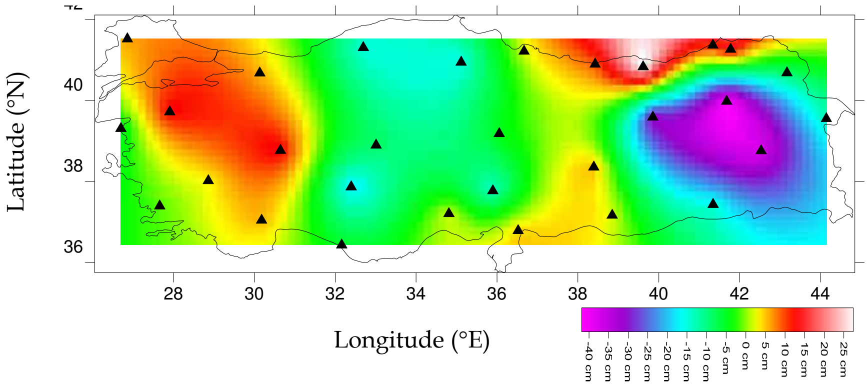


Fig. 2. Geoid height differences between EIGEN-6C model and GNSS/levelling observations

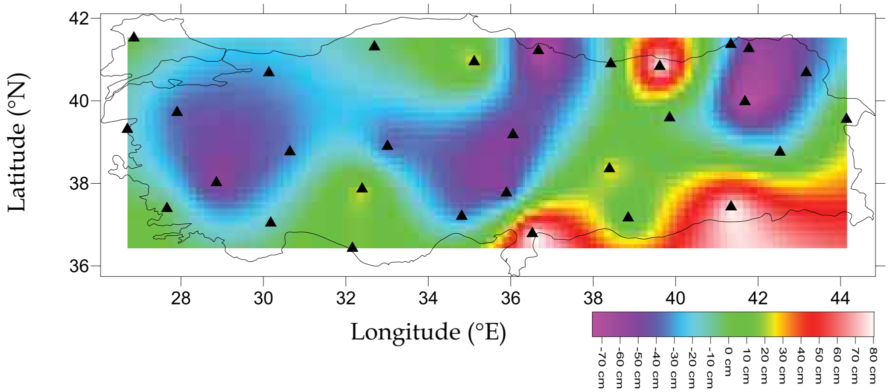


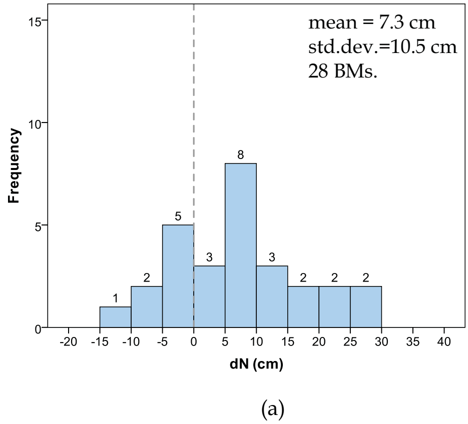
Fig. 3. Geoid height differences between EIGEN-6S model and GNSS/levelling observations

2.2 Regional geoid models in Turkey

In Turkey, various regional geoid models have been computed with different methods, since the 1970's (see e.g. Ayan, 1976; Ayhan, 1993; Ayhan et al., 2002; TNUGG, 2003; TNUGG, 2011), along with the technologic advances and increasing use of GNSS techniques in 1990's, modernization of national geodetic infrastructure, including the vertical datum definition, was required. As a consequence of these developments the geodetic control network was re-established in ITRF96 datum by Turkish Ministry of National Defence-General Command of Mapping between 1997 and 2001, a geoid model (TG99A) as a height transformation surface from GNSS to the regional vertical datum was released in 2000 (Ayhan et al., 2002). Turkey regional geoid model TG99A was gravimetrically determined and fitted to the regional vertical datum at homogeneously distributed GNSS/levelling benchmarks throughout the country. The absolute accuracy of TG99A model is reported between ±12 cm and ±25 cm, however the performance of the model decreases from the central territories through the coastline and boundaries of the country (Ayhan et al., 2002). An updated version of TG99A was released by General Command of Mapping in 2003 (TG03) (TNUGG, 2003). TG03 was computed with R-R method and Least Squares Collocation using terrestrial gravity data in 3-5 km density over the country (at Potsdam gravity datum), marine gravity data (acquired with shipborne and satellite altimetry), terrain based elevation model in 450 m x 450 m resolution and reference global model EGM96, and fitted to the regional vertical datum at 197 high order GNSS/levelling benchmarks (TNUGG, 2003). The accuracy of TG03 is reported as ±8.8 cm by TNUGG (2003) this revealed good improvement when compared the previous TG model.

Release of the Earth Gravitational Model 2008 (EGM08), the collection of new surface gravity observations (~266000), the advanced satellite altimetry-derived gravity over the sea (DNSC08), the availability of the high resolution digital terrain model (90 m resolution) and increased number of GNSS/levelling benchmarks (approximately 2700 benchmarks cover the entire country) enabled the computation of a new regional geoid model for Turkey in 2009, hence TG09 was released by General Command of Mapping as successor of TG03 (TNUGG, 2011). In computations, the quasi geoid model was constructed first using R-R procedure based on EGM08 and RTM reduction of surface gravity data and since the Helmert orthometric heights are used for vertical control in Turkey, the quasi geoid model was then converted to the geoid model. Ultimately, the hybrid geoid model TG09 was derived with combining the gravimetric geoid model and GNSS/levelling heights to be used in GNSS positioning applications. In the test results of TG09 with GNSS/levelling data, the accuracy of the model is reported as ±8.3 cm by TNUGG (2011). This result does not signify much improvement when comparing the TG03.

This section examines the published accuracies of TG03 and TG09 models at 28 GNSS/levelling benchmarks used in the validation of global geopotential models in the previous section. With this purpose in mind, the derived geoid heights at the benchmarks were compared with observations and in the results: TG03 model revealed ±10.5 cm standard deviation with minimum -10.1 cm, maximum 28.9 cm and mean 7.3 cm geoid height differences, whereas the TG09 model has ±9.2 cm standard deviation with minimum -11.3 cm, maximum 36.7 cm and mean of 10.5 cm in geoid height differences. The distribution of geoid height residuals versus the numbers of point are given in histograms in Figure 4. The geoidal height differences for TG03 and TG09 models are illustrated in Figures 5 and 6, respectively.



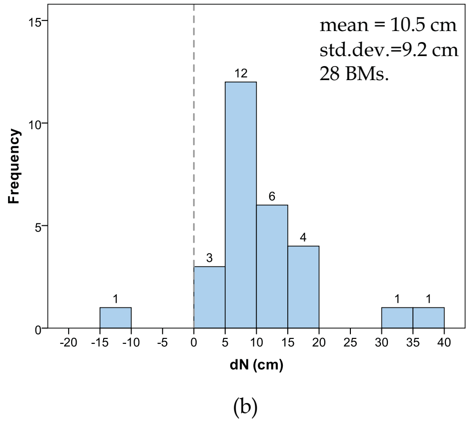


Fig. 4. Validation results of (a) TG03 and (b) TG09 models: geoid height differences (in cm) versus reference benchmark numbers

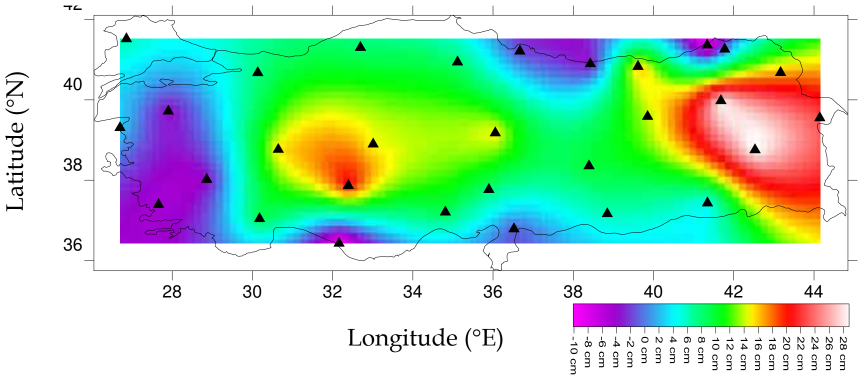


Fig. 5. Geoid height differences between TG03 and GNSS/levelling observations

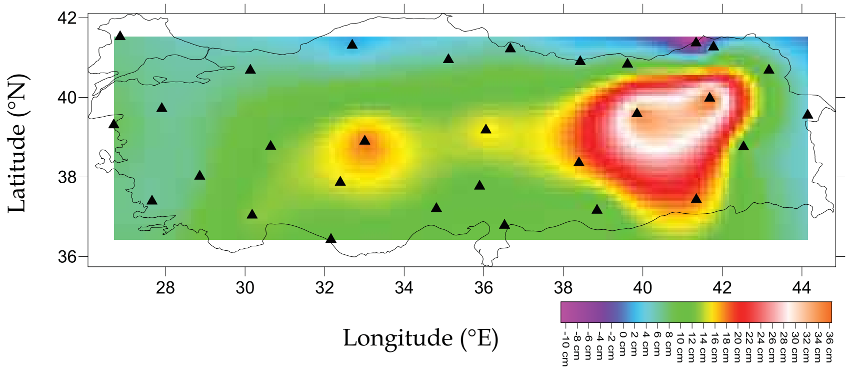


Fig. 6. Geoid height differences between TG09 and GNSS/levelling observations (TG09 data were used from Yılmaz&Karaali (2010))

3. Local GNSS/levelling geoids

Among the computation methods of geoid models (see e.g. Schwarz et al., 1987; Featherstone, 1998; Featherstone, 2001; Hirt and Seeber, 2007; Erol et al., 2008; Erol et al., 2009), geometric approach that GNSS and orthometric heights (h and H, respectively) can be used to estimate the position of the geoid at discrete points (so called geoid reference benchmarks) through a simple relation between the heights ($N \approx h-H$) provides a practical solution to the geoid problem in relatively small areas (typically a few kilometers) (Featherstone et al., 1998; Ayan et al., 2001, 2005; Erol and Çelik, 2006). This method addresses the geoid determination problem as "describing an interpolation surface depending on the reference benchmarks" (Featherstone et al., 1998; Erol et al., 2005; Erol & Çelik, 2006; Erol et al., 2008). The approximate equality in the equation arises due to the disregard for the deflection of the vertical that means the departure of the plumbline from the ellipsoidal normal (Heiskanen and Moritz, 1967). However the magnitude of error steming from this oversight is fairly minimal and therefore acceptable for the height transformation purposes (Featherstone, 1998).

The data quality, density and distribution of the reference benchmarks have important role on the accuracy of local GNSS/levelling geoid model (Fotopoulos et al., 2001; Fotopoulos, 2005; Erol & Çelik, 2006; Erol, 2008, 2011). There are certain criteria on the geoid reference benchmark qualities and locations, as described in the regulations and reference books (LSMSDPR, 2005; Deniz&Çelik, 2007) that will be mentioned in the text that follows. On the other hand using an appropriate surface approximation method in geoid modelling with geometrical approach is also critical for the accuracy of the model. The modelling methods are various but those most commonly employed among are; polynomial equations (of various orders) (Ayan et al, 2001; Erol, 2008; Erol, 2011), least squares collocation (Erol and Çelik, 2004), geostatistical kriging (Erol and Çelik, 2006), finite elements (Çepni and Deniz, 2005), multiquadric or weighted linear interpolation (Yanalak and Baykal, 2001). In addition to these classical methods, soft computing algorithms such as artificial neural networks (either by itself, see e.g. Kavzaoğlu and Saka (2005) or as part of these classical statistical techniques, e.g. Stopar et al. (2006)), adaptive network-based fuzzy inference systems (ANFIS) (Yılmaz and Arslan, 2008) and wavelet neural networks (Erol, 2007) were also evaluated by researchers in the most recent investigations on local geoid modelling.

3.1 Case studies: Istanbul and Sakarya local geoids

In this section, we discuss and explain the handicaps and advantages of geometric approach and local geoid models from the view point of transformation of GNSS ellipsoidal heights. This includes two case studies: Istanbul and Sakarya local geoids, using polynomial equations and ANFIS methods.

3.1.1 Data

One of the case study areas, Istanbul, is located in the North West of Turkey (between 40°30' N – 41°30' N latitudes, 27°30' E – 30°00' E longitudes, see Figure 7). The region has a relatively plain topography and elevations vary between 0 and 600 m. The GNSS/levelling network (Istanbul GPS Triangulation Network 2005, IGNA2005) was established between 2005 and 2006 as a part of IGNA2005 project (Ayan et al., 2006), and the measurement campaigns and data processing strategies adopted to compute benchmark coordinates satisfy the criteria of LSMSDPR (2005), on determination and use of local GNSS/levelling geoids. Accordingly the geoid reference benchmarks must be the common points of C1, C2 and C3 order GNSS benchmarks and high order levelling network points. Thus the GNSS observations of IGNA2005 project were carried out using dual frequency GNSS receivers, with observation durations of at least 2 hours for C1 type network points (for the baselines 20 km in length), and between 45 and 60 minutes for the C2 type network points (for the baselines 5 km in length). The recording interval was set 15 seconds or less during the campaigns. The GNSS coordinates of network benchmarks were determined in ITRF96 datum 2005.000 epoch with ±1.5 cm and ±2.3 cm of root mean square errors in the two dimensional coordinates and heights, respectively (Ayan et al., 2006). The levelling measurements were done simultaneously during the GNSS campaigns and Helmert orthometric heights of geoid reference benchmarks in Turkey National Vertical Control Network 1999 (TUDKA99) datum (Ayhan and Demir, 1993) were derived. Total number of the homogenously distributed reference benchmarks is 1205 with the density of 1 benchmark per 20 km² in the network (see Figure 7).

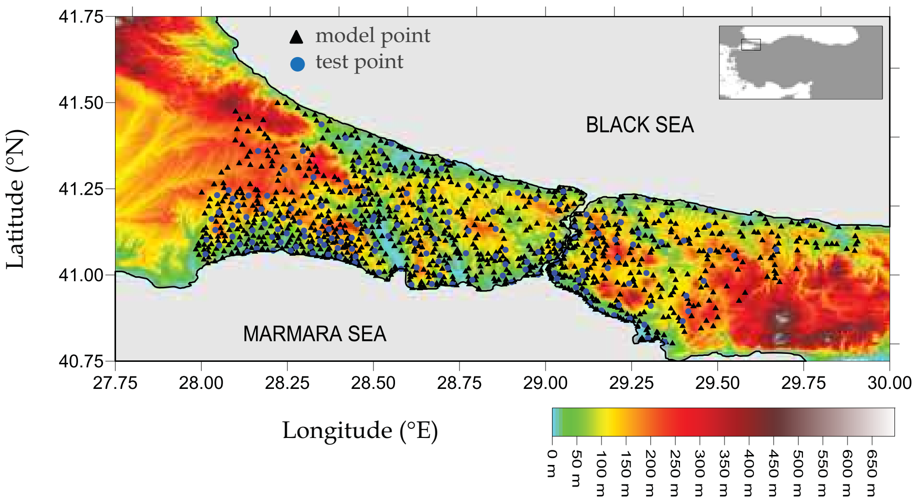


Fig. 7. Geoid reference benchmarks in Istanbul (topographic data from SRTM3 (USGS, 2010)

The second case study on determining local GNSS/levelling geoids was carried out in the Sakarya region situated in the East of Marmara sea and Izmit Gulf (between 40°30' N – 41°30' N latitudes, 28°30' E – 31°00' E longitudes). The GNSS/levelling network was established during the Geodetic Infrastructure Project of the Marmara Earthquake Region Land Information System (MERLIS) in 2002 (Çelik et al., 2002), and overlap with IGNA2005 network. Compared to the Istanbul area, the topography in Sakarya is quite rough and the elevations are between 0 m and 2458 m. The GNSS and levelling observations, and data processes were executed according to the regulation of the project. After the adjustment of GNSS network, the accuracies of ±1.5 cm and ±3.0 cm for the horizontal coordinates and ellipsoidal heights were derived. During the GNSS campaign of the MERLIS project, precise levelling measurements were undertaken, simultaneously, and in the adjustment results of levelling observations the relative accuracy of Helmert orthometric heights is reported as 0.2 ppm by Çelik et al. (2002). The GNSS coordinates are in ITRF96 datum, while the orthometric heights are in TUDKA99 datum.

The distribution of the 109 GNSS/levelling benchmarks is homogenous but rather sparse, and given the rough topography of the region, the coverage of the benchmarks cannot characterize the topographic changes well. The reference point density is 1 benchmark per 165 km². Figure 8 shows the reference network benchmarks on the topographic map of the area.

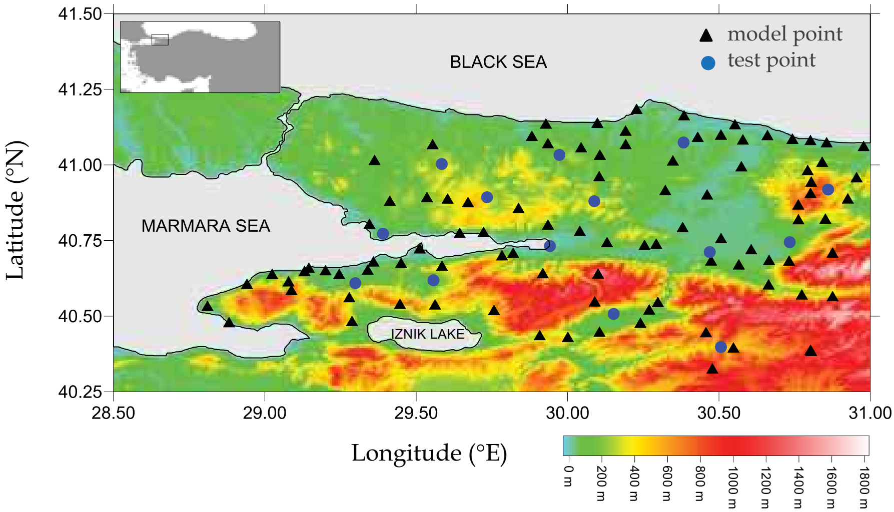


Fig. 8. Geoid reference benchmarks in Sakarya (topographic data from SRTM3 (USGS, 2010)

3.1.2 Methods

Since the computation algorithms, applied for local GNSS/levelling geoid determination in the study, are not able to detect potential blunders in data sets, the geoid heights derived from the observations at the benchmarks were statistically tested and the outliers were cleaned before modelling the data (see Erol (2011) for a case study in screening the reference data before geoid modelling). After removing the outliers from data sets, in Istanbul data, uniformly distributed 200 points of 1205 reference benchmarks (approximately 16% of the entire data) were selected to form the test data, and the remaining 1005 benchmarks were used in computation of the geoid. Similarly, in Sakarya, 14 of the 109 data points (nearly 13% of all data) having homogenous distribution were selected and used for external tests of the geoid model. The model and test points are distinguished with different marks on Figures 7 and 8. The theoretical review of the applied surface interpolation methods and comparisons of their performances by means of the test results are provided in the next section.

3.1.2.1 Polynomials

The polynomial equation for representing a local geoid surface based on the discrete reference benchmarks with known geoid heights in the closed form is:

$$N(u,v) = \sum_{m=0}^{l} \sum_{n=0}^{l-m} a_{mn} u^m v^n \tag{5}$$

where a_{mn} are the polynomial coefficients for m,n = 0 to l, which is the order of polynomial. u and v represent the normalized coordinates, which are obtained by centring and scaling the geodetic coordinates φ and λ. In the numerical tests of this study, the normalized coordinates were obtained by $u = k\,(\varphi - \varphi_o)$ and $v = k\,(\lambda - \lambda_o)$ where φ_o and λ_o are the mean latitude and longitude of the local area, and the scaling factor is $k = 100/\rho°$.

In Equation 5, the unknown polynomial coefficients are determined with least squares adjustment solution. According to this, the geoid height (Ni) and its correction (Vi) at a reference benchmark having (u, v) normalized coordinates as a function of unknown polynomial coefficients is:

$$\begin{aligned} N_i + V_i = {} & a_{00} + a_{10}u + a_{11}v \\ & + a_{20}u^2 + a_{21}uv + a_{22}v^2 \\ & + a_{30}u^3 + a_{31}u^2v + a_{32}uv^2 + a_{33}v^3 \\ & + a_{40}u^4 + a_{41}u^3v + a_{42}u^2v^2 + a_{43}uv^3 + a_{44}v^4 \\ & \ldots \end{aligned} \tag{6}$$

and the correction equations for all reference geoid benchmarks in matrix form is:

$$\begin{bmatrix} N_1 \\ N_2 \\ . \\ . \\ N_i \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_i \end{bmatrix} = \begin{bmatrix} 1 & u_1 & v_1 & \ldots \\ 1 & u_2 & v_2 & \ldots \\ . & . & . & \ldots \\ . & . & . & \ldots \\ 1 & u_i & v_i & \ldots \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{10} \\ . \\ . \\ a_{mn} \end{bmatrix} \tag{7a}$$

$$N + V = AX \tag{7b}$$

and the unknown polynomial coefficients (a_{mn} elements of the X vector, see Equations 7a and 7b):

$$X = \left(A^T A\right)^{-1} A^T \ell \tag{8}$$

and the cofactor matrix of X

$$Q_{XX} = \left(A^T A\right)^{-1} \tag{9}$$

are calculated. In the equations A is coefficients matrix and ℓ is the vector of observations that the elements of the vector are the geoid heights ($N_{GNSS/levelling}$).

One of the main issues of modelling with polynomials is deciding the optimum degree of the expansion, which is critical for accuracy of the approximation as well and its decision mostly bases on trial and error (Erol, 2009). Whilst the use of a low-degree polynomial usually results in an insufficient or rough approximation of the surface, unnecessarily use of a higher degree function may produce an over fitted surface that may reveal unrealistic and optimistic values at the test points. Another critical phase of determining polynomial surface is selecting the significant parameters and hence ignoring the insignificant ones in the model that this decision also bases on statistical criteria. After calculating the polynomials with least squares adjustment, the statistical significance of the model parameters can be analyzed using F-test with the null hypothesis $H_o : X_i = 0$ and the alternative hypothesis $H_1 : X_i \neq 0$ (Draper and Smith, 1998). The F-statistic is used to verify the null hypothesis and computed as a function of observations (Dermanis and Rossikopoulos, 1991):

$$F = \frac{X_i^T Q_{X_i X_i}^{-1} X_i}{t\hat{\sigma}^2} \tag{10}$$

where $\hat{\sigma}^2$ is a-posteriori variance, t is the number of tested parameters. The null hypothesis is accepted if $F \leq F_{t,r}^{\alpha}$, where $F_{t,r}^{\alpha}$ is obtained from the standard statistical tables for a confidence level α and degrees of freedom r that means the tested parameters are insignificant and deleted from the model. If the contrary is true and $F > F_{t,r}^{\alpha}$ is fulfilled, then the parameters remain in the model. After clarifying the optimal form of a polynomial model with significance tests of parameters, the performance of the calculated model is tested empirically, considering the geoid residuals at the benchmarks of the network. The tests are repeated with the polynomials in varying orders and hence an appropriate order of polynomial is determined for the data depending on the comparisons of test results.

3.1.2.2 Adaptive network based fuzzy inference system

ANFIS is an artificial intelligence inspired soft computing method that is first purposed in the late 1960's depending on fuzzy logic and fuzzy set theory introduced by Zadeh (1965). After that this method was used in various disciplines for controlling the systems and modelling non-stationary phenomena, and recently applied in geoid determination, as well (see e.g. Ayan et al, 2005; Yılmaz and Arslan, 2008). The computation algorithm of the method mainly bases on feed-forward adaptive networks and fuzzy inference systems. A fuzzy inference system is typically designed by defining linguistic input and output variables and an inference rule base. Initially, the resulting system is just an approximation for an adequate model. Hence, its premise and consequent parameters are tuned based on the given data in order to optimize the system performance and this process bases on a supervised learning algorithm (Jang, 1993).

In computations with ANFIS, depending on the fuzzy rule structures, there are different neural-fuzzy systems such as Mamdani, Tsukamoto and Takagi-Sugeno (Jang, 1993). Tung and Quek (2009) can be referred for a review on implementation of different neural-fuzzy systems. In Figure 9 a two input, two-fuzzy ruled, one output type 3 fuzzy model is illustrated. In this example Takagi-Sugeno's fuzzy if-then rules are used and the output of each rule is a linear combination of input variables plus a constant term, and the final output is a weighted average of each rule's output.

In the associate fuzzy reasoning in the figure and corresponding equivalent ANFIS structure:

Rule 1: if x is A_1 and y is B_1; then $f_1 = p_1 x + q_1 y + r_1$

Rule 2: if x is A_2 and y is B_2; then $f_2 = p_2x + q_2y + r_2$

where the symbols A and B denote the fuzzy sets defined for membership functions of x and y in the premise parts. The symbols p, q and r denote the consequent parameters of the output functions f (Takagi and Sugeno, 1985; Jang, 1993; Yılmaz, 2010). The Gaussian function is usually used as input membership function μ_i (x) (see Equation 11) with the maximum value equal to 1 and the minimum value equal to 0:

$$\mu_i(x) = \exp\left[-\left(\frac{x-b_i}{a_i}\right)^2\right] \tag{11}$$

where a_i, b_i are the premise parameters that define the gaussian-shape according to their changing values. Yılmaz and Arslan (2008) apply various membership functions and investigate the effect of the each function on the approximation accuracy of the data set.

In the associated ANFIS architecture of Figure 9, the functions of the layers can be explained as such that in *Layer 1*, inputs are divided subspaces using selected membership function, in *Layer 2*, firing strength of a rule is calculated by multiplying incoming signals, in *Layer 3*, the firing strengths are normalised and in *Layer 4*, the consequent parameters (p_i, q_i, r_i) are determined and finally in *Layer 5*, the final output is obtained by summing of all incoming signals.

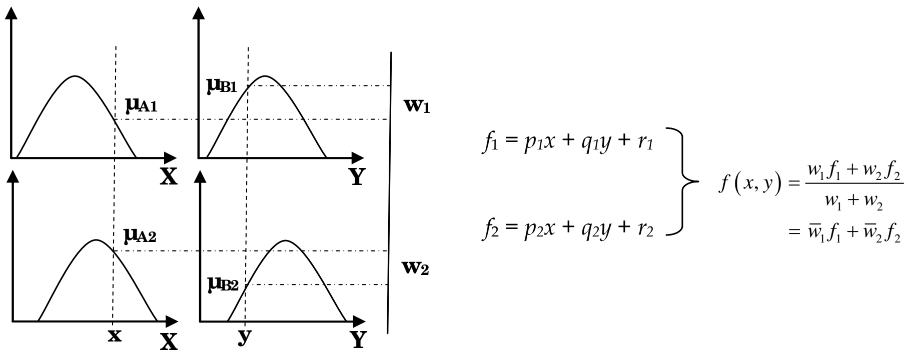
Using the designed architecture, in the running steps of the ANFIS, basically, it takes the initial fuzzy system and tunes it by means of a hybrid technique combining gradient descent back-propagation and mean least-squares optimization algorithms (see Yılmaz and Arslan, 2008). At each epoch, an error measure, usually defined as the sum of the squared difference between actual and desired output, is reduced. Training stops when either the predefined epoch number or error rate is obtained. The gradient descent algorithm is mainly implemented to tune the non-linear premise parameters while the basic function of the mean least-squares is to optimize or adjust the linear consequent parameters (Jang, 1993; Takagi and Sugeno, 1985).

After determination of the local geoid model using either of the methods, the success of the method can be assessed using various statistical measures such as the coefficient of determination, R^2, and the root mean square error, RMSE, of geoidal heights at the reference benchmarks:

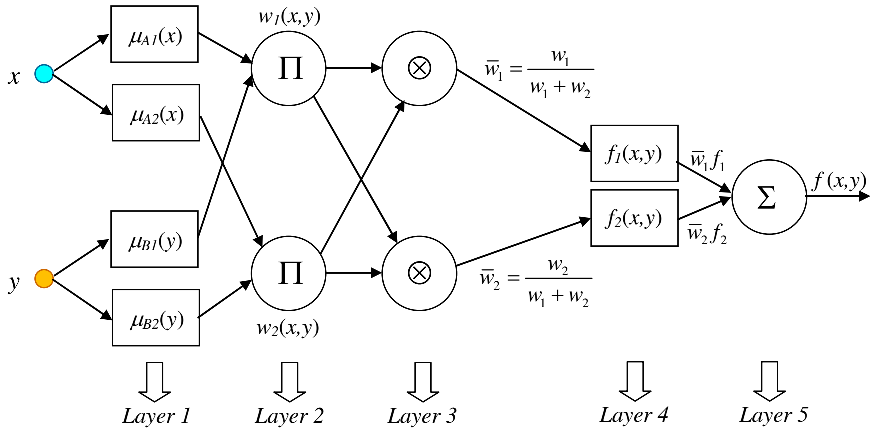
$$R^2 = 1 - \frac{\sum_{i=1}^{j}\left(\ell - \hat{\ell}_i\right)^2}{\sum_{i=1}^{j}\left(\ell_i - \overline{\ell}\right)^2} \tag{12}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{j}\left(\ell_i - \hat{\ell}_i\right)^2}{j}} \tag{13}$$

where $\hat{\ell}_i$ is the geoid height computed with the polynomial or ANFIS N_{model}, and $\bar{\ell}$ is the mean value of observations, and j is the number of observations (Sen and Srivastava, 1990). Coefficient of determination indicates how closely the estimated values ($\hat{\ell}$) from an approximation model corresponds to the actual data (ℓ), and takes values between 0 and 1 (or represented as percentage, and the closer the R^2 is to 1, the smaller the residuals and hence the better the model fit).



(a)



(b)

Fig. 9. (a) type 3 fuzzy reasoning, (b) a simple two-input, two-rule and single-output ANFIS structure (Jang, 1993)

3.1.3 Test results

In the results of the tests, repeated with the varying polynomial orders from first to sixth order, a 5th and 4th order polynomial models (having 21 and 15 coefficients) were determined as optimal for the Istanbul and Sakarya data, respectively. The significance tests of the polynomial parameters revealed the final forms of the models. Evaluation of these polynomials at the reference and test benchmarks, separately, in Istanbul and Sakarya regions, revealed the statistics in Tables 3 and 4. As is seen from the Table 3 for Istanbul area, the accuracy of the fifth order polynomial in terms of RMSE of geoid heights at the test points is ±4.4 cm with a coefficient of determination of 0.992. The geoid height differences of the polynomial model and observations at the benchmarks are mapped in Figure 10a. The test statistics of the polynomial model for Sakarya local geoid are summarized in Table 4 that the evaluation of the model at the independent test points revealed an absolute accuracy of ±20.4 cm in terms of RMSE of the geoid heights. Although the qualities of employed reference data in computations of both local geoid models are comparable (see section 3.1.1), the polynomial surface model revealed much improved results in Istanbul territory than Sakarya. The reasons of low accuracy in local geoid model of Sakarya territory can be told as sparse and non-homogeneous distribution of geoid reference benchmarks and rough topographic character of the territory that makes difficult to access for height measuring. Hence the GNSS/levelling benchmarks whose density and distribution are very critical indeed for precise modelling of the local geoid, are not characterize sufficiently the topographic changes and mass distribution in Sakarya (compare point distribution versus topography in Figure 8). Figure 10b shows the geoid height differences of the polynomial model and observations at the benchmarks for Sakarya.

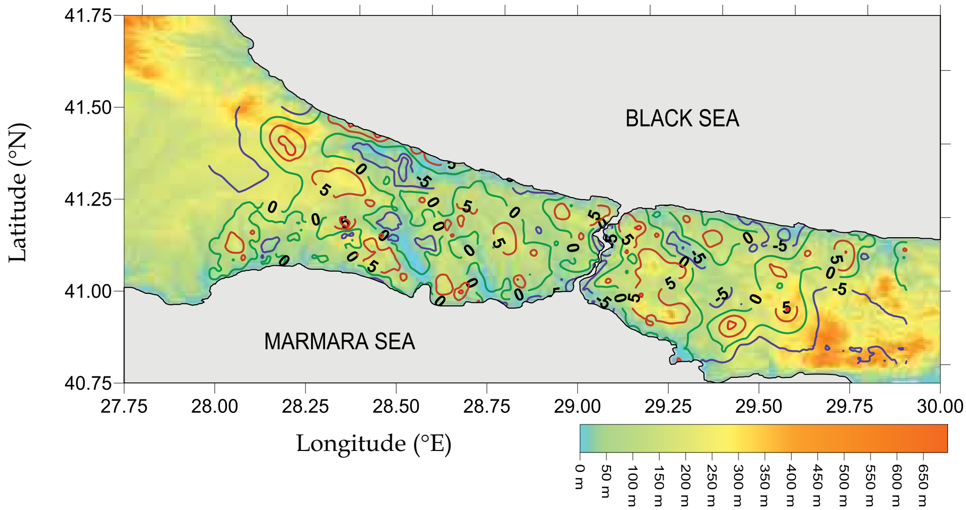
	5th order polynomial		**ANFIS**		**TG03**
	Reference BMs	**Test BMs**	**Reference BMs**	**Test BMs**	
Minimum	-11.2	-11.5	-10.5	-9.7	-32.5
Maximum	11.4	11.5	12.4	9.5	30.0
Mean	0.0	0.0	0.0	0.0	-0.3
RMSE	4.2	4.4	3.6	3.5	10.8
R^2	0.993	0.992	0.996	0.995	0.960

Table 3. Statistical comparison of applied approximation techniques in Istanbul local geoid (units in centimetre, R^2 unitless)

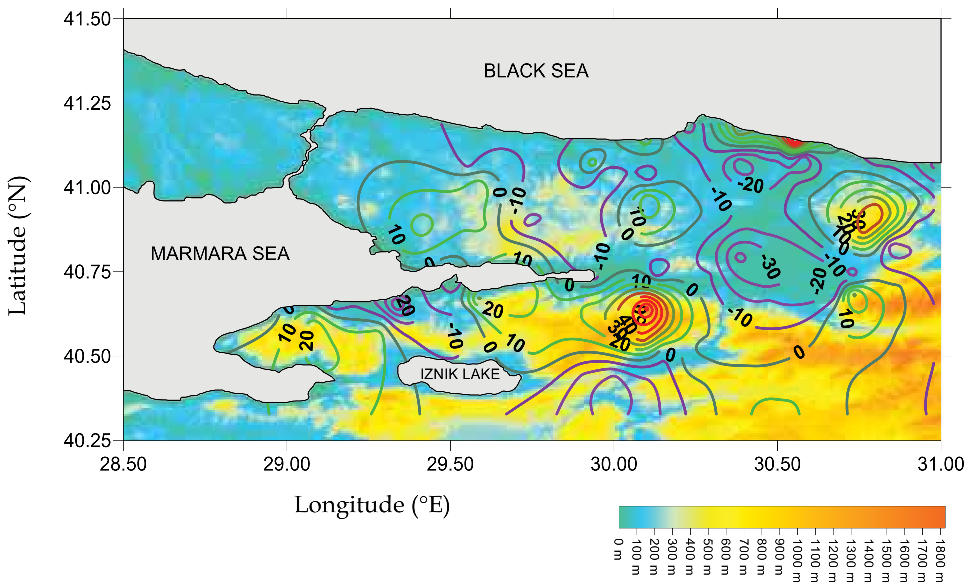
	4th order polynomial		**ANFIS**		**TG03**
	Reference BMs	**Test BMs**	**Reference BMs**	**Test BMs**	
Minimum	-52.0	-36.3	-39.7	-35.4	-53.8
Maximum	82.7	24.1	42.1	19.0	64.3
Mean	-0.3	-7.5	0.0	-11.0	-4.4
RMSE	22.7	20.4	12.0	18.9	18.6
R^2	0.923	0.905	0.978	0.913	0.945

Table 4. Statistical comparison of applied approximation techniques in Sakarya local geoid (units in centimetre, R^2 unitless)

Nonlinear regression structure of ANFIS and its resulting system, based on tuning the model parameters according to local properties of the data may reveal improved results of surface fitting. However one must be careful whilst working with soft computing approaches and pay attention for choosing appropriate design of architecture with optimal parameters such as: (e.g. in ANFIS) the input and rule numbers, type and number of membership functions, efficient training algorithm. Since the prediction capabilities of these algorithms vary depending on adopted architecture, use of unrealistic parameters may reveal optimistic results but, at the same time, produce an over fitted surface model that should be avoided in geoid modelling. While modelling with ANFIS, deciding an optimal architecture for the system is based on trial and error procedure.



(a)



(b)

Fig. 10. Geoid height differences of polynomial models and observations in centimetre ($\Delta N = N_{GNSS/lev.} - N_{poly.}$): (a) Istanbul, (b) Sakarya

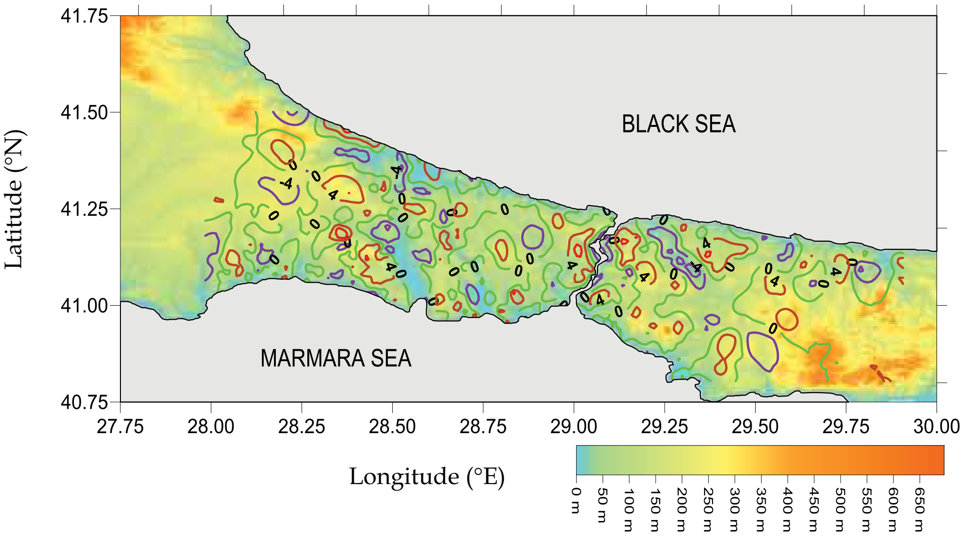
In modelling Istanbul and Sakarya local geoids using the ANFIS approach, training data (the geoid reference benchmarks) were used to estimate the ANFIS model parameters, whereas test data were employed to validate the estimated model. The input parameters are the geographic coordinates of the reference benchmarks, and the output membership functions are the first order polynomials of the input variables. As the number of the output membership functions depends on the number of fuzzy rules, in computations, the latitudes and longitudes were divided into 5 subsets to obtain 5 x 5 = 25 rules in Istanbul, and 4 subsets to obtain 4 x 4 = 16 rules in Sakarya. In both case studies, we adopted the Gaussian type membership function as suggested by Yılmaz (2010). After determining the ANFIS structure, the parameters of both the input and output membership functions were calculated according to a hybrid learning algorithm as a combination of least-squares estimation and gradient descent method (Takagi & Sugeno, 1985). Using the determined ANFIS model parameters for Istanbul and Sakarya data, separately, the geoid heights both at the reference and test benchmarks were calculated. In addition, the statistics of the geoid height differences between the model and observations were investigated in each local area.

In the test results for Istanbul local geoid with ANFIS (Table 3), the geoid height residuals at the test benchmarks vary between -9.7 cm and 9.5 cm with a standard deviation of ±3.5 cm. As the basic statistics in Table 3 provides a comparison between the performances of two methods in Istanbul, ANFIS has a 20% improvement in terms of RMSE of geoid heights comparing the 5th order polynomial model. As the RMSE of the computed geoid heights for the reference benchmarks and the test benchmarks are close values, we can say that the composed ANFIS structure is appropriate for modelling the Istanbul data. The coefficient of determination (R^2), as the performance measure of ANFIS model is 0.996.

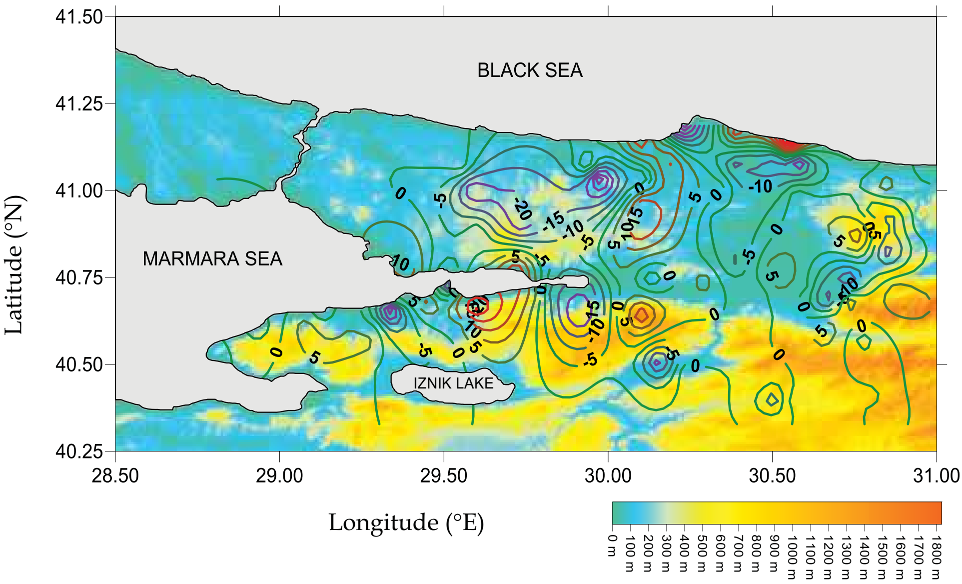
However, in Sakarya, the ANFIS method did not reveal significantly superior results from the 4th order polynomial at the test points with the geoid height residuals between -35.4 cm and 19.0 cm with root mean square error of ±18.9 cm. The improvement of the model accuracy with ANFIS method versus the polynomial is around 7%, considering the RMSE of geoid heights. On the other hand ANFIS revealed much improved test statistics at the reference benchmarks than the polynomial. The inconsistency, observed between the evaluation results at the reference and test benchmarks for ANFIS model may indicate an inappropriateness of this model for Sakarya data. Figure 11 maps the geoid height differences of ANFIS model and observations at the benchmarks in Istanbul and Sakarya.

In addition to the evaluation of surface approximation methods in modelling local GNSS/levelling geoids in case study areas, TG03 model was also evaluated at the reference geoid benchmarks. The statistics of geoid height differences with 0.3 cm mean and ±10.8 cm standard deviation for Istanbul, confirms the reported accuracies of the model by TNUGG (2003) and Kılıçoğlu et al. (2005). Conversely, the validation results of TG03 model in Sakarya GNSS/levelling benchmarks revealed the differences of geoid heights with -4.4 cm mean and ±18.6 cm standard deviations. Considering these validation results, although the performance of TG03 model seems low by means of RMSE of geoid heights, they revealed approximately 44% of improvement when comparing to the performance of previous Turkish regional geoid TG99A in the same region (see the results of TG99A validations in Sakarya region by Kılıçoğlu&Fırat (2003)).

In the conclusion of this section, the Istanbul and Sakarya local GNSS/levelling geoid models by ANFIS approach can be observed in the maps depicted in Figures 12 and 13.



(a)



(b)

Fig. 11. Geoid height differences of ANFIS models and observations in centimetre ($\Delta N = N_{GNSS/lev.} - N_{ANFIS}$): (a) Istanbul, (b) Sakarya

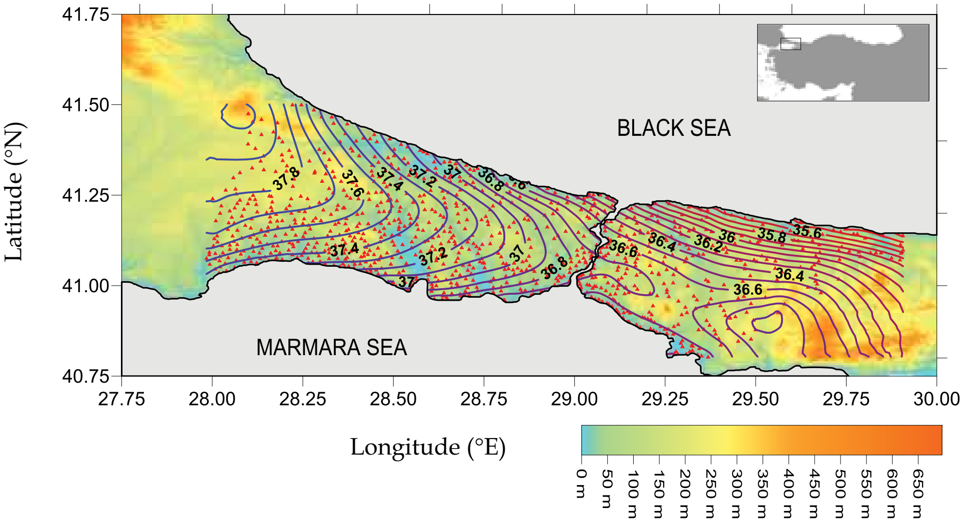


Fig. 12. Istanbul local GNSS/levelling geoid with ANFIS model

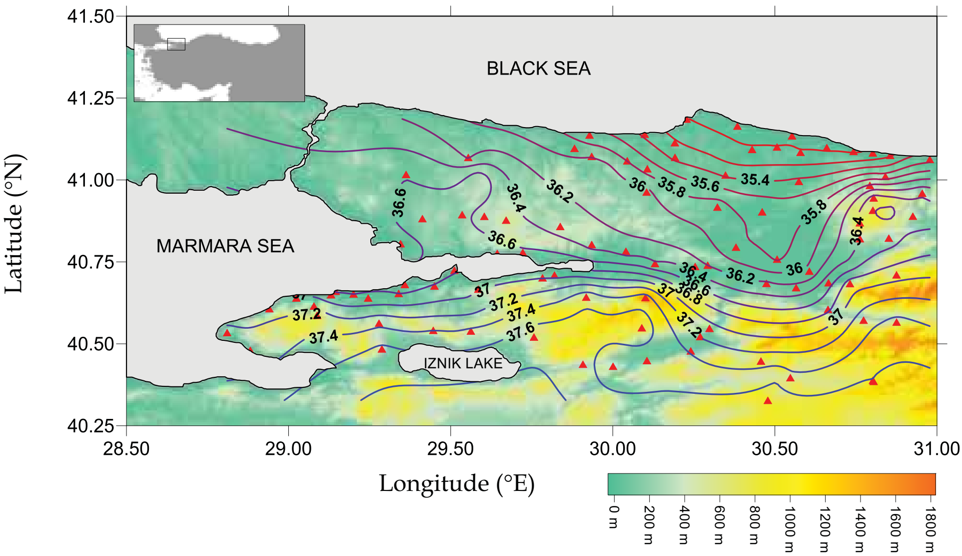


Fig. 13. Sakarya local GNSS/levelling geoid with ANFIS model

3.2 Local improvement of regional geoids

Besides the local GNSS/levelling geoid models, using locally improved regional geoid model with local GNSS/levelling data also provides an applicable solution for transformation of GNSS heights into regional vertical datum. Theoretically, the fundamental relationship between heterogeneous heights: h_{GNSS} - $H_{levelling}$ - N_{model} = 0 should have been satisfied. However, because of physical realities and computational factors that cause discrepancies among the heights, this equation cannot be realised at all in real world. As such, this naturally affects the precision of transformation among the heights in practice. Dealing with these disturbing factors, especially the element caused by the systematic errors and datum inconsistencies as a part of geoid modelling, will reduce the discrepancies among the three heights and hence improve the transformation precision of GNSS ellipsoidal heights. As part of this chapter we therefore explain two methods, which are aimed at minimizing the systematic differences of three heights in terms of optimal combination of the heights, for the improvement of regional geoid models with limited reference data in local areas. In the first approach, the height discrepancies are modelled with a parametric equation, so called corrector surface model, which absorbs inconsistencies of the height sets and allow a direct transformation of GNSS heights to the regional vertical datum. The second method consists of the least squares adjustment of the orthometric height differences, which are derived from ellipsoidal heights and regional geoid model, on the base vectors. Hence the orthometric heights of the new points are derived using the adjusted orthometric height differences. Brief descriptions of these height combination approaches with formulations can be found in the sections below.

3.2.1 Corrector surface model

The corrector surfaces, determined according to combination of GNSS derived heights, orthometric heights from the vertical datum and a gravimetric based geoid model, provides an efficient and practical option to precise GNSS levelling in a local area (see e.g., Featherstone, 1998; Kotsakis & Sideris, 1999; Fotopoulos, 2003). The main idea of modelling the corrector surface is to make the regional model estimate of the geoid coincident with the valid vertical datum at GNSS/levelling benchmarks hence minimising the errors in the regional geoid model and the observed heights at the benchmarks. This provides a practical solution for GNSS users in order to accomplish a direct transformation from GNSS derived ellipsoidal heights to orthometric heights, based on local vertical datum.

Determining an optimal parametric model for discrepancies of three heights follows the similar steps as explained in section 3.1.2 for local GNSS/levelling geoid modelling. These steps basically include: determining an appropriate type for model, selecting the optimum extent (form) of the model, and finally assessing the performance of determined model. Accordingly, although one can find numerous models suggested in the literature for realizing corrector surfaces, selecting procedures of the parametric model is mostly arbitrary and based on comparison of statistical test results that measure the accuracy and numerical stabilities of the various models.

General expression of the discrepancies between GNSS/levelling derived geoid heights and geoid heights from the regional geoid model as a function of geodetic position:

$$h_{GNSS} - H_{lev.} - N_{\mathrm{mod}\,el} - F(\varphi,\lambda) = 0 \tag{14}$$

that $F(\varphi, \lambda)$ function can be presented in various forms in different levels of complexity (e.g. having elements as only a bias, a bias and a tilt, or higher order polynomials), and multiple regression equations generally as low-order polynomials (similar with Equation 5,

$F(\varphi,\lambda)=\sum_{m=0}^{l}\sum_{n=0}^{l-m} a_{mn}u^m v^n$), and four or five parameter similarity transformation equations (see Equations 15 and 16, respectively) are generally used.

$$F(\varphi,\lambda) = a_0 + a_1 \cos\varphi \cos\lambda + a_2 \cos\varphi \sin\lambda + a_3 \sin\varphi \tag{15}$$

and five parameter similarity transformation as an extended version of Equation 15:

$$F(\varphi,\lambda) = a_0 + a_1 \cos\varphi \cos\lambda + a_2 \cos\varphi \sin\lambda + a_3 \sin\varphi + a_4 \sin^2\varphi \tag{16}$$

The coefficients of the parametric models are calculated using least squares adjustment method as described in section 3.1.2.1 with Equations 6-9. The appropriateness of the models are comparable according to the results of empirical tests, and RMSE of the height differences and coefficient of determinations (see Equations 12 and 13), are two of these statistics which provides useful hints on the compatibility of the parametric models as corrector surfaces. Hence the geoidal height at a new point can be determined with better precision as the summation of geoid height derived from the regional model and residual δN_{CS} from the corrector surface model as $N = N_{TG03} + \delta N_{CS}$.

3.2.2 Adjustment of the derived orthometric height differences on the baselines

The second method combines the height differences, which are derived from GNSS ellipsoidal heights (Δh) and regional geoid model (ΔN_{model}), in the least squares adjustment algorithm (Mikhail & Ackermann, 1976) and derives the adjusted orthometric height differences for the baselines between the reference GNSS/levelling benchmarks and new points according to following formulation:

$$\Delta H = \Delta h - \Delta N \tag{17}$$

where ΔH is the orthometric height difference for the baseline between the reference GNSS/levelling benchmark and new computation point, Δh is the ellipsoidal height difference derived from GNSS heights for the same baseline and finally ΔN is the geoid height difference of the baseline derived from the regional geoid model. In the adjustment computations that the orthometric heights of the reference benchmarks are set as ‘known’ to constrain the system, ΔH values are the observations. According to functional model of adjustment:

$$\Delta H + v = H - H^* \tag{18}$$

where H and H^* are approximate and precise orthometric heights of new and reference benchmarks, respectively. And the residual for the orthometric height difference of the baseline:

$$v = -H^* + H - \Delta H \tag{19}$$

and the residuals for all reference benchmarks set the matrix system:

$$v = AX - \ell \tag{20}$$

where the observations matrix is $\ell = \Delta h - \Delta N$, A is the coefficients matrix, and X consists the unknown parameters. The a-priori root mean square error of ΔH of a baseline of S km is $m = m_0 \sqrt{S_{(km)}}$ that m_0 is the a-priori RMSE of unit observation. The unknown parameters from the solution of matrix system in Equation 20 is calculated as

$$X = \left(A^T PA\right) A^T P\ell \tag{21}$$

where P includes the weights of ΔH observations. Hence the adjusted orthometric height differences are:

$$\Delta H^* = \Delta H + v \tag{22}$$

The success of the method can be assessed at the test points where GNSS and levelling observations exist, and in the evaluations the orthometric heights of the test points are compared with their observed orthometric heights.

Furthermore, combining the height sets using the method of least squares, weights of each set are essential to correctly estimate the unknown parameters. Improper stochastic modelling can lead to systematic deviations in the results. Therefore, for the purpose of estimating realistic and reliable variances of the data sets, and therefore constructing the appropriate a−priori covariance matrix of the observations, variance component estimation techniques can be included in combining algorithms of the heights. Numerous solution algorithms suggested for variance component estimation problems can be found in various literature published on the subject however, Rao's Minimum Norm Quadratic Unbiased Estimation is commonly used one of these methods (Rao, 1971.). Sjöberg (1984), Fotopoulos (2003) and Erol et al. (2008) can be referred to for further readings and practicing variance component estimation techniques in the adjustment.

3.2.3 Case study: Local Çankırı geoid

Suggested data combination methods related to local improvement of regional geoids are exemplified and tested in a numerical case study in this title. These results are also included by Erol et al. (2008) to provide a detailed investigation on local performances of the various regional models and their improvement capabilities. The local area covers 154 km x 198 km, and the number of reference benchmarks used in the tests is 31. The GNSS positions of the benchmarks were determined with static measurements using dual frequency GNSS receivers. The accuracies of the latitudes and longitudes in ITRF96 datum is ±1.5 cm, and for the accuracy of ellipsoidal heights is reported as ±3.0 cm (Erol et al., 2008). The adjustment of levelling observations revealed the orthometric heights of the benchmarks with ±2.5 cm in TUDKA99 datum. As can be seen in Figure 14, the benchmarks have quite poor density and non-homogeneous distribution over the area. The approximate density of the benchmarks is 1 point per 900 km². When the poor density of the benchmarks and rough topographic pattern of the area (the heights of the region change between 41 m and 2496 m) are considered, alongside the levelling technique the regional geoid model or its locally improved version can be applied to obtain regional orthometric heights from GNSS. As a result of this the density and distribution of the reference benchmarks do not allow determination of local GNSS/levelling geoid. According to Large Scale Map and Spatial Data Production Regulation of Turkey, legalized by July 2005, the density of the geoid reference benchmarks must be at least 1 benchmark per 15 km^2 for determination of precise local geoid with geometric approach (LSMSDPR, 2005; Deniz&Çelik, 2008), however with the purpose of testing and local improvement of the regional geoid, the density of the reference GNSS/levelling benchmarks are foresighted to be at least 1 benchmark per 200 km^2 by the regulation.

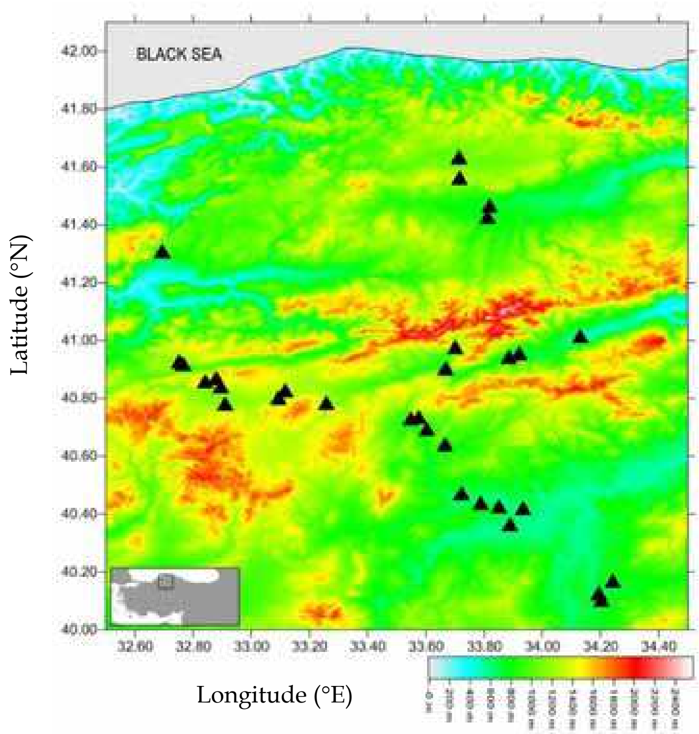


Fig. 14. Çankırı geoid reference benchmarks on topography (Erol et al., 2008)

In respect of the case study carried out with Çankırı local GNSS/levelling network by Erol et al. (2008), Turkish regional geoid TG03 (TNUGG, 2003; Kılıçoğlu et al., 2005) was tested at 31 GNSS/levelling benchmarks, and refined by combining the GNSS/levelling heights using least squares adjustment (LSA) of height differences derived from GNSS ellipsoidal heights and TG03 geoid undulations on the baselines, and simple corrector surface model (CS) with only a bias and a tilt. The performances of the refinement methods were also compared in terms of geoid height residuals at the 9 test points of 31 benchmarks. In addition, LSA of the geoid height differences on the baselines approach was applied with estimated variance components of each height sets, using iterative Minimum Quadratic Unbiased Estimation. The performances of TG03 and its refined versions were also compared with local GNSS/levelling geoid model which were determined with GNSS/levelling heights at 22 reference benchmarks using a 2nd order polynomial equation in the study. Considering the reported results, the accuracy of TG03 model is ±26.2 cm in terms of RMSE of geoid heights and the mean of geoid height differences at the benchmarks is 19.3 cm. When the TG03 is refined with LSA of orthometric height differences on the 58 baselines among the 22 reference and 9 test benchmarks, the accuracy of the refined TG03 model (version 1) is ±15 cm in terms of the RMSE of geoid heights at 9 test points. Hence the improvement of the model is approximately 42%. The refined version of TG03 (version 2) using CS fitting revealed ±19.2 of RMSE of geoid heights at the test points. The third version of refined TG03 was computed using LSA of geoid height differences on the baselines with estimated variance information from iterative MINQUE algorithm, and the internal accuracy of the computed geoid height values having ±4.9 cm RMSE at 31 points were obtained. As expected the 2nd order polynomial type local GNSS/levelling geoid model revealed the worst results with ±46.6 cm RMSE of geoid heights at the test benchmarks. All the results can be compared using the summary statistics at Table 5. For further reading on the applied methods for TG03 local refinement in Çankırı area and the associated case study, Erol et al. (2008) can be referred to.

	refining method		**min.**	**max.**	**mean**	**RMSE**
TG03	-		-10.8	60.4	19.3	26.2
refined TG03_ver.1	LSA of ΔH on Baseline		-28.7	23.1	0.0	15.0
refined TG03_ver.2	CS fit., 1st order polynomial	model	-25.4	43.3	0.0	17.6
		test	-45.4	28.5	0.0	19.2
local geoid model	2nd order polynomial		-120.1	85.7	0.0	46.6

Table 5. Statistical comparison of TG03 and its refined versions in Çankırı (units in centimetre) (Erol et al., 2008)

4. Summary of results and remarks

This chapter compares geoid models from various scales in Turkish territory and aims to provide a road map to GNSS users in practice, with regards to how to choose, compute and use of the geoid model as a tool for transformation of GNSS ellipsoidal heights to the regional vertical datum. As the traditional levelling techniques for obtaining precise height information are left aside, the improved accuracy of the geoid, as a modern technique for vertical control called, known as GNSS(-geoid) levelling, can be contemplated as an alternative for practical height applications. In the numerical evaluations, presented as part of this chapter, the recently released global geoid models, which include the data by the latest gravity field satellite missions, CHAMP, GRACE and GOCE, were tested against the terrestrial data. The results from this indicate the absolute accuracies of the two ultra-high resolution combined global geopotential models, EGM08 ($\ell_{max} = 2190$) and EIGEN-6C ($\ell_{max} = 1420$) in Turkey were calculated around ±17 cm, which means that these global models can directly be used for GNSS levelling in small scale map production and applications that requires regional orthometric heights with decimetre accuracy. A comparison on validation results of satellite only global models put EIGEN-6S and GOCO02S forward that these models were calculated using GOCE and GRACE missions' data until 240, 250 maximum degrees of expansion, with ±44.0 cm absolute accuracy at the test points. Comparing these models, performance of the GGM03S ($\ell_{max} = 180$), the GRACE only model, stayed rough in representation of the local gravity field in the region. Therefore in modelling the regional hybrid geoid, EIGEN-6S and GOCO02S may provide better performances.

In the content of numerical tests, beside the global models, the most recent regional geoids TG03 and TG09 were also validated against the GNSS/levelling heights at the test benchmarks. The validation results showed that although TG09 model provided approximately 12% improvement comparing to TG03 in terms of accuracy of geoid heights, the absolute accuracy of regional geoid models is not yet below 10 cm. This indicates that the regional geoid models remain insufficient to be applied for GNSS levelling purposes in large scale map production and applications that require centimetre level accuracy in heights. Since the lack of a 5-centimetre or higher precision regional geoid in the country; local geoid models, as an alternative solution for height transformation problems, are determined and used. This chapter presents examples of local geoid modelling using geometric approach, at the two case study areas, Istanbul and Sakarya situated in the north east of Turkey, which have precise GNSS/levelling data. Also from the test results of the computed local geoids, it is obvious that the topographic character of the local area, the quality of GNSS and levelling data, the density and distribution of the geoid reference benchmarks are very critical for the accuracy and reliability of the local geoid model. As such the design of the geoid reference network and data acquisition needs to be planned in a specific manner. Applied methodology for modelling the local geoid is another critical parameter that affects the final accuracy. In the numerical tests, the Istanbul and Sakarya local geoids were computed using classical polynomial type multi regression equations and ANFIS method. In Istanbul a fifth order polynomial equation fitted the best the reference geoid data, where as in Sakarya a fourth order polynomial was decided as an optimal model. Evaluation of the polynomial models at the test benchmarks revealed ±4.4 cm and ±20.4 cm absolute accuracies in Istanbul, and Sakarya, respectively. When the topographies and densities of the benchmarks in both local areas are compared, the difference between the accuracies of the polynomial representations of two local geoids can be understood (Figure 7 vs. Figure 8). On the other hand the ANFIS approach provided marked improvements in results with ±3.5 cm and ±18.9 cm accuracies, in Istanbul and Sakarya. TG03 regional geoid model has ±10.8 cm and ±18.6 cm accuracies in Istanbul and Sakarya. When comparing the regional model, in Istanbul, a local geoid model provides much better accuracy but in Sakarya many of the local geoid model solutions did not provide a better alternative to regional geoid for GNSS levelling purposes. The numerical tests on the local geoid modelling also provided an opportunity to compare the two surface approximation techniques. Hence it is concluded that although, ANFIS has a developed computation algorithm and potential to provide more improved results, it has handicaps from a practical point of view: the prediction capability of this method varies depending on the adopted architecture and it is too sensitive to the selection of the reference/test points. Therefore, while geoid modelling with ANFIS, one must be very carefully to employ the appropriate architecture and to decide reference and test data. Otherwise too optimistic and unrealistic statistics can appear with an over fitted surface model.

In the final part of this chapter, local improvement of geoid models is provided as another alternative solution to GNSS levelling. In the case study, local improvement of the TG03 in Çankırı using precise GNSS/levelling data, by corrector surface fitting and adjustment of derived orthometric height differences on the baselines, is presented. The accuracy of TG03 model in the region is ±26.2 cm. Applying least squares adjustment of height differences derived from GNSS and TG03 on the baselines approach provided 42% improvement in the model and the RMSE of the orthometric heights derived from the improved version of TG03 is reported as ±15.0 cm.

5. Conclusion

Numerous advantages of GNSS techniques from a practical perspective and its high precision in geodetic positioning make this satellite based positioning systems on service in a very large spectrum of applications, ranging from routine engineering surveys to scientific researches. On the other hand, the reference system definition of GNSS coordinates separates the geometry from the Earth gravity field, and therefore developing a solution for transition between the ellipsoidal and natural coordinates, especially in heights, constitutes a challenge for geodesists to be solved by a combination of terrestrial and GNSS data in the recent years. As a reflection of advances in computation techniques and improved data resolutions and accuracies, the precisions of geoid models increase and hence GNSS levelling, as a new concept in vertical control, become a consideration as a viable alternative for practical height determination. All these developments lead modernization of geodetic infrastructures in the national and consequently global scale, and cause leaving the traditional onerous surveying techniques aside as a means for obtaining heights. Today, in many countries, the new vertical datum definition is based solely on the geoid and vertical control is provided via GNSS levelling with a precise geoid model (see e.g. Rangelova et al., 2010).

In the light of recent developments on GNSS techniques and their tremendous impacts on definitions of the reference systems and hence geodetic infrastructures, this chapter reviewed the principle geoid models and widely used methodologies for practical determination of regional heights using GNSS. With this purpose, the evaluations on global models validated the improvement of the long and medium wavelength information of the gravity field, as a result of the current state of technologies with modernized GNSS, as well as new LEO missions for dedicated gravity field research (i.e., CHAMP, GRACE, GOCE). The improvements on global models as well as the terrestrial data qualities contribute also to the regional geoid models by reducing their errors in the total budget of hybrid geoid representation. However, according to results drawn from this study, the accuracy of regional geoid model of Turkey is insufficient yet for deriving regional orthometric heights with centimetre precision from GNSS levelling, and therefore local solutions such as modelling local geoid with geometric approach or improving the regional geoid model with local terrestrial data are still required for providing heights with an accuracy under 5 centimetres. Although the local geoids provide high accuracies, there are handicaps related to their determination and use. The determination of local geoid models requires specifically acquired reference data, having good quality and adequate distribution representing the topography well, and an appropriate modelling algorithm, fitting the data. One of the disadvantages related with the use of local geoid models is that they can be applied only in the limited area with high precision and so are not suitable for extrapolation. These local solutions do not contribute to a unified vertical datum definition in the country. In this manner the importance of a precise and reliable regional geoid model in the concept of GNSS levelling for practical determination of precise regional heights is obvious. In Turkey, geoid modelling efforts as a part of modernization of geodetic infrastructure continue, and with the enhanced data qualities, a precise regional geoid model with its time dependent variations for GNSS levelling purposes will be possible in the near future.

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