1. Introduction

The imprisonment of large numbers of drug addicts has the potential to create environments within which social networks that enhance the transmission of infectious diseases form (7–11; 14). About 668,000 men and women are incarcerated in sub-Saharan Africa with South Africa having the highest prison population with 157,402 people behind bars in the region and 335 prisoners per 100,000 of the national population; it has the ninth largest prison population in the world (21). International data show that HIV prevalence among prisoners is between six to fifty times higher than that of the general adult population. For example, in the USA the ratio is 6:1, in France it is 10:1; in Switzerland 27:1 and in Mauritius 50:1 (17). On a global scale, the prison population is growing rapidly, with high incarceration rates leading to overcrowding, which largely stems from national law and criminal justice policies. In most countries, overcrowding and poor physical conditions prevail (20). This phenomenon poses significant health concerns with regard to control of infectious diseases—HIV prevention and care most of all (21). Prisons are high risk settings for HIV transmission. However, HIV prevention, treatment are not adequately developed and implemented to respond to HIV in prisons (13). There is evidence to show that health programmes for the particular needs of imprisoned drug users are not enough in USA and Canada (15; 22). In Russia, a study of intravenous drug users demonstrated the critical role of prisons in the transmission of HIV through high levels of needle (syringes) sharing among the imprisoned (23).

Prison populations are predominantly male and most prisons are male-only institutions, including the prison staff. In such a gender exclusive environment, male-to-male sexual activity (prisoner-to-prisoner and guard-to-prisoner) is frequent (18). While much of the sex among men in prisons is consensual, rape and sexual abuse are often used to exercise dominance in the culture of violence that is typical of prison life (19). Inmate rape, including male rape, is considered one of the most ignored crimes. Sexual and physical abuse in custody remains a tremendous human rights problem (1). Intravenous drug use, tattooing and the following aspects of man-to-man sexual activity in prison make it a high risk for HIV transmission: anal intercourse, rape and the presence of sexually transmitted infections (STIs). Related problems in prisons across Southern Africa include overcrowding, shortages, corruption, and the presence of juveniles alongside adult prisoners. The potential for the spread of HIV is also increased by a lack of information and education, and a lack of proper medical care. STIs, if left untreated, can greatly increase a person’s vulnerability to HIV.

* Visiting Fellow, Clare Hall College, University of Cambridge
through sexual contact, UNAIDS noted (26). Men get tattooed in prison (12). In the absence of proper precautions and access to safe equipment tattooing can be a high-risk activity for the transmission of HIV (24; 25).

The literature and development of mathematical epidemiology is well documented (2; 3; 6). This paper seeks to use mathematical models to gain insights on transmission of HIV among male prisoners while in prison in the context of homosexuality and intravenous drug use. The rest of this paper is organized as follows. In the next section, the model and its basic properties are presented. In Section 3, we determine stability analysis of the equilibria states. Numerical simulations are presented in Section 4 and finally the last section concludes the paper.

2. Model description

The model sub-divides the total male prisoner population into the following sub-populations of susceptible intravenous drug users $S_d(t)$, susceptible non-drug users $S_n(t)$, intravenous drug using HIV-only infected people not yet showing AIDS symptoms $I_n(t)$, non-drug using HIV-only infected people not yet showing AIDS symptoms $I_d(t)$, intravenous drug using AIDS cases $A_d(t)$ and non-drug using AIDS cases $A_n(t)$. There is sexual interaction between intravenous drug users and non-drug users making HIV transmission across different these two distinct distinct groups possible. The population is patterns is heterogeneous mixing with regard to sexual behaviour. The total population is given by;

$$N(t) = N_d(t) + N_n(t), \quad N_d(t) = S_d(t) + I_d(t) + A_d(t), \quad N_n(t) = S_n(t) + I_n(t) + A_n(t), \quad (1)$$

with $N_n(t)$ and $N_d(t)$ being the total number of non-drug using and intravenous drug using male prisoners (intravenous drug users-IDU), respectively. The group $j$ members make $c_j, j = (d, n)$ sexual contacts per unit time, and that a fraction of the contacts made by a member of group $j$ is with a member of group $i$ is $p_{ji}, i = (d, n)$. Then $p_{nn} + p_{nj} = p_{dn} + p_{dn} = 1$. The total number of sexual contacts per unit time by members of group ‘$n$’ (non-drug users) with members of group ‘$d$’ (intravenous drug users) is $c_n p_{pn} N_n$ and because this must be equal to the number of contacts made by members of group ‘$d$’ with members of group ‘$n$’, we have a balance relation

$$\frac{p_{nn} c_n}{N_d} = \frac{p_{dn} c_d}{N_n}. \quad (2)$$

In this case the sexual contact rates (partner acquisition rates) $c_d$ and $c_n$ are saturating terms for the total population and the mixing proportions may change with time. It is worth mentioning here that intravenous drug users are more likely to have more sexual partners than the general population. Therefore, $c_d = B c_n, B \geq 1$. We assume that male prisoners in AIDS stage of the disease are no longer sexually active as they are no longer capable of attracting sexual mates among prisoners. Also drug using AIDS patients no longer share their needles with others as other prisoners do not like sharing needles with someone whose AIDS symptoms are visible. The forces of HIV infection for intravenous drug users and non-drug users in the male prison are:

$$\lambda_{dn} = \frac{p_{dn} c_d \beta_d I_d}{N_d} + \frac{p_{dn} c_d \beta_n I_n}{N_n} + \frac{c_d \beta_{dn} I_d}{N_d},$$

and $\lambda_{dn} = \frac{p_{nn} c_n \beta_n I_n}{N_n} + \frac{p_{nn} c_n \beta_d I_d}{N_d}$, respectively.
with $\beta_i, i = (n,d)$ is probability on individual being infected with HIV by an individual from the n- or d-class per sexual contact; $c_j, j = (n,d)$ are the number of sexual partners an individual acquires per year (partner acquisition rates); $\beta_d$ is the probability an intravenous drug user getting HIV infection through sharing non-sterile needles during drug injections and $c_d$ are the number drug sharing partners an individual acquires.

It is assumed people are recruited into prison at rate $\Lambda$ through committing various crimes and the following proportions $\pi_1, \pi_2, \pi_3$ and $\pi_4$ recruited enter the classes $S_n(t)$, $S_d(t)$, $I_n(t)$ and $I_d(t)$, respectively. We further assume that AIDS cases are too sick to commit a crime, so there are no recruitment of prisoners already in the AIDS stage of disease. Furthermore, it is assumed that intravenous drug using prisoners showing AIDS symptoms are no longer able to exert peer pressure strong enough to make one become a drug user. Individuals in $S_n(t)$ and $I_n(t)$ acquire drug misusing habits at rate $\lambda_n(t)$ due to peer pressure and move into $S_d(t)$ and $I_d(t)$, respectively with

$$\lambda_d = \frac{\beta_d c_h (S_d + I_d)}{N_d},$$

where $\beta_d$ is the probability of becoming an intravenous drug user (IDU) following contact with an IDU and $c_h$ are the number of contacts necessary for one to become an IDU (partner acquisition rate). Individuals in $S_n(t)$ class acquire HIV infection at a rate $\lambda_n(t)$ to move into $I_n(t)$. Individuals in $S_d(t)$ class acquire HIV infection at a rate $\lambda_d(t)$ to move $I_d(t)$ class. Individuals infected with HIV-only not yet displaying symptoms ($I_n(t), I_d(t)$) progress to the AIDS stage ($A_n(t), A_d(t)$) at a rate $\gamma$. Individuals in $A_d(t)$ leave the intravenous drug using habits at a rate $\alpha$ to get into $A_n(t)$ class. Individuals in all classes experience natural death at a rate $\mu$ and those in AIDS stage of the disease experience an additional disease induced death at a rate $\nu$. Individuals in all classes leave the prison at rate $\omega$ upon completion of their sentences. Individuals in the AIDS stage of the disease (final terminal stages) are further released from prison due to sickness at rate $\phi$. The model flow diagram is shown in Figure 1. Based on these assumptions the following system of differential equations describe the model.

$$S'_n(t) = \pi_1 \Lambda - (\lambda_d + \lambda_n) S_n - (\mu + \omega) S_n,$$

$$I'_n(t) = \pi_3 \Lambda + \lambda_n S_n - \lambda_d I_n - (\mu + \omega + \gamma) I_n,$$

$$A'_n(t) = \gamma I_n + \alpha A_d - (\mu + \omega + \phi + \nu) A_n,$$

$$S'_d(t) = \pi_2 \Lambda + \lambda_d S_n - \lambda_n S_d - (\mu + \omega) S_d,$$

$$I'_d(t) = \pi_4 \Lambda + \lambda_d S_d + \lambda_n I_n - (\mu + \omega + \gamma) I_d,$$

$$A'_d(t) = \gamma I_d - (\mu + \alpha + \omega + \phi + \nu) A_n.$$

### 2.1 Model basic properties

In this section, we study the basic results of solutions of model system (5), which are essential in the proofs of stability results.
Fig. 1. Structure of the model.

**Lemma 1.** The equations preserve positivity of solutions.

*Proof.* The vector field given by the right hand side of (5) points inward on the boundary of $\mathbb{R}_+^6 \setminus \{0\}$. For example, if $S_n = 0$ then $S_n' = \pi_1 \Lambda \geq 0$. All the other components are similar.

**Lemma 2.** Each non-negative solution is bounded in $L^1$-norm by $\max \{ N(0), \Lambda/\mu \}$.

*Proof.* The norm $L^1$ norm of each non-negative solution is $N$ and it satisfies the inequality $N' \leq \Lambda - \mu N$. Solutions to the equation $N' = \Lambda - \mu N$ are monotone increasing and bounded by $\Lambda/\mu$ if $M(0) < \Lambda/\mu$. They are monotone decreasing and bounded above if $M(0) \geq \Lambda/\mu$. Since $N' \leq M'$ the claim follows.

**Corollary 1.** The region

$$\Phi = \left\{ (S_n, I_n, A_n, S_d, I_d, A_d) \in \mathbb{R}_+^6 : N \leq \frac{\Lambda}{\mu} \right\}.$$  

is invariant and attracting for system (5).

**Theorem 1.** For every non-zero, non-negative initial value, solutions of model system (5) exist for all times.

*Proof.* Local existence of solutions follow from standard arguments since the right hand side of (5) is locally Lipschitz. Global existence follows from the a-priori bounds.
3. Disease-free equilibrium and stability analysis

The disease free equilibrium of model system (5), $E^0$ is given by

$$
E^0 = \left( S^0, I^0, A^0, S_d^0, I_d^0, A_d^0 \right) = \left( \frac{\pi_1 \Lambda}{\mu + \omega + \beta_d c_d}, 0, 0, \frac{\Lambda (\pi_2 (\mu + \omega) + \beta_d c_d)}{(\mu + \omega)(\mu + \omega + \beta_d c_d)}, 0, 0 \right).
$$

(7)

Following van den Driessche and Watmough (27), the effective reproduction number of model system (5) is given as

$$
R_{SD} = \frac{c_d \beta_d + c_d p_d \beta_d}{2 a_5} + c_H \left( \frac{\pi_1 p_n a_1 c_d \beta_d}{2 a_5 a_4 a_6} + \frac{p_n \beta_n}{2 a_4} \right)
$$

$$
+ \sqrt{\left( \frac{c_d \beta_d + c_d p_d \beta_d}{2 a_5} + c_H \left( \frac{\pi_1 p_n a_1 c_d \beta_d}{2 a_5 a_4 a_6} + \frac{p_n \beta_n}{2 a_4} \right) \right)^2 - \frac{c_n \beta_n (c_d p_n \beta_d + c_d p_d (p_n p_d - p_n p_d))}{a_3 a_4}}
$$

(8)

with $a_1 = \mu + \omega$, $a_2 = \mu + \omega + \phi + \nu$, $a_3 = \mu + \omega + \gamma$, $a_4 = \mu + \omega + \gamma + c_d \beta_d$, $a_5 = \mu + \omega + \phi + \nu$, $a_6 = \pi_2 (\mu + \omega) + c_d \beta_d$ throughout the manuscript. The reproduction number $R_{SD}$ is defined as the number of secondary HIV infections produced by one HIV infected individual during his/ her entire infectious period in a mixed population of non-drug users and intravenous drug male prisoners. Theorem 2 follows from van den Driessche and Watmough (27).

**Theorem 2.** The disease free equilibrium $E^0$ of model system (5) is locally asymptotically stable if $R_{SD} < 1$ and unstable otherwise.

**Analysis of the effective reproduction number, $R_{SD}$**

The reproduction number is differentiated into categories:

**Case 1: No intravenous drug users in the community**

In this case $\beta_d = \beta_d c_d = \beta_d c_d = p_n a_1 = p_n a_6 = 0$, $p_n a_4 = 1$ so that $R_{SD}$ becomes $R_{0s}$ which is given by

$$
R_{0s} = \frac{\beta_n c_n}{a_3}
$$

(9)

which is the number of secondary HIV infections produced by one HIV infected individual through homosexual tendencies in a male prison. It is important to note $R_{0s}$ is a decreasing function of $\omega$, suggesting that increasing the number of prisoners leaving the prison reduces the concentration of HIV cases in prison. Theoretically this is feasible, in reality this begs more questions than answers as sentences communicated cannot be reversed because of HIV. Perhaps, it may be necessary to consider the use of open prison systems where prisoners with less serious crimes can serve their sentences while staying at their homes. This has a further advantage of reducing the high levels of raping of man by man in prisons and the homosexual tendencies which male prisoners resort to in enclosed prisons which is one of the major forces driving HIV/AIDS in male prisons.

**Increase in intravenous drug users**

In this case $(p_n, \beta_d c_d) \to (1, \infty)$ so that $R_{SD}$ becomes $R_{0d}$ which is given by

$$
R_{0d} = \frac{\beta_d c_d + \beta_d c_d}{a_3}
$$

(10)
R₀₀, just like is R₀₅, is a decreasing function of ω, meaning that use of open prison systems will be beneficial in the control of HIV among male prisoners. It is important to note that levels of sexual contact are higher among intravenous drug users than non-drug users, with it increased risk of contracting HIV, so cₙβₙ < cₙβₙ and this translates R₀₅ < R₀₀. This suggest that intravenous drug use enhances HIV transmission in male prisons. Drug using prisoners are at an increased risk of HIV infection than their non-drug using counterparts. May be introducing drug substitution treatment together with introducing needle free exchange programmes will reduce the epidemic in prisons. A reduction in needle in sharing among prisoners result in HIV/AIDS prevalence as the sharing of unsterile needles in a major source of HIV transmission among male prisoners.

4. Numerical simulations

In this section, we carry out detailed numerical simulations using MatLab programming language to assess the effects of HIV transmission among male prisoners in the absence any interventional strategy which is more common in developing countries in Africa for different initial conditions. The parameter values that we use for numerical simulations are in Table 1. In Table 1, NPA denotes National Prison Administration (Zimbabwe). For influence of peer pressure forces influencing one to become an IDU, we used values adapted from Bhunu et al. (4) which are peer pressure forces necessary for one to start smoking, for the sake of illustration. For influence of peer pressure forces including one to become an IDU, we used values adapted from Bhunu et al. (2010) (4) which are peer pressure forces necessary for one to start smoking.

Figure 2 is a graphical representation showing the effect of varying initial conditions when R₅₀ > 1. In Figures 2(a) and 2(b) show the effects of varying the HIV-infected not yet showing symptoms on HIV-infected only and AIDS, respectively. Both graphs show a higher number of HIV-only and AIDS among intravenous drug using male prisoners than non-drug users. This tends to show intravenous drug using male prisoners are at increased risk of HIV infection due to sharing of unsterile needles and increased rates of homosexual sex habits.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recruitment rate</td>
<td>Λ</td>
<td>0.00163yr⁻¹ + 3000000 NPA</td>
<td></td>
</tr>
<tr>
<td>Natural mortality rate</td>
<td>μ</td>
<td>0.02yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Natural rate of progression to AIDS</td>
<td>γ</td>
<td>0.1yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Rate of leaving prison due to AIDS related sickness</td>
<td>φ</td>
<td>0.25 Assumed</td>
<td></td>
</tr>
<tr>
<td>AIDS related death rate</td>
<td>ν</td>
<td>0.4yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Product of effective contact rate for HIV infection and probability of HIV transmission per drug injection</td>
<td>cₙβₙ</td>
<td>0.562yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Probability of HIV transmission per sexual contact</td>
<td>βₙ, βₙ</td>
<td>0.125 (0.01-0.95)yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Sexual acquisition rate</td>
<td>cₙ, cₙ</td>
<td>3yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Product of effective contact rate for becoming a drug user and probability of becoming a drug misuser per contact with a misuser</td>
<td>cₙβₙ</td>
<td>0.4yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Rate of quitting drug misuse of sickness</td>
<td>α</td>
<td>0.3yr⁻¹</td>
<td></td>
</tr>
<tr>
<td>Rate of release from prison</td>
<td>ω</td>
<td>0.25yr⁻¹ Assumed</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Model parameters and their interpretations.

www.intechopen.com
Fig. 2. Effects of varying the HIV infected-only initial conditions when $R_{SD} > 1$. Parameter values used are in Table 1.

Fig. 3. Simulations showing the effects of varying the rate at which non-drug using prisoners become intravenous drug users on the population on drug using HIV-infected prisoners. Parameter values used are in Table 1.
Figure 3 shows the effects of intravenous drug use on HIV/AIDS transmission dynamics among male prisoners is illustrated by varying the rate a male prisoner becoming an intravenous drug user while in prison. It shows an increase in the number of HIV/AIDS cases among drug users cases with increase in drug use while the opposite will be happening among non-drug users. This suggest that effective control of HIV among male prisoners to some extent on the control of intravenous drug use.

5. Discussion

A mathematical model have been presented in attempt to understand the transmission dynamics of HIV/AIDS among male prisoners. Male prisoners are infected with HIV while in prison through intravenous drug use using unsterile needles (syringes) and homosexual tendencies. Intravenous drug use in male prisons act in two way: (i) sharing of unsterile needles/ syringes enhance the transmission of HIV; (ii) flashing blood that drawing is blood from someone who would have injected himself with a drug and inject it into one self which on its own exposes the injector to the HIV infection. Also intravenous drug using prisoners are more likely to engage in homosexual relations with other male prisoners and with it increased risky of HIV transmission. Analysis of the reproduction number have shown that (i) a reduction in drug use results in a decrease of HIV/ AIDS prevalence among male prisoners, (ii) release of prisoners may also act in reducing the concentration HIV/AIDS cases in prisons. The later fact is not feasible, but perhaps implementing opening prison systems where prisoners of less serious crimes are allowed to save their crimes while staying with their families enables male prisoners to cope up with stressful prison conditions. Open prison systems will reduce the influence of peer pressure among prisoners as they will have moral and psychological support from the family which does not exist in enclosed prison systems. Numerical simulations carried also support the analytic results that increase in drug use and tattooing increases HIV/ AIDS prevalence among male prisoners. The result of this study have a public health implication considering high rates of syringe lending and borrowing in prisons. This might explain why there are more HIV cases in prisons than the general population in the case of the USA (8) and this might be the case world wide. HIV infected men in prison pose a risk to their communities upon release from prison, especially in Africa where partners in marriage rarely discuss safe sex so in the absence open prison systems, it may be best to have mandatory HIV/ AIDS screening and specific educational programmes for prisoners. This will reduce the prevalence of high-risk behaviours and lower HIV transmission in male prisons, thus reducing post-release public health threat. Given the high levels of HIV in prisons about three and half times higher among prisoners than the general population, it may be best to consider the introduction of needle/ syringe free exchange programme and drug substitution treatment as a way of keeping in check HIV transmission in men prisons. Additionally provision of condoms might also help given the high levels of homosexuality in male prisoners.

6. References


Global View of HIV Infection
Edited by Dr. Vishwanath Venketaraman

Hard cover, 176 pages
Publisher InTech
Published online 26, October, 2011
Published in print edition October, 2011

Some of the topics covered in this book are: HIV infection HIV transmission Clinical symptoms of AIDS AIDS and opportunistic infection Prevention and treatment of HV Treatment of HIV infection and immune reconstitution

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
