# Power Quality Measurement Under Non-Sinusoidal Condition

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# 1. Introduction

The interest on problems related to non linear devices and their influence on the systems increased considerably since 1980. This is due to the development of new power semiconductor devices and, as a consequence, the development of new converters that increment the non linearity in electric power signals substantially (Arrillaga et al., 1995).

Several research institutions have estimated that seventy percent of all electrical power usage passes through a semiconductor device at least once in the process of being used by consumers. The increase on the utilization of electronic equipment modified the sinusoidal nature of electrical signals. These equipments increase the current waveform distortion and, as a consequence, increment the voltage waveform distortion which causes over voltage, resonance problems in the system, the increase of losses and the decrease in devices efficiency (Dugan et al., 1996).

In general, quantities used in electrical power systems are defined for sinusoidal conditions. Under non sinusoidal conditions, some quantities can conduct to wrong interpretations, and others can have no meaning at all. Apparent power (S) and reactive power (Q) are two of the most affected quantities (Svensson, 1999). Conventional power definitions are well known and implemented extensively. However, only the active power has a clear physical meaning even for non sinusoidal conditions. It represents the average value of the instantaneous power over a fix period. On the other hand, the mathematical formulation of reactive power may cause incorrect interpretation, aggravated when the analysis is extended to three phase systems (Filipski, 1984; Emanuel, 1999).

Although definitions of apparent, active, and reactive power for sinusoidal systems are universally accepted, since IXX century researchers pointed out that the angle difference between voltage and current produces power oscillation between the source and the load. All these research effort remark the importance of the power factor and the reactive power on the optimal economic dispatch. One of the initial proposals consists on dividing the power term into active, reactive and distortion power, and was the most accepted one. In the 80's the discussion about the definitions mentioned above increased because the use of non linear loads incremented considerably. Although many researchers remark the important implications of non sinusoidal conditions, up today it is very difficult to define a unique power definition for electric networks under distorted conditions. The lack of a unique definition makes that commercial measurement systems utilize different definitions, producing different results, and as a consequence, generates significant economic effects (Ghosh & Durante, 1999; Cataliotti, 2008). Therefore, measurement systems, may present different results, not only because of different principle of operation, but because of the adoption of different quantities definitions as well.

This chapter presents a critical review of apparent power, reactive power and power factor definitions. First, the most commonly used definitions for apparent power are presented, after that, reactive power and the power factor definitions are studied. These definitions are reviewed for single phase and three phase systems and are evaluated under different conditions such as sinusoidal, non sinusoidal, one phase, and balanced and unbalanced three phase systems. Then, a methodology to measure power and power quality indexes based on the instant power theory under non sinusoidal conditions is presented. Finally, the most remarkable conclusions are discussed.

#### 2. Electrical power definition under sinusoidal conditions

The classical definition of instant power for pure sinusoidal conditions is:

$$p(t) = v(t) * i(t) \tag{1}$$

Where  $p_{(t)}$ ,  $v_{(t)} \in i_{(t)}$  are the instant power, instant voltage and instant current respectively. Considering sinusoidal voltage and current signals represented by the equations  $v_{(t)} = \sqrt{2} * V * \sin(\omega t)$  and  $i(t) = \sqrt{2} * I * \sin(\omega t - \phi)$  respectively, then Eq. (1) takes the following form:

$$p(t) = V * I * \cos(\phi) - V * I * \cos(\phi) * \cos(2 * \omega t) + V * I * \sin(\phi) * \sin(2 * \omega t)$$

$$(2)$$

$$p(t) = P^* (1 - \cos(2^* \omega t)) + Q^* \sin(2^* \omega t)$$
(3)

The mean value of  $p_{(t)}$  is known as active power *P* and can be represented by:

$$P = V * I * \cos\phi \tag{4}$$

Where *V* and *I* are the root means square (r.m.s.) value of the voltage and current signals respectively and  $\phi$  is the phase shift between  $v_{(t)}$  and  $i_{(t)}$ . In a similar manner, the reactive power *Q* is defined as:

$$Q = V * I * \sin\phi \tag{5}$$

The geometric sum of *P* and *Q* is know as apparent power *S* and can be calculated as follow:

$$S = V * I = \sqrt{P^2 + Q^2} \tag{6}$$

Another important term related to the power definition is the relationship between the active power with respect to the apparent power, it isknown as the system power factor *FP* and gives an indication of the system utilization efficiency:

$$FP = \frac{P}{S} = \cos\phi \tag{7}$$

Analyzing Eq. (1) to (7), the following important properties related to the reactive power can be summarized (Svensson, 1999; Filipski, & Labaj, 1992): a) Q can be represented as a function of  $V * I * \sin(\omega t)$ , b) Q is a real number, c) For a given Bus, the algebraic sum of all reactive power is zero, d) Q is the bidirectional component of the instant power  $p_{(t)}$ , e) Q = 0 means that the power factor *FP* is one, f) Q can be compensated by inductive or capacitive devices, g)The geometric sum of *P* and *Q* is the apparent power *S*, h) The voltage drop through transmission lines is produced mostly by the reactive power Q.

These properties apply exclusively to pure sinusoidal signals; therefore in the case of non sinusoidal conditions not all of these properties are fulfilled. Next section presents different power definitions proposed for that purpose, and discusses for which conditions they meet the above properties.

# 2.1 Electrical power definitions under non-sinusoidal conditions

In order to represent a non-sinusoidal condition, let's consider voltage and current signals with harmonic components, then the apparent power can be represented by the following equation:

$$S^{2} = \sum_{n=0}^{\infty} V_{n}^{2} * \sum_{n=0}^{\infty} I_{n}^{2} = V^{2} * I^{2}$$
(8)

For simplicity, let's assume the case where only harmonic signals are present within the current signals and a voltage signal with only a fundamental component, then:

$$S^{2} = V_{1}^{2} * \sum_{n=0}^{\infty} I_{n}^{2} = V_{1}^{2} * I_{1}^{2} + V_{1}^{2} * \sum_{n\neq 1}^{\infty} I_{n}^{2}$$
<sup>(9)</sup>

By definition, the active power is:

$$P = \frac{1}{T} * \int_{0}^{t} v(t) * i(t) * dt = V_{1} * I_{1} * \cos(\phi_{1})$$
(10)

And the reactive power Q:

$$Q = \sum_{n=1}^{\infty} V_n * I_n * \sin(\phi_n) = V_1 * I_1 * \sin(\phi_1)$$
(11)

Examining the expressions given by Eq. (9) to (11) and comparing them with Eq. (6), can be concluded that if the signals have components in addition to the fundamental sinusoidal component (60Hz or 50 Hz), the following expression obeys:

$$P^{2} + Q^{2} = V_{1}^{2} * I_{1}^{2} \neq S^{2}$$
(12)

From the inequality represented by Eq. (12) it is observed that the sum of the quadratic terms of  $P_{and} Q$  involves only the first term of Eq. (9), therefore property g) does not comply. Hence, definitions of apparent and reactive power useful for sinusoidal conditions may produce wrong results, thus, new definitions for non-sinusoidal conditions are needed. There are proposals to extend apparent power and reactive power formulations for non-sinusoidal situations; the most used ones are described next.

#### 2.2 Reactive power and distortion power definitions

One of the first power definitions that include the presence of harmonics was given by Budenau in 1927 (Budeneau, 1927, as cited in Yildirim & Fuchs 1999), where the active and reactive powers are defined by the following expressions:

$$P = \sum_{h} V_h * I_h * \cos\phi_h \tag{13}$$

$$Q_{B} = \sum_{h} V_{h} * I_{h} * \sin \phi_{h} \tag{14}$$

Where h is the harmonic number. Representing the active and reactive power by Eq. (13) and Eq. (14), the power triangle does not comply, therefore Budenau defined a new term know as distortion power:

$$D = \sqrt{S^2 - P^2 - Q_B^2}$$
(15)

Based on the distortion power, a complementary or fictitious power is also defined:

$$F = \sqrt{S^2 - P^2} = Q_B^2 + D^2 \tag{16}$$

The physical meaning of Eq. (16) is a power oscillation between the source and the sink, however this only stand when all elements are purely linear and reactive (i.e. capacitors and inductors), which means that Eq. (16) can not be used for reactive compensation design.

Based on this initial definition of distortion power, several other authors proposed different definitions of *D* as a function of r.m.s. voltage and current harmonic signals and their phase shift. Reference (Emanuel, 1990) proposes the following definition:

$$D^{2} = \sum_{\substack{m,n=1\\m\neq n}} V_{m}^{2} * I_{n}^{2} + V_{n}^{2} * I_{m}^{2} - 2 * V_{m} * V_{n} * I_{m} * I_{n} * \cos(\phi_{m} - \phi_{n})$$
(17)

where  $V_m$ ,  $I_m$ ,  $V_n$  y  $I_n$  are the r.m.s. harmonic components. The harmonic angles are  $\varphi_m = \alpha_m - \beta_m$ ,  $\varphi_n = \alpha_n - \beta_n$ , with  $\alpha_m$ ,  $\alpha_n$ ,  $\beta_m$  y  $\beta_n$  the angle shift between the voltage and current harmonic components.

Another different definition was proposed by reference (Filipski, 1984):

$$D = \sqrt{\sum_{m} \sum_{n} \left[ V_{m}^{2} * I_{n}^{2} - V_{m} * V_{n} * I_{m} * I_{n} * \cos(\phi_{m} - \phi_{n}) \right]}$$
(18)

After that, Czarnecki (Czarnecki, 1993) recommended the following formulae for D:

$$D = \sqrt{\frac{1}{2} \sum_{m} \sum_{n} \left[ V_m^2 * I_n^2 - 2 * V_m * V_n * I_m * I_n * \cos(\phi_m - \phi_n) \right]}$$
(19)

Similar definition than the one described by Eq. (18) was proposed by the IEEE Std. 100-1996 (Institute of Electrical and Electronic Engineering [IEEE], 1996). Recently, different authors compared them and discussed their advantages and applicability. Yildirim and Fuchs (Yildirim & Fuchs, 1999) compared Eq. (17) to (19) and performed experimental

measurements using different type of voltage and current distortions, recommending the following distortion definition:

$$D^{2} = \sum_{m=0}^{h-1} \sum_{m\neq 1}^{h} \left[ V_{m}^{2} * I_{n}^{2} + V_{n}^{2} * I_{m}^{2} - 2 * V_{m} * V_{n} * I_{m} * I_{n} * \cos(\phi_{m} - \phi_{n}) \right]$$
(20)

The most important conclusions from their studies are that Eq. (17) presents important difference with respect to practical results; results calculated using Eq. (18) to (20) are identical and consistent with experimental results. Eq. (18) and (19) have all terms that multiply variables with the same harmonic order, while in Eq. (20) all terms multiply variables of different harmonic order.

#### 2.3 Reactive power definition proposed by Fryze

The reactive power definition proposed by Fryze is based on the division of the current into two terms; the active current term and the reactive current term (Fryze, 1932, as cited in Svensson 1999):

$$i = i_a + i_r \tag{21}$$

Considering that these terms are orthogonal, the following property applies:

$$\frac{1}{T}\int_{0}^{T} i_{a} * i_{b} * dt = 0 \quad \text{(orthogonal)}$$
(22)

 $i_a$  can be calculated from the active power:

$$i_a(t) = \frac{P}{V^2} * v(t)$$
 (23)

Then, from Eq. (21), the reactive power  $i_r$  is:

$$i_r(t) = i(t) - i_a(t)$$
 (24)

Based on these definitions and considering Eq. (16), the reactive power representation proposed by Fryze is:

$$Q_F = V * I_r = \sqrt{\left(V * I\right)^2 - \left(V * I_a\right)^2} = \sqrt{S^2 - P^2} = \sqrt{Q_B^2 + D^2}$$
(25)

Eq. (25) shows that  $Q_F$  is a function of S and P, therefore, the advantage of this representation is that there is no need to measure the reactive power. However,  $Q_F$  is always a positive magnitude, then, property b) does not apply, hence, it can not be used for power flow analysis. On the other hand, since it is always positive, it can be compensated by injecting a negative current  $-i_r$  which makes it suitable for active filter design.

#### 2.4 Reactive power definition proposed by Emanuel

Emanuel observed that in most cases, the principal contribution to the reactive power is due to the fundamental component of the voltage signal, then, he proposed the following definition for the reactive power term (Emanuel, 1990):

$$Q_1 = V_1 * I_1 * \sin \phi_1 \tag{26}$$

Based on this definition, an additional term named complementary power can be formulated:

$$P_{\rm C}^2 = S^2 - P^2 - Q_1^2 \tag{27}$$

Finally, both active and reactive terms can be represented by two terms; the fundamental and the harmonic component:

$$S^{2} = (P_{1} + P_{h})^{2} + Q_{F}^{2}$$
<sup>(28)</sup>

Where  $Q_F$  is the reactive power defined by Fryze. Expressing  $Q_F$  as a function of the fundamental and harmonic term:

$$Q_F^2 = Q_1^2 + Q_h^2 \tag{29}$$

And replacing Eq. (29) into Eq. (28), the apparent power is:

$$S^{2} = (P_{1} + P_{h})^{2} + Q_{1}^{2} + Q_{h}^{2}$$
(30)

Since  $Q_F$  is defined adding two different terms, the fundamental reactive power  $Q_1$  and the harmonic reactive power  $Q_H$ , this definition became an effective tool for active filters control and monitoring and power factor shift compensation design.

#### 2.5 Definition proposed by Czarnecki

Based on previous definitions, Czarnecki proposed new definitions based on a orthogonal current decomposition that allows to identify different phenomena that cause the efficiency decrease of the electrical energy transmission (Czarnecki, 1993).

The total current is decomposed in active, reactive, harmonic and disperses terms:

$$I^{2} = I_{A}^{2} + I_{R}^{2} + I_{S}^{2} + I_{H}^{2}$$
(31)

The latest three terms are the ones responsible of the efficiency transmission decrease. Where the reactive term is given by:

$$I_R = \sqrt{\sum_{n=N} B_n^2 * V_n^2} \tag{32}$$

Index k is the harmonic component that is not present in the N voltage terms, the harmonic term is calculated as:

$$I_H = \sqrt{\sum_{n=K} I_n^2} \tag{33}$$

And the disperse current can be represented as follow:

$$I_{R} = \sqrt{\sum_{n=N} (G_{n} - G)^{2} * V_{n}^{2}}$$
(34)

Where the equivalent load conductance is:

$$G = \frac{P}{V^2}$$
(35)

And the n-order harmonic component of the load is:

$$Y_n = G_n + jB_n \tag{36}$$

Using this decomposition, the apparent power can be expressed as:

$$S^2 = P^2 + D_S^2 + Q_R^2 + D_H^2$$
(37)

Where the reactive power, the distortion power and the harmonic power are respectivelly:

$$Q_R = V * I_R \tag{38}$$

$$D_s = V * I_s \tag{39}$$

$$D_H = V * I_H \tag{40}$$

One of the main feature of this definition is that is based on suceptances instead of voltages, currents and powers. For systems that contain currents with large harmonic values and voltage with small harmonic values, will present the problem of phase shift uncertainty and, as a concequence, large uncertainty of parameter  $B_{\rm N}$ . This issue may produce errors in the reactive current determination.

#### 2.6 Definition proposed by the IEEE Std 1459-2000

This standard proposes the decomposition of both current and voltage signals into fundamental and harmonic terms (Institute of Electrical and Electronic Engineering [IEEE], 2000):

$$I^2 = I_1^2 + I_H^2 \tag{41}$$

$$V^2 = V_1^2 + V_H^2 \tag{42}$$

Where the harmonic components  $V_H$ ,  $I_H$  include all harmonic terms and the direct current component as well:

$$V_H^2 = \sum_{h=1}^{\infty} V_h^2 \tag{43}$$

$$I_H^2 = \sum_{h\neq 1}^{\infty} I_h^2 \tag{44}$$

Based on these terms, the active power can be represented as the sum of the fundamental and harmonic components:

$$P = P_1 + P_H \tag{45}$$

Where the fundamental and harmonic components are respectivelly:

$$P_1 = \sum_{l_1 \neq 1}^{\infty} V_1 I_1 \cos \phi_1$$
 (46)

$$P_{H} = \sum_{h\neq 1}^{\infty} V_{h} I_{h} \cos\phi_{h}$$
(47)

Similarly, the reactive power can be represented:

$$Q = Q_1 + Q_H \tag{48}$$

Where the fundamental and harmonic components are:

$$Q_{1} = \sum_{h\neq 1}^{\infty} V_{1} I_{1} \sin \phi_{1}$$
(49)

$$Q_H = \sum_{h\neq 1}^{\infty} V_h I_h \sin \phi_h \tag{50}$$

Considering that the square of the apparent power can be represented as a function of the voltage and current terms:

$$S^{2} = (VI)^{2} = (V_{1}I_{1})^{2} + (V_{1}I_{H})^{2} + (V_{H}I_{1})^{2} + (V_{H}I_{H})^{2}$$
(51)

And representing the apparent power *S* as the sum of a fundamental and non fundamental term:

$$S^2 = S_1^2 + S_N^2 \tag{52}$$

It is possible to conclude by comparing Eq. (51) with Eq. (52), that the first term of the square of the apparent power, which is a function of the fundamental components, can be also represented as a function of the fundamental active and reactive components. These terms are:

$$S_1^2 = (V_1 I_1)^2 = P_1^2 + Q_1^2$$
(53)

And term  $S_N$  is composed by the rest of the terms present in Eq. (51):

$$S_N^2 = (V_1 I_H)^2 + (V_H I_I)^2 + (V_H I_H)^2 = D_I^2 + D_V^2 + S_H^2$$
(54)

Where the distortion power due to the harmonic current is:

$$D_I = V_1 I_H \tag{55}$$

And due to the harmonic voltage:

$$D_V = V_H I_1 \tag{56}$$

Finally the last term is known as the harmonic apparent power:

$$S_H = V_H I_H \tag{57}$$

Defining the relationship between the harmonic current and the fundamental current components as the total harmonic current distortion  $I_H/I_1 = THDi$  and similarly for the voltage  $V_H/V_1 = THDv$  then the equations can be represented as a function of the distortion:

$$D_I = S_1 * T H D_I \tag{58}$$

$$D_V = S_1 * THD_V \tag{59}$$

$$S_{H} = S_{1} * THD_{I} * THD_{V} \tag{60}$$

Finally, the apparent power can be decomposed into the active power P and the non-active power N:

$$S^2 = (VI)^2 = P^2 + N^2$$
(61)

Since the harmonic power term is the only one that can have an active component, it can be formulated as follow:

$$S_{H}^{2} = (V_{H}I_{H})^{2} = P_{H}^{2} + N_{H}^{2}$$
(62)

From all these equations, several important observations can be made:  $(P_1 + P_H)$  is the active power, The harmonic power  $S_H$  has (n-1) terms as a function of  $V_h * I_h * \cos \varphi_h$ , these terms can have the following values: Null, if  $V_h$  or  $I_h$  are null, or the phase shift is 90°. Positive, if  $V_h$  and  $I_h$  are not null and the phase shift verifies the following inequalities–90°  $< \varphi_h <$ 90°. Negative, if  $V_h$  and  $I_h$  are not null and the phase shift verifies the following inequalities90°  $< \varphi_h < 270°$ . Some harmonic component can produce and others can consume power, and in general  $P_H$  is negative. Relationship  $S_N/S_1$  is a good indicator of harmonic distortion. The following inequality stand:

$$S_N \ge S_H \ge P_H \tag{63}$$

The power factor due to the fundamental component, also known as shift power factor is:

$$PF_1 = \cos\phi_1 = \frac{P_1}{S_1}$$
(64)

The total power factor is given by the following expression:

$$PF = \frac{P}{S} = \frac{(P_1 + P_H)}{S} = \frac{\binom{P_1}{S_1} * \left[1 + \binom{P_H}{P_1}\right]}{\sqrt{1 + \binom{S_N}{S_1}^2}} = \frac{\left[1 + \binom{P_H}{P_1}\right] * PF_1}{\sqrt{1 + THD_1^2 + THD_2^2 + \left(THD_1 * THD_V\right)^2}}$$
(65)

In summary, the discussion related to the different definitions is focused on which of the property is complied and which one is not (Filipski & Labaj, 1992). Nevertheless, it is also

important to undertand the meaning of the different expressions and to select the correct index for the specific application such as compensation, voltaje control, identify the source of the harmonic perturbation, or to evaluate the power losses determinado (Balci & Hocaoglu, 2004). The same type of analysis can be extended for multiphase systems, the apparent power definitions for three phase systems is described next.

# 3. Electric power definitions for three phase systems

Similarly to a single phase system, the definition of apparent power for a three phase system under non sinusoidal conditions has no physical meaning, therefore may drives to wrong interpretations. The measurement, analysis and definition of the different terms of three phase power signal, where voltages and currents are unbalanced and distorted, have been studied in order to standardize the correct indexes that quantify the level of harmonic and distortion (Emanuel, 1999, 2004). An incorrect interpretation or error measurements may produce the wrong operation of the system and as a consequence, a high economic impact. The normal indicators such as apparent power and nominal voltage that are very important for equipment selection (i.e. transformers, machines) are set for balanced, symmetric and sinusoidal signals. Moreover, they are used by utilities to design the tariff scenario. The power factor index quantifies the energy utilization efficiency (Catallioti et al., 2008, 2009a). As a consequence, nowadays, to have an accurate and consensual definition of apparent, reactive power and power factor for non-sinusoidal three phase systems becomes relevant. In the next section the most used definitions are discussed.

# 3.1 Apparent power definition for three phase systems

There are several definitions related to the calculation of apparent power for unbalanced three phase systems. In this section the most relevant ones are reviewed (Pajic & Emanuel, 2008; Eguiluz & Arrillaga, 1995; Deustcher Industrie Normen [DIN], 2002; Institute of Electrical and Electronic Engineering [IEEE], 2000).

Based on the single phase definitions, in a multiphase system, the apparent power vector is:

$$S_{V} = \sqrt{\left(\sum_{k=a}^{c} P_{k}\right)^{2} + \left(\sum_{k=a}^{c} Q_{bk}\right)^{2} + \left(\sum_{k=a}^{c} D_{k}\right)^{2}}$$
(66)

The arithmetic apparent power can be represented as de sum of all phase's apparent power:

$$S_A = \sum_{k=a}^{c} \sqrt{P_k^2 + Q_{bk}^2 + D_k^2}$$
(67)

For a phase k,  $P_k$  is the active power, and  $Q_{bk}$  and  $D_k$  are de reactive and distortion power defined by Budeanu, respectively. The definitions described by Eq. (66) and Eq (67) are identical and produce correct results for balanced load and sinusoidal voltage and current signals. However, for general unbalanced and/or distorted signals, it can be proved that:

$$S_{V} \le S_{A} \tag{68}$$

In addition, the power factor index will also produce different results depending on which definition is used:

$$FP_{V} = \frac{P}{S_{V}} \ge FP_{A} = \frac{P}{S_{A}}$$
(69)

Where  $FP_V$  and  $FP_A$  are the power factor using the apparent power vector and the arithmetic definition respectively.

The following expression to calculate the apparent power is proposed in (Goodhue, 1933 cited in Depenbrock, 1992; Emanuel, 1998):

$$S = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ac}^2}{3}} \sqrt{I_a^2 + I_b^2 + I_c^2}$$
(70)

Conceptually, Eq (70) illustrates that for a given three phase system it is possible to define an equivalent apparent power known as the effective apparent power that is defined as follow:

$$S_{e} = 3 * V_{e} * I_{e}$$
 (71)

Where  $V_e$  y  $I_e$  are the r.m.s. effective voltage and current values respectively.

Recently, several authors proposed different mathematical representation based on Eq. (71). The most important ones are the one described by the standard DIN40110-2 (Deustcher Industrie Normen [DIN], 2002) and the one developed by the IEEE Working Group (Institute of Electrical and Electronic Engineering [IEEE], 1996) that was the origin of the IEEE Standard 1459-2000 (Institute of Electrical and Electronic Engineering [IEEE], 2000). These two formulations are described next.

## 3.2 Definition described in the standard DIN40110-2

This method, known as FBD method (from the original authors Fryze, Buchholz, Depenbrock) was developed based on Eq. (71) (Depenbrok, 1992, 1998; Deustcher Industrie Normen [DIN], 2002). It defines the effective values of currents and voltages based on the representation of an equivalent system that shares the same power consumption than the original system.

Then, the effective current can be calculated by the following expression:

$$I_{eT} = \sqrt{\frac{1}{3} \left( I_r^2 + I_s^2 + I_t^2 + I_n^2 \right)} = \sqrt{\left( I_+ \right)^2 + \left( I_- \right)^2 + 4 * \left( I_0 \right)^2}$$
(72)

Where  $i_r$ ,  $i_s$ ,  $i_t$  are the line currents and  $i_n$  the neutral current. Similarly, the effective voltage is:

$$V_{e} = \sqrt{\frac{(V_{r}^{2} + V_{s}^{2} + V_{t}^{2}) + (V_{rs}^{2} + V_{rt}^{2} + V_{ts}^{2})}{12}} = \sqrt{(V_{+})^{2} + (V_{-})^{2} + \frac{1}{4} * (V_{0})^{2}}$$
(73)

This method allows decomposing both currents and voltages into active and non active components. Moreover, it allows distinguishing each component of the total non active term, becoming a suitable method for compensation studies.

# 3.3 Definition proposed by the IEEE Standard 1459-2000

This standard assumes a virtual balanced system that has the same power losses than the unbalanced system that it represents. This equivalent system defines an effective line current  $I_e$  and an effective phase to neutral voltage  $V_e$ .

$$I_{e} = \sqrt{\frac{1}{3} \left( I_{r}^{2} + I_{s}^{2} + I_{t}^{2} + \rho * I_{n}^{2} \right)}$$
(74)

Where the factor  $\rho = r_n/r$  can vary from 0.2 to 4.

Similar procedure can be followed in order to obtain a representation for the effective voltage  $V_e$ . In this case, the load is represented by three equal resistances conected in a star configuration, and three equal resistances connected in a delta configuration, the power relationship is defined by factor  $\varepsilon = P_{\Delta}/P_{Y}$ .

Considering that the power losses are the same for both systems, the effective phase to neutral voltage for the equivalent system is:

$$V_{e} = \sqrt{\frac{3*(V_{r}^{2} + V_{s}^{2} + V_{t}^{2}) + \varepsilon*(V_{rs}^{2} + V_{rt}^{2} + V_{ts}^{2})}{9*(1+\varepsilon)}}$$
(75)

In order to simplify the formulations, the standard assumes unitary value of  $\rho$  and  $\varepsilon$ , then Eq. (74) and (75) can be represented as:

$$I_{e} = \sqrt{\frac{1}{3} \left( I_{r}^{2} + I_{s}^{2} + I_{t}^{2} + I_{n}^{2} \right)}$$
(76)

$$V_e = \sqrt{\frac{3*(V_r^2 + V_s^2 + V_t^2) + (V_{rs}^2 + V_{rt}^2 + V_{ts}^2)}{18}}$$
(77)

These effective current and voltage can also be represented as a function of sequence components:

$$I_e = \sqrt{(I_+)^2 + (I_-)^2 + 4^* (I_0)^2}$$
(78)

$$V_e = \sqrt{(V_+)^2 + (V_-)^2 + \frac{1}{2} * (V_0)^2}$$
(79)

Since one of the objectives of these formulations is to separate the funtamental term from the distortion terms, the effective values can be further decomposed into fundamental and harmonic terms:

$$V_{e}^{2} = V_{e1}^{2} + V_{eH}^{2}$$
(80)

$$I_e^2 = I_{e1}^2 + I_{eH}^2$$
(81)

Where the fundamental terms are:

$$V_{e1} = \sqrt{\frac{3*(V_{r1}^2 + V_{s1}^2 + V_{t1}^2) + (V_{rs1}^2 + V_{t1}^2 + V_{ts1}^2)}{18}} \quad (\varepsilon = 1)$$
(82)

$$I_{e1} = \sqrt{\frac{1}{3} \left( I_{r1}^2 + I_{s1}^2 + I_{t1}^2 + I_{n1}^2 \right)} \quad (\rho = 1)$$
(83)

And the harmonic terms:

$$V_{eH}^2 = V_e^2 - V_{e1}^2$$
(84)

$$I_{eH}^2 = I_e^2 - I_{e1}^2$$
(85)

Considering these definitions, the effective apparent power can be calculated as follow:

$$S_{e}^{2} = (3 * V_{e1} * I_{e1})^{2} + (3 * V_{e1} * I_{eH})^{2} + (3 * V_{eH} * I_{e1})^{2} + (3 * V_{eH} * I_{eH})^{2}$$
(86)

Where the fundamental term of the effective apparent power is:

$$S_{e1} = 3 * V_{e1} * I_{e1} \tag{87}$$

The fundamental term can also be represented as a function of active and reactive sequence powers:

$$\left(S_{1}^{+}\right)^{2} = \left(P_{1}^{+}\right)^{2} + \left(Q_{1}^{+}\right)^{2}$$
(88)

Where:

$$P_1^+ = 3 * V_1^+ * I_1^+ * \cos \phi_1^+ \tag{89}$$

$$Q_1^+ = 3 * V_1^+ * I_1^+ * \sin \phi_1^+ \tag{90}$$

Then, the square of the fundamental effective apparent power can be represented as the addition of two terms:

$$S_{e1}^{2} = \left(S_{1}^{+}\right)^{2} + \left(S_{U1}\right)^{2}$$
(91)

Where the term  $S_{U1}$  is due to the system unbalance. Similarly, the non fundamental term  $S_{eN}$  can be represented by:

$$S_{eN}^{2} = (3 * V_{e1} * I_{eH})^{2} + (3 * V_{eH} * I_{e1})^{2} + (3 * V_{eH} * I_{eH})^{2}$$
(92)

Where the three terms can be represented as a function of the total harmonic distortion, defining the distortion power due to the current as:

$$D_{eI} = 3 * V_{e1} * I_{eH} = 3 * S_{e1} * THD_I$$
(93)

The distortion power due to the voltage:

$$D_{eV} = 3 * V_{eH} * I_{e1} = 3 * S_{e1} * THD_{V}$$
(94)

And the effective harmonic apparent power:

$$S_{eH} = 3 * V_{eH} * I_{eH} = 3 * S_{e1} * THD_V * THD_I$$
(95)

Finally, the harmonic active power can be calculated:

$$P_{H} = \sum_{\substack{h \neq 1 \\ i = r, s, t}} V_{ih} * I_{ih} * \cos \phi_{ih} = P - P_{1}$$
(96)

The main features of the formulations proposed by this standard are:  $P_1^+$  can be separated from the rest of the active power component. In general  $P_H$ ,  $P_1^- y P_1^0$  can be neglected since are small with respect to  $P_1^+$ , therefore results obtained by measuring only this term is accurate enough. Identify  $Q_1^+$  from the rest of the reactive power componets, it allows to design the appropriate capacitor bank in order to compensate the power factor shift  $\cos \phi_1^+$ . The non fundamental apparent power  $S_{eN}$  allows evaluating the distortion severity and becomes a usefull parameter to estimate the harmonic filter size to compensate the distortion.

Analyzing the apparent power definitions for three phase systems can be observed that the apparent power may have different values depending on the system conditions and the selected definition, being (Eguiluz & Arrillaga, 1995; Emanuel, 1999):

$$S_v \le S_a \le S_e \tag{97}$$

Similarly observation stands for the power factor values:

$$FP_{V} = \frac{P}{S_{VEC}} \ge FP_{A} = \frac{P}{S_{AVA}} \ge FP_{e} = \frac{P}{S_{e}}$$
(98)

Method FBD is replaced by the one proposed by the IEEE because it is simpler and is more related to the network parameters

#### 4. Power measurement under non-sinusoidal conditions

The determination of the different power terms such as active power, reactive power, distortion, fundamental component of the positive sequence, and other important parameters (*i.e.FP,DFP,THDv,THDi*) are becoming relevant. The power measurement algorithms included in the electronic devices are based on these definitions. Therefore, it is always a concern to implement the most accurate methodology, since errors in power measurement may translate into huge economic losses (Filipsky & Labaj, 1992; Cook & Williams, 1990). In general, these errors are negligible if the system is sinusoidal and balanced; however, this is not the scenario when the system has harmonic or/and unbalanced signals (Cataliotti et al., 2009a, 2009b; Gunther & McGranaghan, 2010).

New technology allows the use of accurate, fast, and low cost measurement systems, however the lack of a unique apparent power definition for unbalanced and distorted systems makes the results of the measurements if not wrong, at least a controversial issue (Morsi & El-Hawary, 2007; Cataliotti et al., 2009a).

In this section, based on the power definitions explained in previous sections and the instantaneous power theory, a methodology to measure power under unbalanced conditions is proposed.

The algorithm is based on the standard IEEE 1459 – 2000 (Institute of Electrical and Electronic Engineering [IEEE], 2000), the instant power theory, currently used for active filter design, is used for the signal processing phase (Akagi et al, 1983; Herrera & Salmerón, 2007; Watanabe et al., 1993; Akagi et al., 2007; Czarneky, 2006, 2004; Seong-Jeub, 2008). The Fundamentals of this theory is explained next.

#### 4.1 Instant power theory

A three phase system can be represented by three conductors where the voltage are  $v_r$ ,  $v_s$ ,  $v_t$  and the line currents are  $i_r$ ,  $i_s$ ,  $i_t$  then, this system can be represented by an equivalent two phase system with the following voltages and currents:

$$\begin{bmatrix} v_{\alpha} \\ v_{\beta} \end{bmatrix} = T * \begin{bmatrix} v_{r} \\ v_{s} \\ v_{t} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} = T * \begin{bmatrix} v_{r} \\ v_{s} \\ v_{t} \end{bmatrix}$$
(99)

Where *T* is known as the Park transformation matrix:

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$
(100)

Then, the active and reactive power can be calculated as follow:

$$p = v_{\alpha} * i_{\alpha} + v_{\beta} * i_{\beta} \tag{101}$$

$$q = v_{\alpha} * i_{\beta} - v_{\beta} * i_{\alpha} \tag{102}$$

where:  $v_{\alpha}$ : instant voltage, direction  $\alpha$ ,  $v_{\beta}$  : instant voltage, direction  $\beta$ ,  $i_{\alpha}$  : instant current direction  $\alpha$ ,  $i_{\beta}$  : instant current, direction  $\beta$ , p: instant active power [W], q: instant reactive power [VA].

Using matrix notation, the power equation can be represented as:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_{\alpha} & v_{\beta} \\ -v_{\beta} & v_{\alpha} \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix}$$
(103)

And the following expression stands:

$$P_{3\phi} = v_r * i_r + v_s * i_s + v_t * i_t = v_\alpha * i_\alpha + v_\beta * i_\beta$$
(104)

Where  $P_{3\phi}$  is the three phase instant power.

These expressions can be extended for a four conductor system; in this case a zero sequence term is needed:

$$\begin{bmatrix} v_{0} \\ v_{\alpha} \\ v_{\beta} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_{r} \\ v_{s} \\ v_{t} \end{bmatrix}$$
(105)

Where:  $v_0$ : Instant zero sequence voltage,  $i_0$ : Instant zero sequence current. Similar expressin can be obtained for the currents. Defining the instant zero sequence power  $p_0$  as:

$$p_0 = v_0 * i_0 \tag{106}$$

Then, the power vector can be calculated as follow:

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & -v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}$$
(107)

And the three phase power can be represented by:

$$P_{3\varphi} = v_r * i_r + v_s * i_s + v_t * i_t = = v_\alpha * i_\alpha + v_\beta * i_\beta + v_0 * i_0$$
(108)

From Eq. (106) and (108) the three phase power can be calculated:

$$P_{3\phi} = p + p_0 \tag{109}$$

Power terms p,  $q \ge p_0$  can be decomposed using Fourier series and rearranged as a constant power and an harmonic power with a zero mean value:

$$p = \overline{p} + \widetilde{p};$$

$$q = \overline{q} + \widetilde{q};$$

$$p_0 = \overline{p}_0 + \widetilde{p}_0$$
(110)

Where:  $\bar{p}$ : pmean value,  $\tilde{p}$ : Oscillatory component of p,  $\bar{q}$ : qmean value,  $\tilde{q}$ : Oscillatory component of q,  $\overline{p_0}$ : mean value of  $p_0$  and  $\tilde{p_0}$ : Oscillatory component of  $p_0$ . Assuming a three phase system distorted, the line current can be represented as a function of the symmetrical components:

$$i_{r}(t) = \sum_{n=1}^{\infty} \sqrt{2} * I_{0n} * \sin(\omega_{n}t + \phi_{0n}) + \\ + \sum_{n=1}^{\infty} \sqrt{2} * I_{+n} * \sin(\omega_{n}t + \phi_{+n}) + \\ + \sum_{n=1}^{\infty} \sqrt{2} * I_{-n} * \sin(\omega_{n}t + \phi_{-n})$$
(111)

Same formulae can be obtained for  $i_{s}$ ,  $i_t$ . Similar expressions can be formulated for the voltages. Then, the power equation described in Eq. (110) can be written as a function of symmetrical currents and voltages:

$$\overline{p} = \sum_{n=1}^{\infty} 3V_{+n} * I_{+n} * \cos(\varphi_{+n} - \delta_{+n}) + 3V_{-n} * I_{-n} * \cos(\varphi_{-n} - \delta_{-n})$$
(112)

$$\tilde{p} = \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} 3V_{+m} * I_{+n} * \cos((\omega_m - \omega_n)t + \varphi_{+m} - \delta_{+n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} 3V_{-m} * I_{-n} * \cos((\omega_m - \omega_n)t + \varphi_{-m} - \delta_{-n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} -3V_{+m} * I_{-n} * \cos((\omega_m - \omega_n)t + \varphi_{+m} - \delta_{-n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} -3V_{-m} * I_{+n} * \cos((\omega_m - \omega_n)t + \varphi_{-m} - \delta_{+n}) \right]$$
(113)

$$\overline{q} = \sum_{n=1}^{\infty} -3V_{+n} * I_{+n} * \sin(\varphi_{+n} - \delta_{+n}) + 3V_{-n} * I_{-n} * \sin(\varphi_{-n} - \delta_{-n})$$
(114)

$$\tilde{q} = \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} -3V_{+m} * I_{+n} * \sin((\omega_m - \omega_n)t + \varphi_{+m} - \delta_{+n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} 3V_{-m} * I_{-n} * \sin((\omega_m - \omega_n)t + \varphi_{-m} - \delta_{-n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} 3V_{+m} * I_{-n} * \sin((\omega_m - \omega_n)t + \varphi_{+m} - \delta_{-n}) \right] + \\ + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[ \sum_{n=1}^{\infty} -3V_{-m} * I_{+n} * \sin((\omega_m - \omega_n)t + \varphi_{-m} - \delta_{+n}) \right]$$
(115)

$$\overline{p}_{0} = \sum_{n}^{\infty} 3V_{on} * I_{0n} * \cos(\varphi_{0n} - \delta_{0n})$$
(116)

$$\tilde{p}_{0} = \sum_{\substack{m=1\\m\neq n}} \left[ \sum_{n=1}^{\infty} 3V_{om} * I_{0n} * \cos((\omega_{m} - \omega_{n})t + \varphi_{0n} - \delta_{0n}) \right] + \sum_{m=1} \left[ \sum_{n=1}^{\infty} -3V_{om} * I_{0n} * \cos((\omega_{m} - \omega_{n})t + \varphi_{0n} - \delta_{0n}) \right]$$
(117)

Finally, based on the oscillatory terms, reference (Watanabe et al, 1993) defined the distortion power terms as follow:

$$H = \sqrt{\overline{p^2 + q^2}} \tag{118}$$

# 4.2 Measurement algorithm evaluation

These previous equations define expressions for real, imaginary and zero sequence power terms as a function of symmetrical components. Based on them, expressions that allow the apparent power calculation considering different conditions are explained next.

**Case 1 -** Balanced and sinusoidal system: In this case, the oscillatory, the negative sequence, and the zero sequence terms of both the real and the imaginary power components are zero, therefore the following expression can be proposed for apparent power calculation:

$$S^{2} = \overline{p}^{2} + \overline{q}^{2} + H^{2} = \overline{p}^{2} + \overline{q}^{2}$$
(119)

Then the apparent power can be calculated as follow:

$$S^{2} = \left(3 * V_{+1} * I_{+1}\right)^{2} = S_{e}^{2}$$
(120)

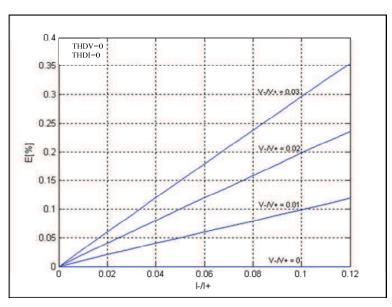
Case 2 - Unbalanced load: From Eq. (119), for this case the apparent power is:

$$S^{2} = \left[3 * V_{+1} * \left(I_{+1}^{2} + I_{-1}^{2}\right)\right]^{2} = \left(3 * V_{+1} * I_{+1}\right)^{2} + \left(3 * V_{+1} * I_{-1}\right)^{2} = S_{e}^{2}$$
(121)

Case 3 - Unbalanced voltages and currents: In this case, the apparent power is:

$$S^{2} = S_{e}^{2} + 18 * (V_{+1} * V_{-1} * I_{+1} * I_{-1}) * \cos(\varphi_{+1} - \delta_{+1} + \varphi_{-1} - \delta_{-1})$$
(122)

Figure 1 shows the error of **S** with respect to  $S_e$  as a function of the current unbalance  $(I = (I_-/I_+))$ . Each curve is parameterized for different values of voltage unbalance  $(DV = (V_-/V))$ . For simplicity, the harmonic distortion, and the phase shift between voltage and current are zero. The relative error is calculated as follow:



$$E[\%] = \frac{S - S_e}{S_e} \tag{123}$$

Fig. 1. Measurement error for an unbalanced system

**Case 4** - Balanced and non sinusoidal system: The apparent power for this case can have different formulations depending on where the distortion is present; in the current or in the voltage signals.

a. Distorted current

$$S^{2} = 9 * V_{+1}^{2} * \sum_{n=1}^{\infty} I_{+n}^{2} = S_{e}^{2}$$
(124)

b. Distorted voltaje and current

$$S^{2} = S_{e}^{2} + 18 * \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} \left( V_{+n} * V_{+m} * I_{+n} * I_{+m} * \cos(\varphi_{+n} - \varphi_{+m} - \delta_{+n} + \delta_{+m}) \right)$$
(125)

**Case 5 -** Non sinusoidal system with unbalanced and distorted load: Similar to the previous case, the formulation is different depending on where the distortion is present. a. Distorted current, sinusoidal voltage

$$S^{2} = 9 * V_{+1}^{2} * \sum_{n=1}^{\infty} \left( I_{+n}^{2} + I_{-n}^{2} \right) = S_{e}^{2}$$
(126)

b. Distorted voltaje

$$S^{2} = S_{e}^{2} + 18 * \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} \left\{ V_{+n} * V_{+m} * \left[ I_{+n} * I_{+m} * \cos(\varphi_{+n} - \varphi_{+m} - \delta_{+n} + \delta_{+m}) + I_{-n} * I_{-m} * \cos(\varphi_{+n} - \varphi_{+m} + \delta_{-n} - \delta_{-m}) \right] \right\}$$
(127)

**Case 6** - Non sinusoidal system, with unbalanced currents and voltages: This is the most general case where the apparent power, based on Eq.(119) *S* is determined by the Eq. (128). Eq. (128) is the most general formulation of the apparent power and can be used to calculate the apparent power in all practical cases; all other expressions are a subset of this general one.

$$S^{2} = S_{e}^{2} + 18 * \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} \{ V_{+n} * V_{+m} * [I_{+n} * I_{+m} * \cos(\varphi_{+n} - \varphi_{+m} - \delta_{+n} + \delta_{+m}) + I_{-n} * I_{-m} * \cos(\varphi_{+n} - \varphi_{+m} + \delta_{-n} - \delta_{-m}) ] \} + I_{-n} * I_{-m} * \cos(\varphi_{+n} - \varphi_{-m} + \delta_{-n} - \delta_{-m}) ] \} + I_{-n} * I_{-m} * \cos(\varphi_{-n} - \varphi_{-m} - \delta_{-n} + \delta_{-m}) ] \} + I_{-n} * I_{-m} * \cos(\varphi_{-n} - \varphi_{-m} - \delta_{-n} + \delta_{-m}) ] \} + I_{-n} * I_{-m} * \cos(\varphi_{-n} - \varphi_{-m} - \delta_{-n} + \delta_{-m}) ] \} + I_{-n} * I_{-m} * \cos(\varphi_{+n} + \varphi_{-m} - \delta_{-n} - \delta_{-m}) + I_{+m} * I_{-n} * \cos(\varphi_{+n} + \varphi_{-m} - \delta_{-n} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+n} + \varphi_{-m} - \delta_{-n} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m}) + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * \cos(\varphi_{+m} + \varphi_{-n} - \delta_{-m} - \delta_{-m} - \delta_{-m}) ] \} + I_{+m} * I_{-n} * O(\varphi_{+m} + \varphi_{-n} - \delta_{-m} ] ] \} + I_{+m} * I_{-n} * O(\varphi_{+m} + \varphi_{-n} - \delta_{-m} ] ] \} + I_{+m} * I_{-n} * I_{+n} * I_{-n} * O(\varphi_{+m} - \delta_{-m} ] ] \} + I_{+m} * I_{-n} * I_{+n} * I_{-n} * I_{+n} * I_{-n} * O(\varphi_{+m} - \delta_{-m} - \delta_{-m} - \delta_{-m} - \delta_{-m} - \delta_{-m} ] ] \} + I_{+m} * I_{-m} * I_{-m} + \delta_{-m} - \delta_{-m} ] ] \} + I_{+m} * I_{-m} * I_{+m} * I_{-m} * I_{+m} * I_{-m} * I_{+m} * I_{-m} * I_{-m} * I_{+m} * I_{-m} + \delta_{-m} - \delta_{-m} ] ] \} + I_{+m} * I_{-m} * I_{+m} * I_{-m} + I_{-m} * I_{-m} + I_{-m} * I_{-m} + I_{-m} + I_{-m} * I_{-m} *$$

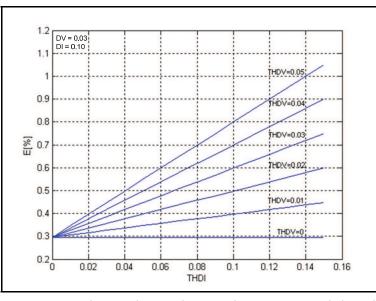


Fig. 2. Measurement error due to voltage and current distortion in an unbalanced system

As an illustrative example, in the case of a three phase system with distortion in both current and voltage signals but with unbalanced load only, the first two terms of Eq. (128) are not zero, then the equation becomes Eq. (125). Figure 2 shows the apparent power deviation with respect to  $S_e$  for different values of *THDv* as a function of *THDi*. These curves are parameterized for a voltage distortion of 3% and a current distortion of 10%. The figure describes that if THDv is zero, the error is constant regardless of the value of THDi, moreover, the deviation is due to the current and voltage unbalance.

From these results can be seen that the apparent power value is different from the effective apparent power defined by the IEEE only if there is unbalanced or distortion in both current and voltage signals. The maximum relative error can be evaluated from Fig.1 and 2, where the error in the presence of voltage and current unbalance can be determined by the following relationship:

$$E_{\rm p}\% \cong DI^*DV^*100\tag{129}$$

Similarly for the case of voltage and current distortion:

$$E_{R} \% \cong THDI * THDV * 100 \tag{130}$$

Therefore, in a general situation, the maximum error is:

$$E_{R}\% \cong (DI * DV + THDI * THDV) * 100$$
(131)

These relationships mean that the máximum deviation of S can be evaluated knowing the system condition at the measurement point. In addition to the the formulation presented in this section, the signals can be further processed in order to obtain additional indexes such as the total harmonic distortion, power factor, fundamental power terms, and the active and

reactive harmonic power terms ( $P_1$ ,  $Q_1$ ,  $S_1$ ,  $P_H$ ,  $Q_H$ ,  $S_H$ ).by using low pass filters and matrix transformations, the proposed general methodology is described in Figure 3.

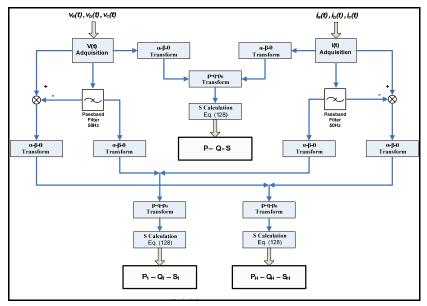


Fig. 3. Flow chart for power components calculations

The proposed methodology uses instant power value instead of r.m.s values and filters; band pass filters for fundamental components, and low pass filters for mean values. The total active, fundamental and harmonic powers can be calculated from the mean real power even for unbalanced conditions. Finally, additional indexes such as THDv, THDi, power phase shift, power factor are also calculated.

# 5. Conclusion

In this chapter a critical review of the most commonly used apparent power, reactive power and power factor definitions for both single phase and three phase systems were presented. The utilization of these definitions was evaluated under different conditions such as sinusoidal, non sinusoidal, one phase, and balanced and unbalanced three phase systems. Finally a methodology to measure power and power quality indexes based on the instant power theory under non sinusoidal conditions is proposed. Results demonstrate that using this methodology, accurate values of the different power quality indexes can be obtained in a simpler manner even for the worst case scenario that may include unbalance and distortion signals.

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# Power Quality â€" Monitoring, Analysis and Enhancement

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This book on power quality written by experts from industries and academics from various counties will be of great benefit to professionals, engineers and researchers. This book covers various aspects of power quality monitoring, analysis and power quality enhancement in transmission and distribution systems. Some of the key features of books are as follows: Wavelet and PCA to Power Quality Disturbance Classification applying a RBF Network; Power Quality Monitoring in a System with Distributed and Renewable Energy Sources; Signal Processing Application of Power Quality Monitoring; Pre-processing Tools and Intelligent Techniques for Power Quality Analysis; Single-Point Methods for Location of Distortion, Unbalance, Voltage Fluctuation and Dips Sources in a Power System; S-transform Based Novel Indices for Power Quality Disturbances; Load Balancing in a Three-Phase Network by Reactive Power Compensation; Compensation of Reactive Power and Sag Voltage using Superconducting Magnetic Energy Storage; Optimal Location and Control of Flexible Three Phase Shunt FACTS to Enhance Power Quality in Unbalanced Electrical Network; Performance of Modification of a Three Phase Dynamic Voltage Restorer (DVR) for Voltage Quality Improvement in Distribution System; Voltage Sag Mitigation by Network Reconfiguration; Intelligent Techniques for Power Quality Enhancement in Distribution Systems.

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