Optimal Design of Cooling Towers

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1. Introduction

Process engineers have always looked for strategies and methodologies to minimize process costs and to increase profits. As part of these efforts, mass (Rubio-Castro et al., 2010) and thermal water integration (Ponce-Ortega et al. 2010) strategies have recently been considered with special emphasis. Mass water integration has been used for the minimization of freshwater, wastewater, and treatment and pipeline costs using either single-plant or inter-plant integration, with graphical, algebraic and mathematical programming methodologies; most of the reported works have considered process and environmental constraints on concentration or properties of pollutants. Regarding thermal water integration, several strategies have been reported around the closed-cycle cooling water systems, because they are widely used to dissipate the low-grade heat of chemical and petrochemical process industries, electric-power generating stations, and refrigeration and air conditioning plants. In these systems, water is used to cool down the hot process streams, and then the water is cooled by evaporation and direct contact with air in a wet-cooling tower and recycled to the cooling network. Therefore, cooling towers are very important industrial components and there are many references that present the fundamentals to understand these units (Foust et al., 1979; Singham, 1983; Mills, 1999; Kloppers & Kröger, 2005a).

The heat and mass transfer phenomena in the packing region of a counter flow cooling tower are commonly analyzed using the Merkel (Merkel, 1926), Poppe (Pope & Rögner, 1991) and effectiveness-NTU (Jaber & Webb, 1989) methods. The Merkel’s method (Merkel, 1926) consists of an energy balance, and it describes simultaneously the mass and heat transfer processes coupled through the Lewis relationship; however, these relationships oversimplify the process because they do not account for the water lost by evaporation and the humidity of the air that exits the cooling tower. The NTU method models the relationships between mass and heat transfer coefficients and the tower volume. The Poppe’s method (Pope & Rögner, 1991) avoids the simplifying assumptions made by Merkel, and consists of differential equations that evaluate the air outlet conditions in terms of enthalpy and humidity, taking into account the water lost by evaporation and the NTU. Jaber and Webb (Jaber & Webb, 1989) developed an effectiveness-NTU method directly applied to counterflow or crossflow cooling towers,
basing the method on the same simplifying assumptions as the Merkel’s method. Osterle (Osterle, 1991) proposed a set of differential equations to improve the Merkel equations so that the mass of water lost by evaporation could be properly accounted for; the enthalpy and humidity of the air exiting the tower are also determined, as well as corrected values for NTU. It was shown that the Merkel equations significantly underestimate the required NTU. A detailed derivation of the heat and mass transfer equations of evaporative cooling in wet-cooling towers was proposed by Kloppers & Kröger (2005b), in which the Poppe’s method was extended to give a more detailed representation of the Merkel number. Cheng-Qin (2008) reformulated the simple effectiveness-NTU model to take into consideration the effect of nonlinearities of humidity ratio, the enthalpy of air in equilibrium and the water losses by evaporation.

Some works have evaluated and/or compared the above methods for specific problems (Chengqin, 2006; Nahavandi et al., 1975); these contributions have concluded that the Poppe’s method is especially suited for the analysis of hybrid cooling towers because outlet air conditions are accurately determined (Kloppers & Kröger, 2005b). The techniques employed for design applications must consider evaporation losses (Nahavandi et al., 1975). If only the water outlet temperature is of importance, then the simple Merkel model or effectiveness-NTU approach can be used, and it is recommended to determine the fill performance characteristics close to the tower operational conditions (Kloppers & Kröger, 2005a). Quick and accurate analysis of tower performance, exit conditions of moist air as well as profiles of temperatures and moisture content along the tower height are very important for rating and design calculations (Chengqin, 2006). The Poppe’s method is the preferred method for designing hybrid cooling towers because it takes into account the water content of outlet air (Roth, 2001).

With respect to the cooling towers design, computer-aided methods can be very helpful to obtain optimal designs (Oluwasola, 1987). Olander (1961) reported design procedures, along with a list of unnecessary simplifying assumptions, and suggested a method for estimating the relevant heat and mass transfer coefficients in direct-contact cooler-condensers. Kintner-Meyer and Emery (1995) analyzed the selection of cooling tower range and approach, and presented guidelines for sizing cooling towers as part of a cooling system. Using the one-dimensional effectiveness-NTU method, Söylemez (2001, 2004) presented thermo-economic and thermo-hydraulic optimization models to provide the optimum heat and mass transfer area as well as the optimum performance point for forced draft counter flow cooling towers. Recently, Serna-González et al. (2010) presented a mixed integer nonlinear programming model for the optimal design of counter-flow cooling towers that considers operational restrictions, the packing geometry, and the selection of type packing; the performance of towers was made through the Merkel method (Merkel, 1926), and the objective function consisted of minimizing the total annual cost. The method by Serna-González et al. (2010) yields good designs because it considers the operational constraints and the interrelation between the major variables; however, the transport phenomena are oversimplified, the evaporation rate is neglected, the heat resistance and mass resistance in the interface air-water and the outlet air conditions are assumed to be constant, resulting in an underestimation of the NTU.

This chapter presents a method for the detailed geometric design of counterflow cooling towers. The approach is based on the Poppe’s method (Pope & Rögener, 1991), which
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rigorously addresses the transport phenomena in the tower packing because the evaporation rate is evaluated, the heat and mass transfer resistances are taken into account through the estimation of the Lewis factor, the outlet air conditions are calculated, and the NTU is obtained through the numerical solution of a differential equation set as opposed to a numerical integration of a single differential equation, thus providing better designs than the Merkel’s method (Merkel, 1926). The proposed models are formulated as MINLP problems and they consider the selection of the type of packing, which is limited to film, splash, and tickle types of fills. The major optimization variables are: water to air mass ratio, water mass flow rate, water inlet and outlet temperatures, operational temperature approach, type of packing, height and area of the tower packing, total pressure drop of air flow, fan power consumption, water consumption, outlet air conditions, and NTU.

2. Problem statement

Given are the heat load to be removed in the cooling tower, the inlet air conditions such as dry and wet bulb temperature (to calculate the inlet air humidity and enthalpy), lower and upper limits for outlet and inlet water temperature, respectively, the minimum approach, the minimum allowable temperature difference, the minimal difference between the dry and wet bulb temperature at each integration interval, and the fan efficiency. Also given is the economic scenario that includes unit cost of electricity, unit cost of fresh water, fixed cooling tower cost, and incremental cooling tower cost based on air mass flow rate and yearly operating time. The problem then consists of determining the geometric and operational design parameters (fill type, height and area fill, total pressure drop in the fill, outlet air conditions, range and approach, electricity consumption, water and air mass flow rate, and number of transfer units) of the counterflow cooling tower that satisfy the cooling requirements with a minimum total annual cost.

3. Model formulation

The major equations for the heat and mass transfer in the fill section and the design equations for the cooling tower are described in this section. The indexes used in the model formulation are defined first: \textit{in} (inlet), \textit{out} (outlet), \textit{j} (constants to calculate the transfer coefficient), \textit{k} (constants to calculate the loss coefficient), \textit{r} (makeup), \textit{ev} (evaporated water), \textit{d} (drift), \textit{b} (blowdown), \textit{m} (average), \textit{w} (water), \textit{a} (dry air), \textit{wb} (wet-bulb), \textit{n} (integration interval), \textit{fi} (fill), \textit{fr} (cross-sectional), \textit{misc} (miscellaneous), \textit{t} (total), \textit{vp} (velocity pressure), \textit{f} (fan), \textit{ma} (air-vapor mixture), \textit{e} (electricity), \textit{s} (saturated) and \textit{v} (water vapor). In addition, the superscript \textit{i} is used to denote the type of fill and the scalar NTI is the last interval integration. The nomenclature section presents the definition of the variables used in the model. The model formulation is described as follows.

3.1 Heat and mass transfer in the fill section for unsaturated air

The equations for the evaporative cooling process of the Poppe’s method are adapted from Poppe & Rögener (1991) and Kröger (2004), and they are derived from the mass balance for the control volume shown in Figures 1 and 2. Figure 1 shows a control volume in the fill of a counter flow wet-cooling tower, and Figure 2 shows an air-side control volume of the fill illustrated in Figure 1.
where $w$ is the humidity ratio through the cooling tower, $T_w$ is the water temperature, $cp_v$ is the specific heat at constant pressure at water temperature, $m_w$ is the water flow rate through the cooling tower, $m_a$ is the air flow rate, $i_{wa,v}$ is the enthalpy of saturated air evaluated at water temperature, $i_{wa,v}$ is the enthalpy of the air-water vapor mixture per mass.
of dry-air, \( \dot{m}_i \) is the enthalpy of the water vapor, \( w_{v,w} \) is the humidity saturated ratio evaluated at the water temperature, \( NTU \) is the number of transfer units, and \( Lef \) is the Lewis factor. This relationship is an indication of the relative rates of heat and mass transfer in an evaporative process, which for unsaturated air can be determined by (taken from Kloppers & Kröger, 2005b):

\[
Lef = 0.865^{-0.665} \left( \frac{w_{v,w} + 0.622}{w + 0.622} - 1 \right) \left[ \ln \left( \frac{w_{v,w} + 0.622}{w + 0.622} \right) \right]^{0.865}
\]  (4)

The ratio of the mass flow rates changes as the air moves towards the top of the fill, and it is calculated by considering the control volume of a portion of the fill illustrated in Figure 3.

\[
\frac{\dot{m}_w}{\dot{m}_a} = \frac{\dot{m}_{w,in}}{\dot{m}_{a,in}} \left( 1 - \frac{\dot{m}_w}{\dot{m}_{w,in}} (w_{out} - w) \right)
\]  (5)

where \( \dot{m}_{w,in} \) is the water flow rate inlet to the cooling tower and \( w_{out} \) is the outlet humidity ratio from the cooling tower.

Fig. 3. Control volume of the fill

The Poppe model consists of the above set of coupled ordinary differential and algebraic equations, which can be solved simultaneously to provide the air humidity, the air enthalpy, the water temperature, the water mass flow rate and the NTU profiles in the cooling tower. Also, the state of the outlet air from the cooling tower can be fully determined with this model. The Merkel model can be derived from the Poppe model by assuming a Lewis factor equal to one (\( Lef = 1 \)) and negligible water evaporation (i.e., \( \dot{d}m_w = 0 \)).

A model with ordinary differential equations and algebraic equations is quite complex for MINLP optimization purposes. Therefore, the set of ordinary differential equations comprising the Poppe model is converted into a set of nonlinear algebraic equations using a fourth-order Runge-Kutta algorithm (Burden & Faires, 1997; Kloppers & Kröger, 2005b), and the physical properties are calculated with the equations shown in Appendix A. Note that the differential equations (1-3) depend of the water temperature, the mass fraction humidity and the air enthalpy, which can be represented as follow,
\[
\frac{dw}{dT_w} = f(i_{w_t}, w, T_u) \quad (1')
\]
\[
\frac{di_{w_t}}{dT_w} = f(i_{w_t}, w, T_u) \quad (2')
\]
\[
\frac{dNTU}{dT_w} = f(i_{m_a}, w, T_u) \quad (3')
\]

To convert these differential equations into algebraic equations using the Runge-Kutta algorithm, the first step is to divide the range of water temperature in the fill into a number of intervals,

\[
\Delta T_w = \frac{T_{w_{in}} - T_{w_{out}}}{N} \quad (6)
\]

Here, \(\Delta T_w\) is the increase of the water temperature in the integration intervals, \(T_{w_{in}}\) is the water inlet temperature on the cooling tower, \(T_{w_{out}}\) is the water outlet temperature on the cooling tower and \(N\) is the number of intervals considered for the discretization of the differential equations. Figure 4 shows a graphical representation of the Runge-Kutta algorithm using five intervals; once the conditions at level 0 that corresponds to the bottom of the cooling tower are known, the conditions at level N+1 can be calculated successively to reach the last level corresponding the top of the tower with the following set of algebraic equations,

\[
w_{n+1} = w_n + \left(J_{(n+1)} + 2J_{(n+2)} + 2J_{(n+3)} + J_{(n+4)}\right) / 6 \quad (7)
\]

\[
i_{m_a,n+1} = i_{m_a,n} + \left(K_{(n+1,1)} + 2K_{(n+1,2)} + 2K_{(n+1,3)} + K_{(n+1,4)}\right) / 6 \quad (8)
\]

\[
NTU_{n+1} = NTU_n + \left(I_{(n+1)} + 2I_{(n+2)} + 2I_{(n+3)} + I_{(n+4)}\right) / 6 \quad (9)
\]

where

\[
I_{(n+1, 1)} = \Delta T_w \cdot f(T_{w,n}, i_{m_a,n}, w_n) \quad (10)
\]

\[
K_{(n+1,1)} = \Delta T_w \cdot g(T_{w,n}, i_{m_a,n}, w_n) \quad (11)
\]

\[
L_{(n+1,1)} = \Delta T_w \cdot h(T_{w,n}, i_{m_a,n}, w_n) \quad (12)
\]

\[
I_{(n+1, 2)} = \Delta T_w \cdot f\left(T_{w,n} + \frac{\Delta T_w}{2}, i_{m_a,n} + \frac{K_{(n+1,1)}}{2}, w_n + \frac{I_{(n+1,1)}}{2}\right) \quad (13)
\]

\[
K_{(n+1,2)} = \Delta T_w \cdot g\left(T_{w,n} + \frac{\Delta T_w}{2}, i_{m_a,n} + \frac{K_{(n+1,1)}}{2}, w_n + \frac{I_{(n+1,1)}}{2}\right) \quad (14)
\]
Here $J$, $K$ and $L$ are the recursive relations to determine the increase of the air ratio humidity, air enthalpy and number of transfer units, respectively. Notice that the differential equations are now represented by a set of algebraic equations, whose solution gives the profiles of the air humidity ratio, air enthalpy and number of transfer units through the fill. In addition, the number of algebraic equations and variables depends of the number of intervals and sub-intervals considered to get the above profiles, and the start point for their solution. Figure 5 represents one interval of Figure 4 divided into subintervals; the conditions at the bottom of the fill provide the starting N point for the calculations of the conditions in the next level N+1. In addition, the specifications of the conditions at the bottom and the top of the cooling tower should be included for estimating the design variables, which include the water inlet temperature, water outlet temperature, water inlet mass flow rate, inlet mass-fraction humidity of air stream, outlet mass-fraction humidity of air stream, inlet dry bulb temperature of air stream, outlet dry bulb temperature of air stream, inlet wet bulb temperature of air stream, outlet wet bulb temperature of air stream and the inlet and outlet enthalpy of air stream,

$$L_{(n+1,2)} = \Delta T_w \cdot h \left( T_{w,n} + \frac{\Delta T_w}{2}, i_{na,n} + \frac{K_{(n+1,1)}}{2}, w_n + \frac{I_{(n+1,1)}}{2} \right)$$  \hfill (15)

$$I_{(n+1,3)} = \Delta T_w \cdot f \left( T_{w,n} + \frac{\Delta T_w}{2}, i_{na,n} + \frac{K_{(n+1,2)}}{2}, w_n + \frac{I_{(n+1,2)}}{2} \right)$$  \hfill (16)

$$K_{(n+1,3)} = \Delta T_w \cdot g \left( T_{w,n} + \frac{\Delta T_w}{2}, i_{na,n} + \frac{K_{(n+1,2)}}{2}, w_n + \frac{I_{(n+1,2)}}{2} \right)$$  \hfill (17)

$$L_{(n+1,3)} = \Delta T_w \cdot h \left( T_{w,n} + \frac{\Delta T_w}{2}, i_{na,n} + \frac{K_{(n+1,2)}}{2}, w_n + \frac{I_{(n+1,2)}}{2} \right)$$  \hfill (18)

$$I_{(n+1,4)} = \Delta T_w \cdot f \left( T_{w,n} + \Delta T_w, i_{na,n} + K_{(n+1,3)}, w_n + I_{(n+1,3)} \right)$$  \hfill (19)

$$K_{(n+1,4)} = \Delta T_w \cdot g \left( T_{w,n} + \Delta T_w, i_{na,n} + K_{(n+1,3)}, w_n + I_{(n+1,3)} \right)$$  \hfill (20)

$$L_{(n+1,4)} = \Delta T_w \cdot h \left( T_{w,n} + \Delta T_w, i_{na,n} + K_{(n+1,3)}, w_n + I_{(n+1,3)} \right)$$  \hfill (21)

$$T_{w,n} = T_{w,n,N+1}$$  \hfill (22)

$$T_{w,\text{out}} = T_{w,\text{in}}$$ \hfill (23)

$$m_{w,n} = m_{w,n,N+1}$$  \hfill (24)

$$m_{w,\text{out}} = m_{w,\text{in}}$$ \hfill (25)

$$w_n = w_{\text{in}}$$  \hfill (26)
The system of equations above described is only valid for unsaturated air; one should keep in mind that only this region is considered in the design of wet-cooling towers because the air exiting from the tower cannot be saturated before leaving the packing section.

3.2 Design equations
The relationships to obtain the geometric design of the cooling tower are presented in this section; they are used in conjunction with a numerical technique for the solution of the Poppe’s equations.

\[
\begin{align*}
\text{w}_{\text{out}} &= \text{w}_{\text{e-NTI}} \\
\text{T}_{\text{e,in}} &= \text{T}_{\text{e,0}} \\
\text{T}_{\text{a,in}} &= \text{T}_{\text{a,NTI}} \\
\text{T}_{\text{w,in}} &= \text{T}_{\text{w,0}} \\
\text{T}_{\text{w,out}} &= \text{T}_{\text{w,NTI}} \\
\text{i}_{\text{ma,in}} &= \text{i}_{\text{ma,0}} \\
\text{i}_{\text{ma,out}} &= \text{i}_{\text{ma,NTI}}
\end{align*}
\]
3.2.1 Heat load

The heat of the water stream removed in the cooling tower \((Q)\) is calculated as follows:

\[
Q = cp_{w,in}m_{w,in}T_{w,in} - cp_{w,out}m_{w,out}T_{w,out}
\]

(34)

where \(cp_{w,in}\) is the specific heat at constant pressure at inlet water temperature, \(cp_{w,out}\) is the specific heat at constant pressure at outlet water temperature, \(T_{w,in}\) is the inlet water temperature to the cooling tower, \(T_{w,out}\) is the outlet water temperature from the cooling tower, and \(m_{w,out}\) is the outlet water flow rate, which is obtained from the following relationship:

\[
m_{w,out} = m_{w,in} - m_a(w_{out} - w_{in}) - m_{w,d}
\]

(35)

where \(m_{w,d}\) is the drift water for air flow rate. Notice that equation (34) is an improved equation for the heat rejection rate, according to the Merkel or effectiveness-NTU methods; it is used when there are water losses by evaporation and it is included in the energy balance (Kloppers & Kröger, 2005a), situation modeled in Poppe’s method.

3.2.2 Transfer and loss coefficients

The transfer coefficients are related to the NTU and they depend on the fill type (Kloppers & Kröger, 2005c). The value of Merkel’s number at the last level \((NTI)\) is given by:

\[
NTU_{w=NTI} = c_1 \left( \frac{m_{w,in}}{A_f} \right)^2 \left( \frac{m_a}{A_g} \right)^3 \left( L_s \right)^{\gamma-4} \left( T_{w,in} \right)^5
\]

(36)
where $A_p$ is the packing area, $L_p$ is the height of packing, $c_1$ and $c_5$ are constants that depend on the type of fill, and $m_{w,n}$ is the average water flow rate, calculated as follows:

$$m_{w,n} = \frac{m_{w,\text{in}} + m_{w,\text{out}}}{2}$$  \hspace{1cm} (37)

Table 1 shows the values for the $a_i'$ constants (Kloppers & Kröger, 2005c) for different types of fills.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$a_i'$</th>
<th>$i$ = 1</th>
<th>$i$ = 2</th>
<th>$i$ = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(splash fill)</td>
<td>0.249013</td>
<td>1.930306</td>
<td>1.019766</td>
</tr>
<tr>
<td>2</td>
<td>(trickle fill)</td>
<td>-0.464089</td>
<td>-0.568230</td>
<td>-0.432896</td>
</tr>
<tr>
<td>3</td>
<td>(film fill)</td>
<td>0.653578</td>
<td>0.641400</td>
<td>0.782744</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0</td>
<td>-0.352377</td>
<td>-0.292870</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
<td>-0.178670</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Constants for transfer coefficients

The following disjunction and its reformulation through the convex hull technique (Vicchietti, et al., 2003) is used for the optimal selection of fill type:

$$Y^i = c_j' + c_j $\quad j = 1,\ldots,5$$

$$Y^3 = c_j $\quad j = 1,\ldots,5$$

$$Y^3 = Y^1 + Y^2 + Y^3 = 1$$  \hspace{1cm} (38)

$$c_j = c_j' + c_j + c_j' $\quad j = 1,\ldots,5$$  \hspace{1cm} (39)

$$c_j = a_i' y_i $\quad i = 1,\ldots,3. j = 1,\ldots,5$$  \hspace{1cm} (40)

The loss coefficients ($K_p$) in cooling towers are analogous to the friction factors in heat exchangers; they are used to estimate the pressure drop through the fill using the following correlation for different types of fills (Kloppers & Kröger, 2003):

$$K_p = \left[ d_i \left( \frac{m_{w,n}}{A_p} \right)^{a_2} \left( \frac{m}{A_p} \right)^{a_3} \right] L_p$$

$$\left[ d_i \left( \frac{m_{w,n}}{A_p} \right)^{a_2} \left( \frac{m}{A_p} \right)^{a_3} \right] L_p$$

The following disjunction is used to select the fill type:

$$Y^i = d_i d_i' $\quad k = 1,\ldots,6$$

$$Y^2 = d_i d_i' $\quad k = 1,\ldots,6$$

$$Y^3 = d_i d_i' $\quad k = 1,\ldots,6$$
The disjunction is reformulated as follows:

\[ d_k = d_k^1 + d_k^2 + d_k^3, \quad k = 1,...,6 \]  

\[ d_k^i = b_k^i y^i, \quad i = 1,...,3, \quad k = 1,...,6 \]  

Values for \( b_k^i \) coefficients for different fill types are shown in Table 2 (Kloppers & Kröger, 2003).

### 3.2.3 Pressure drop in the cooling tower

According to Li & Priddy (1985), the total pressure drop (\( \Delta P \)) in mechanical draft cooling towers is the sum of the static and dynamic pressure drops (\( \Delta P_{sp} \)). The first type includes the pressure drop through the fill (\( \Delta P_f \)) and the miscellaneous pressure drop (\( \Delta P_{misc} \)). The pressure drop through the fill is calculated from (Kloppers & Kröger, 2003):

\[
\Delta P_f = K_f L_f \frac{m_{av}^2}{2 \rho_n A_f^2} 
\]

Table 2. Constants for loss coefficients

<table>
<thead>
<tr>
<th>( k )</th>
<th>( i=1 )</th>
<th>( i=2 )</th>
<th>( i=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(splash fill)</td>
<td>(trickle fill)</td>
<td>(film fill)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.179688</td>
<td>7.047319</td>
<td>3.897830</td>
</tr>
<tr>
<td>2</td>
<td>1.083916</td>
<td>0.812454</td>
<td>0.777271</td>
</tr>
<tr>
<td>3</td>
<td>-1.965418</td>
<td>-1.143846</td>
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</tr>
<tr>
<td>4</td>
<td>0.639088</td>
<td>2.677231</td>
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</tr>
<tr>
<td>5</td>
<td>0.684936</td>
<td>0.294827</td>
<td>0.215975</td>
</tr>
<tr>
<td>6</td>
<td>0.642767</td>
<td>1.018498</td>
<td>0.079696</td>
</tr>
</tbody>
</table>

Here \( \rho_n \) is the harmonic mean air vapor flow rate through the fill, \( m_{av} \) is an average air-vapor flow rate, calculated from:

\[
m_{av} = \frac{m_{av_s} + m_{av_w}}{2}
\]

\[
\rho_n = \frac{1}{\frac{1}{\rho_{in}} + \frac{1}{\rho_{out}}}
\]

\[
m_{av_w} = m_a + w_a m_a
\]

\[
m_{av_w} = m_a + w_a m_a
\]

where \( \rho_{in} \) and \( \rho_{out} \) are the inlet and outlet air density, respectively. The miscellaneous pressure drop is calculated as follows:

\[
\Delta P_{misc} = 6.5 \frac{m_{av}^2}{2 \rho_n A_f^2}
\]
The other part is the dynamic pressure drop. According to Li & Priddy (1985), it is equal to 2/3 of the static pressure drop,

$$\Delta P_{dp} = \left(\frac{2}{3}\right)\left(\Delta P_{fi} + \Delta P_{misc} \right)$$  \hspace{1cm} (50)

Combining equations (44), (49) and (50), the total pressure drop is,

$$\Delta P = 1.667 \left(\Delta P_{fi} + \Delta P_{misc} \right)$$  \hspace{1cm} (51)

### 3.2.4 Power demand

The power requirements for the fan (HP) can be calculated by multiplying the total pressure drop times the volumetric flow rate, which depends on the localization of the fan. For mechanical draft cooling towers we have (Serna-González et al., 2010):

$$HP = \frac{m_{av} \Delta P}{\rho_w \eta_f}$$  \hspace{1cm} (52)

where $\eta_f$ is the fan efficiency.

### 3.2.5 Water consumption

In cooling towers, water losses are due to the water evaporated ($m_{w,v}$), the drift water for air flow rate ($m_{w,d}$), and the blowdown ($m_{w,b}$) to avoid salts deposition,

$$m_{w,v} = m_w \left( w_{aw} - w_a \right)$$  \hspace{1cm} (53)

$$m_{w,b} = \frac{m_{w,v}}{n_{cycle}} - m_{w,d}$$  \hspace{1cm} (54)

where $n_{cycle}$ is the number of concentration cycles that are required. Usually $n_{cycle}$ has a value between 2 and 4 (Li & Priddy, 1985). For an efficient design, the loss for drift should not be higher than 0.2% of the total water flow rate (Kemmer, 1988),

$$m_{w,d} = 0.002 m_{w,net}$$  \hspace{1cm} (55)

Combining Equations (53), (54) and (55), we can calculate the water consumption ($m_{w,v}$) as,

$$m_{w,v} = \frac{n_{cycle} m_{w,v}}{n_{cycle} - 1}$$  \hspace{1cm} (56)

### 3.2.6 Feasibility constraints

The temperature difference between the water at the outlet and the wet-bulb temperature of the air entering the tower is called the tower approach. In practice, the water outlet temperature should be at least 2.8°C above the wet-bulb temperature (Li & Priddy, 1985),
The dry bulb air temperature should be higher than the wet bulb air temperature through the packing at least in the last integration interval ($NTI$),

$$T_{a,n} \geq T_{wb,n} + \Delta T_{n \neq NTI}$$  \hspace{1cm} (58)

From thermodynamic principles, the outlet water temperature from the cooling tower should be lower than the lowest outlet process stream of the cooling network, and the inlet water temperature to the cooling tower cannot be higher than the hottest inlet process stream in the cooling network. Additionally, to avoid pipe fouling, a maximum temperature of 50°C is usually specified for the water entering the cooling tower (Douglas, 1988),

$$T_{w,o} \leq TMPO - DMIN$$  \hspace{1cm} (59)

$$T_{w,o} \leq TMPO - DMIN$$  \hspace{1cm} (60)

$$T_{w,i} \leq 50^\circ C$$  \hspace{1cm} (61)

Here $TMPO$ is the outlet temperature of the coldest hot process streams in the cooling network, $TMP$ is the inlet temperature of the hottest hot process stream in the cooling network, and $DMIN$ is the minimum allowable temperature difference. Although cooling towers can be designed for any ratio of the mass flow rate, designers suggest the following limits (Singh, 1983):

$$0.5 \leq \frac{m_{w,n}}{m_w} \leq 2.5$$  \hspace{1cm} (62)

The correlations for the transfer and loss coefficients are limited to (Kloppers & Kröger, 2003, 2005c),

$$2.90 \leq \frac{m_{w,n}}{A_f} \leq 5.96$$  \hspace{1cm} (63)

$$1.20 \leq \frac{m}{A_f} \leq 4.25$$  \hspace{1cm} (64)

### 3.2.7 Objective function

The objective function is the minimization of the total annual cost ($TAC$), which consists of the capital annualized cost ($CAP$) and operational costs ($COP$),

$$TAC = K_f \cdot CAP + COP$$  \hspace{1cm} (65)

where $K_f$ is an annualization factor. Water consumption and power requirements determine the operational costs, and they are calculated using the following relationship,

$$COP = H_c u_v m_{w,f} + H_c u_s HP$$  \hspace{1cm} (66)
where $H_y$ is the annual operating time, $cu_w$ is the unit cost of fresh water, and $cu_e$ is the unit cost of electricity. The capital cost for the cooling tower depends on the fixed cooling tower cost ($CTF_C$), packing volume and air flow rate (Kintner-Meyer & Emery, 1995),

$$CAP = C_{CTF} + C_{CTV}A_yL_y + C_{CTMA}m_a$$

(67)

$C_{CTV}$ depends on the type of packing; therefore, the following disjunction, along with a convex hull reformulation (Vicchietti et al., 2003), is used:

$$C_{CTV} = C_{CTV}^1 + C_{CTV}^2 + C_{CTV}^3$$

(68)

$$C_{CTV}^i = e_{CTV}^i \ y_i^{'} \quad i = 1,\ldots,3$$

(69)

Common values for unit costs $e_i$ are reported in Table 3.

<table>
<thead>
<tr>
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<th>(splash fill)</th>
<th>(trickle fill)</th>
<th>(film fill)</th>
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<td>1,812.25</td>
<td>1,606.15</td>
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</table>

Table 3. Cost coefficients $C_{CTV}^i$ for each type of fill

The proposed model consists of equations (4) to (69), plus the discretization of the governing equations and the relationships to estimate physical properties presented in Appendix A. The model was implemented in the software GAMS (Brooke et al., 2006) and it was solved using the DICOPT solver.

4. Results and discussion

To demonstrate the application of the proposed model, six case studies taken from Serna-González et al. (2010) were considered. The values for the parameters $H_y$, $K_F$, $\eta_{cycle}$, $cu_w$, $cu_e$, $C_{CTF}$, $C_{CTMA}$, $\eta_f$, and $P_a$, are 8150 hr/year, 0.2983 year$^{-1}$, 4, 5.283 x 10$^{-4}$ US$/kg-water$, 0.085 US$/kWh$, 31185 US$, 1097.5 US$/kg-dry-air$, 0.75 and 101325 Pa, respectively. In addition, 25 intervals to discretize the differential equations were used. The results obtained are compared with the ones reported by Serna-González et al. (2010), where the Merkel method was used to represent the behavior of the cooling tower. Tables 4 and 5 show the results obtained using the Merkel (Merkel, 1926) and Poppe models (Pope & Rögener, 1991). For examples 1, 3, 4 and 6, the designs obtained using the Poppe’s method are cheaper because of low operating costs, which depend on the makeup water cost and power cost. The effect of the air flowrate and ranges over evaporated water rate is shown in Figures 6a and 6b; it can be observed how the relation between air flowrate and the range generates the optimum evaporative rate. Figure 7 presents a sensibility analysis on the evaporative rate with respect to the air flowrate and range; notice the higher impact of the range factor.
The prediction of power fan cost using the Poppe’s method is higher than with the Merkel’s method because more air is estimated for the same range; this means that the cooling capacity of the inlet air in the Merkel’s method is overestimated and the outlet air is oversaturated. This is proved by the solution of Equations (1)-(3) using the results obtained ($T_{\text{in}}$, $T_{\text{out}}$, $m_{\text{in}}$, and $m_{\text{a}}$) from the Merkel’s method, and plotting the dry and wet bulb air temperatures for the solution intervals. Notice in Figure 6 that the air saturation ($T_{\text{wb}} \geq T_{\text{a}}$) is obtained before of the outlet point of the packing section.

![Graphs](a) vs (b) m_{\text{w, cv}} vs m_{\text{a}} vs Range

**Fig. 6. Evaporate profile respect to air flow rate and range**

![Graph](m_{\text{s}} vs % decrease)

**Fig. 7. Sensitivity analysis of the evaporate rate with respect to air flowrate and range**

With respect to the capital cost for cases 1 and 6, the estimations obtained using the Poppe’s method are more expensive because of the higher air flowrate, area and height packing. However, for examples 3 and 4 both capital and operating costs are predicted at lower levels with the Poppe’s method; the capital cost is lower because the inlet air is relatively dry and therefore it can process higher ranges with low air flowrates, which requires a lower packing volume. This can be explained because of the effect that the range and air flow rate have in the packing volume, and the effect that the range has in the capital cost of the towers.
(see Figure 8a, 8b and 8c). Notice in Figure 9 that there exists an optimum value for the range to determine the minimum capital cost.

Fig. 8. Air temperature profile in the packing section
### Table 4. Results for Examples 1, 2 and 3

<table>
<thead>
<tr>
<th>DATA</th>
<th>1</th>
<th>2</th>
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<th>Poppe</th>
<th>Merkel</th>
<th>Poppe</th>
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<tr>
<td>T(_{a,in}) (ºC)</td>
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<td>17</td>
<td>17</td>
<td>22</td>
<td>22</td>
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<tr>
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<td>(w_{in}) (kg water/kg dry air)</td>
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<td>(m_{w,in}) (kg/s)</td>
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<td>$m_a$ (kg/s)</td>
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<td>43.2373</td>
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</table>

| Makeup water cost (US$/year) | 23983.4 | 17412.4 | 23901.7 | 19435.6 | 23865.9 | 16988.0 |
| Power fan cost (US$/year) | 13882.8 | 32785.4 | 50190.5 | 63783.4 | 5559.9 | 5635.7 |
| Operation cost (US$/year) | 37866.2 | 32785.4 | 74092.2 | 83218.9 | 29425.8 | 22623.7 |
| Capital cost (US$/year) | 32667.7 | 29866.7 | 43186.5 | 67320.6 | 25030.3 | 25202.8 |
| Total annual cost (US$/year) | 70533.9 | 62652.1 | 117278.7 | 150539.6 | 54456.0 | 47826.5 |

Table 5. Results for Examples 4, 5 and 6
For Examples 2 and 5, the designs obtained using the Merkel’s method are cheaper than the ones obtained using the Poppe’s model; this is because the lower capital cost estimation. In Example 2 there is a high inlet wet air temperature and therefore air with poor cooling capacity, whereas in Example 5 there is a low outlet water temperature with respect to the wet bulb air temperature, which reduces the heat transfer efficiency (see Figure 10).

To demonstrate that the Merkel’s method is less accurate, one can see cases 1 and 4, in which the inlet air conditions are the same but the maximum allowable temperatures are 50°C and 45°C. For the Merkel’s method the designs show the maximum possible range for each case; however, the design obtained from the Poppe’s method are the same because the inlet air conditions determine the cooling capacity.
5. Conclusions

A mixer integer nonlinear programming model for the optimal detailed design of counter-flow cooling towers has been presented. The physical properties and the transport phenomena parameters are rigorously modeled for a proper prediction. The objective function consists of the minimization of the total annual cost, which considers operating and capital costs. Results show that low wet temperatures for the air inlet and high ranges favor optimal designs. The operating costs are proportional to the range, and the capital costs require an optimal relation between a high range and a low air flow rate; therefore, the strongest impact of the physical representation of the transport phenomenon is over the capital cost. For all cases analyzed here the minimum possible area was obtained, which means that the packing area is a major variable affecting the total annual cost. The cooling capacity of the inlet air determines the optimum relation between range and air flow rate. Since the model here presented is a non-convex problem, the results obtained can only guaranty local optimal solutions. Global optimization techniques must be used if a global optimal solution is of primary importance.

6. Appendix A

The relationships for physical properties were taken from Kröger [25]. All temperatures are expressed in degrees Kelvin. The enthalpy of the air-water vapor mixture per unit mass of dry-air is:

\[ i_w = cp_a (T_a - 273.15) + w [i_{gw} + cp_v (T_v - 273.15)] \] (A.1)

The enthalpy for the water vapor is estimated from:

\[ i_v = i_{gw} + cp_v (T_v - 273.15) \] (A.2)

The enthalpy of saturated air evaluated at water temperature is:

\[ i_{w,s} = cp_{w,s} (T_a - 273.15) + w_s [i_{gw} + cp_v (T_v - 273.15)] \] (A.3)

The specific heat at constant pressure is determined by:

\[ cp_a = 1.045356 \times 10^1 - 3.161783 \times 10^{-1} T + 7.083814 \times 10^{-3} T^2 - 2.705209 \times 10^{-5} T^3 \] (A.4)

Specific heat of saturated water vapor is determined by:

\[ cp_v = 1.3605 \times 10^1 + 2.31334 T - 2.46784 \times 10^{-3} T^2 + 5.91332 \times 10^{-5} T^4 \] (A.5)

The latent heat for water is obtained from:

\[ i_{gw} = 3.4831814 \times 10^{-1} - 5.8627703 \times 10^{-4} T + 12.139568 T^2 - 1.40290431 \times 10^{-3} T^3 \] (A.6)

The specific heat of water is:

\[ cp_w = 8.15599 \times 10^1 - 2.80627 \times 10 T + 5.11283 \times 10^{-3} T^2 - 2.17582 \times 10^{-5} T^3 \] (A.7)

The humidity ratio is calculated from:
The vapor pressure is:

$$ P_v = 10^z \quad \text{(A.9)} $$

$$ z = 10.79586 \left( 1 - \frac{273.16}{T} \right) + 5.02808 \log_{10} \left( \frac{273.16}{T} \right) + 1.50474 \times 10^{-1} \left[ 1 - 10^{-0.268 \left( \frac{273.16}{T} \right) \left( \frac{T}{273.16} \right)^{0.5}} \right] + 4.2873 \times 10^{-6} \left[ 10^{-0.268 \left( \frac{273.16}{T} \right) \left( \frac{T}{273.16} \right)^{0.5}} - 1 \right] + 2.786118 \quad \text{(A.10)} $$

### 7. Nomenclature

- $a_i'$: disaggregated coefficients for the estimation of NTU
- $A_{fr}$: cross-sectional packing area, m²
- $b_i'$: disaggregated coefficients for the estimation of loss coefficient
- $c_1c_5$: correlation coefficients for the estimation of NTU
- $CAP$: capital cost, US$/year
- $C_{CTF}$: fixed cooling tower cost, US$
- $C_{CTMA}$: incremental cooling tower cost based on air mass flow rate, US$/s/kg
- $C_{CTV}$: incremental cooling tower cost based on tower fill volume, US$/m³
- $C_{ctv}$: disaggregated variables for the capital cost coefficients of cooling towers
- $COP$: annual operating cost, US$/year
- $c_i$: variables for NTU calculation
- $c_i'$: disaggregated variables for NTU calculation
- $c_p$: specific heat at constant pressure, J/kg·K
- $c_p^v$: specific heat of saturated water vapor, J/kg·K
- $c_p^w$: specific heat of water, J/kg·K
- $c_p^{w,in}$: specific heat of water in the inlet of cooling tower, J/kg·K
- $c_p^{w,out}$: specific heat of water in the outlet of cooling tower, J/kg·K
- $cu_e$: unitary cost of electricity, US$/kW-h
- $cu_w$: unitary cost of fresh water, US$/kg
- $d_1$-$d_6$: correlation coefficients for the estimation of loss coefficient, dimensionless
- $d_i$: variables used in the calculation of the loss coefficient
\( d_i' \) disaggregated variables for the calculation of the loss coefficient

\( D_{TMIN} \) minimum allowable temperature difference, °C or K

\( e' \) coefficient cost for different fill type

\( H_Y \) yearly operating time, hr/year

\( HP \) power fan, HP

\( i_{fw} \) heat latent of water, J/kg

\( i_{ma} \) enthalpy of the air-water vapor mixture per mass of dry-air, J/kg
dry-air

\( i_{ma,s,w} \) enthalpy of saturated air evaluated at water temperature, J/kg
dry-air

\( i_v \) enthalpy of the water vapor, J/kg dry-air

\( J \) recursive relation for air ratio humidity

\( K \) recursive relation for air enthalpy

\( K_{fi} \) loss coefficient in the fill, m^{-1}

\( K_F \) annualization factor, year^{-1}

\( K_{misc} \) component loss coefficient, dimensionless

\( L \) recursive relation for number of transfer units

\( L_f \) fill height, m

\( Le_f \) Lewis factor, dimensionless

\( m_a \) air mass flow rate, kg/s

\( mav_{in} \) inlet air-vapor flow rate, kg/s

\( mav_{m} \) mean air-vapor flow rate, kg/s

\( mav_{out} \) outlet air-vapor flow rate, kg/s

\( m_w \) water mass flow rate, kg/s

\( m_{w,b} \) blowdown water mass flow rate, kg/s

\( m_{w,d} \) drift water mass flow rate, kg/s

\( m_{w,ev} \) mass flow rate for the evaporated water, kg/s

\( m_{w,la} \) inlet water mass flow rate in the cooling tower, kg/s

\( m_{w,m} \) average water mass flow rate in the cooling tower, kg/s

\( m_{w,la} \) outlet water mass flow rate from the cooling tower, kg/s

\( m_{w,r} \) makeup water mass flow rate, kg/s

\( NTU \) number of transfer units, dimensionless

\( n_{cycle} \) number of cycles of concentration, dimensionless

\( P \) vapor pressure, Pa

\( P_l \) total vapor pressure, Pa

\( P_{C,wb} \) saturated vapor pressure, Pa

\( Q \) heat load, W or kW

\( T_a \) dry-bulb air temperature, °C or K

\( TAC \) total annual cost, US$/year

\( T_{a,n} \) dry-bulb air temperature in the integration intervals, °C or K

\( TMPI \) inlet of the hottest hot process stream, °C or K

\( TMPO \) inlet temperature of the coldest hot process streams, °C or K

\( T_w \) water temperature, °C or K

\( T_{wb} \) wet-bulb air temperature, °C or K
$T_{wb,in}$ inlet wet-bulb air temperature in the cooling tower, °C or K
$T_{wb,n}$ wet-bulb air temperature in the integration intervals, °C or K
$T_{w,in}$ inlet water temperature in the cooling tower, °C or K
$T_{w,out}$ outlet water temperature in the cooling tower, °C or K
$w$ mass-fraction humidity of moist air, kg of water/kg of dry-air
$w_{in}$ inlet humidity ratio in the cooling tower, kg of water/kg of dry-air
$w_{out}$ outlet humidity ratio in the cooling tower, kg of water/kg of dry-air
$w_{s,w}$ humidity saturated ratio, kg of water/kg of dry-air

### 7.1 Binary variables

$y_k$ used to select the type of fill

### 7.2 Greek symbols

$\Delta P_t$ total pressure drop, Pa
$\Delta P_{cp}$ dynamic pressure drop, Pa
$\Delta P_{fi}$ fill pressure drop, Pa
$\Delta P_{misc}$ miscellaneous pressure drop, Pa
$\eta_f$ fan efficiency, dimensionless
$\rho_{in}$ inlet air density, kg/m$^3$
$\rho_m$ harmonic mean density of air-water vapor mixtures, kg/m$^3$
$\rho_{out}$ outlet air density, kg/m$^3$

### 7.3 Subscripts

a dry air
b blowdown water
d drift water
e electricity
ev evaporated water
f fan
fi packing or fill
fr cross-sectional
in inlet
j constants to calculate the transfer coefficient depending of the fill type
k constants to calculate the loss coefficient depending of the fill type
m average
ma air-vapor mixture
 misc miscellaneous
n integration interval
out outlet
r makeup
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$s$ saturated
$t$ total
$v$ water vapor
$vp$ velocity pressure
$w$ water
$wb$ wet-bulb temperature

7.4 Superscripts

$i$ fill type, $i=1, 2, 3$

8. References


This book covers a number of topics in heat and mass transfer processes for a variety of industrial applications. The research papers provide advances in knowledge and design guidelines in terms of theory, mathematical modeling and experimental findings in multiple research areas relevant to many industrial processes and related equipment design. The design of equipment includes air heaters, cooling towers, chemical system vaporization, high temperature polymerization and hydrogen production by steam reforming. Nine chapters of the book will serve as an important reference for scientists and academics working in the research areas mentioned above, especially in the aspects of heat and mass transfer, analytical/numerical solutions and optimization of the processes.

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