Final Results of Assembly Line Balancing Problem

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1. Introduction

The manufacturing assembly line was first introduced by Henry Ford in the early 1900’s. It was designed to be an efficient, highly productive way of manufacturing a particular product. The basic assembly line consists of a set of workstations arranged in a linear fashion, with each station connected by a material handling device. The basic movement of material through an assembly line begins with a part being fed into the first station at a predetermined feed rate. A station is considered any point on the assembly line in which a task is performed on the part. These tasks can be performed by machinery, robots, and/or human operators. Once the part enters a station, a task is then performed on the part, and the part is fed to the next operation. The time it takes to complete a task at each operation is known as the process time (Sury, 1971). The cycle time of an assembly line is predetermined by a desired production rate. This production rate is set so that the desired amount of end product is produced within a certain time period (Baybars, 1986). In order for the assembly line to maintain a certain production rate, the sum of the processing times at each station must not exceed the stations’ cycle time (Fonseca et al, 2005). If the sum of the processing times within a station is less than the cycle time, idle time is said to be present at that station (Erel et al, 1998). One of the main issues concerning the development of an assembly line is how to arrange the tasks to be performed. This arrangement may be somewhat subjective, but has to be dictated by implied rules set forth by the production sequence (Kao, 1976). For the manufacturing of any item, there are some sequences of tasks that must be followed. The assembly line balancing problem (ALBP) originated with the invention of the assembly line. Helgeson et al (Helgeson et al, 1961) were the first to propose the ALBP, and Salveson (Salveson, 1955) was the first to publish the problem in its mathematical form. However, during the first forty years of the assembly line’s existence, only trial-and-error methods were used to balance the lines (Erel et al., 1998). Since then, there have been numerous methods developed to solve the different forms of the ALBP. Salveson (Salveson, 1955) provided the first mathematical attempt by solving the problem as a linear program. Gutjahr and Nemhauser (Gutjahr & Nemhauser, 1964) showed that the ALBP problem falls into the class of NP-hard combinatorial optimization problems. This means that an optimal solution is not guaranteed for problems of significant size. Therefore, heuristic methods have become the most popular techniques for solving the problem.
2. Heuristic methods in assembly line balancing problem

The heuristic approach bases on logic and common sense rather than on mathematical proof. Heuristics do not guarantee an optimal solution, but results in good feasible solutions which approach the true optimum. Most of the described heuristic solutions in literature are the ones designed for solving Single Assembly Line Balancing Problem. Moreover, most of them are based on simple priority rules (constructive methods) and generate one or a few feasible solutions. Task-oriented procedures choose the highest priority task from the list of available tasks and assign it to the earliest station which is assignable. Among task-oriented procedures we can distinguish immediate-update-first-fit (IUFF) and general-first-fit methods depending on whether the set of available tasks is updated immediately after assigning a task or after the assigning of all currently available tasks. Due to its greater flexibility immediate-update-first-fit method is used more frequently. The main idea behind this heuristic is assigning tasks to stations basing on the numerical score. There are several ways to determine (calculate) the score for each tasks. One could easily create his own way of determining the score, but it is not obvious if it yields good result. In the following section five different methods found in the literature are presented along with the solution they give for our simple example. The methods are implemented in the Line Balancing program as well. From the moment the appropriate score for each task is determined there is no difference in execution of methods and the required steps to obtain the solution are as follows:

**Step 1.** Assign a numerical score \( n(x) \) to each task \( x \).

**Step 2.** Update the set of available tasks (those whose immediate predecessors have been already assigned).

**Step 3.** Among the available tasks, assign the task with the highest numerical score to the first station in which the capacity and precedence constraints will not be violated. Go to **Step 2**.

The most popular heuristics which belongs to IUFF group are:
- IUFF-RPW Immediate Update First Fit – Ranked Positional Weight,
- IUFF-NOF Immediate Update First Fit – Number of Followers,
- IUFF-NOIF Immediate Update First Fit – Number of Immediate Followers,
- IUFF-NOP Immediate Update First Fit – Number of Predecessors,
- IUFF-WET Immediate Update First Fit – Work Element Time.

In the literature we can often find the implementation of Kilbridge & Wester or Moodie & Young methods, too. Both of them base on precedence graph or precedence matrix of produced items.

3. Measures of final results of assembly line balancing problem

Some measures of solution quality have appeared in line balancing problem. Below are presented three of them (Scholl, 1999).

Line efficiency (LE) shows the percentage utilization of the line. It is expressed as ratio of total station time to the cycle time multiplied by the number of workstations:

\[
LE = \frac{\sum_{i=1}^{K} ST_i}{c \cdot K} \cdot 100\%
\]

where: \( K \) - total number of workstations, 
\( c \) - cycle time.
Smoothness index (SI) describes relative smoothness for a given assembly line balance. Perfect balance is indicated by smoothness index 0. This index is calculated in the following manner:

\[
SI = \frac{1}{\sum_{i=1}^{K} (ST_{\text{max}} - ST_{i})^2}
\]

where:
- \(ST_{\text{max}}\) - maximum station time (in most cases cycle time),
- \(ST_{i}\) - station time of station i.

Time of the line (LT) describes the period of time which is need for the product to be completed on an assembly line:

\[
LT = c \cdot (K - 1) + T_K
\]

where:
- \(c\) - cycle time,
- \(K\) - total number of workstations,
- \(T_K\) - processing time of last station.

Two-sided assembly lines (Fig. 1.) are typically found in producing large-sized products, such as trucks and buses. Assembling these products is in some respects different from assembling small products. Some assembly operations prefer to be performed at one of the two sides (Bartholdi, 1993).

![Fig. 1. Two-sided assembly line structure](www.intechopen.com)

The final result estimation of two-sided assembly line balance needs some modification of existing measures (Grzechca, 2008).

Time of line for TALBP

\[
LT = c \cdot (Km - 1) + \text{Max}\{t(S_K), t(S_{K-1})\}
\]

where:
- \(Km\) - number of mated-stations
- \(K\) - number of assigned single stations
- \(t(S_K)\) - processing time of the last single station

As far as smoothness index and line efficiency are concerned, its estimation, on contrary to LT, is performed without any change to original version. These criterions simply refer to each individual station, despite of parallel character of the method.
But for more detailed information about the balance of right or left side of the assembly line additional measures will be proposed:

Smoothness index of the left side

\[
SI_L = \sqrt{\sum_{i=1}^{K} (ST_{\text{max}L} - ST_{iL})^2}
\]

(5)

where:

- \(SI_L\) - smoothness index of the left side of two-sided line
- \(ST_{\text{max}L}\) - maximum of duration time of left allocated stations
- \(ST_{iL}\) - duration time of i-th left allocated station

Smoothness index of the right side

\[
SI_R = \sqrt{\sum_{i=1}^{K} (ST_{\text{max}R} - ST_{iR})^2}
\]

(6)

where:

- \(SI_R\) - smoothness index of the right side of two-sided line,
- \(ST_{\text{max}R}\) - maximum of duration time of right allocated stations,
- \(ST_{iR}\) - duration time of i-th right allocated station.

4. Line and station efficiency

Efficiency line was introduced to the assembly line balancing problem by Salveson. It was the optimization goal of ALBP and the best solution was when it achieved 100%. Unfortunately this measure is only useful when number of stations or cycle time are changed. If for many final results we obtain the same number of stations and cycle time the line efficiency does not deliver us the detailed knowledge about quality of the line balance.

Example of 12 tasks will be discussed. In Table 1 processing task times are presented. Figure 2 shows the relations between tasks (technology of assembly).

Fig. 2. Precedence graph of 12 tasks numerical example
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<table>
<thead>
<tr>
<th>task</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>20</td>
<td>40</td>
<td>70</td>
<td>10</td>
<td>30</td>
<td>11</td>
<td>32</td>
<td>60</td>
<td>27</td>
<td>38</td>
<td>50</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 1. Operation times for numerical example

Ranked Positional Weight (Halgeson et al, 1961) and Immediate Update First Fit – Working Element Time heuristic for obtaining the balance of assembly line were chosen. As we can see (Fig. 3 ÷ 5) final results are different. For time-oriented balance we got 5 stations and LE for different heuristics is the same. Author proposes an additional station efficiency measurement which allows to find “bottleneck” in the production system and helps to estimate a good feasible line balance. The new measure describes more detailed the efficiency of each workstation and helps to find the worst point in whole assembly structure. Station efficiency ($LE_{STi}$) shows the percentage utilization of each workstation. It is expressed as ratio of station time to the cycle time:

$$LE_{STi} = \frac{ST_i}{c} \cdot 100\%$$

<table>
<thead>
<tr>
<th>Measures</th>
<th>RPW heuristic</th>
<th>IUFF-WET heuristic</th>
<th>Measures</th>
<th>RPW heuristic</th>
<th>IUFF-WET heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>40,42</td>
<td>59,08</td>
<td>LE_{ST1}</td>
<td>90 %</td>
<td>81 %</td>
</tr>
<tr>
<td>LT</td>
<td>462</td>
<td>462</td>
<td>LE_{ST1}</td>
<td>92 %</td>
<td>98 %</td>
</tr>
<tr>
<td>LE</td>
<td>80 %</td>
<td>80 %</td>
<td>LE_{ST1}</td>
<td>65 %</td>
<td>59 %</td>
</tr>
<tr>
<td>LE_{ST1}</td>
<td>90 %</td>
<td>100 %</td>
<td>LE_{ST1}</td>
<td>62 %</td>
<td>62 %</td>
</tr>
</tbody>
</table>

Table 2. Detailed measures of final results of RPW and IUFF-WET heuristics

Fig. 3. Final balance of 12 tasks example using RPW heuristic
5. Last station problem

Below are presented two heuristic solutions. We consider the case of single assembly line (Fig. 6.) where all tasks can be assigned to any position (E). The cycle time is 16. Fig. 7. presents solution where three stations have efficiency 100 % or very close to the value. Last station has an idle time which is a consequence of completed number of tasks. Fig. 8. shows solution where the last station is utilized in 100 %. The contribution of tasks causes idle times of second and third station. As we can observe both solutions are feasible but with different assignment of tasks and different station efficiency. RPW solution is near optimal but the last station idle time is the biggest of all stations. WET solution is feasible and not so close to optimal but smoothness index of this final result is smaller (balancing or equalizing problem).
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Fig. 6. Numerical example of 12 tasks – time duration and positional constrains are given in braces (L – left, R – right, E – any position)

Fig. 7. Final solution of assembly line balancing problem – RPW heuristic

Fig. 8. Final solution of assembly line balancing problem – IUFF - WET heuristic
6. Assembly line structure problem

We consider numerical example from Fig. 8. The positional constrains are respected. As we can observe not only assigning of task becomes a problem (Fig. 9. ÷ Fig. 14.). The structure of assembly line can changed for different cycle times (Grzechca, 2010).

Fig. 9. Assembly line balance (c=16)

Fig. 10. Assembly line structure (c=16)

Fig. 11. Assembly line balance (c=15)
In two-sided assembly line balancing problem it is very difficult to obtain a complete station structure. This type of line very hard depends on precedence and position constraints.
7. Conclusions

Assembly lines are a popular manufacturing structure. Assembly line balancing problem is known more than 50 years. There are hundreds exact and heuristic methods. It is very important to obtain the feasible and acceptable results. It is very important to analyze and estimate the final results and to implement the best one. Author of the chapter hopes that the presented knowledge helps to understand the problem.

8. References


An assembly line is a manufacturing process in which parts are added to a product in a sequential manner using optimally planned logistics to create a finished product in the fastest possible way. It is a flow-oriented production system where the productive units performing the operations, referred to as stations, are aligned in a serial manner. The present edited book is a collection of 12 chapters written by experts and well-known professionals of the field. The volume is organized in three parts according to the last research works in assembly line subject. The first part of the book is devoted to the assembly line balancing problem. It includes chapters dealing with different problems of ALBP. In the second part of the book some optimization problems in assembly line structure are considered. In many situations there are several contradictory goals that have to be satisfied simultaneously. The third part of the book deals with testing problems in assembly line. This section gives an overview on new trends, techniques and methodologies for testing the quality of a product at the end of the assembling line.

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