1. Introduction

One of the most relevant characteristics of inventory management methods is the amplification phenomenon called the “bullwhip-effect”, defined as the upstream increasing of production variability, caused by a supply chain’s demand variability at the retail level. This effect has been extensively studied, both from industrial and theoretical points of view (Takahashi et al., 1994). Among the multiple reasons mentioned in literature (Lee et al., 2000; Takahashi et al., 1994; Warburton, 2004; Wu and Meixell, 1998), four features are frequently reported at the origin of this phenomenon (Lee et al., 1997): demand signal processing, strategic ordering behavior, ordering batching and price variations.

Geary et al. (2006) pointed out ten common causes of bullwhip-effect and the subsequent re-engineering principles to eliminate or prevent amplification. Among them, the time compression principle suggests that the most relevant principle to achieve this goal is the existence of an optimal minimum lead time. Takahashi and Myreshka (2004) extensively studied sources of bullwhip-effect and proposed several counter-measures for the demand, ordering process and supply sides. Some of these counter-measures are: sharing information about inventory and production levels among stages along the chain, controlling inventory replenishment by a single method, reducing the lead-time, designing appropriate forecasting methods (or eliminating forecasting practices) and implementing pull or hybrid methods.

It has been argued that the lack of flexibility in a supply chain is a consequence of the bullwhip-effect. In order to analyze this argument, Pereira (1999) developed general expressions for the amplification measure in the case of three ordering methods: push, pull and hybrid. In a further contribution, Pereira and Paulre (2001) introduced the adjustment degree of production to demand rate, a flexibility measure evaluating the distance between the demand and production signals on each supply chain stage. Considering the ordering methods above, an AR(1) demand process and a non-capacity-restricted supply chain model, it was found that the adjustment degree behaves as a bullwhip-effect, especially in push systems. More importantly, it was found that the bullwhip-effect is structurally due to the upstream propagation of the demand forecasting. Chen et al. (2000) also studied the increments of variability in a generic supply chain structure, for the specific case of a stationary AR(1) process, finding that the demand forecasting importantly impacts the amplification level in the supply chain. However, they did not explain how it is produced by forecasting methods.
In a recent work Pereira et al. (2009) have shown that for an AR(1) demand stochastic process, flexibility on each stage of the supply chain strongly depends on the manager’s belief about the downstream forecasting processes. Beliefs affect the decision rules in ordering methods, structurally defining the adaptation capability in the supply chain. Then, flexibility could be used as a strategy to keep amplification under control. In this chapter we present some analytical results that explore this insight, considering the modeled supply chain and demand process. Moreover, an introductory analysis of inventory amplification is presented, in order to inspect the effect of manager’s belief on it. We propose that belief-based regulation may improve the amplification levels both in production and inventory sides. But, it strongly depends on the adopted forecasting method and the assumed demand process.

The remainder of this chapter is organized as follows. In section 2.1, the supply chain model and ordering equations are presented. In section 2.2, the flexibility framework is introduced and relevant preliminary results on adjustment degree for the modeled supply chain are presented for push, pull and hybrid methods. In section 3, we introduce one of the amplification acceptability criteria proposed in literature, which indicates the requirements for control of the bullwhip effect. Further, the mathematical relation between the adjustment degree and the amplification is presented, which allows us to express the amplification acceptability criteria in terms of flexibility conditions. In section 3.3, a fading variable, representing the manager’s belief on estimates, is analyzed in terms of its impact both on production and inventory amplification measures. Conclusions are presented in section 4.

2. Preliminaries

2.1 The supply chain model

Consider a multi-echelon, single-item, supply chain composed by production stages $P_i$ ($i = 1, \ldots, n$), stock sites $B_i$ ($i = 0, \ldots, n$), and a supplier stage “Supplier”, as shown in Fig. 1. This will be called the reference model $M$ (Pereira and Paulre, 2001).

Let us consider a periodic ordering method managing the production levels on each stage of the supply chain (Takahashi and Myreshka, 2004). Then, the $i$-th production stage periodically receives an order $O_i$, which defines how many units of the item stocked in $B_i$ need to be processed and further stocked in $B_{i-1}$. The elapsed time between the instant when an order is calculated and the moment where the ordered units are ready to be delivered (i.e., the lead time) is considered an exogenous variable, identical in all stages: $L^i = L_0$ ($i = 1, \ldots, n$). A period is defined here as a unitary interval of time. Thus, $t \in \mathbb{Z}$ starts the $t$-th period; $t + 1$ starts the $(t + 1)$-th, and so forth.

![Fig. 1. Serial configuration of production stages](https://www.intechopen.com)

The following variables are defined in the model $M$:

Furthermore, the production rate at stage $P_i$ is given by

$$P_i^t = O_i^t - L, \quad (i = 1, \ldots, n),$$

which means that the manufacturing lead time between stages $P_i$ and $P_1$ can be written as $LT^t = iL$. 

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Inventory management systems differ in the way the production order on each stage is defined. In the case of push, hybrid and pull management methods, the ordering equation any stage \( i \) is expressed as (Pereira and Paulre, 2001):

\[
\text{Push:} \quad O_{i}^{t} = D_{t-(i-1)L} + \sum_{j=1}^{i} \Delta D_{t-(i-j)L}',
\]

(2)

\[
\text{Hybrid:} \quad O_{i}^{t} = D_{t-(i-1)L} + \Delta D_{i-1}' + L_{i},
\]

(3)

\[
\text{Pull:} \quad O_{i}^{t} = D_{t-(i-1)L}.
\]

(4)

Notice that (3) characterizes a system where only the first stage operates in push.

### 2.2 Evaluating flexibility in the supply chain

A system is said flexible whenever it has the capability to self-adjust in response to changes in its environment. The design of a flexible system implies control of three dimensions (Pereira and Paulre, 2001): degree, effort and time of adjustment. More precisely, let a system and its environment be characterized by the trajectories they take in the state spaces \( \mathcal{S} \) and \( \mathcal{E} \), respectively. In addition, let us assume an observer is able to recognize the environment and the system states \( e_{t} \in \mathcal{E} \) and \( s_{t} \in \mathcal{S} \), at time \( t \); she/he also identifies a logic \( \mathcal{L} \) such that

\[
\mathcal{L}(e_{t-l}, s_{t}) = (s_{t}^{*}, \|s_{t}^{*} - s_{t}\|).
\]

(5)

This means that, given \( e_{t-l} \) and \( s_{t} \), \( \mathcal{L} \) allows the observer to define an expected state \( s_{t}^{*} \in \mathcal{S} \) and its distance to the current state \( s_{t} \). Thus, the system responsiveness remains characterized by \( l_{t} \geq 0 \), indicating that the expected state depends on information provided to \( \mathcal{L} \) in \( t \), but occurring in \( t-l_{t} \). The considered system is said to be in partial equilibrium when \( \mathcal{L}(e_{t-l_{t}}, s_{t}) = (s_{t}^{*}, 0) \). Whenever \( \|s_{t}^{*} - s_{t}\| \neq 0 \), flexibility is the property that tends to realize the partial equilibrium in the system. In order to do this, the system must expend a specific effort and time. Thus, in given times \( t_{1}, t_{2}, \ldots, t_{n} \), we assume that a flexible system dynamically adjusts to demanded changes defined in a succession of states \( \mathcal{D} = s_{t_{1}}^{*}, \ldots, s_{t_{n}}^{*} \).

<table>
<thead>
<tr>
<th>Stage</th>
<th>Push</th>
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<tbody>
<tr>
<td>( i = 1 )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>0</td>
</tr>
<tr>
<td>( i &gt; 1 )</td>
<td>( \theta^{i-1} + H_{i} )</td>
<td>( \theta^{i-1} )</td>
<td>0</td>
</tr>
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</table>

Table 1. Adjustment degree \( \theta^{i} \) for the three management methods

Now, we argue that the flexibility analysis provides a convenient framework to study the supply chain bullwhip-effect. In fact, let us consider that \( \mathcal{D} \) may be represented by the demand process \( D_{t} \) and the system states, on each stage, by \( P_{t} \). Then, given a stage \( i \), a deviation...
variable is defined as $\theta^i_t = P^i_t - D^i_{t-iL}$, which means that a demand signal received by the stage $i$ at time $t - (i + 1)L$ has a response at $t - iL$, i.e. within a leadtime $L$. This delay may be considered the responsiveness capability (adjustment time) of this stage. The adjustment degree on $i$ is expressed as follows,

$$\vartheta^i = \frac{V[\theta^i]}{V[D_{t-iL}]} \quad \forall i \geq 1,$$

where $V[\cdot]$ denotes the variance of the argument. Notice that, as $\vartheta^i$ decreases, the stage-$i$'s adjustment of the production level to the delayed demand signal improves. Thus, the optimal adjustment is reached when $\vartheta^i = 0, \forall i$.

It has been shown that, when the model $M$ is considered, $\vartheta^i$, as measured for pull, push and hybrid management methods, has the structure presented in Table 1 (Pereira and Paulre, 2001), where $G$ and $H_i$ depend on the demand forecasting strategy (see section 3.2). This result reveals that push-type stages propagate adjustment variability upstream in the supply chain, scaling up or down the adjustment degree, in a very similar way to the bullwhip-effect behavior.

### 3. Flexibility and amplification

#### 3.1 The amplification of production

The bullwhip-effect in a supply chain is usually evaluated by an amplification measure, defined as follows (Muramatsu et al., 1985),

$$Amp^i = \frac{V[P^i]}{V[D_t]}.$$  

This metric may be interpreted as the scaling effect of demand variability, from the first to upstream stages. It has been proposed that an adequate ordering method should satisfy the following inequality (Muramatsu et al., 1985), called here the Muramatsu Amplification Condition (MAC):

$$1 \geq Amp^1 \geq Amp^2 \geq \ldots \geq Amp^n.$$  

Hereinafter, let us see the relation between the amplification and the adjustment degree measures. Indeed, expanding the expression for (6), it follows that

$$\frac{V[P^i_t - D^i_{t-iL}]}{V[D_t]} = \frac{V[P^i_t]}{V[D_t]} + \frac{V[D^i_{t-iL}]}{V[D_t]} - 2 \frac{V[D_t]}{V[D_t]} \text{cov} \left[ P^i_t, D^i_{t-iL} \right].$$

Stationarity assumption allows us to write

$$\vartheta^i = Amp^i + 1 - 2 \frac{V[D_t]}{V[D_t]} \text{cov} \left[ P^i_t, D^i_{t-iL} \right].$$

Defining $\gamma^i = 2 \frac{V[D_t]}{V[D_t]} \text{cov} \left[ P^i_t, D^i_{t-iL} \right]$, we have

$$Amp^i = \vartheta^i + \gamma^i - 1.$$
In consequence, the MAC inequality may be written in terms of the adjustment degree of production as follows:

\[
1 \geq \vartheta^1 + \gamma^1 - 1 \geq \vartheta^2 + \gamma^2 - 1 \geq \ldots \geq \vartheta^n + \gamma^n - 1. \tag{12}
\]

This is an interesting result because, since \(Amp^i\) measures the bullwhip-effect of a given management system, when faced to a specific demand behavior, it suggests that monitoring of \(\vartheta^i\) yields a more adequate feedback to the supply chain manager. In fact, it furnishes her/him with a control variable in the supply chain. In the next section, this idea is explored for the three ordering methods.

### 3.2 Flexibility conditions for an AR(1) demand process

A simple observation of Table 1 exposes the way that the adjustment behavior propagates upstream in the supply chain. Inspecting the expression (12), a manager could rapidly establish a control condition, when implementing a particular method. For instance, it is easy to see that a hybrid method satisfies

\[
2 \geq \vartheta^1 + \gamma^1 \geq \vartheta^2 + \gamma^2 \geq \ldots \geq \vartheta^1 + \gamma^n, \tag{13}
\]

whilst in a pull method with \(\vartheta^i = 0 \ (\forall i)\), we have

\[
2 \geq \gamma^1 \geq \gamma^2 \geq \ldots \geq \gamma^n. \tag{14}
\]

However, for a push method this condition needs to be found for every specific demand process. Therefore, for sake of analysis, let us assume that the demand rate can be accurately modeled by an i.i.d stationary AR(1) stochastic process with mean \(\mu\), variance \(\sigma^2\) and autocorrelation coefficient \(\lambda \in (-1, 1)\).

When a pull ordering method is adopted, using (1) and (4), we have \(P^i_t = D^t_{t-iL}\). Hence, for a stationary stochastic demand process it follows,

\[
\gamma^i = \frac{2}{\sqrt{\text{Var}[D_t]}} \left( E \left[ (D^t_{t-iL})^2 \right] - (E [D^t_{t-iL}])^2 \right) = 2. \tag{15}
\]

Thus, the relation between \(\vartheta^i\) and \(Amp^i\) is

\[
Amp^i = \vartheta^i + 1. \tag{16}
\]

But \(\vartheta^i = 0, \forall i\) (see Table 1), which implies \(Amp^i = 1\). In consequence, a pull inventory management simultaneously minimizes \(\vartheta^i\) and accomplishes the MAC criteria. Differently, when a push ordering method is considered, using (1) and (2), we have

\[
P^i_t = D^t_{t-iL} + \sum_{j=1}^{i} \Delta D^i_{(i+1-j)L} = D^t_{t-iL} + \theta^i_t. \tag{17}
\]
Therefore,
\[
\gamma^i = \frac{2}{V[D_t]} \{V[D_t] + E[D_t-iL \left( \sum_{j=1}^{i} \Delta \hat{D}_{i-(i+1)}^j \right) ] - E[D_t] E[ \sum_{j=1}^{i} \Delta \hat{D}_{i-(i+1)}^j ] \},
\] (18)

This equation shows that in the push method, the relation between \( \vartheta^i \) and \( \text{Amp}^i \) depends on the first and second order statistics of the demand stochastic process able to describe the requested units. A closed expression can be found for some specific demand stochastic processes. In particular, given an AR(1) stochastic demand process, a straightforward analysis shows that
\[
\Delta \hat{D}_i = (D_t - D_{t-1}) \sum_{j=1}^{L+1} \lambda^{LT(j-1)+j} = (D_t - D_{t-1}) \lambda^{LT(i-1)} \phi.
\] (19)

where \( \phi = \lambda \frac{L+1}{\lambda-1}, \lambda \neq 1 \). Knowing that \( E[D_{t-k}D_{t-j}] = \lambda^{k-j} \sigma^2 + \mu^2, \ \forall k > j \), we find an expression for \( \gamma^i \), expressed as
\[
\gamma^i = 2 + 2(\lambda - 1) \phi \sum_{j=1}^{i} \lambda^{LT(j-1)-(i-j)L-1} = 2 + 2 \left( \lambda^{L+1} - 1 \right) \frac{1 - \lambda^{2L}}{1 - \lambda^{2L}}.
\] (20)

From this equation, \( \gamma^i - \gamma^{i-1} \leq 0 \). In addition, (11) and Table 1 imply \( \vartheta^i = \text{Amp}^{i-1} - \gamma^{i-1} - 1 \) and \( \vartheta^i = \vartheta^{i-1} + H_i \), respectively. Then
\[
\text{Amp}^i = \text{Amp}^{i-1} + \gamma^i - \gamma^{i-1} + H_i.
\] (21)

Now, let us restrict \( \vartheta^i \) such that
\[
\vartheta^1 \geq \vartheta^2 \geq \ldots \geq \vartheta^n,
\] (22)
meaning that \( H_i \leq 0, \forall i \). In such case, (21) implies \( \text{Amp}^{i-1} \geq \text{Amp}^i, \forall i \), and the MAC condition would be satisfied. Unfortunately, in a previous publication we have shown that \( H_i \leq 0 \) is rarely satisfied and for most of \( \lambda \) values we have \( \vartheta^i \geq \vartheta^{i-1} \) (Pereira and Paulre, 2001). For this reason, a different strategy needs to be explored. Actually, given that the MAC condition is immediately satisfied by a pull method, it could be interesting to know how amplification is reduced when a push or hybrid method moves closer to the pull case. In the next section such idea is analyzed, introducing a fading variable which models the manager’s belief on demand forecasting.

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3.3 The manager’s belief effect
In Pereira et al. (2009) we proposed an alternative to control the bullwhip-effect, using a learning variable representing the manager’s belief on the forecasted demand change. This learning was modeled by a factor $\alpha$, included in the ordering equation as $O_i^t = P_i^{t-1} + \alpha \Delta \hat{D}_i^t$, which conveys $\theta_i^t = \alpha \Delta \hat{D}_i^t$. Applying the same procedure yielding the results on Table 1 (Pereira and Paulre, 2001), it is straightforward to prove that the amplification value on stage $i$, $Amp_i^\alpha$, is expressed as follows,

$$ Amp_i^\alpha = \begin{cases} 1 + A_\alpha & i = 1, \\ Amp_{i-1}^\alpha + F_i^\alpha & i > 1. \end{cases} $$

(23)

In particular, when the AR(1) process is considered, we find

$$ A_\alpha = 2\alpha \phi (1 - \lambda) (\alpha \phi + 1), $$

(24)

$$ F_i^\alpha = 2\alpha \phi (1 - \lambda) \lambda^{2(i-1)L} \{ \alpha \phi - \frac{1}{\lambda} - \phi \frac{1-\lambda}{\lambda} (i-1) \} \quad (i = 2, \ldots, n). $$

(25)

In Fig. 2 amplification for $\alpha \in [0, 1]$, $L = 1$, $\lambda \in (-1, 1)$ and $i \in \{2, 8\}$ is presented. Notice that for $i = 2$ and the region $\lambda \geq 0$, the more $\alpha$ increases the more the bullwhip-effect is important, but the greatest amplification value is not reached as $\lambda$ approaches 1. On the other hand, results for $i = 8$ (Fig. 2(b)) are not intuitive and suggest that the improvement strategy consisting on the progressive reduction of the adjustment degree, by decreasing $\alpha$, does not necessarily reduce the bullwhip-effect. Even though, one may conclude that in push or hybrid methods, the bullwhip-effect is robustly reduced when stages approaches a pull-type ordering method. In other words, a manager is not necessarily enforced to abandon the push strategy to obtain acceptable amplification levels, but she/he should make a careful analysis in order to appreciate the consequences of his beliefs about the demand behavior and estimates.

Now, it is interesting to know how the inventory amplification level is shaped by the demand process. In particular, the way that the belief variable influences such level. Therefore, let us define $\text{InvAmp}_{i-1}(i = 1, \ldots, n)$ as the inventory amplification of the stock site $B_{i-1}$, that is

$$ \text{InvAmp}_{i-1} = \frac{V(B_i^{(i-1)})}{V(D_i)}. $$

(26)

It has been demonstrated that the production amplification impacts the inventory fluctuation, in the way depicted in Table 2 (Pereira, 1995). In general, $\psi_i$ and $\nu_i$ ($i = 1, \ldots, n$) are complex expressions depending on the forecasted and real demand processes. Instead, let us consider the expression (27), which represents the amplification level of the marginal inventory change,

$$ Amp_{\Delta B} = \frac{V(B_i^{(i-1)} - B_{i-1}^{(i-1)})}{V(D_i)}. $$

(27)

<table>
<thead>
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</tr>
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<tbody>
<tr>
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<td>$Amp_i^1 + \psi_i^1$</td>
<td>$Amp_i^1 + \nu_i^1$</td>
</tr>
<tr>
<td>$i &gt; 1$</td>
<td>$Amp_i^1 + \psi_i^1$</td>
<td>$Amp_i^1 + \psi_i^1$</td>
<td>$Amp_i^1 + \nu_i^1$</td>
</tr>
</tbody>
</table>

Table 2. Amplification of inventory $\text{InvAmp}_{i-1}$ for the three management methods
This variable measures how sensitive the inventory is to the demand process. Intuitively, the more sensitive it is, the less smooth the inventory signal, when faced to the demand process. Restricting ourselves to the case $i = 1$ and given that $B_{0}^t = B_{0}^{t-1} + P_{1}^{t-1} - D_{t}$, a straightforward analysis reveals that, when the learning variable $\alpha$ is included in the model, the following expression is obtained

$$Amp_{\alpha} B_{0}^{\triangle} = Amp_{\alpha}^1 + 1 - 2 \left[ \lambda^{L+1} + \alpha \phi (\lambda^{L+1} - \lambda^{L+2}) \right]$$

$$= 2 \left[ 1 + \alpha \phi ((1 - \lambda)(\alpha \phi + 1) - \lambda^{L+1} + \lambda^{L+2}) - \lambda^{L+1} \right].$$

Figure 3 shows $Amp_{\alpha}^1 B_{0}^{\triangle}$ for $\alpha \in [0, 1]$ and $\lambda \in (-1, 1)$, when $L = 1$. This indicates that the inventory on stock site $B_{0}$ is actually sensitive to the belief variable meaning that a smoothing effect should be expected if $\alpha$ is decreased for a given $\lambda$ value. As qualitatively observed, effectiveness of $\alpha$ is low for negative values of autocorrelation. Notice that the same kind of phenomenon is observed in Figure 2: the more $\alpha$ decreases, the less the amplification improves.

We may conclude that a fading action, implemented via the manager’s belief variable, may be a sound strategy for reduction of the bullwhip effect, both on the production and inventory sides, but only for specific values of autocorrelation. In particular, this kind of management should be surely applied for low positive values of $\lambda$.

4. Conclusions

In a previous paper we proposed that flexibility aids in reduction of the bullwhip-effect in a multi-echelon, single-item, supply chain model. In this chapter we have found a flexibility condition that guarantees the control of the bullwhip-effect in the supply chain (expression
Fig. 3. Marginal inventory change amplification on stock site $B_0$, when $\alpha \in [0, 1]$.

This is an interesting result because it asks the manager for an ordering strategy that synchronizes the flexibility among stages in the chain. However, such condition being difficult to fulfill when an AR(1) demand process is considered, a different strategy has been explored. Control of a learning variable, representing the manager’s belief on demand forecasting, has been proposed here as an alternative strategy to regulate the bullwhip-effect. We have seen that, although this strategy does not necessarily assure fulfillment of the MAC condition, it may be an effective way to smooth production and inventory fluctuation. Our results indicate that, under the model assumptions, the pull ordering method is highly robust, in the sense of reduction of the amplification effect. Thus, the fading strategy suggested invites the supply chain manager to improve synchronization among stages in the supply chain, becoming closer to the pull method. Nevertheless, a manager is not necessarily enforced to abandon the push strategy in order to obtain acceptable amplification levels, but she/he should make a careful analysis assessing the consequences of his beliefs about the demand and estimates behavior.

Results presented in this chapter open to new ideas about the way that different fading strategies impact the bullwhip-effect behavior. Even if an early study was proposed by Pereira et al. (2009), the focus was rather mathematical and no framework was suggested as a specific analytical grid. In consequence, future research concerns the hypothesis that decision makers evidence limited rationality bias when facing an ordering method. Although this idea has been already analyzed (Oliva and Gonçalves, 2005), we think that the availability heuristic proposed by Tversky and Kahneman (1974), in our case concerning the overreaction to the downstream information, could be successfully explored using our supply chain model.

5. Acknowledgment

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6. References


Challenges faced by supply chains appear to be growing exponentially under the demands of increasingly complex business environments confronting the decision makers. The world we live in now operates under interconnected economies that put extra pressure on supply chains to fulfil ever-demanding customer preferences. Relative attractiveness of manufacturing as well as consumption locations changes very rapidly, which in consequence alters the economies of large scale production. Coupled with the recent economic swings, supply chains in every country are obliged to survive with substantially squeezed margins. In this book, we tried to compile a selection of papers focusing on a wide range of problems in the supply chain domain. Each chapter offers important insights into understanding these problems as well as approaches to attaining effective solutions.

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