

Stability Analysis of a Simple Active Biped Robot with a Torso on Level Ground Based on Passive Walking Mechanisms

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1. Introduction

This study focuses on the passive dynamic walking to enable a biped robot on level ground to walk efficiently with simple mechanisms. To build an efficient bipedal robot, utilizing the dynamical property of the robot system is a useful approach. McGeer studied passive-dynamic walking, and showed a biped robot without actuators and controllers can walk stably down a shallow slope in simulations and experiments (McGeer, 1990). The simplest passive walker, which has only two mass-less links with hip mass, still can walk (Garcia et al., 1998). Collins et al. built the three-dimensional passive-dynamic walker which has knees and arms (Collins et al., 2001).

Passive-dynamic walking is useful to study efficient level-ground walking robots, (e.g. Collins et al. 2005), but passive walking has some limitations. The walking motion of the passive walker depends on the slope angle. The walking speed decreases with the slope angle. On the other hand, increasing the slope angle brings about a period doubling bifurcation leading to chaotic gaits and there are only unstable gaits in high speed region (Garcia et al., 1998). Biped robots based on the passive walking mechanisms were proposed (e.g. Goswami et al., 1997; Asano et al., 2004; Asano et al., 2005; Spong & Bullo, 2005), but the robots are mainly controlled by ankle torque, which has drawback from the viewpoints of Zero Moment Point (ZMP) condition, discussed in (Asano et al., 2005). The limitations of the passive-dynamic walkers and the ankle-torque controlled walkers should be addressed.

We propose the level-ground walking by using a torso and swing-leg control. Although using a torso for energy supply replacing potential energy, used in the case of the passive-dynamic walking, was proposed by McGeer (McGeer, 1988), there are few studies to use a torso explicitly for energy supply. Wisse et al. showed that the swing-leg motion is important to avoid falling forward (Wisse et al. 2005). From this viewpoint, we introduce a swing-leg control depending on the stance-leg motion. To modify the pendulum motion of the swing-leg by using the swing-leg control, the impact condition between the swing-leg and the ground will be satisfied before falling down.

In this paper, we study a knee-less biped robot with a torso on level ground. This paper presents a stability analysis of the biped robot to demonstrate the effectiveness of the swing-leg control. We use a Poincaré map to analyze walking motions which is a common tool in the study of the passive walking (McGeer, 1990; Goswami et al., 1996; Garcia et al., 1998;

Garcia et al., 2000). Walking analysis is as follows. First, using Newton-Raphson method, we search a periodic gait. Even though we know the existence of the periodic gait, we should know whether it is stable or unstable. Then we numerically approximate the Jacobian matrix of the Poincaré map of the periodic gait. If the Jacobian has all of its eigenvalues inside the unit circle, the gait is stable. Furthermore we search a set of initial conditions leading to stable walking. The stability analysis shows that the swing-leg control enables the robot to walk stably over a wide range of speed.

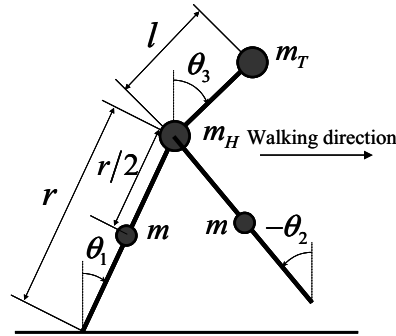


Figure 1. Biped knee-less walking robot

2. Biped walking model

2.1 Biped walking robot and model assumptions

The level-ground walking based on passive walk proposed in this paper needs a torso. In this paper, a simple biped robot with a torso shown in Fig. 1, is considered. This walking model is adding compass-like walking model (Goswami et al., 1996) to a torso, and has been studied in (Grizzle et al., 2001). The robot is composed of a torso, hips, and two legs. All masses are lumped. Dynamic variable values are measured from ground normal. Two torques u_1 and u_2 , between the torso and the stance-leg, and between the torso and the swing-leg are applied, respectively. The motion of the robot is constrained to the sagittal plane. The scuffing problem of the swing-leg, which is inevitable in the case of a biped knee-less robot of which motion is constrained to the sagittal plane, is neglected during the swing phase, see in detail (McGeer, 1990; Grizzle et al., 2001).

2.2 Swing phase model

During the swing phase, the stance-leg acts as a pivot joint. By the method of Lagrange, the swing phase model is written as (Grizzle et al., 2001)

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + G(\boldsymbol{\theta}) = B\mathbf{u} \quad (1)$$

where $\boldsymbol{\theta} = [\theta_1 \ \theta_2 \ \theta_3]^T$, $\mathbf{u} = [u_1 \ u_2]^T$. The details of the matrices are

$$M(\boldsymbol{\theta}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & 0 \\ M_{13} & 0 & M_{33} \end{bmatrix}$$

$$M_{11} = \left(\frac{5}{4}m + m_H + m_T\right)r^2, \quad M_{12} = -\frac{1}{2}mr^2 \cos(\theta_1 - \theta_2)$$

$$M_{13} = m_T r l \cos(\theta_1 - \theta_3), \quad M_{22} = \frac{1}{4}mr^2, \quad M_{33} = m_T l^2.$$

$$C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} 0 & -\dot{\theta}_2 C_{12} & \dot{\theta}_3 C_{13} \\ \dot{\theta}_1 C_{12} & 0 & 0 \\ -\dot{\theta}_1 C_{13} & 0 & 0 \end{bmatrix}.$$

$$C_{12} = \frac{1}{2}mr^2 \sin(\theta_1 - \theta_2), \quad C_{13} = m_T r l \sin(\theta_1 - \theta_3).$$

$$G(\boldsymbol{\theta}) = \begin{bmatrix} -\frac{1}{2}g(3m + 2m_H + 2m_T)r \sin \theta_1 \\ \frac{1}{2}gmr \sin \theta_2 \\ -gm_T l \sin \theta_3 \end{bmatrix}.$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}.$$

2.3 Impact phase model

An impact occurs when the swing-leg touches the ground, which is called heel-strike. The condition of the impact, heel-strike, is given by

$$\theta_1 + \theta_2 = 0 \quad (2)$$

The impact is assumed to be inelastic and without slipping, and the stance-leg lifts from the ground without interaction (Hurmuzlu & Marghitu, 1994; Grizzle et al., 2001), and the actuators cannot generate impulses. Angular momentum is conserved at the impact for the whole robot about the new stance-leg contact point, for the torso about the hip, and for the new swing-leg about the hip. The conservation law of the angular momentum leads to the following compact equation between the pre- and post-impact angular velocities (Goswami et al. 1996):

$$Q^+(\boldsymbol{\theta}^+) \dot{\boldsymbol{\theta}}^+ = Q^-(\boldsymbol{\theta}^-) \dot{\boldsymbol{\theta}}^- \quad (3)$$

The superscripts “-” and “+” respectively denote pre- and post-impact. During the impact phase, the configuration remains unchanged. The pre- and post-impact angles are identified with

$$\boldsymbol{\theta}^+ = J\boldsymbol{\theta}^- \quad (4)$$

where

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (3) and (4) we have

$$\dot{\boldsymbol{\theta}}^+ = H(\boldsymbol{\theta}^-) \dot{\boldsymbol{\theta}}^- \quad (5)$$

The detail of the matrix is

$$H(\theta^-) = \frac{1}{H_d} \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{21} & H_{22} & 0 \\ H_{31}/l & H_{32}/l & H_d \end{bmatrix}$$

$$H_d = -3m + 2m \cos(2\theta_1^- - 2\theta_2^-) - 4m_H - 2m_T + 2m_T \cos(2\theta_2^- - 2\theta_3^-)$$

$$H_{11} = (-2m - 4m_H - 2m_T) \cos(\theta_1^- - \theta_2^-) + 2m_T \cos(\theta_1^- + \theta_2^- - 2\theta_3^-), \quad H_{12} = m$$

$$H_{21} = m - (4m + 4m_H + 2m_T) \cos(2\theta_1^- - 2\theta_2^-) + 2m_T \cos(2\theta_1^- - 2\theta_3^-), \quad H_{22} = 2m \cos(\theta_1^- - \theta_2^-)$$

$$H_{31} = (-m - m_H - m_T) 2r \cos(\theta_1^- - \theta_3^-) + (m + m_H + m_T) 2r \cos(\theta_1^- - 2\theta_2^- + \theta_3^-) + mr \cos(3\theta_1^- - 2\theta_2^- - \theta_3^-)$$

$$H_{32} = -mr \cos(\theta_2^- - \theta_3^-)$$

Equation (5) can be also obtained by another method (Grizzle et al., 2001).

3. Simple control scheme

3.1 Torso and Swing-leg control

To hold the torso around a desired angle, the simple PD control scheme given by

$$T_T = -k_T^P (\theta_3 - \theta_3^d) - k_T^D \dot{\theta}_3 \quad (6)$$

is considered (McGeer, 1988). θ_3^d is the desired torso angle, k_T^P and k_T^D are control gain. k_T^P and k_T^D are determined as follows (McGeer, 1988). If the legs are firmly planted on the ground, the linearized equation of the torso motion about $\theta_3 = 0$ with the PD control becomes

$$m_T l^2 \ddot{\theta}_3 + k_T^D \dot{\theta}_3 + (k_T^P - m_T g l) \theta_3 = 0 \quad (7)$$

The frequency of the torso is

$$\hat{\omega}_T = \sqrt{\frac{k_T^P - m_T g l}{m_T l^2}} \quad (8)$$

The damping ratio is

$$\hat{\zeta}_T = \frac{k_T^D}{2m_T l^2 \hat{\omega}_T} \quad (9)$$

On the other hand, if the stance-leg is firmly planted on the ground, the linearized equation of the swing-leg motion about $\theta_2 = 0$ becomes

$$mr \ddot{\theta}_2 + 2mg \theta_2 = 0 \quad (10)$$

The natural frequency of the swing-leg is

$$\omega_s = \sqrt{\frac{2g}{r}} \quad (11)$$

In this paper, we determine the torso control parameters, k_T^p and k_T^d , to satisfy

$$\hat{\omega}_T = 3\omega_s \quad (12)$$

$$\hat{\zeta}_T = 0.7 \quad (13)$$

In order to satisfy the transition condition (Eq. (2)) before the robot falls down, we apply the simple control law given by

$$T_s = -k_s^p(\theta_2 - (-\theta_1)) \quad (14)$$

In the control law, the desired angle of the swing-leg depends on the stance-leg angle. $-\theta_1$ is the desired angle of the swing-leg which is opposed to the spring model between the legs (Kuo, 2002; Wisse et al., 2004). The swing-leg control will result in modifying the natural motion of the swing-leg. If the stance-leg angle is constant, the linearized equation of the swing-leg motion about $\theta_2 = 0$ with the swing-leg control becomes

$$mr^2\ddot{\theta}_2 + (2mgr + 4k_s^p)\theta_2 = 0 \quad (15)$$

The frequency is

$$\hat{\omega}_s = \sqrt{\frac{4k_s^p + 2mgr}{mr^2}} \quad (16)$$

k_s^p is determined by

$$\hat{\omega}_s = K\omega_s \quad (17)$$

K is a new swing-leg control parameter which shows the ratio between the frequencies of the swing-leg with the swing-leg control and without the swing-leg control. Then, we have

$$k_s^p = \frac{1}{4}mr^2K^2\omega_s^2 - \frac{1}{2}mgr \quad (18)$$

3.2 Control inputs for biped robot

From the torso control and the swing-leg control mentioned in the previous section, the control inputs are given by

$$u_1 = T_T + T_s \quad (19)$$

$$u_2 = -T_s \quad (20)$$

4. Stability analysis

4.1 Poincaré map

Poincaré map is commonly used to study the passive walking and quite useful to analysis biped locomotion. We follow the procedure to analysis the active biped robot on level ground.

The state just after the impact, heel-strike, is usually used as the Poincaré section. The Poincaré section removes one state. The Poincaré map is denoted as

$$P({}^i \mathbf{q}^+) = {}^{i+1} \mathbf{q}^+ \quad (21)$$

where the superscript “ i ” denotes step number, and “ $+$ ” denotes post-impact between the swing-leg and the ground. Then ${}^i \mathbf{q}^+$ is the state just after the heel-strike of step i . A fixed point of the Poincaré map, \mathbf{q}^* , satisfies

$$P(\mathbf{q}^*) = \mathbf{q}^* \quad (22)$$

The fixed point represents a periodic (period-one) gait.

4.2 Periodic gaits

We can obtain a periodic gait to find the fixed point which is not only stable but also unstable, as follows (Garcia, 1999). Equation (22) corresponds to

$$\mathbf{g}({}^i \mathbf{q}) = 0 \quad (23)$$

where

$$\mathbf{g}({}^i \mathbf{q}) = P({}^i \mathbf{q}) - {}^i \mathbf{q} \quad (24)$$

To search for \mathbf{q}^* such that $\mathbf{g}(\mathbf{q}^*) = 0$, Newton-Raphson method is used.

Given an initial guess at a fixed point, \mathbf{q}_0 , the Jacobian of \mathbf{g} is found numerically to perturb one state, i th element of \mathbf{q} by ε and evaluate \mathbf{g}_ε^i . An estimate of the i th column of Jacobian is given by

$$\frac{\mathbf{g}_\varepsilon^i - \mathbf{g}(\mathbf{q}_0)}{\varepsilon} \quad (25)$$

Repeating this procedure find a numerical approximation to the Jacobian of \mathbf{g} .

Assuming that $\mathbf{g}(\mathbf{q}^*) = 0$, Newton-Raphson method provides the next approximation, \mathbf{q}_1 , which is given by

$$\mathbf{q}_1 = \mathbf{q}_0 - \frac{\partial \mathbf{g}}{\partial \mathbf{q}}^{-1} \mathbf{g}(\mathbf{q}_0) \quad (26)$$

If a periodic gait exists and initial guess is sufficiently close, this search will converge to the fixed point \mathbf{q}^* .

4.3 Stability of the gait

By adding a small perturbation $\hat{\mathbf{q}}$ from the fixed point \mathbf{q}^* , Poincaré map P can be expressed as

$$P(\mathbf{q}^* + \hat{\mathbf{q}}) = P(\mathbf{q}^*) + J(\mathbf{q}^*)\hat{\mathbf{q}} \quad (27)$$

where J is the Jacobian of the Poincaré map.

$$J = \frac{\partial P(\mathbf{q})}{\partial \mathbf{q}} \quad (28)$$

J is determined approximately by performing the procedure described in Section 4.2. Note that instead of evaluating J , we can use the relationship Eq. (24). From Eq. (24), we obtain (Garcia et al., 1998)

$$\frac{\partial P(\mathbf{q})}{\partial \mathbf{q}} = \frac{\partial g(\mathbf{q})}{\partial \mathbf{q}} + \mathbf{I} \quad (29)$$

where \mathbf{I} is the identity matrix.

Since $P(\mathbf{q}^*) = \mathbf{q}^*$, we can rewrite Eq. (27) as

$$P(\mathbf{q}^* + \hat{\mathbf{q}}) - \mathbf{q}^* = J(\mathbf{q}^*)\hat{\mathbf{q}} \quad (30)$$

Then we obtain

$${}^{i+1}\hat{\mathbf{q}} = J(\mathbf{q}^*)^i \hat{\mathbf{q}} \quad (31)$$

If all of its eigenvalues of the Jacobian are inside the unit circle, all sufficiently small perturbations $\hat{\mathbf{q}}$ will converge to 0, and the gait is asymptotically stable. If any eigenvalues of the Jacobian are outside the unit circle, the gait is unstable.

5. Simulation results

5.1 Simulation method

Values of the system parameters for the biped robot (Fig.1) are shown in Table 1.

To analysis the walking motion, we use numerical simulations. In swing phase, the angular accelerations are solved as functions of the angles and the angular velocities to invert M in Eq. (1).

$$\ddot{\boldsymbol{\theta}} = M^{-1}(\boldsymbol{\theta}) \left(-C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} - G(\boldsymbol{\theta}) + B\mathbf{u} \right) \quad (32)$$

The simulations were run by using MATLAB®/SIMULINK®. We use ODE45 in MATLAB®/SIMULINK®, and specify a scalar relative error tolerance of 1e-8 and an absolute error tolerance of 1e-8. The heel strike of the biped robot was detected by zero-crossing detection in SIMULINK®. At the heel strike, the post-impact angular velocities and angles are calculated by Eq. (4) and (5).

Parameter	Unit	Value	Parameter	Unit	Value
m	kg	5	r	m	1
m_H	kg	10	l	m	0.5
m_r	kg	10	g	m/s ²	9.80665

Table 1. Values of the system parameters

In addition to the search of periodic gaits by using the Newton-Raphson method as mentioned in Section 4.2, by increasing the torso angle from 0.01rad in steps of 0.001rad, we find period-doubling bifurcations and chaotic gaits, which are demonstrated with the simplest model (Garcia et al., 1998), the compass-like model (Goswami et al., 1996) kneed models (Garcia et al., 2000), and level-ground walking (Howell & Baillieul, 1998)

5.2 Stability results

First, we search a stable walking while the swing-leg is left free, that is $K = 1.0$, where $\theta_3^d = 0.01[\text{rad}]$. In our search, a periodic gait could not be found. Then we introduce the swing-leg control.

Figure 2 shows the evolution of the walking speed as a function of the desired torso angle where the swing-leg control parameter $K = 1.4, 1.7, 1.95$ and 2.0 . Figure 2 demonstrates the walking speed increases with the desired torso angle and the maximum walking speed of the stable gait increases with the swing-leg control parameter. But when we increase further the swing-leg control parameter K , period-doubling bifurcations occur and we didn't find stable gaits. Figure 2 shows that the maximum walking speed of stable gaits doesn't necessarily increase with the swing-leg control parameter K . Figure 3 shows the evolution of the absolute eigengvalues of the Jacobian J as a function of the desired torso angle where $K = 1.95$ and 2.0 .

5.3 Stable region

Even if a stable periodic walking is achieved, we want to know when the biped robot keeps walking, and when it falls down from a disturbance and incorrect launch. To answer it, yet partially, a set of initial conditions leading to stable walking is searched for. The more initial conditions result in the fixed point without falling down, the more stable the walking is. To find the initial conditions, we perturb one state of \mathbf{q}^* . For example, we perturb θ_1^+ from the fixed point and the other states $(\theta_3^+, \dot{\theta}_1^+, \dot{\theta}_2^+, \dot{\theta}_3^+)$ remain unchanged at the equilibrium position. Figure 4 shows the initial conditions of the perturbed state leading to continuous walking where $\theta_3^d = 0.2$.

From Fig. 4, increasing the swing-leg control parameter K results in the increase of the range of the initial conditions leading to stable walking. The simulation results show that the swing-leg control enlarges the stable region.

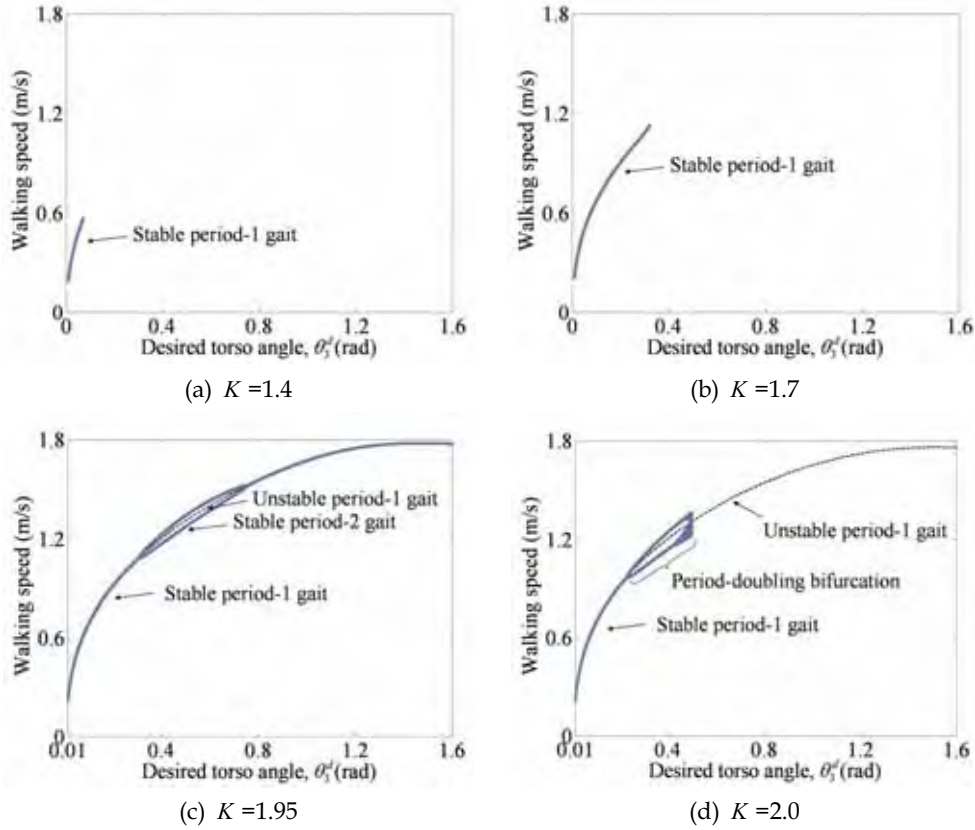


Figure 2. Walking speed versus desired torso angle

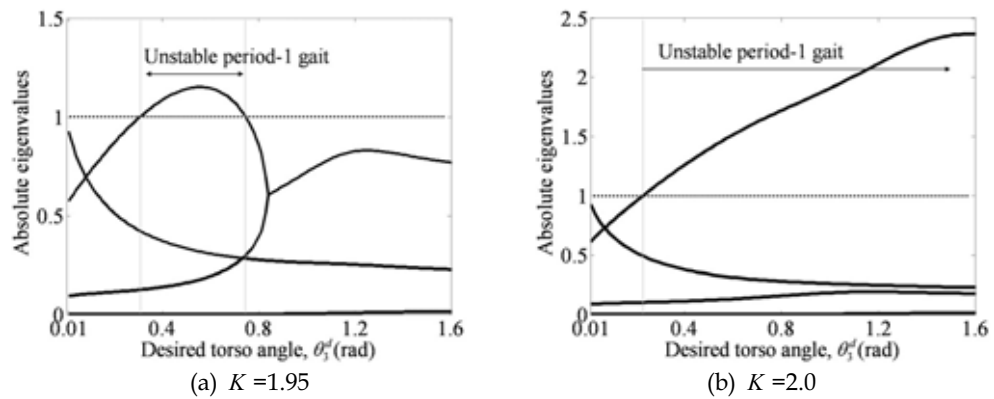


Figure 3. Absolute eigenvalues versus desired torso angle

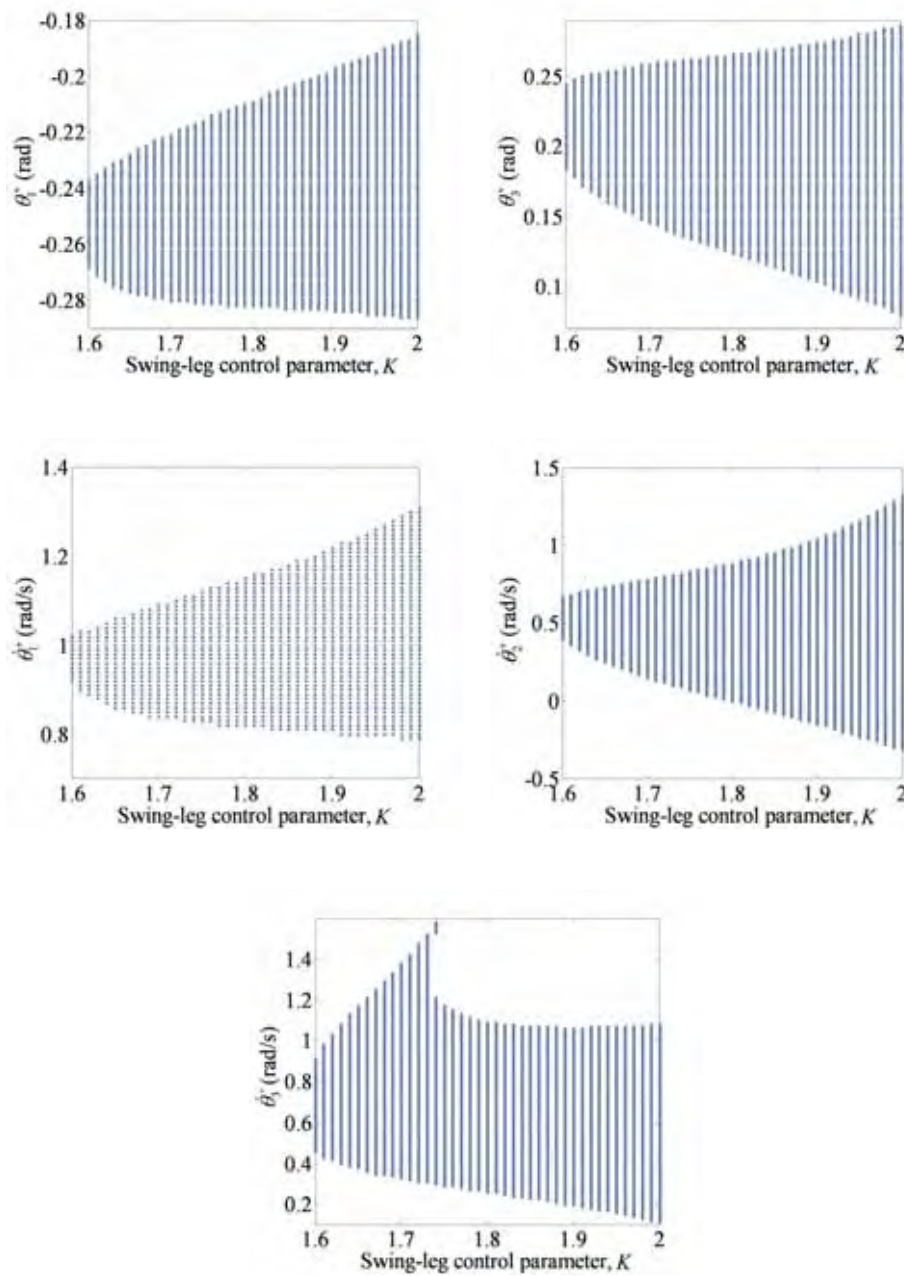


Figure 4. Stable range as a function of the swing-leg control parameter K where $\theta_3^d = 0.2$

6. Conclusion

We study a simple active biped robot on level ground to walk over a wide range of speed. The use of the torso for energy supply and the swing-leg control for stable walking are introduced. Numerical simulations show that the swing-leg control enables the biped robot on level ground to walk stably over a wide range of speed, and enlarges the stable region.

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8. References

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