Interaction of Electromagnetic Radiation with Substance

Andrey N. Volobuev
Samara State University
Russia

1. Introduction

1.1 Distribution of Electromagnetic Field Momentum in dielectrics in stipulation of self-induced transparency

Interaction of quantums of electromagnetic radiation with substance can be investigated both from a wave position, and from a quantum position. From a wave position under action of an electromagnetic wave there are compelled fluctuations of an electronic orbit and nucleus of atoms. The energy of electromagnetic radiation going on oscillation of nucleus passes in heat. Energy of fluctuations of an electronic orbit causes repeated electromagnetic radiation with energy, smaller, than initial radiation.

From a quantum position character of interaction is more various. Interaction without absorption of quantums is possible: resonant absorption, coherent dispersion. The part of quantums is completely absorbed. Quantums can be absorbed without occurrence secondary electrons. Thus all energy of quantums is transferred fonons - to mechanical waves in a crystal lattice, and the impulse is transferred all crystal lattice of substance. At absorption of quantums can arise secondary electrons, for example, at an internal photoeffect. Absorption of quantums with radiation of secondary quantums of smaller energy and frequency is possible, for example, at effect of Compton or at combinational dispersion.

All these processes define as formation of impulses of electromagnetic radiation in substance, and absorption of radiation by substance.

1.2 Coordination of the electromagnetic impulse with the substance

Firstly, consider the one-dimensional task the electric part of electromagnetic field momentum with the dielectric substance, which posses a certain numerical concentration $n$ of centrosymmetrical atoms - oscillators. For the certainty of the analysis we suggest the atom to be one-electronic. It is also agreed, that no micro current or free charge are present in the medium. The peculiarities of interaction between magnetic aspect of momentum and the atoms will be considered later.

We accept that there takes place the interaction of quantum of electromagnetic radiation with nuclear electrons, thus quantum are absorbed by the electrons. By gaining the energy of quantum the electrons shift to the advanced power levels. Further, by means of resonate shift of electrons back, appears the quantum radiation forward. The considered medium lacks non-radiating shift of electrons, i.d. the power of quantum is not transfered to the atom.
Thus, the absorption of electromagnetic radiation in the case of its power dissipation in the substance, owing to SIT, is disregarded. There appears the atomic sypraradiation of quantum. Thus, the forefront of momentum passes the power on to the atomic electrons of the medium, forming its back front.

The probabilities of quantum's absorption and radiation by the electrons in the unity of time, with a large quantity of quantum in the impulse, according to Einstein, can be referred to as the approximately identical [6]. For the separate interaction of the with the electron this very probability is the same and is proportional to the cube of the fine-structure constant $\sim (1/137)^3$ [7]. Consider a random quantity – the number of interactions of quantum with atomic electrons in the momentum. In accordance with the Poisson law of distribution, the probability of that will not be swallowed up any quantum atomic's electrons (will not take place any interaction), at rather low probability of separate interaction, is equal an exponent from the mathematical expectation of a random variable – an average quantity of interactions $\lambda$ of quantaums and electrons in impulse, taken with the minus $p = \exp(-\lambda)$. Therefore, as it will be explained further, it is possible that the intensity of non-absorbed power of impulse by the atomic electrons of the medium in it forefront is determined by the exponential Bouguer law [3] (in German tradition - Beer law)

$$I = I_0 \exp(-\alpha l),$$

(1.1)

where $\alpha$ - index of electromagnetic wave and substance interaction, $l$ - length of interaction layer, $I_0$ - intensity of incident wave. Thus, the intensity of atomic electron's power recoil into impulse on its back front could be described with the help of the Bouguer law with the negative index of absorption [8].

The index of interaction is $\alpha = \sigma n$, where $\alpha$ – effective section of atom-oscillator interaction with the wave. Hence,

$$\alpha l = \sigma nl = n V_{\text{eff}} = n V \frac{V_{\text{eff}}}{V} = M \frac{V_{\text{eff}}}{V} = MN,$$

(1.2)

where $V_{\text{eff}}$ - the effective volume of interaction. In defining (1.2) the right part of the formula is multiplied and divided by the geometric volume $V$, in which there is $M$ of particles interacting with the radiation. The ratio $\frac{V_{\text{eff}}}{V} = N$. The ratio of effective volume of interaction to the geometric volume characterizes the medium possibility of electromagnetic radiation's interaction with the atom. Hence, by exponential function in the Bouguer law (1.1) the mathematical expectation of random variable is supposed, which subdues to the Poisson law distribution – average variable of atoms interacting with the electromagnetic radiation in the area of impulse influence $\lambda = NM$.

Taking into account that the wave intensity is $I \sim \left(\frac{E^2}{H^2}\right)$ we shall have

$$\left(\frac{|E|}{|H|}\right) = \left(\frac{|E_0|}{|H_0|}\right) \exp\left(-\frac{\alpha l}{2}\right),$$

(1.3)
where \(|E_0|, |H_0|\) – the amplitudes of electric and magnetic fields' strength of the impulse on longitudinal coordinate \(X = 0\).

In the formula (1.3) and further the upper variables in parentheses are referred to electric field, and lower – to the magnetic field of impulse.

By the ratio (1.2) it is possible to find

\[
N = -\frac{2}{M} \ln \left| \frac{E}{E_0} \right| = -\frac{2}{M} \ln \left| \frac{H}{H_0} \right|.
\]  

(1.4)

The formula (1.4) demands some further consideration. If \(E<E_0\) that reflects the process of wave absorption by atomic electrons \(N>0\) and classical consideration of electromagnetic wave interaction with the atom is quite admissible. The case when \(E>E_0\) reflects the process of wave over-radiation. Thus, \(N<0\) and variable \(N\) can not be considered as the probability of electromagnetic wave interaction with the atom. In this case we speak about the quantum-mechanical character of the process of interaction between the quantum and the bi-level power system of the atom, provided that the power transition's radiation is reversed. Variable \(N\) in this case possess the notion of united average of filling by atom \((-1<N<1\)). Due to the use of the average of filling to raise the atom and bend of its magnetic moment in the magnetic field of the impulse, the existence of bi-level quantum system by magnetic quantum numbers. Thus, the variable \(N\) provides with the measure of inversion of the system of atom-radiators by the raised atoms [2] as well as the measure of inversion of the magnetic moment of the atom's system by magnetic quantum numbers. If \(N=-1\) all the atoms occur in the basic condition [3].

Fig. 1. Dependence of volumetric density of energy of electromagnetic radiation impulse \(w\) (curve 1) and average on atoms of number of filling \(N\) (curve 2) from time; 3 and 4 - points of an excess of function \(w(t)\)

We consider the dependence of the average of filling on the time \(N(t)\). If to accept the proportion of polarization of separate bi-level atom to the intensity of electric field in the impulse, then, in accordance with the Maxwell-Bloch equations, the average by atoms of considered volume, the filling number is proportional to the volumetric density of electromagnetic wave power \(N\sim w\) [3]. However such a monotonous dependence between these variables can not remain on the whole extent of the impulse. Firstly, by the high
volumetric density of impulse power \( w \), typical of SIT, when the central part of impulse power is higher than any variable \( w \), there exists energetic saturation of the medium. The average filling number thus \( N=1 \), all the atoms are raised, fig. 1 (curve 1 - the dependence \( w \) of time, thicker curve 2 – the considered dependence \( N \) of time). The violation of proportion \( N \sim w \) in the central part of impulse is the basic drawback of frequently used system of Maxwell-Bloch equations for the SIT description.

Secondly, the period of variable \( N \) relaxation is not less than \( 1 \text{ ns} \) [2] that is why the dependence \( N(t) \) can not repeat high-frequently oscillations on both fronts of the impulse. The dependence \( N \sim w \) could characterize the proportion of average filling number and envelope \( w \) (curve 1) in the impulse. However, in two points of the fold (3 and 4 fig. 1) on the sites of increase and decrease if the envelope \( w \) the variable \( \frac{\partial^2 w}{\partial t^2} = 0 \) hence, also \( \frac{\partial^2 N}{\partial t^2} = 0 \). Besides, the dependence \( N(t) \) has the symmetrical character as at the SIT impulse becomes the conservative system (there is no reverse dispersion and dissipation of power) [2]. Therefore, it could be thoroughly concerned that on the whole extent of impulse, except the points of curve’s \( N(t) \) fold, the condition remains

\[
\frac{\partial^2 N}{\partial t^2} = 0,
\]

while the dependence \( N(t) \) has the character as shown on the fig. 1, curve 2. it could be also highlighted the high generality of formula (1.5), which is possible for any piecewise linear function \( N(t) \). Thus, the points of function break are excluded, as the derivates undergo the break.

1.3 Non-linear Schrödinger equation

One-dimensional wave equation for electric and magnetic aspects of electromagnetic field for the considered problem is [2]

\[
\frac{\partial^2}{\partial X^2} (E) - \frac{1}{c^2} \left( \frac{\mu}{\varepsilon} \right) \frac{\partial^2}{\partial t^2} (E) = \frac{1}{c^2} \left( \frac{\mu}{\varepsilon_0} \right) \frac{\partial^2}{\partial t^2} (P),
\]

where \( E \equiv E_y \) or \( E \equiv E_z \), \( H \equiv H_y \) or \( H \equiv H_z \), \( X \) and \( t \) – accordingly the coordinate alongside of which the impulse and the time are distributed, \( P \) – polarization of substance, \( J \) – its magnetization, \( \varepsilon_0 \) and \( \mu_0 \) – electrical and magnetic constant, \( \varepsilon \) – relative static permittivity of substance, \( \mu \) – relative magnetic permittivity, \( c = 1 / \sqrt{\varepsilon_0 \mu_0} \) – speed of light in vacuum.

We introduce the transformation of electric field intensity be formula

\[
\left( \begin{array}{c} E(X,t) \\ H(X,t) \end{array} \right) = \Phi(X,t) \exp(-i\omega t).
\]

The function \( \Phi(X, t) \) is less rapidly changing one in time then \( E(X,t) \) or \( H(X,t) \), \( \omega_0 \) – aspect of cyclic frequency of high-frequent oscillations of the field.

By substituting (1.7) and (1.6) we get

\[
\left[ \frac{\partial^2 \Phi}{\partial X^2} - \frac{1}{c^2} \left( \frac{\mu}{\varepsilon} \right) \frac{\partial^2 \Phi}{\partial t^2} - 2i \omega_0 \frac{\partial \Phi}{\partial t} - \omega_0^2 \Phi \right] \exp(-i\omega t) = \frac{1}{c^2} \left( \frac{\mu}{\varepsilon_0} \right) \frac{\partial^2}{\partial t^2} (P).
\]
We estimate the relative variable of first and second items in the parenthesis of the left side (1.8). for this purpose we would introduce the scales of variables time \( t \) and \( \Phi \)

\[ T = t / t', \quad \Phi_0 = \Phi / \Phi^* \]

where the asterisk designates dimensionless parameters. For the time scale the duration (period) of impulse \( T \) should be logically chosen. The scale \( \Phi_0 \) is chosen from a condition that dimensionless second derivative \( \frac{\partial^2 \Phi^*}{\partial t'^2} \) and the dimensionless function \( \Phi^* \) are in the same order. Hence, the the first item in round brackets (1.8) is \( \frac{\Phi_0 \partial^2 \Phi^*}{T^2 / \partial t'^2} \), and the last one \( \omega^2 \Phi_0 \Phi^* \). Instead of impulse \( T \) period we introduce cyclic frequency of impulse \( \omega = \frac{2\pi}{T} \). By comparing these items, it is realized, that \( \Phi_0 \omega^2 \frac{\partial^2 \Phi^*}{4\pi^2 / \partial t'^2} \ll \omega^2 \Phi_0 \Phi^* \) as the cyclic frequency of impulse is far less than infrequencies of field's oscillations, especially when \( \omega^2 \ll \omega^2 \).

Similarly, it can be presented that the second item in the round brackets (1.8) is far more that the first one.

Hence, by disregarding the small item in (1.8), we observe

\[
\left[ \frac{\partial^2 \Phi}{\partial X^2} - \frac{1}{c^2} \left( \mu \frac{\partial \Phi}{\partial t} + \omega^2 \Phi \right) \right] \exp(-i\omega_0 t) = \frac{1}{c^2} \left( \mu / \varepsilon_0 \varepsilon \right) \frac{\partial^2}{\partial t'^2} \left( P \right).
\]

(1.9)

By accepting vector of polarization \( P \) or magnetizing \( J \) to be directly proportional, accordingly, to the electric and magnetic fields strength, we could derive the wave equation from (1.6), which is possible to any form of the wave. However, there exists a physical mechanism, which restrict the wave form. This mechanism is connected with the way of over-radiating of electromagnetic impulse with the atomic electrons. This process is precisely considered further.

We consider the strength of electric and magnetic fields of impulse as

\[
\left( \begin{array}{c}
E(X,t) \\
H(X,t)
\end{array} \right) = \left( \begin{array}{c}
|E(X,t)| \\
|H(X,t)|
\end{array} \right) \exp[i(rX - \delta t)],
\]

(1.10)

where \( r \) and \( \delta \) - are constants, \( |E(X,t)| \) and \( |H(X,t)| \) are the modules of functions \( E(X,t) \) and \( H(X,t) \).

Formulas (1.4) and (1.5) reflect the offered physical model of electric and magnetic field of impulse interaction with atoms in SIT.

Hence, taking into account (1.4) and (1.5) there is

\[
\frac{\partial^2 \ln |E|}{\partial t^2} = \frac{\partial^2 \ln |H|}{\partial t^2} = 0.
\]

(1.11)

By transforming (1.11) we have

\[
\frac{\partial^2 |E|}{\partial t^2} = \left( \frac{\partial \ln |E|}{\partial t} \right)^2 |E|.
\]

(1.12)
The similar ratio can be also referred to the function $|H|$. These ratios should not be regarded as the equations to define the module of electric and magnetic aspect of impulse. It is the approximate expression of the second derivative $\frac{\partial^2 |E|}{\partial t^2}$ or $\frac{\partial^2 |H|}{\partial t^2}$ for the considered physical model and reflects several non-linear effects of interaction between electromagnetic radiation and substance. The approximate ratio (1.12) defines the connection of medium polarization $P$ with the strength of impulse electric field (similarly to the magnetization $J$ with the magnetic field strength), that would be considered further. The electromagnetic field impulse strengths should be estimated from the equation (1.6) taking into account the ratio (1.12).

In accordance with (1.10),
$$
\left[\begin{array}{c}
\frac{|E(X,t)|}{|H(X,t)|} = \left( \frac{E(X,t)}{H(X,t)} \right) \exp[-i(rX - \delta t)]
\end{array}\right],
$$

hence, from (1.12) we estimate equation for the electromagnetic field impulse

$$\frac{\partial^2 E}{\partial t^2} = -2i\delta \frac{\partial E}{\partial t} + \left[ \delta^2 + \left( \frac{\partial \ln \left| \frac{E}{E_0} \right|}{\partial t} \right)^2 \right] E . \quad (1.13)$$

The same ratio exists for the magnetic field also. Passing over to (1.13) to the function $\Phi(X,t)$ by formula (1.7) and by concerning $P = \varepsilon_0 \chi E$, where $\chi$ - relative dielectric permittivity of substance, we have

$$\frac{\partial^2 P}{\partial t^2} = \left[ -2i\delta\varepsilon_0 \chi \frac{\partial \Phi}{\partial t} - 2i\delta\varepsilon_0 \chi \omega_0 \Phi + \varepsilon_0 \chi \left[ \delta^2 + \left( \frac{\partial \ln \left| \frac{\Phi}{\Phi_0} \right|}{\partial t} \right)^2 \right] \right] \Phi \exp(-i\omega_0 t) , \quad (1.14)$$

For the variable $\frac{\partial^2 J}{\partial t^2}$ by using $J = \chi H$, where $\chi$ - relative magnetic permittivity of substance, we get the ratio, similar to (1.14), except that the right part lacks $\varepsilon_0$.

The variables
$$\left( \begin{array}{c}
\frac{E_0}{H_0}
\end{array} \right) = \Phi_0 \exp(-i\omega_0 t) .$$

By comparing (1.7) and (1.10) we state

$$\left( \begin{array}{c}
\frac{|E|}{|H|} = |\Phi|, \quad \left( \begin{array}{c}
\frac{|E_0|}{|H_0|}
\end{array} \right) = |\Phi_0| = \text{const} .
\end{array}\right)$$

By substituting (1.14) into (1.9)

$$2i(\omega_0 + \chi \delta) \frac{\partial \Phi}{\partial t} + c^2 \left( \frac{1}{\mu} \right) \frac{\partial^2 \Phi}{\partial \chi^2} + \left( \omega_0^2 + 2 \chi \delta \omega_0 - \chi \delta^2 \right) \Phi = \chi \left( \frac{\partial \ln \left| \frac{\Phi}{\Phi_0} \right|}{\partial t} \right)^2 \Phi . \quad (1.15)$$
In the equation (1.15) the variable $\chi$ is meaningful to dielectric permittivity for electric and magnetic permittivity for the magnetic aspects of electromagnetic field. The non-linear Schrödinger equation with complicated type of linearity is received. We introduce the signs: $\alpha = \omega_0 + \chi \delta$, $\gamma = \alpha_0 + 2 \chi \delta \omega_0 - \chi \delta^2 = \alpha^2 - \left( \frac{\varepsilon}{\mu} \right) \chi \delta^2$, where $\left( \frac{\varepsilon}{\mu} \right) = 1 + \chi$ - relative permittivities of the substance. Hence, the equation (1.15) will be

$$2i\alpha \frac{\partial \Phi}{\partial t} + c^2 \left( \frac{1}{\varepsilon} \right) \frac{\partial^2 \Phi}{\partial X^2} + \chi \Phi = \chi \left( \frac{\partial \ln \left| \Phi \right|}{\partial t} \right)^2 \Phi.$$  \hspace{1cm} (1.16)

We shall find the solution to the non-linear Schrödinger equation (1.16) as in [9]

$$\Phi = \Phi_0 f(kX - \omega t) \exp \left[ i(rX - \delta^* t) \right],$$  \hspace{1cm} (1.17)

where the type of the function $f(kX - \omega t)$ is still unknown. The variables $k$, $\omega$ and $\delta^*$ - constants. By marking $\zeta = kX - \omega t$, and substituting (1.17) in (1.16) and concerning $|\Phi| = \Phi_0 f(\zeta)$ we get

$$c^2 \left( \frac{1}{\varepsilon} \right) k^2 \frac{d^2 f}{d \zeta^2} + 2i \frac{df}{d \zeta} \left( kr \left( \frac{1}{\mu} \right) - \alpha \omega \right) = f \left( \gamma + 2\alpha \delta^* - r^2 c^2 \left( \frac{1}{\varepsilon} \right) \right) = \chi \omega^2 \left( \frac{d \ln f}{d \zeta} \right)^2 f.$$  \hspace{1cm} (1.18)

If to permit that $kr c^2 = \alpha \left( \frac{\mu}{\varepsilon} \right) \omega$ as there should not be any imaginary items in (1.18), this equation is transformed to

$$c^2 \left( \frac{1}{\varepsilon} \right) k^2 \frac{d^2 f}{d \zeta^2} + f \left( \gamma + 2\alpha \delta^* - r^2 c^2 \left( \frac{1}{\varepsilon} \right) \right) = \chi \omega^2 \left( \frac{d \ln f}{d \zeta} \right)^2 f.$$  \hspace{1cm} (1.19)

We consider the solution of the equation (1.19) by

$$f = C_1 \exp \left[ \frac{C_2 \zeta^2}{4} \right],$$  \hspace{1cm} (1.20)

where $C_1$ and $C_2$ - constants. By substituting (1.20) into (1.19) we get that the constant $C_1$ could be the arbitrary variable, $\chi \left( \frac{\mu}{\varepsilon} \right) \omega^2 = k^2 c^2$.

The constant $C_2$ could not depend upon the parameters of equation. It is accepted that $C_2 = -1$. Then the frequency and the wave number in (1.17), accordingly, are

$$\delta^* = \frac{c^2 \left( \frac{1}{\varepsilon} \right) (r^2 + k^2 / 2) - \gamma}{2\alpha}; \hspace{1cm} r = \frac{\alpha \left( \frac{\mu}{\varepsilon} \right) \omega}{kc^2}. \hspace{1cm} (1.21)$$
The formulas (1.21) associate the frequency and the wave number of oscillations of function \( \Phi(X,t) \) with the parameters of substance and electromagnetic field impulse. The most simple ratios between the parameters are gained, when \( \delta = \omega_0 \). In this case 

\[
\alpha = \left( \frac{\epsilon}{\mu} \right) \delta, \quad \gamma = \left( \frac{\mu}{\epsilon} \right) \delta^2.
\]

From the equations in (1.21), and concerning \( \chi^{\frac{\mu}{\epsilon}} \omega_0^2 = k^2 c^2 \) there is

\[
r = \frac{\alpha \left( \frac{\mu}{\epsilon} \right) \omega}{\chi^2 \omega} = \frac{\alpha k}{\chi \omega} = \frac{\epsilon \delta k}{\epsilon \omega}, \quad \delta = \frac{\alpha}{2 \chi} + \frac{\chi^2 \omega_0^2}{\alpha} - \frac{\gamma}{2 \alpha} = \frac{2 \left( \frac{\mu}{\epsilon} \right) \delta^2 + \chi^2 \omega_0^2}{4 \epsilon \left( \frac{\mu}{\epsilon} \right) \delta}.
\]

(1.22)

By concerning that \( 2 \left( \frac{\mu}{\epsilon} \right) \delta^2 >> \chi^2 \omega_0^2 \), we have \( \delta' = \frac{\delta}{2 \chi} \). This inequality is true, as for the rarefied gas \( n < 10^{18} \text{ atoms/cm}^3 \) \( \left( \frac{\epsilon}{\mu} \right) >> \chi \) and the frequency of wave filling of impulse \( \delta \) is far more than frequency of impulse envelope \( \omega \).

Taking into account (1.10), (1.20) and the \( \left( \frac{|E|}{|H|} \right) = |\Phi| \), we can find the laws of electromagnetic field strengths shifting by

\[
\left( \frac{E}{H} \right) = \left( \frac{E_0}{H_0} \right) \exp \left[ \frac{-(kX - \omega t)^2}{4} \right] \exp \left[ i(rX - \delta t) \right].
\]

(1.23)

It should be stressed, that though, the ratios for the electric aspect of impulse in [1] and (1.23) are similar to each other and feature the same phases of oscillations, that is possible on some distance from the over-radiating atom, the non-linear Schrödinger equations are differ in type of non-linearity. The reason of this lies in the fact that in [1] the impulse was considered with regard to low intensity, the one that does not lead to the energetic saturation of medium, in which it is disseminated.

For the estimation, like in [1] we have \( k = \frac{\omega}{c} = 2,1 \cdot 10^4 \text{ m}^{-1} \), \( \delta = \frac{\gamma}{c} = 2,1 \cdot 10^5 \text{ m}^{-1} \), \( \omega = 6,28 \cdot 10^{12} \text{ s}^{-1} \), \( \delta = 6,28 \cdot 10^{13} \text{ s}^{-1} \).

For instance, the result of strength estimation of the electric filed impulse by the coordinate \( X \), calculated with the MathCAD system by formula (1.23), is shown in fig. 2.

Taking in to account the reciprocal orthogonality of planes of vectors' envelopes of electric and magnetic fields impulse, we could gain the type of electromagnetic soliton, fig. 3.

Figure 4 shows the envelopes of electric field impulse in the SIT, based on formula (1.23), curve 1, and by formula (1.24), being the consequence of Maxwell-Bloch theory, curve 2. the impulse envelope of electric field strength in this theory is expressed as the first derivative of the \textit{Sin}-Gordon equation solving and is

\[
E = \frac{E_0}{\text{ch}(kX - \omega t)}.
\]

(1.24)
Fig. 2. Calculation of the electric component of electromagnetic radiation impulse in dielectric

Fig. 3. Intensity of electric and magnetic fields electromagnetic soliton in dielectric in conditions of the self-induced transparency

Fig. 4. Comparison bending around of the electromagnetic field impulse, received on the basis of the offered theory, a curve 1, and the equations the Maxwell - Bloch, curve 2
Evidently, the first derivative of Sin-Gordon equation solving is similar to the soliton envelope in the non-linear Schrödinger equation with cube non-linearity solving (27). Curves 1 and 2 in fig. 4 are designed for the same parameters as the function in fig. 2. We can infer from fig. 4 that impulse, referred to formula (1.23), curve 1, is broader in its central part, but asymptotically shorter than impulse, inferred by the Maxwell-Bloch theory, curve 2. Evidently, its is bound with the energetic permittivity of medium in the central part of impulse.

2. Angular distribution of photoelectrons during irradiation of metal surface by electromagnetic waves

There is the problem of achieving of the maximum photoelectric flow during irradiation of the metal by flow of electromagnetic waves while designing of photoelectrons. The depth of radiation penetration into metal during irradiation of its surface is defined by the Bouguer low [10]:

\[ I = I_0 \exp \left( -\frac{4\pi}{\lambda} n\chi z \right), \]

where \( I_0 \) – is the intensity of the incident wave, \( I \) – is the intensity on z-coordinate, directed depthward the metal, \( \lambda \) – is the wavelength of radiation, \( n\chi \) – is the product of refractive index by extinction coefficient.

Let's estimate the thickness of the metal at which intensity of light decreases in \( e = 2,718 \) times:

\[ z = \frac{\lambda}{4\pi n\chi} \]

Average wavelength of a visible light for gold \( \lambda = 550 \text{ nm}, \ n\chi = 2,83 \), therefore \( z = 15,5 \text{ nm} \). Considering [11] that lattice constant for gold \( a = 0,408 \text{ nm} \), it is possible to deduce that electromagnetic radiation penetrates into the metal on 40 atomic layers. Therefore radiation interaction occurs basically of the top layers of atoms and angular distribution of electron escape from separate atoms, i.e. during the inner photoemissive effect, it will appreciably have an impact on distribution of electron escape from the metal surface.

As a result it is interesting to consider angular distribution of photoelectrons during the inner photoemissive effect.

2.1 Nonrelativistic case

Although Einstein has explained the photoeffect nature in the early 20th century, various aspects of this phenomenon draw attention, till nowadays for example, the role of tunnel effect is investigated during the photoeffect [12].

In the description of angular distribution of the photoelectrons which are beaten out by photons from atoms, there are also considerable disagreements. For example it is possible to deduce that the departure of photoelectrons forward of movement of the photon and back in approach of the main order during the unitary photoeffect is absent, using the computational method of Feynman diagrams [13]. It is marked that photoelectrons don't take off in the direction of distribution of quantum [14]. This conclusion is made on the basis of positions which in the simplified variant are represented by the following.
The momentum of the taken off electron is defined basically by action produced by the electric vector of quantum of light on electron. If electron takes off in the direction of an electric vector of quantum it gets the momentum. On a plane set at an angle to a plane of polarization of quantum of light, (fig. 5) electron momentum value will be \( p_{1m} = p \cos \phi \).

\[ \text{Fig. 5. Direction of vectors of particle momentum at the inner photoemissive effect} \]

Besides, if the electron momentum is set at an angle \( \theta \) to the direction of quantum of light its value will be:

\[ p_1 = p \cos \phi \sin \theta \]  \hspace{1cm} (2.1)

Therefore, photoelectron energy is equal to:

\[ E_1 = \frac{p_1^2}{2m_1} = \frac{p^2 \cos^2 \phi \sin^2 \theta}{2m_1}, \]  \hspace{1cm} (2.2)

where \( m_1 \) – is the electronic mass.

If \( \theta = 0 \) then photoelectron energy \( E_1 = 0 \). Photoelectrons take off readies its maximum in the direction of a light vector or a polarization vector, i.e. an electric field vector of quantum of light. The same dependence is offered in the work [7]. The formula (2.2) has the simplified nature in comparison with [7, 14], but convey correctly the basic dependence of distribution energy of a photoelectrons escape from the corners \( \phi \) and \( \theta \).

\[ \text{Fig. 6. Angular distribution of photoelectrons during interaction of orbital electron with the electromagnetic wave} \]
The lack of dependence (2.2) is that at its conclusion the law of conservation of momentum, wasn't used and therefore there is no electron movement to the direction $\theta = 0$. Usage of the momentum conservation equation in [7, 14] can't be considered satisfactory since in the analysis made by the authors it has an auxiliary character. At the heart of the analysis [7, 14] is the passage of electron from a discrete energy spectrum to a condition of a continuous spectrum under the influence of harmonious indignation, i.e. the matrix element of the perturbation operator is harmonious function of time. In other words, the emphasis is on the wave nature of the quantum cooperating with electron. Angular distribution of electron energy in the relative units, made according the formula (2.2) is shown on fig. 6, a curve 1. Let's illustrate the correction to the formula (2.2) connected with presence of photon momentum, following [15].

Fig. 7 demonstrates change of photoelectron momentum in the presence of a photon momentum. The conclusion made on the basis of the is $\theta = \theta + \delta$. Let's find $\sin \theta = \sin \theta \cos \delta + \sin \delta \cos \theta$. Considering that $\delta$ is too small we find $\sin \theta = \sin \theta \left(1 + \frac{\sin \delta \cos \theta}{\sin \theta}\right) = \sin \theta \left(1 + \frac{p}{p_e} \cos \theta\right)$. The law of sines for a triangle on fig. 7 is used.

Further consideration $\beta = \frac{p}{p_e} = \frac{\hbar \nu}{m V_c} = \frac{1}{2} \beta + \frac{W}{m V_c}$, where $\beta = \frac{V}{c}$ - is the relation of photoelectron speed to a speed of light in vacuum, $W$ - is the work function of electrons from atom, we have $\sin \theta = \sin \theta \left(1 + \beta' \cos \theta\right)$. Taking for granted that $\beta'$ is small we will transform (2.2) into $E_i = \frac{p_e^2 \cos^2 \phi \sin^2 \theta}{2 m_i} \left(1 + 2 \beta' \cos \theta\right)$. Angular distribution of electron energy for $\beta = 0,15$, made according to the (2.2) taking into account the correction is shown on fig. 7, a curve 2.

Fig. 7. The account of the momentum of quantum $\vec{p}$ using wave approach of electron interaction with the electromagnetic wave

Thus scattering indicatrix of photoelectrons has received some slope forward, but to the direction of quantum momentum, i.e. at $\theta = 0$ electrons don't take off as before.

The formula (2.2) is accounted as a basis of the wave nature of light. For the proof of this position we will consider interaction of an electromagnetic wave with orbital electron. The description of orbital movement electron is done on the basis of Bohr semiclassical theory since interacting process of electron with an electromagnetic wave is investigated from the positions of classical physics, fig. 8.
Interaction of Electromagnetic Radiation with Substance

Fig. 8. Attitude of components velocity of orbital electron during its interaction with the electromagnetic wave

By the sine law from a triangle of speeds we find:

\[
\sin \alpha = \frac{V_1}{V_t} \cos \theta ,
\]

where \( V_t \) – is the speed of electron movement round the nucleus, \( V_1 \) – is the total speed of electron considering the influence on it of an electromagnetic wave.

By the law of cosines we have:

\[
V_i^2 = V_n^2 + V_t^2 - 2V_nV_t \cos \alpha = V_n^2 + V_t^2 - 2V_nV_t \sqrt{1 - \left( \frac{V_t}{V_i} \cos \theta \right)^2} ,
\]

where \( V_n \) – is the component of the general speed of electron movement after its detachment from a nucleus which arises under the influence of dielectric field intensity \( E \) in the electromagnetic wave.

Solving (2.4) rather \( V_t \), we find:

\[
(V_i^2) = (V_n^2 + V_t^2 - 2V_n^2 \cos^2 \theta) + \sqrt{(V_n^2 + V_t^2 - 2V_n^2 \cos^2 \theta)^2 - (V_n^2 - V_t^2)^2} .
\]

The condition of detachment electron from atom at any position of electron \( V_n \geq V_t \).

In case of equality of speeds \( V_n = V_t \), we have:

\[
V_i = 2V_n \sin \theta .
\]

Distribution of speeds (2.6) corresponds to (2.2) and fig. 6, a curve 1. Thus, the parity (2.6) arises if to consider only the wave nature of the electromagnetic wave cooperating with orbital electron.

In [6] distribution of an angle of the electron escape is investigated only for a relativistic case. It is thus received that electrons are emanated mainly to a direction of photon distribution. However the done conclusion is also actually based on the formula (2.1). Therefore the drawback of the conclusion [6] is in absence in definitive formulas of angular
distribution of electrons of nuclear mass \( m_2 \). And after all the nuclear mass defines a share of the photon momentum which can incur a nuclear.

Let’s consider the phenomenon of the inner photoemissive effect from positions of corpuscular representation of quantum of light, fig. 5. The quantum of light by momentum \( \vec{p} \) and energy \( E \) beats out electron from atom, making \( A \) a getting out. Thus both laws of conservation of energy should be observed:

\[
E = A + E_1 + E_2,
\]

(2.7)

Where \( E_1 \) – is the kinetic energy of taken off electron, \( E_2 \) – is the kinetic energy of nucleus as well as the law of conservation of momentum:

\[
\vec{p} = \vec{p}_1 + \vec{p}_2,
\]

(2.8)

Where \( \vec{p}_1 \) – is the momentum of taken off electron, \( \vec{p}_2 \) – is the momentum transferred to a nucleus.

The formula (2.7) differs from Einstein’s standard formula \( E = A + E_1 \). The point is that Einstein’s formula means the absence of angular distribution of photoelectrons speed. Really, if energy of photon \( E \) is set and work function \( A \) for the given chemical element is determined certain speed of the electron escape from atom is thereby set. It means that speeds of electrons, taking off to every possible directions are identical, and the problem of finding out their angular distribution is becoming incorrect.

The value of the momentum transferred to a nucleus can be found using the formula, following (2.8):

\[
p_2^2 = p^2 + p_1^2 - 2pp_1\cos \theta .
\]

(2.9)

The system of equations (2.7) and (2.9) to obtain a combined solution and the equation (2.9) are convenient to express through energy. Taking into account \( E = pc \), where \( c \) – is the speed of light in vacuum, \( p_1^2 = 2m_1E_1 \) and \( p_2^2 = 2m_2E_2 \), we find:

\[
2m_2E_2 = \left( \frac{E}{c} \right)^2 + 2m_1E_1 - 2 \frac{E}{c} \sqrt{2m_1E_1\cos \theta} ,
\]

(2.10)

where \( m_1 \) – is the electronic mass, \( m_2 \) – is the nuclear mass.

Substituting in (2.10) kinetic energy of nuclear \( E_2 \) by (2.7), we have:

\[
E - A - E_1 = \frac{1}{2m_2} \left( \frac{E}{c} \right)^2 + m_1E_1 - \frac{E}{m_2c} \sqrt{2m_1E_1\cos \theta} .
\]

(2.11)

Let us introduce the following notation \( G = \sqrt{E_1} \), \( \alpha = \frac{E}{m_2c} \sqrt{2m_1} \), \( \sigma = 1 + \frac{m_1}{m_2} \), \( \gamma = \frac{1}{2m_2} \left( \frac{E}{c} \right)^2 + A - E \). Then the equation (2.11) will be transformed into:

\[
\beta G^2 - \alpha G\cos \theta + \gamma = 0 .
\]

(2.12)
Solving quadratic equation (2.12) provided $\sigma = 1$ (electronic mass is much less that nuclear mass), we find:

$$G_{1,2} = \frac{\alpha}{2} \cos \theta \pm \sqrt{\frac{\alpha^2}{4} \cos^2 \theta - \gamma}. \quad (2.13)$$

Substituting in (2.13) accepted notation we have:

$$G_{1,2} = \sqrt{\frac{m_1}{2} E - m_2 \cos \theta \pm \sqrt{\frac{E^2}{2m_2c^2} \left( \frac{m_1}{m_2} \cos^2 \theta - 1 \right) + E - A}}. \quad (2.14)$$

Considering that $G = \sqrt{E_1} = V_1 \sqrt{\frac{m_1}{2}}$, where $V_1$ - speed of photoelectrons provided $\frac{m_1}{m_2} \cos^2 \theta << 1$, we find:

$$V_1 = \frac{E}{m_c} \cos \theta \pm \sqrt{\frac{m_1}{m_2} \cos^2 \theta - 1} \left( \frac{m_2}{m_1} \left( \frac{E}{m_c} \right)^2 - \frac{2(E - A)}{m_1} \right). \quad (2.15)$$

Provided that nuclear mass is aiming to infinity $m_2 \to \infty$ the formula (2.15) is transformed into Einstein’s standard law for the photoeffect. Besides, this, as if it has been specified earlier, angular distribution of speed of photoelectrons disappears.

For the inner photoemissive effect in the formula (2.15) it is necessary to use effective nuclear mass $m_{2e} > m_2$, considering attractive powers between atoms in substance. Therefore for an outer photoemissive effect, i.e. for interaction of the solid and the photon, Einstein’s formula $E = A + E_1$ is applicable absolutely.

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Transforming the formula (2.15), we get:

$$V_1 = \frac{E}{m_c} \left( \cos \theta \pm \sqrt{\cos^2 \theta + \frac{m_2}{m_1} \left( 2(E - A) \frac{m_2 c^2}{E^2} - 1 \right)} \right). \quad (2.16)$$

Let us nominate $\Delta = E - A$. Distribution of photoelectrons will arise at $\Delta \geq \frac{E^2}{2m_2c^2}$. In the right part of the received inequality there is a very small value, therefore distribution of photoelectrons will arise practically at $E > A$.

Let us nominate $\Delta = \eta \frac{E^2}{2m_2c^2}$, where $\eta \geq 1$ characterizes the value of exceedance of photon energy over work function in relative units. Thus the formula (2.16) takes the form:

$$V_1 = \frac{E}{m_c} \left( \cos \theta \pm \sqrt{\cos^2 \theta + \frac{m_2}{m_1} \left( \eta - 1 \right)} \right). \quad (2.17)$$
The analysis of the formula (2.17) shows that the root must to taking a plus since otherwise electron scattering basically goes aside, contrary to the direction of a falling photon. Angular distribution of the electron escape during the inner photoemissive effect in the relative units \( \frac{V_1}{E/m_c} \) is shown on fig. 9, made according to the formula (2.17) with several values \( \eta \) for copper.

<table>
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<th>( \eta )</th>
<th>( \frac{V_1}{E/m_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.0001</td>
</tr>
<tr>
<td>3</td>
<td>1.001</td>
</tr>
<tr>
<td>4</td>
<td>1.001</td>
</tr>
</tbody>
</table>

![Angular distribution of photoelectrons](image)

Fig. 9. Angular distribution of photoelectrons during the inner photoemissive effect depending on parameter \( \eta \) during interaction of orbital electrons with light quantum. The results of experiments [16] are shown by black small squares.

The figure makes it evident that speeds of photoelectrons become almost identical in all directions already at \( \eta \geq 1.01 \). Then Einstein's formula \( E = A + E_1 \) becomes fair and for the inner photoemissive effect. Considering that, for example, for copper the relation \( \frac{E^2}{2m_c} = 4.2 \cdot 10^{-11} \) equivalent as far as the order of value is concerned \( \frac{E^2}{2m_c} \eta = \frac{1}{\Delta} \) in the field of red photoelectric threshold (\( \lambda_r = 250 \text{ nm} \)), it is possible to draw the conclusion that the evident difference of distribution of photoelectrons speeds from spherical, i.e. actually formula is violated \( E = A + E_1 \), can be observed only in very short wave part of spectrum \( \gamma \)-radiations.

The observed data of angular distribution of the photoelectrons which have been beaten out from a monolayer of atoms of copper by covering the nickel surface are shown in fig. 9 by black small squares [16]. The wavelength of quanta allowed observing the photoeffect with 2p-atom shell of copper, but the photoeffect on nickel thus was absent. Experimental distribution of photoelectrons contradicts calculated distribution in fig. 6. Moreover, in distinction in fig. 6, small maxima of indicatrix of the distributions directed to an opposite direction of flight of light quanta at an angle of approximately 45° to the direction of light flux are observed. In [16] these maxima are explained by focusing properties of all population of atoms of the surface. The amplitude of maxima ascends with the increase of quantity of the monolayers of copper atoms on nickel.

Thus, angular distribution of photoelectrons will be absolutely various depending on whether what properties, wave or corpuscular are reveal by the light quantum in interaction with orbital electron. Only experiment can give the answer to the question what distribution
it is true, fig. 6 or fig. 9. However existence of electron flux from an illuminated surface at normal light incidence [16], in the direction opposite to intensity of light, shows at the prevalence of corpuscular properties of light in its interaction with atoms.

2.2 Relativistic case
Dealing with relativistic case of the inner photoemissive effect, the law of conservation of energy needs to be written down as:

\[ E = A + E_k + E_z, \]  

(2.18)

Where \( E_k \) – is the kinetic energy of photoelectron.

The law of conservation of momentum remains in the form (2.9). Using relativistic relation between the energy and the momentum for electron:

\[ E_i^2 = p_i^2c^2 + m_1^2c^4, \]  

(2.19)

where \( E_i \) – is the total energy of electron, \( m_1 \) – is the electron rest mass, we will express the momentum of electron from (2.19) and we will substitute in (2.9). For convenience of the further transformations we will write down (2.19) into:

\[ p_i^2 = \frac{E_i^2 - m_1^2c^4}{c^2} = \frac{(E_1 + m_1c^2)E_k}{c^2}. \]  

(2.20)

Formulating (2.20) the relation has been used:

\[ E_k = E_i - m_1c^2. \]  

(2.21)

The equation (2.9) will be transformed into:

\[ 2m_1c^2E_z = E_i^2 + (E_1 + m_1c^2)E_k - 2E \sqrt{(E_1 + m_1c^2)E_k \cos \theta}. \]  

(2.22)

Because of that the nucleus that has a big mass and a relatively low speed after interaction with the photon, expression for relation of the momentum of the nucleus with its kinetic energy \( E_z \) is used in the nonrelativistic form.

Substituting value \( E_z \) in (2.22) from the equation (2.18), we get:

\[ 2m_1c^2(E - A - E_k) = E_i^2 + (E_1 + m_1c^2)E_k - 2E \sqrt{(E_1 + m_1c^2)E_k \cos \theta}. \]  

(2.23)

Let us nominate:

\[ \alpha = \frac{E \sqrt{E_1 + m_1c^2}}{m_1c^2}; \quad G = \sqrt{E_k}. \]  

(2.24)

As a result (2.23) will be transformed into:

\[ \delta G^2 - \alpha G \cos \theta + \gamma = 0. \]  

(2.25)
The notation \( \gamma = \frac{1}{2m_2} \left( \frac{E}{c} \right)^2 + A - E = \frac{1}{2m_2} \left( \frac{E}{c} \right)^2 - \Delta = \frac{1}{2m_2} \left( \frac{E}{c} \right)^2 (1 - \eta) \) corresponds to item 1 section.

The value

\[
\delta = 1 + \frac{m_2c^2}{2} \left( \frac{\alpha}{E} \right)^2 = 1 + \frac{m_1}{2m_2} + \frac{E_1}{2m_2c^2} = 1 + \frac{m_1}{2m_2} + \frac{E_k - m_1c^2}{2m_2c^2} = 1 + \frac{E_k}{2m_2c^2} = 1.
\]

It is thus accounted for that \( E_k << 2m_2c^2 \).

Solving the equation (2.25), we get:

\[
G_{1,2} = \frac{\alpha}{2} \cos \theta \pm \sqrt{\frac{\alpha^2}{4} \cos^2 \theta - \gamma}.
\]

(2.26)

Substituting notations, we find:

\[
E_k = \left( \frac{\alpha}{2} \right)^2 \left( \cos \theta + \sqrt{\cos^2 \theta + 2 \left( \frac{E_1^3}{m_2c^2} \right)(\eta - 1)} \right)^2.
\]

(2.27)

In contrast to the nonrelativistic case, the formula (2.17), formula (2.27) possesses in its right part value \( \alpha = \frac{E \sqrt{E_1 + m_2c^2}}{m_2c^2} \) which depends on the total energy of electron \( E_1 \) the structure of which includes also kinetic energy \( E_k \). But dependence of value \( \alpha \) on \( E_k \) not strong as the total energy structure includes rather big rest energy of electron \( m_1c^2 \).

Considering that \( E_1 = \frac{m_2c^2}{\sqrt{1 - \beta^2}} \), where \( \beta = \frac{V_1}{c} \) is the relative speed of the photoelectron, we find:

\[
\frac{\alpha^2}{2} = \left( \frac{E}{m_2c} \right)^2 \frac{m_1}{2} \left( \frac{1}{\sqrt{1 - \beta^2}} + 1 \right).
\]

(2.28)

Substituting the equation (2.28) in the equation (2.27) and considering that \( E_k = m_1c^2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \), we get:

\[
2c \sqrt{1 - \mu} = \frac{E}{m_1c} \left( \cos \theta + \sqrt{\cos^2 \theta + \mu \frac{m_1}{m_2}} (\eta - 1) \right),
\]

(2.29)

where \( \mu = \frac{2}{\sqrt{1 - \beta^2}} + 1 \).
Considering that \( 2c\sqrt{1-\mu} = \frac{2E_k}{m_1} \sqrt{1 + \frac{E_k}{2m_1c^2}} = V_1 \), at \( 1 >> \frac{E_k}{2m_1c^2} \), we find:

\[
V_1 = \frac{E}{m_2c} \left( \cos \theta + \sqrt{\cos^2 \theta + \mu \frac{m_2}{m_1}(\eta - 1)} \right)
\]  
(2.30)

The formula (2.30) allows to consider relativistic effects at the photoeffect, in case of rather big speeds of photoelectrons. Thus, in contrast to (2.17), relativistic coefficient \( \mu \) is introduced under the root. The calculation of dependence \( \mu(\beta) \) shows on fig. 10, relativistic effects while calculating distribution of photoelectrons escape, can be neglected and (2.17) can be used while the photoelectron speeds read approximately half the value of the light speed in the vacuum.

![Fig. 10. Dependence of relativistic coefficient \( \beta \) on relative speed of photoelectrons \( \beta = \frac{V}{c} \)](image)

### 3. Conclusion

The laws of formation of the impulse of electromagnetic radiation in dielectric environment for conditions self-induced transparency are considered. The insufficiency of the description of such impulse with the help of the equations Maxwell - Bloch are shown. The impulse of electromagnetic radiation in conditions of a self-induced transparency submits to nonlinear equation of Schrödinger with logarithmic nonlinearity. The way of connection of an average number filling and energy of the impulse taking into account energy saturation of environment are offered. The calculation of a electrical component of the impulse is submitted.

Angular distribution of photoelectrons is investigated during the inner photoemissive effect for two variants: quantum of light basically reveals wave and basically corpuscular properties interacting with orbital electron. Distinction in angular distribution of photoelectrons for these variants is demonstrated. If electromagnetic radiation shows basically quantum properties during a photoeffect there is an emission of photoelectrons on a direction of movement of quanta. It corresponds Einstein’s to formula. In Einstein’s formula there is no corner of a start of photoelectrons. Angular distribution in the second variant is investigated for the nonrelativistic and relativistic cases.
4. References


