1. Introduction

The recognition of grain boundaries in deformed rocks from images of thin-sections or polished slabs is an essential step in describing and quantifying various features and analysis, which is usually achieved by image processing procedures. Recently, researchers from geoscience and computer science focused on this issue and many methods have been proposed.

We begin with an application of mineral grain boundary detection of thin section in geoscience: strain analysis[1]. Strain analysis plays an important role in the study of structural geology, especially when investigating the tectonic history of a region(see [2] and [3]). The following scheme illustrates main steps of strain analysis. The critical step in the above scheme is identifying mineral with image segmentation techniques based on edges or regions. High sample intensity is required to provide adequate raw data for precious strain analysis. But in recent publications, the samples used for analysis seem to be insufficient. The primary reason maybe is the laborious and time-consuming methods to obtain the raw data required for strain analysis[4]. Though many methods based on the technology of image processing have been proposed, there is still much room for improving in efficiency and accuracy.

In image processing and computer version, segmentation or boundary detection is still a challenging problem and is motivated by some new certain applications. As an interdisciplinary problem, grain boundary detection provides much room to improve well established method to tackle with the special situations in geosciences. Three aspects should be paid more attention when designing the new approaches. The two of them are derived from image capture, and the other is the features of objects to be identified.

- Light phenomenon: under plane-polarised light, many mineral grains appear as colorless; under cross-polarised light, mineral grains show varying colors and intensities. In cross-polarized light the interference color displayed depends on the mineral type, the orientation of the indicatrix of the grain with respect to the polarizers and the thickness of the thin section.
• Rotating stage: grains may appear different light with different angels. To get a better visual effect, thin section must be rotated and several images are captured. Each of these image contains the important available for image segmentation. But how many images should be collected and how to fuse the information from these image have been completely answered yet.

• Grain features: every kind of grains has its own special shape features. However, in a thin section image, some grains are with a certain orientation, which is determined by the way how it is cut and polished. These features provide important cues for image segmentation and data match.

In the previous publications, various techniques have been reported for some certain grains identify. Threshold is used to identify grains of thin section image, while interview is often necessary to select different thresholds to separate several kinds of mineral. Edge detection has a simple control structure and provides more precise location of grain boundaries, but cannot guarantee closure of boundaries. If a boundary is discontinuous, pixels belonging to adjacent grains may be connected and therefore identified as belonging to a single mineral grain. Thus, grain identification will be finished completely. Several techniques, such as heuristic search and artificial neural net can be used to correct such edge detection errors. Instead of directly identifying the grain boundary, region based method is proposed to identify a region, i.e. a set of points that purportedly belong to the clast. Region-based segmentation uses image features to map individual pixels to sets of pixels that correspond to objects. Closed boundaries are always segmented but the positions of the boundaries may not be as accurate as those obtained from edge detection. Seeds are always selected by human interaction, which can be replaced by an automated complex algorithm. Holes may appear in the identified region, but for extracting features like major and minor axes, orientation, centroid, etc., the presence of these holes has a far diminished role to play than incorrectly identified edges. Watershed usually produce over-segmentation results for boundary detection. An important tool of post-processing, it is often used to separate touching grains in binary map created by other arithmetic.

In the past two decades, methods based on the partial differential equation (PDE) have been widely used in image segmentation and other image science fields recently. Impressive effects have been obtained with various PDE models. In [5] and [6], Chan and Verge proposed a successful segmentation model (C-V model) using level set, which derived from the classical active contour (snake) model[7]. The essential idea of the snake model is to evolve an initial contour governed by a PDE until it stops on the edge of the objects. To represent the evolving curve, level set technique is adopted. Level set method is based on the description of the curve as the zero crossing of a higher-dimensional function and allows major simplifications. It offers a nature representation for the contour of the object, which can deal with complicated structures with many advantages, especially when the curve undergoes complex topological change. The key of level set method is to identify grain boundaries and to represent them as closed outlines. A framework of level set for mineral boundary detection was reported recently[8]. Level set method identifies all the objects simultanetly instead of identifying all the interesting objects chosen by clicking them using mouse one by one.

In this paper, we present a framework for boundary detection with level set for different kinds of input images. After reviewing related work, we first introduce the level set for boundary detection with a single gray scale image as input. Then level set for color image is presented. For processing two color polarising images as input, a novel energy functional with a curve represented with level set is constructed and a new mathematical model for mineral
identification is obtained. The curve evolves driven by the structures of two color polarizing images and stops at the region edge of the grains.

2. Previous work: Brief view

Boundary detection of mineral from thin section image is not a trivial task. Various method, including threshold, gradient operator, region grow, watershed, level set, artificial neural network or a combination of them, have been proposed to address this problem. We give a detailed review of previous work in order of publication.

Lumbreras and Serrat considered the segmentation of grains in digital image of thin marble section[10]. An over-segmentation of the image with watershed method was performed and a region-merging procedure was carried out with some parameters determined by a sequence of images of the same sample with polarized light. In [11], polished rock samples were scanned by a color image scanner and an automatic mineral classification approach was presented. Fueten devised a computer-controlled rotating polarizer stage for the petrographic microscope. In this pioneer work, Fueten presented an important system allowed a thin section to remain fixed while the polarizing filters were rotated by stepper motors. This approach permitted a better integration between the processing software and the microscope and hence better data gathering possibilities. In 1998, Goodchild and Fueten proposed a boundary detection procedure which calculated closed boundary with a series of image processing routines[12]. A color RGB image was converted into a gray intensity image and seven steps were performed to produced accurate and closed edges for mineral grains. Whereas this algorithm was not perfect effective it presented a significant improvement over existing routines of that time. In fact, only intensity information was utilized in the original publication and some of color information was missed. The work of Nail and Murthy in [13], in which they constructed a standardization of edge magnitude in color image, may be helpful to improve the algorithm in [12]. To depict the geometrical structures of rock, boundary was defined as pixel with high gradient and fragmentation were rebuilt and reconnected ro form an uninterrupted boundary net[14]. Orientation contrast (OC) images represent a useful starting point to develop an automated technique able to assess grain boundaries in a completely objective and reproducible way. The method in [15] defined boundaries as high brightness gradient features on an OC image of a quartz mylonite through a specifically designed sequence of detection and filter algorithms that minimize the effect of local background noise. The initial boundaries produced by edge detection methods were with many imperfections. They employed a detection-filtering algorithm to automatically rebuild the real boundary net. When quantifying microstructures of fine grained carbonate mylonites, manually setting the threshold value was adopted to select as much of the dark grain-boundary area as possible without selecting the grey values of the grain interiors[16]. Heilbronner presented a simple procedure for creating grain boundary maps in a crystalline aggregate from multiple input images: for each image of a given input set, only the most significant grain boundaries were detected with gradient filtering and by combining those, a single grain boundary map was obtained[17]. Thompson, Fueten and Bockus used an artificial neural network(ANN) for the classification of minerals[18]. Based on a set of seven primary images during each sampling, a selected set of parameters were estimated and a three-layer feed forward neural network was trained on manually classified mineral samples. This is a beginning of ANN for mineral classification. Ross, Fueten and Yashkir proposed an automatic mineral identification with another important evolutionary computation: genetic algorithm[19]. Touching grains in digital images of thin sections is a hot potato when considering segmentation. Van den Berg and his collaborators tried to deal with it with a
algorithm by separating touching grain sections in binary images of granular material[20]. The algorithm detected characteristic sharp contact wedges in the outline of touching grain sections and created an intersection after checking if the angle of the contact wedge was smaller than a user-defined threshold value. When making analysis of deformation of rock analogues, Fueten et.al applied gradient filtering for different types of movies with frames[21]. Zhou et al. proposed a segmentation method of petrographic images by integrating edge detection and region grow method[22]. They employed a boundary detection method to get the edge information, with which the seeds for region grow were selected automatically. Gradient filtering and threshold were also adopted step by step to pick several minerals in olivine–phyric basalts[23]. A similar method with multi-threshold was presented to extract several kinds of objects and to produce the corresponding binary images[24]. A recent method was presented by Roy Choudhury et al. , which they called CASRG, to identify a ‘region’, a set of points belonging to the grain[4]. The authors selected the seeds manually and chose the optimal threshold separately for each grain instead of using a single threshold for the whole image. The modification was very efficient and the accuracy has been validated for low and high strain samples in their contribution. The imperfections of this method was that all the seeds should be chosen by clicking the mouse. The working of choosing the seeds was also onerous for the large or high stress samples when the clasts needed to be clicked are too much or the clasts were deformed badly. In fact, complex optical properties of plagioclase, such as twinning, present a particularly difficult challenge to thin section image processing. Gradient-based boundary detection method are likely to classify optical twin zones as different grains. Barraud shown an example of textural analysis of thin sections of rocks with a Geographic Information System (GIS) program, in which boundary was obtained by watershed segmentation on digital pictures of the thin section[25]. Region-grow method have also been improved to identify both twinned and un-twinned plagioclase areas as seeds[26]. To overcome this problem, a set of plane polarised light images, taken at 51 intervals with 18 polarizer rotations were used to create an average grey level image with high resolution. Homogeneous zones were detected and they were classified manually as seeds to form the basis of further grain boundary recognition. Obara presented a registration algorithm for reversed transformation of rock structure images taken with polarizing microscope stage rotations[27]. The idea behind this algorithm was based on finding the optimal rotation angle and optimal translation vector, thus ensuring the best correspondence between analysed images. The criteria for optimization was formulated on the basis of the shapes of edges located on images, in which edges were detected with gradient filtering. To identify transcrystalline microcracks in microscope images of a dolomite structure, Obara devised a polarizing system using two nicols: one was fixed and the other could be rotated, while the thin section was kept fixed. 12 colour images were taken: 11 images with two crossed nicols and one with one nicol[28]. Based on the dolomite structure, CIELab color system was used and some image processing techniques, including gradient, threshold and mathematical morphology functions utilizing linear structuring elements, were performed for different components to detect transcrystalline microcracks. The maximum of standard deviation values of 11 α components were fused to be a single image for consequent operations. Obara also used a similar method to detection of the morphological anisotropy of calcite grains in marble[29]. Filtering with a rotated stencil consisting of two linear structuring elements preserved fine structure in thin section image. Fueten and Mason proposed to edit edges with an artificial neural net assisted approach[30]. The goal of this method was to produce close boundaries and enclose areas were considered to be grains. This method significantly improved the speed with which edges could be edited in preparation for other
studies, although training of the neural network and manual correction of the results were still necessary. Using GIS, Li, Onasch and Guo presented an automated method to detect grain boundaries and construct a boundary database in which the shape, orientation, and spatial distribution of grains could be quantified and analyzed in a reproducible manner[31]. In this procedure, they calculated the difference between the maximum and minimum value within a specified neighborhood, large values were recognized as grain boundaries.

In [8], level set method (LSM) was introduced to detect the grain boundary. The major advantage of the LSM is that the grain boundary detected by the LSM is a closed curve, which is preferred for features extraction and data analysis. To quantify microstructures of coarse-grained marbles, Ebert et al. presented a new approach based on the intensity of light reflectance in dependence of the crystallographic orientation of calcite grains[32]. As filters could not distinguish between twin boundaries and different phases (especially in the case of thin micas), and the grain boundaries were compiled from a stack of images (one image for each sample rotation increment), grain boundaries was traced with Photoshop manually.

To determine crystal size distributions of olivine in kimberlite, scanned images of polished rock slab were analysed and region of interesting (ROI) was determined by combining texture, colour and grey intensity analysis outputs. Adjoined crystals were separated by adapting and extending the marker-based watershed algorithm. In a study in [33], the application of 2D and 3D textural analysis to the quantification of olivine populations in kimberlites was investigated. Olivine grains were segmented with a threshold filter selecting grey-values from 50 to 73 connected to grey-values from 50 to 66 (seeded threshold) after the data set was subjected to a median filter for noise reduction. Using ANN for image processing seemed to be more popular and is a highly researched area. Baykan proposed an ANN for the classification of minerals using color spaces without boundary detection[36]. A microscopic information system (MIS) for petrographic analysis was presented with GIS and applied to transmitted light images[37]. Two region functions were developed and embedded in the GIS environment. GIS software provided optimal management of the MIS database, allowing the cumulative measurement of more than 87,000 grains.

The methods mentioned above are mainly based on the traditional image segmentation and edge detection technology in image processing. The initial boundaries produced by edge detection methods have many imperfections. The boundary may be open, discontinuous, which do not coincide with the realistic boundary. Grain boundary, specifically a close one, provides fundamental information about material properties, such as area, orientation, percent, microstructural analyse [32] and crystal size distributions [33]. Such boundaries can not be used to feature extraction and other image processing tasks and a post processing arithmetic is needed to obtain more realistic boundaries. So post-processing is necessary to get a closed boundary. Some techniques, as suggested in [30], have been reported to link edges. Concomitantly, the subjectivity and the modification on raw data introduced by the post-processing make them unsuitable for data analysis. Region grow method tries to identify the region of the grain by ‘absorbing’ all the points similar with the ‘seed’ point. It’s best advantage is that the boundary of the region is closed, which is preferred for measurements and analysis. A critical point in this method is to select the seeds, which often involves human interventions to avoid the seeds failing to grow according to the given rules. In CASGR[4], seed was selected by human interaction and the threshold was set adaptive to data automatically. Zhou et al. tried to overcome this problem by introducing a hybrid method that the results of edge-detection provided clues for automated seed select[22]. Apart from the human intervention in seeding, another defect is the occur of the holes within the region. The size of the hole depends on the noise distribution and the area of
the heterogeneity of color information within the clasts. The small holes can be filled by employing simple morphological operations, but the larger ones remains almost unchanged. Such post-processing ineluctably affects the extracted boundaries as well as the extracted features, such as major and minor axes, orientation, centroid, etc[4].

3. Level set method for grain boundary detection

Methods based on the partial differential equation (PDE) have been widely used in image segmentation and other image science fields recently. Impressive effects have been obtained with various PDE models. In [6], Chan and Verse proposed a successful segmentation model (C-V model) using level set, which derived from the classical active contour (snake) model [7]. The essential idea of snake model is to evolve an initial contour governed by a PDE until it stops on the edge of the objects. For initial snake model, the edge is defined where have larger magnitude of gradient. As imposed with a second order derivative constraint, the curve have a good smoothing shape, which is contrast to the boundaries produced by edge-detection operator. Level set method is based on the description of the curve as the zero crossing of a higher-dimensional function and allows major simplifications[38]. In level set, a closed curve is seen as the zero level set of a function in high dimension. It offers a nature representation for the contour of the object, which can deal with complicated structures with many advantages, especially when the curve undergoes complex topological change. It is a thriving method in image science for its following advantages:

• easy to implement numerically;
• the outline of the object is closed;
• tackle with the topology change easily, such as merge and split.
• some geometric quantities can be expressed directly.

With this method, initial curve can be anywhere or with any shape in the image plane. In the following, we will give a framework of level set for grain boundary detection. First we introduce level set for gray scale image. Then, active contour for vector image is given. We end this section by consider a level set model with two polarising images as input.

3.1 Level set for gray scale image

For a grayscale image \( u \), considering the following energy functional:

\[
E(c_1, c_2, C) = \int_{\text{inside}(C)} |u(x, y) - c_1|^2 dxdy + \int_{\text{outside}(C)} |u(x, y) - c_2|^2 dxdy + \mu \cdot \text{Length}(C),
\]

where \( c_1 \) and \( c_2 \) are constant unknowns representing the average value of \( u \) inside and outside the curve \( C \).

For curve evolution, the level set has been used widely. It can deal with cusps, corners and automatic topological changes. Now, we rewrite the original model (1) in the level set formulation. The curve \( C \) is defined as the zero level set as follows: \( C = \{ (x, y) \in \Omega | \phi(x, y) = 0 \} \). Assuming that \( \phi \) has opposite signs on each side of \( C \), the energy can be rewritten as:

\[
E(c_1, c_2, \phi) = \int_{\Omega} ((u(x, y) - c_1)^2 H(\phi) + (u(x, y) - c_2)^2 (1 - H(\phi))) dxdy + \mu \int_{\Omega} |\nabla H(\phi)| dxdy,
\]

(2)
where $\phi$ is level set function, $\nu$ is a positive coefficient, $H$ denotes the Heaviside function:

$$H(z) = \begin{cases} 
1, & z \geq 0; \\
0, & z < 0. 
\end{cases}$$

In order to compute the associated Euler–Lagrange equation for the unknown function, we consider slightly regularized versions of the function $H$, denoted here by $H_\epsilon$ as $\epsilon \to 0$. One example of such approximations is given by

$$H_\epsilon(z) = \begin{cases} 
1, & z > \epsilon, \\
0, & z < \epsilon, \\
\frac{1}{2} \left[ 1 + \frac{z}{\epsilon} + \frac{1}{\pi} \sin \left( \frac{n\pi}{\epsilon} \right) \right], & |z| \leq \epsilon,
\end{cases}$$
as proposed in [6].

For minimizing the functional defined in (2), fixing $c_1$ and $c_2$, we obtain the following Euler-Lagrange equation:

$$\delta_\epsilon(\phi)[\nu \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - (u_0 - c_1)^2 + (u_0 - c_2)^2] = 0$$

(3)

where $\delta$ is one-dimensional Dirac measure and $\delta_\epsilon = H_\epsilon'(z)$. Using gradient descent method by an artificial time $t$, we yield the following evolution equation:

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi)[\nu \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - (u_0 - c_1)^2 + (u_0 - c_2)^2]$$

(4)

An alternative way to improving the above model is to replace $\delta_\epsilon(\phi)$ by $|\nabla u|$ to extend the evolution to all level set of $\phi$.

Keeping $\phi$ fixed and minimizing the energy yields the following expressions for $c_1$ and $c_2$:

$$c_1(\phi) = \frac{\int_{\Omega} u(x,y)H_\epsilon(\phi(x,y))dxdy}{\int_{\Omega} H_\epsilon(\phi(x,y))dxdy},$$

(5)

$$c_2(\phi) = \frac{\int_{\Omega} u(x,y)(1 - H_\epsilon(\phi(x,y)))dxdy}{\int_{\Omega}(1 - H_\epsilon(\phi(x,y)))dxdy}.$$  

(6)

To solve this evolution problem, we use a finite differences scheme, as suggested in [6].

As the thin section image contain abundant structure, the initial contour we obtain is so complex that it is almost impossible to make strain analysis directly, though all the valuable ones have been identified. The following we have to do is to select the useful grains suitable for analysis. It is a difficult task as the noise disturbs the segmented results, and the small objects are too much, so we should set constrains to discard the useless ones. The select depends on the grain to be identified and the question tackled with. Here we provide a simple way, which could be improved by the Recalling when the image is processed manually or using other automated methods, the intensity, area and shape of the grain are dominant factors for human inspection and segmentation criterion. Level set provides the boundaries of the objects by utilizing the intensity distribution, so area and shape criterions are adopted for a tough tentative strains to abandon the unsuitable objects. The theory of level set method shows that the contour is a closed curve, so the objects whose area is ranging between two
thresholds \( k_1 \) and \( k_2 \) are selected. As for the shape information, the ration of major axis and minor axis could be adopted as another important criterion for high formed minerals. It is a simple attempt to devise such constraints, more precious constraints, mainly decided by the previous knowledge of the the grains, will be proposed in future work.

### 3.2 Level set for color image

Color information is important factor for image segmentation and edge detection. Though luminance could be extracted from color image and processed as gray scale image, but it is still necessary to consider color components for grain boundary detection. In the following we will discuss the level set for color micrographic of thin section.

Let \( u_{ij} \) be the jth channel of an color image \( u \) on \( \Omega \), with \( j = 1, 2, 3 \) channels and \( C \) the evolving curve. Each channel would contain the same image with some differences. Let \( c^+ = (c_1^+, c_2^+, c_3^+) \) be a unknown constant vector. The extension of the C-V model to vector images is

\[
E(c^+, c^-, C) = \int_{\text{inside}(C)} \frac{1}{3} \sum_{j=1}^{3} \lambda_j^+ |u_j(x, y) - c_j^+|^2 dxdy + \int_{\text{outside}(C)} \frac{1}{3} \sum_{j=1}^{3} \lambda_j^- |u_j(x, y) - c_j^-|^2 dxdy + \mu \cdot \text{Length}(C),
\]

(7)

The curve \( C \) is defined as the zero level set as follows: \( C = \{(x, y) \in \Omega | \phi(x, y) = 0\} \).

In this model, the active contour \( C \) is the boundary between two regions defined over all channels. Rewrite it in level set form, we obtain

\[
E(c^+, c^-, \phi) = \int_{\Omega} \frac{1}{3} \sum_{j=1}^{3} \lambda_j^+ |u_j(x, y) - c_j^+|^2 H_\epsilon((\phi(x, y))dxdy \\
+ \int_{\Omega} \frac{1}{3} \sum_{j=1}^{3} \lambda_j^- |u_j(x, y) - c_j^-|^2 (1 - H_\epsilon((\phi(x, y))dxdy + \mu \int_{\Omega} |\nabla H_\epsilon(\phi)|dxdy, (8)
\]

for \( j = 1, 2, 3 \). The parameters \( \mu \) is the weight for the length term of the curve. \( \lambda^{+, -} = (\lambda_j^{+, -}) \) are the weights for the noise error. \( H \) denotes the Heaviside function, again.

Minimizing the energy with respect to the constant \( c_j^+, c_j^- \), for \( j = 1, 2, 3 \), we obtain:

\[
c_j^+ = \frac{\int_{\Omega} u_j H_\epsilon(\phi(x, y))dxdy}{\int_{\Omega} H_\epsilon(\phi(x, y))dxdy}, \quad \text{average}(u_j)_{\Omega \geq 0},
\]

\[
c_j^- = \frac{\int_{\Omega} u_j (1 - H_\epsilon(\phi(x, y)))dxdy}{\int_{\Omega} H_\epsilon(\phi(x, y))dxdy}, \quad \text{average}(u_j)_{\Omega < 0}.
\]

Assuming that \( c_j^+ \) are constant vectors, and minimizing the energy functional with respect to \( \phi \), we obtain the following Euler-lagrange equation for \( \phi \):

\[
\delta_{e}(\phi) \left[ -\frac{1}{3} \sum_{j=1}^{3} \lambda_j^+ (u_j - c_j^+)^2 + \frac{1}{3} \sum_{j=1}^{3} \lambda_j^- (u_j - c_j^-)^2 + \mu \cdot \text{div} \left( \phi \frac{\nabla \phi}{|\nabla \phi|} \right) \right] = 0,
\]

(9)
Mineral Grain Boundary Detection With Image Processing Method: From Edge Detection Operation To Level Set Technique

3.3 Level set for polarised images

Polarised images captured under different light contain important clues for mineral boundary detection. For a thin section, two images can be captured from plane- or cross-polarised light in microscopy. The main difference between polarizing images and ordinary images is polarised light phenomenon: under plane-polarised light, many mineral grains appear as colorless; under cross-polarised light, mineral grains show varying colors and intensities. The colors and intensity rely the mineral, the thickness of the grain and its crystallographic orientation. In the previous publications, gradient filtering has been proposed to deal with polarised images captured from different light contain important clues for mineral boundary detection. For a thin section, two images can be captured from plane- or cross-polarised light in microscopy. The main difference between polarizing images and ordinary images is polarised light phenomenon: under plane-polarised light, many mineral grain appear as colorless; under cross-polarised light, mineral grains show varying colors and intensities. The colors and intensity rely the mineral, the thickness of the grain and its crystallographic orientation. In the previous publications, gradient filtering has been proposed to deal with polarised images captured from different light. Figure 1 shows the two images of a thin section. Figure 1(a) is of a less contrast and the edges of Fig. 1(b) seems to be more legible. It is not proper to detect the grain boundary with any image. To segmentation the polarised images, we construct a new active contour model below. The energy as well as the evolving is similar to the C-V model to polarised images.

Let $u_{i,j}$ be the $j$th channel of an image $u_i$ on $\Omega$, with $j = 1, 2, 3$ channels, $i = 1, 2$ images and $C$ the evolving curve. Each channel would contain the same image with some differences. Let $c^+ = (c^{1+}_{1,1}, c^{1+}_{1,2}, c^{1+}_{1,3})$ and $c^- = (c^{-}_{2,1}, c^{-}_{2,2}, c^{-}_{2,3})$ be two unknown constant vectors. The extension of the C-V model to polarised images is

$$E(c^+, c^-, C) = \int_{\text{inside}(C)} \frac{1}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda^+_{ij} |u_{i,j}(x,y) - c^+_{ij}|^2 dxdy + \int_{\text{outside}(C)} \frac{1}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda^-_{ij} |u_{i,j}(x,y) - c^-_{ij}|^2 dxdy + \mu \cdot \text{Length}(C).$$

In this model, the active contour $C$ is the boundary between two regions defined over all channels. Rewrite it in level set form, we obtain

$$E(c^+, c^-, \phi) = \int_{\Omega} \frac{1}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda^+_{ij} |u_{i,j}(x,y) - c^+_{ij}|^2 H_c(\phi(x,y)) dxdy$$

$$+ \int_{\Omega} \frac{1}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda^-_{ij} |u_{i,j}(x,y) - c^-_{ij}|^2 (1 - H_c(\phi(x,y))) dxdy + \mu \int_{\Omega} |\nabla H_c(\phi)| dxdy,$$

for $i = 1, 2$ and $j = 1, 2, 3$. The parameters $\mu$ is the weight for the length term of the curve. $\lambda^{+, -} = (\lambda^{+, -}_{ij})$ are the weights for the noise error.
Fig. 1. Two polarized thin section images.

Minimizing the energy with respect to the constant $c_{ij}^+, c_{ij}^-$ for $i = 1, 2, j = 1, 2, 3$, we obtain:

$$c_{ij}^+ = \frac{\int_{\Omega} u_{ij} H_e(\phi(x,y)) dx dy}{\int_{\Omega} H_e(\phi(x,y)) dx dy}, \quad \text{average}(u_{ij}) \text{on} \Omega \geq 0,$$

$$c_{ij}^- = \frac{\int_{\Omega} u_{ij} (1 - H_e(\phi(x,y))) dx dy}{\int_{\Omega} H_e(\phi(x,y)) dx dy}, \quad \text{average}(u_{ij}) \text{on} \Omega < 0.$$

Assuming that $c_{ij}^+$ are constant vectors, and minimizing the energy functional with respect to $\phi$, we obtain the following Euler-lagrange equation for $\phi$ (introducing an artificial time as gradient descent method):

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left[ -\frac{1}{6} \sum_{i=1}^{2} \sum_{j=1}^{3} \lambda_{ij}^+(u_{ij}(x,y) - c_{ij}^+)^2 + \frac{1}{6} \sum_{i=1}^{2} \sum_{j=1}^{3} \lambda_{ij}^-(u_{ij}(x,y) - c_{ij}^-)^2 + \mu \cdot \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \right],$$

(13)

in $\Omega$ with the boundary condition

$$\frac{\delta_e(\phi) \partial \phi}{|\nabla \phi| \partial n} = 0$$

on $\partial \Omega$, where $\partial n$ denotes the unit normal at the boundary of $\Omega$. To solve this evolution problem, we use a finite differences scheme, as suggested in [6].
The main difference between the proposed model and the classical model in [6] is the input images. Our model is strongly inspired by the work of [6]. The three methods evolve the curve depending on the information with multichannel. The main difference is the number of the input image and channel. Our model deals with the two polarised images of a thin section of the same view. Under plane- and cross-polarised light, different color information is captured. Some grains can be observed in one image while they may have a unconspicuous edges or regions. In the new model, the active contour evolves according to the structure of the two polarised images instead of any single image. More information is concerned to locate a more reasonable and accurate edge position. Similarly, the contour produce by the above model need a post-processing to remove unmeaning objects.

**4. Experimental results**

We present in this section some numerical results obtained with the models from the previous section. For the details of the numerical schemes and for other numerical results, we refer the reader to [6],[8],[39]. As we will see in this section, these models have the abilities of automatic detection of grain boundaries. Every grain boundary is represented with a closed curve. In Fig.2–Fig.4, we illustrate the whole procedures of low strain grain boundaries detection using level set with gray scale as input. We also compare the grains detected by level set with that of hand-drawn method. Comparisons of features and strain analysis results can be found in [8]. In Fig.5, an example of high strain sample boundary detection is given and the result is compared with that of a region growing method: CASGR[4]. A note to this example is that our detection is performed on gray scale image and CASGR is performed with color image. In Fig.6, we show the result of color grain boundary detection with equation (10). We skip the preprocessing of TV flow and the we refer to [40] for more details. In Fig.7, we demonstrate the segmentation of polarising images of a thin section. The thin section contains some grains and other clasts. The red contours represent the edges of the identified grains. In Fig.7(a), segmentation of plane-polarising image with classical vector-valued image is shown. Some small clasts are identified as grains wrongly. Segmentation of cross-polarising image is shown in Fig.7(b). In the left of the image, a large part of ares is recognized as inner of a grain according to the structure of the cross-polarsing image, which is not coincide with the fact of plane-polarising image. The edges of grains detected with the proposed model is shown in Fig.7(c). The time space is 0.1 and the total iter number is set 200. The other parameter are set as follows: \( \mu = 0.2, \lambda_{i,j} = 1, i,j = 1,2,3 \). Some clasts are excluded from the identified grains. The reason for a new segmenting result is that the information in six channel is used to determine the regions of the grains. With more clues concerned, the disturb of the noise and false edges is suppressed.

**5. Conclusion**

We have presented in this paper a framework for grain boundary detection using level set with different input images: gray scale image, color image, plane-polarising and cross-polarising images. The obtained variational level set models yield closed grain boundary which is preferred for feature extraction and data analysis. Application to grain boundary detection have been illustrated. In future, we will focus on the grain identification and classification with multi-phase level set approach for thin section image of complex topologies.
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Fig. 2. Thin section image of mylonite.

Fig. 3. Curve involving of thin section image.
Fig. 4. Grain boundaries: level set VS hand-drawn.
(a) Grains boundaries by level set in color image.
(b) Contours of grains by level set in corresponding binary image.
(c) Hand-drawn boundaries of grains in color image.
(d) Hand-drawn boundaries of grains in corresponding binary image.

Fig. 5. Grain boundaries detection of low strain sample.
(a) Color image.
(b) Gray scale image.
(c) Grain boundaries by CASGR.
(d) Grain boundaries by level set.
Fig. 6. Grain boundary detection of andalusite with color image.
Fig. 7. Grain boundary detection with different input images

(a) Segmentation of plane-polarising image

(b) Segmentation of cross-polarising image

(c) Segmentation of two polarising images
7. References


It was estimated that 80% of the information received by human is visual. Image processing is evolving fast and continually. During the past 10 years, there has been a significant research increase in image segmentation. To study a specific object in an image, its boundary can be highlighted by an image segmentation procedure. The objective of the image segmentation is to simplify the representation of pictures into meaningful information by partitioning into image regions. Image segmentation is a technique to locate certain objects or boundaries within an image. There are many algorithms and techniques have been developed to solve image segmentation problems, the research topics in this book such as level set, active contour, AR time series image modeling, Support Vector Machines, Pixon based image segmentations, region similarity metric based technique, statistical ANN and JSEG algorithm were written in details. This book brings together many different aspects of the current research on several fields associated to digital image segmentation. Four parts allowed gathering the 27 chapters around the following topics: Survey of Image Segmentation Algorithms, Image Segmentation methods, Image Segmentation Applications and Hardware Implementation. The readers will find the contents in this book enjoyable and get many helpful ideas and overviews on their own study.

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