Sliding Mode Control of DC Drives

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1. Introduction

Variable structure control (VSC) with sliding mode control (SMC) was first proposed and elaborated in early 1950 in USSR by Emelyanov and several co researchers. VSC has developed into a general design method for wide spectrum of system types including nonlinear system, MIMO systems, discrete time models, large scale and infinite dimensional systems (Carlo et al., 1988; Hung et al., 1993; Utkin, 1993). The most distinguished feature of VSC is its ability to result in very robust control systems; in many cases invariant control systems results. Loosely speaking, the term "invariant" means that the system is completely insensitive to parametric uncertainty and external disturbances.

In this chapter the unified approach to the design of the control system (speed, Torque, position, and current control) for DC machines will be presented. This chapter consists of parts: dc motor modelling, sliding mode controller of dc motor i.e. speed control, torque, position control, and current control. As will be shown in each section, sliding mode control techniques are used flexibly to achieve the desired control performance. All the design procedures will be carried out in the physical coordinates to make explanations as clear as possible. Drives are used for many dynamic plants in modern industrial applications.

The simulation result depicts that the integral square error (ISE) performance index for reduced order model of the system with observer state is better than reduced order with measured state.

Simulation results will be presented to show their agreement with theoretical predications. Implementation of sliding mode control implies high frequency switching. It does not cause any difficulties when electric drives are controlled since the "on –off" operation mode is the only admissible one for power converters.

2. Dynamic modelling of DC machine

Fig. 1 shows the model of DC motor with constant excitation is given by following state equations (Sabanovic et al., 1993; Krause, 2004).

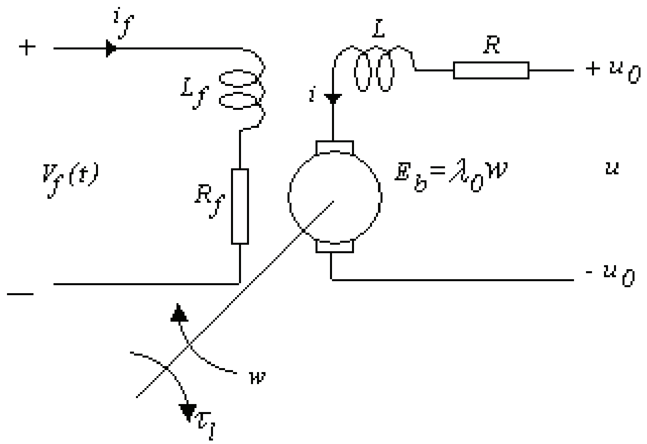

Fig. 1. Model of DC motor with constant excitation.

$$
\begin{aligned}
L\frac{di}{dt} &= u - Ri - \lambda_0 w \\
j\frac{dw}{dt} &= k_t i - \tau_l
\end{aligned}
\tag{1}
$$

where

i armature current
w shaft speed
R armature resistance
λ_0 back emf constant
τ_l load torque
u terminal voltage
j inertia of the motor rotor and load
L armature inductance
k_t torque constant.

Its motion is governed by second order equations (1) with respect to armature current i and shaft speed w with voltage u and load torque k_t. A low power-rating device can use continuous control. High power rating system needs discontinuous control. Continuously controlled voltage is difficult to generate while providing large current.

3. Sliding mode control design

DC motors have been dominating the field of adjustable speed drives for a long time because of excellent operational properties and control characteristics. In this section different sliding mode control strategies are formulated for different objectives e.g. current control, speed control, torque control and position control.

3.1 Current control

Let i^* be reference current providing by outer control loop and i be measured current. Fig. 2 illustrates Cascaded control structure of DC motors.

Consider a current control problem, by defining switching function

$$s = i^* - i \tag{2}$$

Design a discontinuous control as

$$u = u_0 sign(s) \tag{3}$$

Where u_0 denotes the supplied armature voltage.

$$s\dot{s} = s(\frac{di^*}{dt} + \frac{R}{L}i + \frac{\lambda_0}{L_0}w) - \frac{1}{L}u_0|s| \tag{4}$$

Choice of control u_0 as

$$u_0 > \left| L\frac{di^*}{dt} + Ri + \lambda_0 w \right| \quad \text{Makes (4)}$$

$s\dot{s} < 0$ Which means that sliding can happen in $s = 0$.

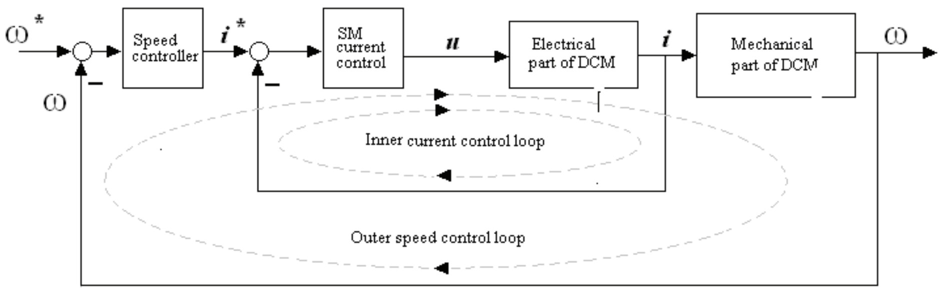

Fig. 2. Cascaded control structure of DC motors.

3.2 Speed control

w^* be the reference shaft speed, then the second order motion equation with respect to the error $e = w^* - w$ is of form. State variable $x_1 = e$ & $x_2 = \dot{e}$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_1 x_1 - a_2 x_2 + f(t) - bu \end{aligned} \tag{5}$$

Where

$$a_1 = \frac{k_t \lambda_0}{jL}, a_2 = \frac{R}{L} \;\&\; b = \frac{k_t}{jL}$$

$$f(t) = \ddot{w} + a_2\dot{w}^* + a_1 w^* + \frac{R\tau_l}{jL} + \frac{\dot{\tau}_l}{j}$$

are constant values.

The sliding surface and discontinuous control are designed as

$$s = c(w^* - w) + \frac{d}{dt}(w^* - w) \tag{6}$$

$$u = u_0 sign(s)$$

This design makes the speed tracking error e converges to zero exponentially after sliding mode occurs in $s = 0$, where c is a positive constant determining the convergence rate for implementation of control (6), angle of acceleration $x_2 = \dot{e}$ is needed. The system motion is independent of parameters a_1, a_2, b and disturbances in g(t).
Combining (1) & (6) produces

$$\begin{aligned} \dot{s} &= c\dot{w}^* + \ddot{w} - \frac{c}{j}(k_t - \tau_l) + \frac{1}{j}\tau_l + \frac{k_t}{jL}(Ri + k_t w) - \frac{k_t}{jL}u \\ &= g(t) - \frac{k_t}{jL}u \end{aligned} \tag{7}$$

$$g(t) = c\dot{w}^* + \ddot{w} - \frac{c}{j}(k_t - \tau_l) + \frac{1}{j}\tau_l + \frac{k_t}{jL}(Ri + k_t w) - \frac{k_t}{jL}u \tag{8}$$

$$if\ \ u_0 > \frac{jL}{k_t}|g(t)|,\ \ s\dot{s} < 0 \tag{9}$$

Then sliding mode will happen (Utkin,1993).
The mechanical motion of a dc motor is normally much slower then electromagnetic dynamics. It means that $L << j$ in (1).
a. Reduced -order Speed control with measured speed
Following reduced order control methods proposed below will solve chattering problem without measuring of current and acceleration (x_2).
Speed tracking error is $w_e = w^* - w$. The dc motor model (1) in terms of w_e:

$$\begin{aligned} L\frac{di}{dt} &= u - Ri - \lambda_0(w^* - w_e) \\ j\frac{dw_e}{dt} &= -k_t i + \tau_l + j\dot{w}^* \end{aligned} \tag{10}$$

Let L be equal to zero due to $L << j$. Then (10) becomes with $L = 0$

$$i = -\frac{\lambda_0}{R}(w^* - w_e) - \frac{k_t}{R}u + \tau_l + j\dot{w}^* \tag{11}$$

Substituting (11) into (10) results in

$$j\frac{dw_e}{dt} = \frac{k_t\lambda_0}{R}(w^* - w_e) - \frac{k_t}{R}u + \tau_l + j\dot{w}^* \tag{12}$$

Equation (12) is a reduced order (first order) model of dc motor. The discontinuous control is designed as

$$u = u_0 sign(w_e) \tag{13}$$

and the existence condition for the sliding mode $w_e = 0$ will be

$$u_0 > \left| \lambda_0 (w^* - w_e) + \frac{\tau_l R}{k_t} + \frac{jR\dot{w}^*}{k_t} \right| \tag{14}$$

The principle advantage of the reduced order based method is that the angle acceleration $(x_2 = \dot{e})$ is not needed for designing sliding mode control (Carlo et al.,1988).

b. Reduced -order Speed control with observer speed

The unmodelled dynamics (1) may excite non-admissible chattering.

Let us design an a asymptotic observer to estimate w_e (Utkin,1993).

$$j\frac{d\hat{w}_e}{dt} = \frac{k_t \lambda_0}{R}(w^* - \hat{w}_e) - \frac{k_t}{R}u + \hat{\tau}_l + j\dot{w}^* - l_1(\hat{w} - w_e)$$

$$j\frac{dw_e}{dt} = \frac{k_t \lambda_0}{R}(w^* - w_e) - \frac{k_t}{R}u + \tau_l + j\dot{w}^* \tag{15}$$

$$\frac{d\hat{\tau}_l}{dt} = -l_2(w^* - w_e)$$

where

$\hat{w}_e$	Estimated error
$\overline{w}_e = \hat{w}_e - w_e$	Speed tracking error
l_1, l_2	Observer gain

The discontinuous control designed using estimate state $\hat{w}_e$ (Utkin,1993) will be

$$u = u_0 sign(\hat{w}_e) \tag{16}$$

The sliding mode will happen if

$$u_0 > \left| \lambda_0 (w^* - \hat{w}_e) + \frac{\hat{\tau}_l R}{k_t} + \frac{jR\dot{w}^*}{k_t} + \frac{l_1 R}{k_t}(w^* - w_e) \right| \tag{17}$$

And $\hat{w} = 0 \,\&\, \hat{\tau} = 0$.

Under the control scheme, chattering is eliminated, but robustness provided by the sliding mode control is preserved within accuracy of $\frac{L}{j} << 1$. The observer gains l_1, l_2 should be chosen to yields mismatch dynamics slower than the electrical dynamics of the dc motors to prevent chattering. Since the estimated $\hat{w}$ is close to w , the real speed w tracks the desired speed w^* . Fig. 3 shows the control structure based on reduced order model and observed state. Chattering can be eliminated by using reduce observer states. The sliding mode occurs in the observer loop, which does not contain unmodelled dynamics.

3.3 Position control

To consider the position control issue, it is necessary to augment the motor equations (1) with

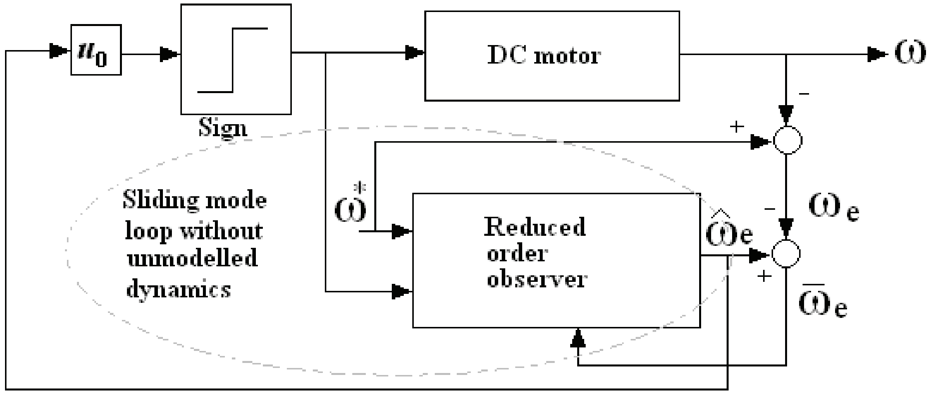

Fig. 3. Speed control based on reduced order model and observed state.

$$\frac{d\theta}{dt} = w \tag{18}$$

where θ denotes the rotor position.
The switching function s for the position control is selected as

$$\dot{s} = (\ddot{\theta}^* - \ddot{\theta}) + c_1(\dot{\theta}^* - \dot{\theta}) + c_2(\theta^* - \theta) \tag{19}$$

and the discontinuous control is

$$u = u_0 sign(s) \tag{20}$$

Combining (1), (18), (19)

$$\dot{s} = h(t) - \frac{k_t}{jL}u \tag{21}$$

where

$$h(t) = \ddot{w}^* + c_1\dot{w}^* + c_2 w^* - \frac{c_1}{j}(k_t i - \tau_l) - c_2 w + \frac{1}{j}\dot{\tau}_l + \frac{k_t}{jL}(Ri + k_t w) \tag{22}$$

Choice of u_0 as

$$u_0 > \frac{jL}{k_t}|h(t)| \tag{23}$$

Makes $s\dot{s} < 0$ which means that sliding mode can happen $s = 0$ with properly chosen c_1, c_2 .We can make velocity tracking error $e = w^* - w$ converges to zero.

3.4 Torque control

The torque control problem by defining switching function

$$s = \tau^* - \tau \tag{24}$$

As the error between the reference torque τ^* and the real torque τ developed by the motor. Design a discontinuous control as

$$u = u_0 sign(s) \tag{25}$$

Where u_0 is high enough to enforce the sliding mode in $s = 0$, which implies that the real torque τ tracks the reference torque τ^*.

$$\begin{aligned} \dot{s} &= \dot{\tau}^* - k_t i \\ &= \dot{\tau}^* + \frac{k_t R i}{L} + \frac{k_t \lambda_0 w}{L} - \frac{k_t}{L} u \\ &= f(t) - \frac{k_t}{L} u \end{aligned} \tag{26}$$

Where

$$f(t) = \dot{\tau}^* + \frac{k_t R i}{L} + \frac{k_t \lambda_0 w}{L}$$

depending on the reference signal for

$$\begin{aligned} u_0 &> \frac{L}{k_t} |f(t)| \\ s\dot{s} &= sf(t) - \frac{k_t}{L} u_0 |s| < 0 \end{aligned} \tag{27}$$

So sliding mode can be enforced in $s = 0$.

4. Simulation results

4.1 Simulation results of current control

Examine inequality (2 & 4), if reference current is constant, the link voltage u_0 needed to enforce sliding mode should be higher than the voltage drop at the armature resistance plus back emf, otherwise the reference current i^* cannot be followed.

Figure 4 depicts a simulation result of the proposed current controller. The sliding mode controller has been already employed in the inner current loop thus, if we were to use another sliding mode controller for speed control, the output of speed controller i^* would be discontinuous, implying an infinite $\frac{di^*}{dt}$ and therefore destroying in equality (4) for any implement able u_0

4.1 Simulation results of speed control

To show the performance of the system the simulation result for the speed control of dc machine is depicted. Rated parameters of the dc motor used to verify the design principle are 5hp, 240V, R=0.5Ω, L=1mH, j=0.001kgm2, $k_t = 0.008$NmA-1 , $\lambda_0 = 0.001$v rad s^{-1} and $\tau_l = Bw$ where B=0.01 Nm rads-1.

Figure 5 depicts the response of sliding mode reduced order speed control with measured speed. It reveals that reduced order speed control with measured speed produces larger overshoot & oscillations.

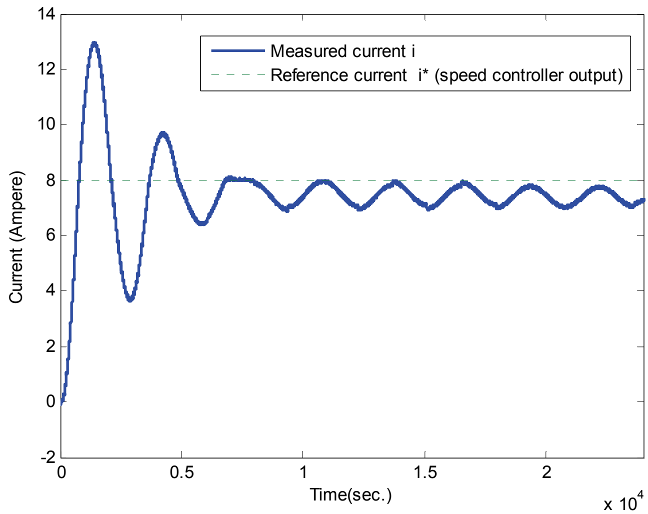

Fig. 4. Cascade current control of dc motor.

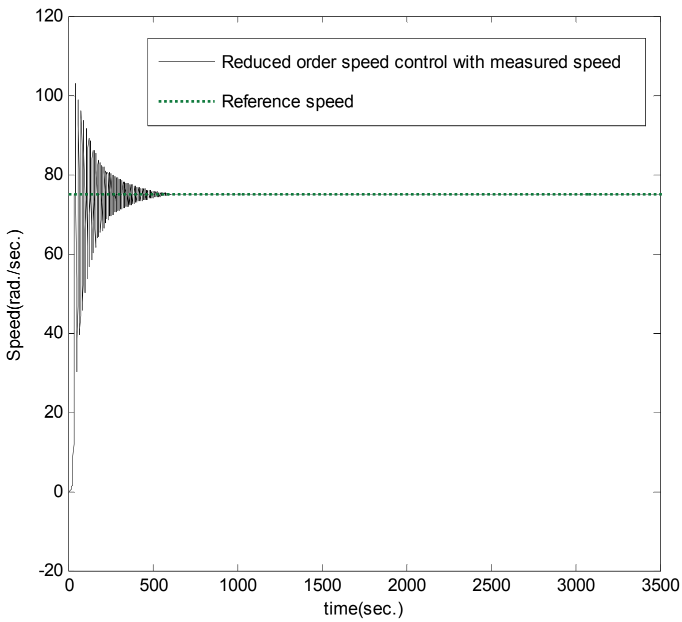

Fig. 5. Response of sliding mode reduced order speed control with measured speed.

Figure 6 depicts the response of sliding mode reduced order speed control with observer speed. It reveals that reduced order speed control with observer speed produces smaller overshoot & less oscillation.
Fig. 7(a), 7(b), 7(c) & 7(d) depicts the simulation result of response of sliding mode speed control, variation of error, squared error and integral square error (ISE) with reference speed of 75 radian per sec respectively. Fig. 8 reveals that variation of the controller for reduced order speed control with observed speed at load condition. Fig. 9 reveals that variation of the controller for reduced order speed control with measured speed at load condition. Fig. 10 depicts the response of sliding surface in sliding mode control. Fig. 11 depicts the robustness (insensitivity) to parameters (+10%) variation. Fig. 12 depicts the robustness (insensitivity) to parameters (-10%) variation. The high frequency chatter is due to neglecting the fast dynamics i.e. dynamics of the electric of the electric part. In order to reduce the weighting of the large initial error & to Penalise small error occurring later in response move heavily, the following performance index is proposed.The integral square error (ISE) is given by (Ogata,1995).

$$\text{ISE} = \int_0^T e^2(t)dt \tag{28}$$

The minimum value of ISE is obtained as gain tends to infinity.

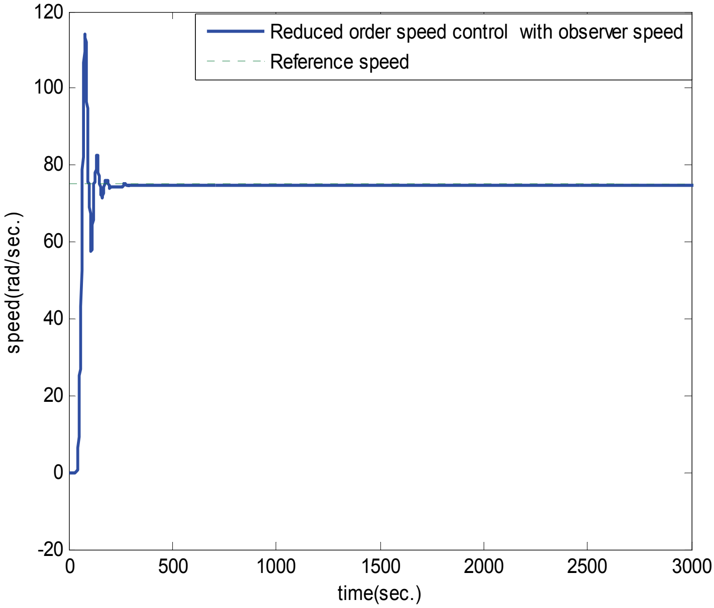

Fig. 6. Response of sliding mode reduced order speed control with observer speed.

5. Conclusions

The SMC approach to speed control of dc machines is discussed. Both theoretical and implementation result speed control based on reduced order with measured speed and reduced order with observer speed, using simulation are conducted. Besides, reduced order observer deals with the chattering problem, en-counted often in sliding mode. Control area Selection of the control variable (angular position, speed, torque) leaves basic control structures unchanged. Inspection of Fig. 7(a), 7(b), 7(c) & 7(d) reveals that reduced order speed control with observer speed produces smaller overshoot & oscillation than the reduced order speed with measure speed (Panchade et al.,2007). The system is proven to be robust to the parameters variations, order reduction, fast response, and robustness to disturbances.

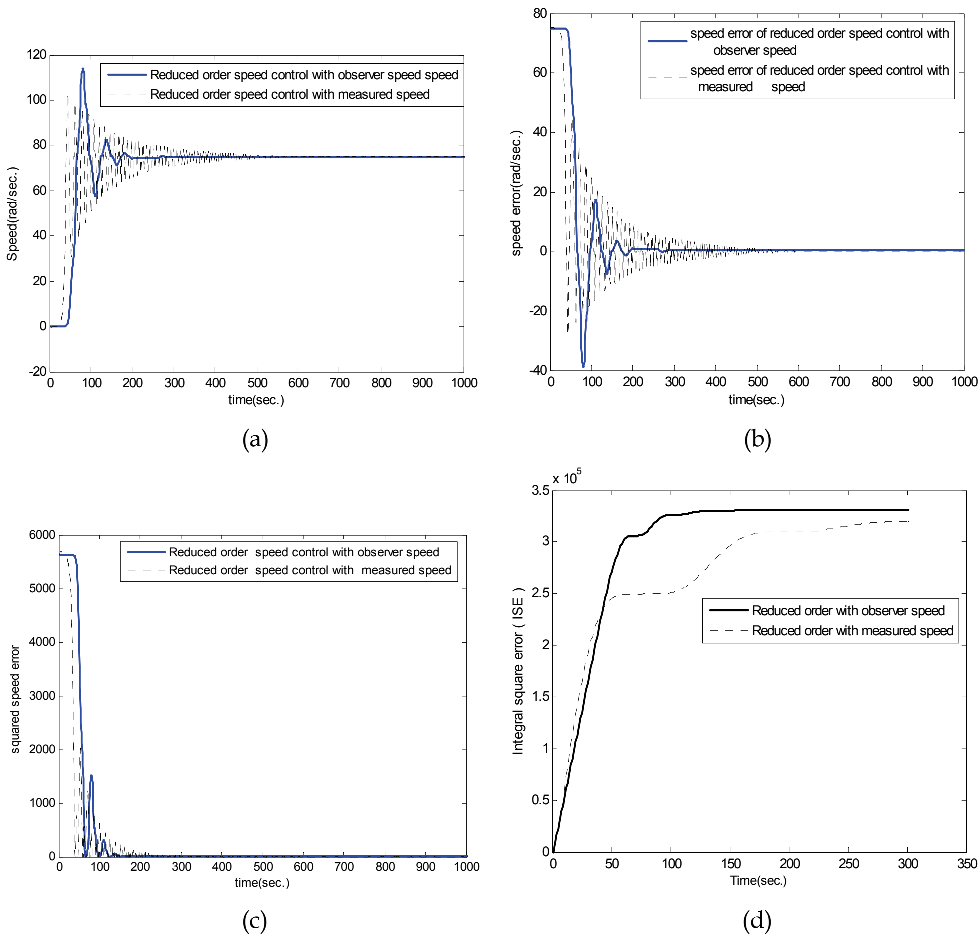

Fig. 7. (a) Response of the sliding mode speed control with reference speed 75 radian per sec. (b) The variation error with reference speed 75 radian per sec. (c) The variation of squared error with reference speed 75 radian per sec (d) Integral square error (ISE)

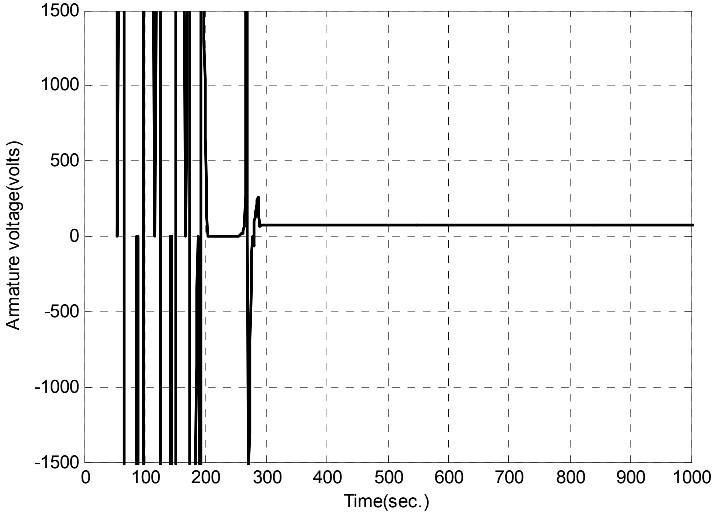

Fig. 8. Variation of the controller for reduced order speed control with observed speed at load condition

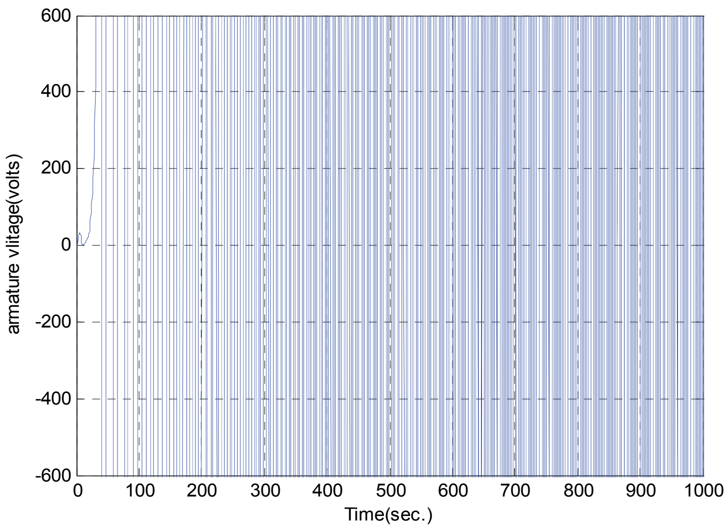

Fig. 9. Variation of the controller for reduced order speed control with measured speed at load condition

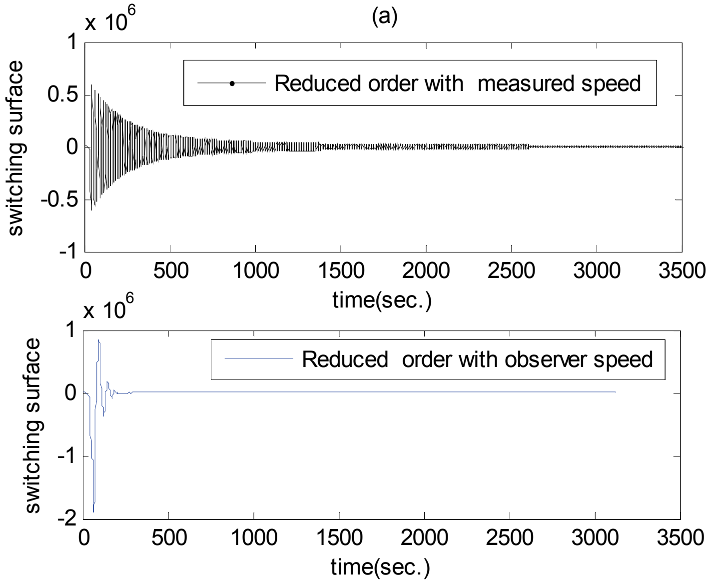

Fig. 10. Response of sliding surface in sliding mode speed control.

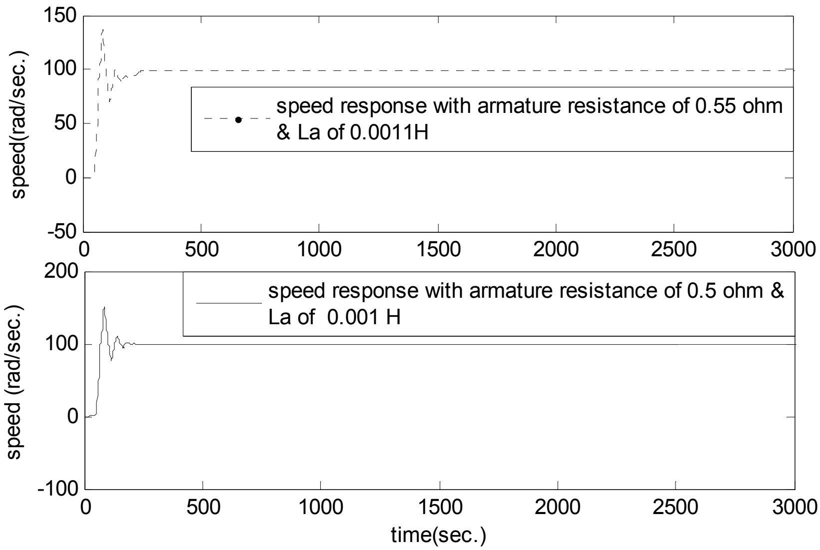

Fig. 11. Robust (Insensitivity) to parameters (+10%) variation.

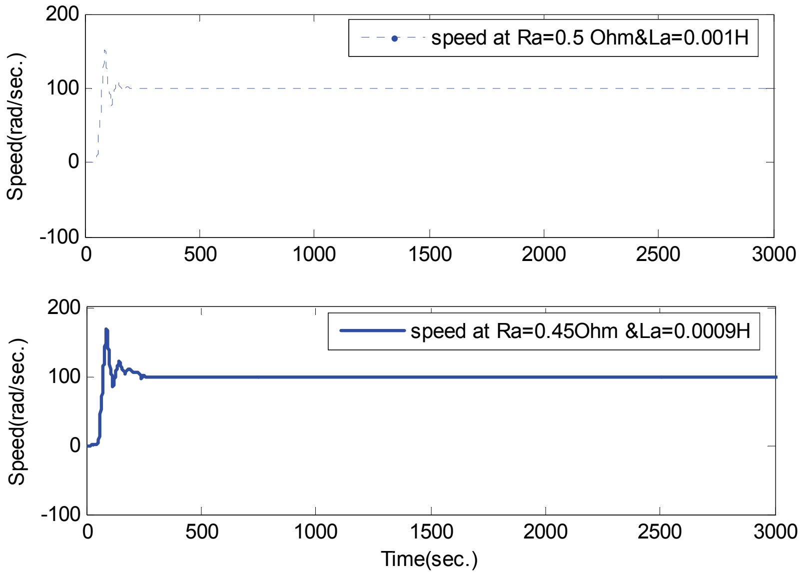

Fig. 12. Robust (insensitivity) to the parameters (-10%) variation

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The main objective of this monograph is to present a broad range of well worked out, recent application studies as well as theoretical contributions in the field of sliding mode control system analysis and design. The contributions presented here include new theoretical developments as well as successful applications of variable structure controllers primarily in the field of power electronics, electric drives and motion steering systems. They enrich the current state of the art, and motivate and encourage new ideas and solutions in the sliding mode control area.