

Low-sensitivity design of allpass based fractional delay digital filters

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1. Introduction

Conventional linear digital circuits are providing usually a delay response that is equal to an integer number of sampling intervals (as in linear-phase FIR (finite-impulse-response) realizations) or is changing uncontrollably with the frequency (for all IIR (infinite-impulse-response) digital filters). It appeared, however, that we might often need a circuit with a delay response that is a fraction of the sampling interval and is fixed or variable (or only adjustable). Design and implementation of such circuits with given and properly controlled fractional delay (FD) is the hottest digital filters topic in the last ten years. These circuits are invaluable in many telecommunications applications, like time adjustment and precise jitter elimination in digital receivers, echo cancellation, phase-array antenna systems, transmultiplexers, sample-rate converter and software radio. They are needed in speech synthesis and processing, image interpolation, sigma-delta modulators, time-delay estimation, in some biomedical applications and for modeling of musical instruments. Most of these applications are overviewed in (Laakso et al., 1996) and (Valimaki & Laakso, 2001).

1.1 FIR fractional delay filters

The design of fixed FIR FD filters (FDF) is well developed and quite a mature field, because it is relatively easy to formulate the design problem and to obtain an optimal solution. Many methods, so far, have been advanced and most of them are well summarized in (Laakso et al., 1996) and (Valimaki & Laakso, 2001). They include a least squared (LS) integral error design, often combined with properly selected window functions or other methods for smoothing the filter transition band; weighted LS (WLS) integral error approximation of the frequency response (Laakso et al., 1996); maximally-flat FD design based on Lagrange interpolation (very popular and widely used, but with several drawbacks (Deng & Nakagawa, 2004); (Deng, 2009a)); minimax design, achieving lower than LS and Lagrange filters maximal error (Valimaki & Laakso, 2001); splines-based FDF design (Laakso et al., 1996). Most of these methods are used to design also variable FD (VFD) FIR filters. There are many other VFD FIR filters design methods like a constrained minimax optimization method (Vesma & Saramaki, 2000), a singular value decomposition method (Deng & Nakagawa, 2004), a Taylor series expansion method (Johanson & Lovenborg, 2003), and the WLS design (Tseng, 2004); (Huang et al., 2009). Recently a new method (Tseng & Lee, 2009)

and a new criterion (Shyu et al., 2010) for design of such filters have been proposed. Most of the VFD FIR filters are using the Farrow structure (Farrow, 1988), its modifications (Yli-Kaakinen & Saramaki, 2006) or transformations (Deng, 2009a). In (Deng, 2010a) several new hybrid structures with reduced complexity have been developed. Common disadvantages of all the FIR FDFs are their higher complexity (higher order transfer function (TF) and too many multipliers and delays), very high overall delay and not constant for all frequencies magnitude response, varying additionally when the delay is tuned.

1.2 General IIR fractional delay filters

Recently, several methods for design and implementation of general IIR variable FDFs have been proposed. The method in (Zhao & Kwan, 2007) is based on a two-steps procedure, where in the first step a set of fixed delay general IIR filters are designed by minimizing a quadratic objective function defined by integrated error criterion; in the second step the TF coefficients of the fixed delay filters are represented as polynomials and are fitted for any given FD. The method in (Tsui et al., 2007) is based on a new model reduction technique and is applicable to IIR TFs that are decomposable to sub-filters with a common denominator (which will stay fixed when the filter is tuned), realized then as Farrow structures. These methods are further generalized and expanded to FIR, allpass, Hilbert transformers and other devices in (Kwan & Jiang, 2009); (Pei et al., 2010). Both methods are achieving an impressive FD variability, but at a price of too higher TF order (30 or 55 in (Zhao & Kwan, 2007)) and calculation of too many multiplier coefficients (for example 426 in (Zhao & Kwan, 2007)), to be practical. The interest in general IIR VFD realizations, will grow, however, because they may offer a lower overall group delay time compared to the allpass realizations (Kwan & Jiang, 2009) and also could be used for a simultaneous magnitude and phase approximation.

1.3 Allpass-based fractional delay filters

There are IIR FDFs (fixed and variable), avoiding all the disadvantages of the FIR and of the general IIR FDFs, and they are based on allpass structures. The main advantage of the allpass-based FDF is that their magnitude is unity for all frequencies and it remains unity when the FD is tuned. The TF order of these filters is low and so are the circuit complexity and the total delay time compared to those of the FIR realizations. Many methods for design of allpass based FDF have been described in (Laakso et al., 1996) and (Valimaki & Laakso, 2001) and many more new methods (mainly for variable FDFs) have been proposed after that.

One group in (Laakso et al., 1996) and (Valimaki & Laakso, 2001) consists of several WLS methods. Recently (Tseng, 2002) a new iterative WLS method was developed, but it was shown (Deng, 2006) that very often it is not converging. A new noniterative approach solving the minimization problem by using a matrix equation and thus avoiding the convergence problems was advanced in (Deng, 2006). Both methods are rigorously proven and are producing very impressive results (very low frequency response error), but as with the general IIR methods, the TF order is very high (35 for example), each of the multiplier coefficients is represented by polynomial of 5th or 6th order (making thus the total number of the coefficients higher than 200). Then 100 sets of coefficients are calculated to cover the frequency range from 0 to 0.9π , and another 30 sets are calculated to cover the range of FD

from -0.5 to 0.5. And, if the required FD is not coinciding with some of these 30 sets, new coefficients are calculated using a polynomial interpolation. The method in (Deng, 2006) was further generalized in (Deng, 2009b) throughout an optimization of the range of the variable part of the delay-time, a usage of different order subfilters (canceling thus the application of the matrix approach), and a reformulation of the WLS design. As a result, the complexity of the final structure was additionally reduced (to only 158 filter coefficients, compared to 210 and 175 for the example with the three methods), making this the best in the group. The structure complexity and the computational load, however, are still very high and we consider this approach to realize allpass-based VFDFs quite unpractical and not permitting a real time tuning.

Another group of design methods encompasses all the minimax approaches to allpass FDFs design in terms of minimal phase error, phase-delay or group-delay error (Laakso et al., 1996). An improved optimization method was proposed in (Yli-Kaakinen & Saramaki, 2004) to overcome the problems with the convergence when designing VFDFs. It is based on a gradual increase of the filter order and optimization in minimax sense to obtain optimal values for the adjustable parameters. This method is addressing the famous “gathering structure” (Makundi et al., 2001), widely used for realization of allpass-based VFDFs. Recently another method, approximately formulating the minimax design as a linear programming problem, solved noniteratively or iteratively, was advanced (Deng, 2010b). These methods are efficient and the results are impressive, but the design procedures, including complicated optimizations, are quite difficult to be applied in an engineering design.

The third and most popular group of methods is the maximally-flat design of allpass FDFs based on Thiran approximation (Thiran, 1971), giving a closed-form solution for the TF coefficients. The Thiran-based design of VFDF is somehow connected to the gathering structure, which permits very easy real-time tuning by recalculating and reprogramming a single coefficient value. This structure was criticized recently for its long critical path and big difference between the coefficient values (requiring longer wordlength) and an improved structure was proposed in (Cho et al., 2007). Another way to use Thiran approximation but to avoid usage of gathering structure to realize VFDF (and thus to avoid the division operation in the recalculation of the coefficients) was proposed in (Hacıhabiboğlu et al., 2007) and it is called “root displacement interpolation (RDI) method” (See Sect. 6.1). The resulting structure, however, is quite complicated, the range of tuning is narrow and the tuning error is quite high.

All general IIR and allpass-based VFD filters are having a common drawback, consisting of considerable transients appearing every time when the filter is tuned. Suppression of these transients is a difficult problem, several methods to solve it are discussed in (Valimaki & Laakso, 1998); (Valimaki & Laakso, 2001); (Makundi et al., 2002) and (Hacıhabiboğlu et al., 2007), but publications on this topic are very few and a lot more remains to be done.

The main aim of the present chapter is to investigate and compare the existing and to develop new methods of design, realization and tuning of allpass-based FDFs and to increase the accuracy throughout minimization of their sensitivities. It will permit more efficient multiplierless realizations, shorter wordlength and lower power consumption. The design procedures should be straightforward, without iterative and complicated optimization steps, in order to be easily used by practicing engineers and the structures have to be with the lowest possible TF order and complexity, in order to be easily tuned in real time.

2. Low-Sensitivity Design Principles

It is clear from the above considerations that allpass based FDFs (with fixed and variable FD) are most appropriate for almost all practical applications, providing lower order TF, low complexity and low total delay-time realizations, permitting an easy real-time FD tuning.

We select to use the Thiran approximation procedure (Thiran, 1971) for designing allpass based FD digital filters with maximally flat group delay response. This procedure gives an easy way to express the TF coefficients a_k as a function of the desired fractional delay parameter value D :

$$a_k = (-1)^k \binom{N}{k} \prod_{n=0}^N \frac{D-N+n}{D-N+k+n}, \text{ for } k=0, 1, 2, \dots, N, \quad (1)$$

for every allpass TF of N -th order

$$H_{AP}(z) = \frac{a_N + a_{N-1}z^{-1} + \dots + a_1z^{N-1} + a_0z^{-N}}{a_0 + a_1z^{-1} + a_2z^{-2} + \dots + a_Nz^{-N}} = \frac{B(z)}{A(z)}. \quad (2)$$

In the literature very often this allpass TF is realized as a direct form ($2N + 1$ multipliers and N delays are needed for the realization) or a lattice structure ($2N$ multipliers and N delays), which are by far non-canonic with respect to the multipliers number (a canonic allpass structure of N -th order should contain only N multipliers) and the direct structure is also very sensitive to the changes of the coefficient values. The strategy to achieve our aim is based on our approach, described in (Stoyanov et al., 2007) and using (when possible) a cascade realization of the allpass TF. It is well known that a cascade realization of the allpass TF will decrease considerably the overall sensitivity and will open the way for further sensitivity reduction. To achieve this we propose, after decomposing the allpass TF to first- and second-order terms, to minimize the sensitivities of the individual first- and second-order allpass sections, realizing each real pole or couple of complex-conjugate poles. This minimization may consist of a careful selection of proper sections (there are too many allpass sections already known) according to the position of the poles in the z -plane or of development of new allpass sections when there is no low sensitivity realizations readily available for given pole positions. These sections should be with canonic structures with respect to the number of the multipliers and the delay elements. The new low-sensitivity sections could be developed using the coefficient conversion method, proposed by Nishihara (Nishihara, 1984) or some other known methods.

We choose to use the classical (normalized) sensitivity of the phase response $\theta(\omega)$ to the changes of the multiplier coefficients m_k

$$S_{m_k}^{\theta(\omega)} = \frac{\partial \theta(\omega)}{\partial m_k} \frac{m_k}{\theta(\omega)}. \quad (3)$$

For evaluation of the sensitivity to the changes of all the multiplier coefficients, necessary as a figure of merit in a case of sensitivity minimization or as a measure when different realizations are compared, we can use the worst-case sensitivity

$$WS_m^{\theta(\omega)} = \sum_{k=1}^N |S_{m_k}^{\theta(\omega)}| \quad (4)$$

or the so called Schoeffler (statistical) sensitivity, employing squared addends in (4). Both sensitivities are easily calculated for every given section topology by using the package PANDA (Sugino & Nishihara, 1990).

Very convenient tool to evaluate the sensitivity of second-order sections when realizing poles in different areas within the unit-circle is the pole-density for given multiplier coefficients wordlength, but there are some problems in calculating this density of sections obtained throughout a coefficient conversion.

Decreasing the sensitivity (throughout a proper design) would reduce the error of the fixed FD filter realizations in a limited wordlength environment especially when a fixed-point arithmetic is used. In a case of variable FD filters it will improve additionally the accuracy of tuning, as lower sensitivity means more possible values of the FD for given multiplier coefficients wordlength. Instead of higher accuracy, the low sensitivity could be used to decrease the power consumption and the computational load by using a shorter wordlength and this is of a prime importance when realizing different portable devices.

Many low-sensitivity filter (and allpass) sections have been developed through the years, but mainly to improve the performance of different narrowband and very selective amplitude filters, having their TF poles usually situated in the area near unity in the z -plane. These sections might not be useful to realize low-sensitivity phase and FD filters because their TF poles could be located in some other areas of the unit-circle. Because of that, our consideration starts with a study of the typical pole positions of the TFs obtained using the Thiran approximation.

3. FD Allpass Transfer Functions Poles Loci Investigations

The sensitivities of the realizations are strongly depending on the position of their TF poles in the z -plane, so it is important to know how the poles of the allpass-based FD filters are situated there.

3.1 Real poles behavior

The possible FD TF real poles are positioned differently depending on N and D as follows:

1. Odd order FD TF and $N-1 < D < N$ - the real pole is negative. When the FD parameter values are increasing from $N-1$ to N , the possible pole positions are moving from $z = -1$ to the area near $z = 0$ (as case 1 in Fig. 1).

2. Odd order FD TF and $D > N$ - the real pole is positive and increasing D to infinity moves the pole from the area near $z = 0$ to the area near $z = 1$ (as case 2 in Fig. 1).

3. Even order FD TF and $N-1 < D < N$ - there are one negative and one positive real poles as shown in the Fig. 1 for sixth order FD TF. When the FD is increasing from $N-1$ to N , these two poles are moving as in the above mentioned cases 1 and 2.

3.2 Complex-conjugate poles behavior

The complex-conjugate poles behavior falls into two categories regarding the range of the FD parameter values.

1. $N - 1 < D < N$ - the complex-conjugate poles pairs are situated around the area $z = 0$ and can be either with positive or negative real part depending of a given FD parameter value as can be seen from Fig. 1.

2. $D > N$ - the behavior of the poles is more dynamic. The complex-conjugate poles are positioned mainly in the right half of the unit circle and only the higher order TFs have poles in the left half, as illustrated in Fig. 1. The dashed line with number 3 shows the poles movement when increasing the FD parameter values to infinity.

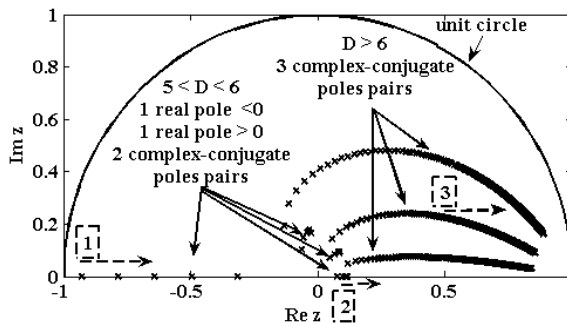


Fig. 1. Possible poles position of real poles (for odd-order TF) and of all the poles of sixth order allpass FD TF.

4. Allpass Sections Sensitivities Study

4.1 First order allpass sections

It follows from Fig. 1 that if a cascade realization of the FD allpass filters would be used, as the possible real pole positions are scattered all around the real axes, first-order allpass sections with low sensitivities for all these positions will be needed. About 20 such sections, including several newly developed, have been investigated and compared in (Stoyanov & Clausert, 1994) and it was shown that several low-sensitivity sections for every real pole-position could be found. We select to use four of them, shown in Fig. 2, namely the ST1 section, providing low-sensitivity for poles near $z=1$, MH1 and SC, having low sensitivity for poles near $z=0$ and SV section for poles near $z=-1$. Their TFs are:

$$H_{ST1}(z) = \frac{-(1-a) + z^{-1}}{1 - (1-a)z^{-1}}; \tag{5}$$

$$H_{MH1}(z) = \frac{-b + z^{-1}}{1 - bz^{-1}}; \tag{6}$$

$$H_{SC}(z) = \frac{-b - z^{-1}}{1 + bz^{-1}}; \tag{7}$$

$$H_{SV}(z) = \frac{1 - c + z^{-1}}{1 + (1 - c)z^{-1}} \tag{8}$$

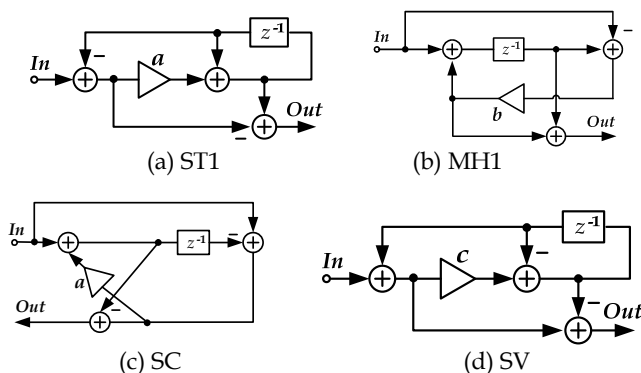


Fig. 2. Different first-order allpass sections.

The closed form solutions for their TF coefficients for given FD parameter D are:

$$a_{ST1} = \frac{2}{D+1}; \quad b_{MH1} = \frac{D-1}{D+1}; \tag{9}$$

$$b_{SC} = -\frac{D-1}{D+1}; \quad c_{SV} = \frac{2D}{D+1}. \tag{10}$$

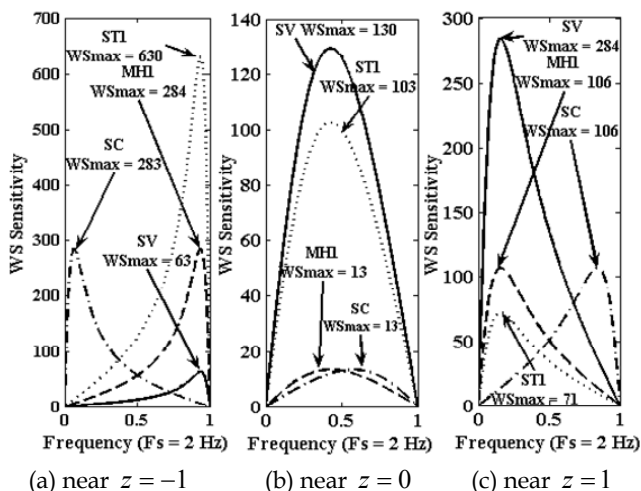


Fig. 3. Worst-case phase-sensitivities of first order allpass sections for different pole-positions.

In Fig. 3 the worst-case phase-response-sensitivities of these four sections are given for realizations with different TF pole positions. It is clearly seen that there exists a proper

choice of sections for every possible pole position and the difference between the maximal values of the sensitivities may reach 10 times.

4.2 Second order allpass sections

There are a great number of second order allpass sections in the literature and we need some preliminary selection among them before starting deeper study. The complex-conjugate poles are positioned mainly in the right half of the unit circle and only rarely (for higher TFs order) in the left half, as illustrated in Fig. 1. Our extensive investigations show that the study, the classification and the selection of second order allpass sections will be eased if those complex-conjugate poles are grouped into 11 zones as shown in Fig. 4 for the upper half of the unit circle. The poles positions of tenth order allpass based FD filter, for example, for values of D in the range $N < D < 50$ will scatter as shown in Fig. 4, but for $N < D < N+1$ (the most typical case) they all will concentrate only in zones 1, 2, 5, 6. This is valid also for TFs of any order. Thus, we will need most often second-order allpass sections with minimized sensitivities for complex-conjugate poles pairs positioned in these zones in order to obtain low-sensitivity FD realization and better FD time accuracy. These zones are not typical for conventional selective filters, whose poles are situated usually near $z = 1$, so we selected initially the most popular sections, having canonic structures and known with low sensitivities. They are the Gray-Markel section (GM2), the Mitra and Hirano sections (MH2A and MH2B), the Kwan sections (KW2A and KW2B) and the low sensitivity section ST2A, shown in Fig. 5 and developed or discussed (together with many other sections with similar sensitivities) in (Topalov & Stoyanov, 1991); (Stoyanov & Nishihara, 1995); (Stoyanov & Kawamata, 1998); (Stoyanov & Kawamata, 2003); (Stoyanov et al., 2005) and in the references there-in. These sections are realizing the following TFs:

$$H_{GM2}(z) = \frac{-a_1 - a_2(1-a_1)z^{-1} + z^{-2}}{1 - a_2(1-a_1)z^{-1} - a_1z^{-2}}; \quad (11)$$

$$H_{MH2A}(z) = \frac{b_1b_2 - b_1z^{-1} + z^{-2}}{1 - b_1z^{-1} + b_1b_2z^{-2}}; \quad (12)$$

$$H_{MH2B}(z) = \frac{b_2 - b_1z^{-1} + z^{-2}}{1 - b_1z^{-1} + b_2z^{-2}}; \quad (13)$$

$$H_{KW2A}(z) = \frac{1 + a_1 - a_2 - (a_1 + a_2)z^{-1} + z^{-2}}{1 - (a_1 + a_2)z^{-1} + (1 + a_1 - a_2)z^{-2}}; \quad (14)$$

$$H_{KW2B}(z) = \frac{d_1 + d_2 - 1 - (d_1 - d_2)z^{-1} + z^{-2}}{1 - (d_1 - d_2)z^{-1} + (d_1 + d_2 - 1)z^{-2}}; \quad (15)$$

$$H_{ST2A}(z) = \frac{1 - 2b - 2(1-b)(1-2a)z^{-1} + z^{-2}}{1 - 2(1-b)(1-2a)z^{-1} + (1-2b)z^{-2}}. \quad (16)$$

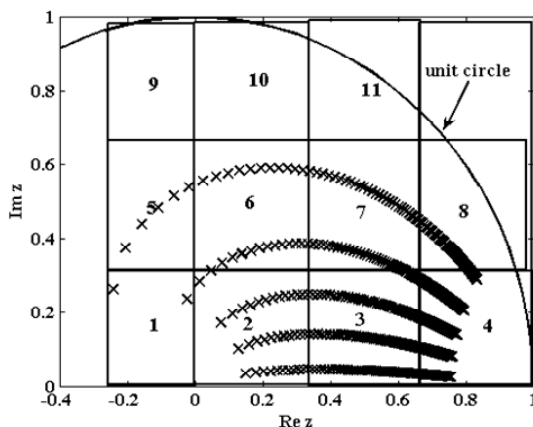


Fig. 4. Zoning of the z-plane for allpass FD TFs pole positions.

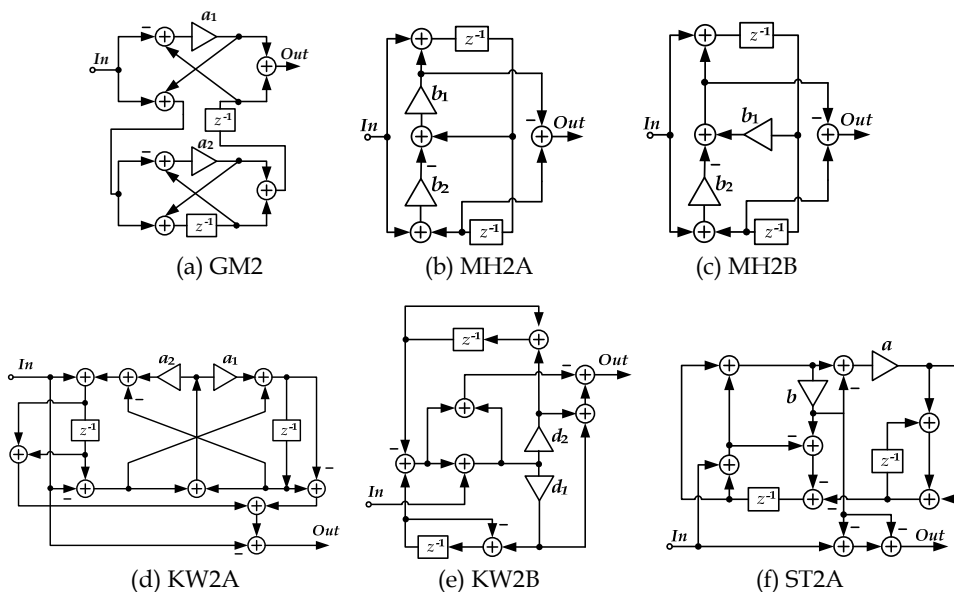


Fig. 5. Different popular canonic second-order allpass sections.

It appeared, however, that all these sections, developed for selective filters applications, are not having enough low sensitivities for poles in zones 1, 2, 5, 6, as shown in Fig. 7, where especially wrong choice is ST2A. We have developed in (Ivanova & Stoyanov, 2007); (Nikolova et al., 2009) a new section, shown in Fig. 6 (we shall call it IS-section) and with minimized sensitivity for the TF poles situated exactly in zone 2. Its transfer function is

$$H_{IS}(z) = \frac{b + (-a - 2b + ab)z^{-1} + z^{-2}}{1 + (-a - 2b + ab)z^{-1} + bz^{-2}}, \tag{17}$$

it is canonic with respect to the number of the multipliers and the delays, its round-off noises are constant and very low and it is structurally lossless and structurally bounded real.

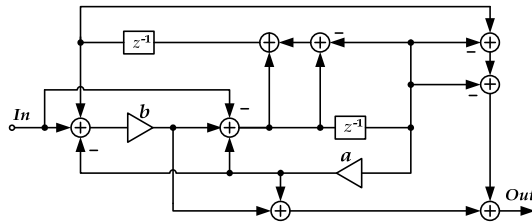


Fig. 6. IS allpass section, suitable for FD filter realizations with TF poles in zone 2.

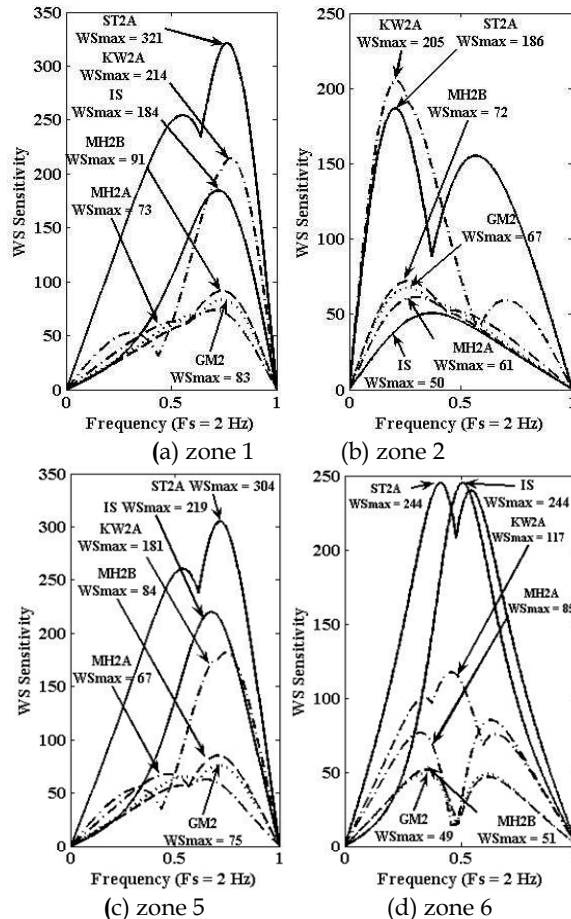


Fig. 7. Worst-case phase-sensitivities of second order allpass sections for TF poles in different zones.

The phase sensitivities of the new allpass section together with these of the other second-order allpass sections were investigated for complex-conjugate pole pairs in zones 1, 2, 5 and 6. The results for the worst-case phase sensitivities are given in Fig. 7. It is obvious that the worst case phase sensitivity of the IS section is the lowest for small values of the FD parameter D ($D \approx N$) which correspond to TF poles situated in zone 2. The other allpass sections suitable for realizations of small values of FD are GM2 and MH2B (zone 6) and GM2 and MH2A (zone 1 and zone 5). KW2A, KW2B and ST2A (and the other numerous known sections) generally cannot be recommended and have to be investigated in every specific case. The TF coefficients as function of D are given in Tables 1-3.

IS		GM2	
a	b	a_1	a_2
$\frac{D-2}{D}$	$\frac{(D-1)(D-2)}{(D+1)(D+2)}$	$\frac{(D-1)(D-2)}{(D+1)(D+2)}$	$\frac{(D-2)(D+2)}{(D^2+2)}$

Table 1. IS and GM2 FD transfer function coefficients.

MH2A		MH2B	
b_1	b_2	b_1	b_2
$2 \frac{(D-2)}{(D+1)}$	$\frac{(D-1)}{2(D+2)}$	$2 \frac{(D-2)}{(D+1)}$	$\frac{(D-1)(D-2)}{(D+1)(D+2)}$

Table 2. MH2A and MH2B FD transfer function coefficients.

KW2A		KW2B	
a_1	a_2	d_1	d_2
$\frac{(D^2-3D-4)}{(D+1)(D+2)}$	$\frac{(D^2+3D-4)}{(D+1)(D+2)}$	$2 \frac{(D-1)}{(D+2)}$	$\frac{6}{(D+1)(D+2)}$

Table 3. KW2A and KW2B FD transfer function coefficients.

5. Low-Sensitivity Design of fixed FD Filters

Having in mind the principles of high-accuracy design from Sect. 2 and taking into account the results obtained here-above, we propose the following design procedure:

1. Apply the Thiran approximation to obtain an allpass TF with order N ensuring a phase-delay error within given limits over the required frequency range. Broadening excessively this range will increase considerably the order N .
2. Decompose the TF to first and second-order terms and check in which zones the poles of these terms are situated.
3. Select or develop new first and second-order allpass sections providing lowest sensitivities for each real or couple of complex-conjugate poles.

4. For poles in some zones, as seen in Figs. 3 and 7, several allpass sections are equally good possible candidates. In such case compose several sets of allpass sections and investigate the overall sensitivity of each set to select the one with the lowest sensitivity. This procedure was applied to obtain an FD allpass structure realizing $D=11.2$. The 11th order TF has five pairs of complex-conjugate poles (two pairs in zone 1 and three - in zone 2) and one real pole, as shown in Fig. 8. The most recommendable (from what follows from Figs. 3 and 7) set of allpass sections is suggested in the same figure, but the other possible four sets have also been considered. The worst-case phase sensitivities of the realizations, corresponding to all the five sets, are shown in Fig. 9.

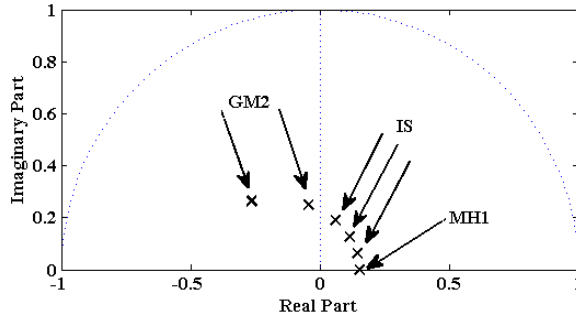


Fig. 8. Pole-position plot of 11th order allpass FD filter realizing $D = 11.2$.

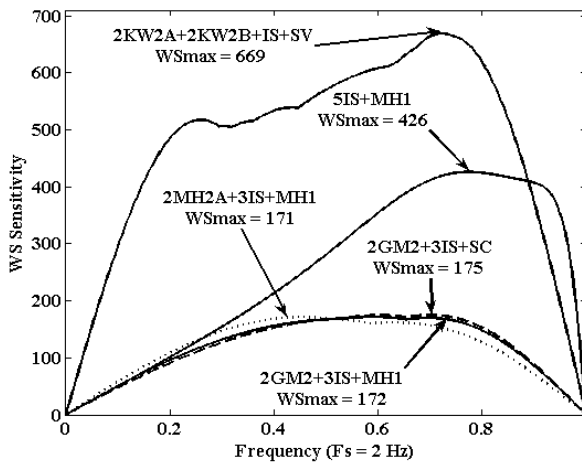


Fig. 9. Worst-case phase-sensitivities of different sets of sections realizing an 11th order allpass-based FD TF with $D = 11.2$.

It is seen from Fig. 9 that the method is working properly and two of the sets are by far worst than the other three. It is amazing that for this specific example there are three sets of allpass sections that are having very similar overall worst-case sensitivity and the final choice has to be made after considering other details, like total number of adders, range of

values of multiplier coefficients and deterioration of the delay response after the coefficients quantization.

The reduction of the overall sensitivity permits a considerable shortening of the coefficients wordlength followed by more efficient multiplierless implementation. We have applied this approach in (Ivanova et al., 2005) and after deriving closed form expressions for the coefficients of the allpass sections given in Sect. 4, we have obtained multiplierless realizations with no more than three adders per coefficient. A further improvement of the multiplierless design was achieved in (Stoyanov et al., 2009) by applying a genetic algorithm to optimize the values of the coefficients within the set of possible values limited by the quantization.

6. Low-Sensitivity Design and Implementation of Variable FD Filters

6.1 Design procedure

The calculation of the coefficients obtained by Thiran approximation (1) include too many division operations that are making difficult tuning of such circuit in real time. In (Makundi et al., 2001) the coefficients (1) have been presented as:

$$\begin{aligned} \hat{a}_k &= (-1)^k \binom{N}{k} \frac{\prod_{n=0}^{k-1} (d+n)}{\prod_{n=1}^k (d+N+n)} \prod_{n=1}^N (d+N+n) = \\ &= (-1)^k \binom{N}{k} \prod_{n=0}^{k-1} (d+n) \prod_{n=k+1 \leq N}^N (d+N+n) = \sum_{l=1}^N \hat{e}_{lk} d^l, \text{ for } k=1, 2 \dots N, \end{aligned} \tag{18}$$

where d is the fractional part of the phase-delay and $d = D - N$. Then, the allpass TF (2) was given in the form

$$H_{AP}(z) = \frac{g(d)[\hat{a}_N + \dots + \hat{a}_1 z^{-(N-1)}] + z^{-N}}{1 + g(d)[\hat{a}_1 z^{-1} + \dots + \hat{a}_N z^{-N}]}, \tag{19}$$

and the coefficient $g(d)$ was approximated using the truncated Maclaurin series as

$$\begin{aligned} g(d) &= \frac{1}{\prod_{n=1}^N (d+N+n)} = \\ &\cong \frac{N!}{(2N)!} \prod_{n=1}^N \left[1 + \sum_{k=1}^I (-1)^k \left(\frac{d}{N+n} \right)^k \right] \cong \sum_{i=0}^I g_i d^i, \end{aligned} \tag{20}$$

where I is the order of the approximating polynomial. The structure obtained through this method is called "gathering structure". Even though very famous, this structure has many drawbacks:

- (a) it contains a great number of multipliers and adders leading to long critical paths;
- (b) as any direct structure it has higher sensitivity;
- (c) for higher TF order N there is a big difference between the smallest and the biggest coefficient (about 10^2 for $N = 2, I = 2$; about 10^3 for $N = 2, I = 3$ and 10^5 for $N = 3, I = 3$), requiring very large wordlength.

To avoid them, the following representation was proposed in (Cho et al., 2007):

$$\begin{aligned}
 a_k &= \sum_{i=0}^I g_i d^i \cdot \sum_{l=1}^N \hat{e}_{lk} d^l = \sum_{m=1}^{I+N} \left[\sum_{l=1}^m g_{m-l} \hat{e}_{lk} \right] d^m = \\
 &\cong \sum_{m=1}^P \left[\sum_{l=1}^m g_{m-l} \hat{e}_{lk} \right] d^m = \sum_{m=1}^P \left[\sum_{n=1}^N c_{mn} \right] d^m,
 \end{aligned} \tag{21}$$

where P is the order of the approximating polynomial and it is in the range $N \leq P \leq N + I$. We shall call the variable structure obtained by using (21) "Cho-Parhi-structure". It has less multipliers and shorter critical path, compared to gathering structure, and similar values of the coefficients c_{mn} (21).

We found in (Nikolova & Stoyanov, 2008) that it is possible to obtain even more efficient variable realizations by expressing each transfer function coefficients a_k (2) as a Taylor series expansion with respect to d and then to truncating after the linear, quadratic or cubic term ($T = 1, 2, 3$) depending on the desired accuracy. To achieve the tuning in real time we propose the following design procedure:

1. Select of the allpass TF order corresponding to given requirements (desired fractional delay value d and/or the bandwidth with maximally flat phase delay response).
2. Obtain an allpass FD filter using Thiran approximation.
3. Taylor series expansion of each TF coefficient and truncation after the linear (when only adjustment of the phase delay is required), quadratic or cubic term (if tuning over larger range of values of the phase delay is required).
4. Realize all the multiplier coefficients as composite multipliers (see Figs. 10, 11).

The proposed design procedure is simple to use and the obtained structures have no critical path. The method can be applied for an arbitrary TF order but in the cases of first and second order TFs it allows to implement structures different from direct form and to minimize the sensitivity of the realizations. For the low-sensitivity structure IS (Fig. 6), for example, the coefficients are expressed by d as

$$a = \frac{d}{d+2}; \quad b = \frac{d(d+1)}{(d+3)(d+4)}. \tag{22}$$

After expanding (22) to Taylor series and truncating after the quadratic or the cubic term, we get correspondingly:

$$a = \frac{1}{2}d - \frac{1}{4}d^2; \quad b = \frac{1}{12}d + \frac{5}{144}d^2; \tag{23}$$

$$a = \frac{1}{2}d - \frac{1}{4}d^2 + \frac{1}{8}d^3; \quad b = \frac{1}{12}d + \frac{5}{144}d^2 - \frac{47}{1728}d^3. \tag{24}$$

All these coefficients have homogenous structure, they do not include division operation and can be realized as composite multipliers containing fixed and variable multipliers. The composite multiplier realizations for second and third order Taylor approximation of a are shown in Fig. 10 and Fig. 11.

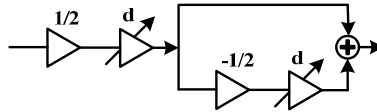


Fig. 10. Composite variable multiplier realization of a (23) after a second order Taylor approximation.

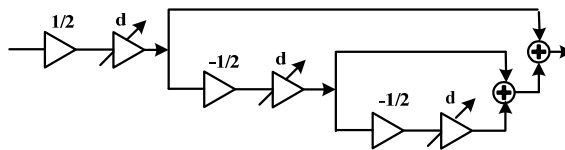


Fig. 11. Composite variable multiplier realization of a (24) after a third order Taylor approximation.

It is worth mentioning that some of the fixed multiplier coefficients values, obtained after the Taylor series expansions (23), (24), are machine representable (they have values $\pm 2^{\pm i}$) and will be realized by using only shifts and adds. In fact, in Fig. 10 and 11 all fixed multipliers are of this type and thus the complexity of the composite multipliers is kept very low. The RDI-method (Hachhabiboğlu et al., 2007), is using two N^{th} order allpass FD TFs approximating different FD values D_1 and D_2 to obtain a new allpass FD filter with phase delay time D_i such that $D_1 < D_i < D_2$. The denominator of (2) (the denominators of the two initial allpass transfer functions) is represented as (Hachhabiboğlu et al., 2007):

$$A_i(z) = \begin{cases} [1 - r_i z^{-1}] \times \prod_{k=1}^{(N-1)/2} [1 - c_{i,k}^2 z^{-2}], & N \text{ odd} \\ \prod_{k=1}^{N-1} [1 - c_{i,k}^2 z^{-2}], & N \text{ even} \end{cases} \quad (25)$$

where $\{c_{i,k}, c_{i,k}^*\}$ is k^{th} complex-conjugate pole pair and r_i is the real pole of the filter with TF $H_i(z)$ (2). The complex-conjugate poles (for real pole is the same procedure) are sorted with respect to their angles and are paired according to their angular proximity. The interpolated complex poles are calculated from the paired poles as

$$c_{\text{int},k} = [1 - \rho]c_{1,k} + \rho c_{2,k}, \quad (26)$$

where ρ is a constant between 0 and 1. This can be realized using only adders and multipliers, as shown in (Hacıhabiboğlu et al., 2007), and the phase-delay time D_i can be tuned within the range $D_1 < D_i < D_2$ by trimming only the constant ρ . This method is not connected to any particular realization of the initial allpass filters of order N , so the sensitivity cannot be an object of consideration in this case. Two disadvantages are readily seen, however: quite complicated circuitry (two allpass filters plus four additional multipliers) and narrow range of tuning of D with growing error of tuning in the middle of this range.

6.2 Accuracy investigations

To compare the accuracy of the first three methods, considered in Sect. 6.1, we have designed and investigated realizations and tuning in the range $1.5 < D < 2.5$ (i.e. $d = \pm 0.5$) of second order allpass FD filters. For the polynomial approximation of the TF coefficients truncation after the third order term was used, i.e. $I = 3$ (20), $P = 3$ (21) and $T = 3$ (for

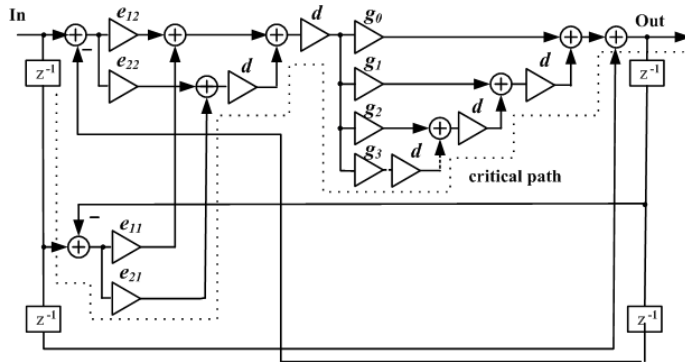


Fig. 12. Gathering structure realizing a second-order variable FD allpass filter (with $I = 3$).

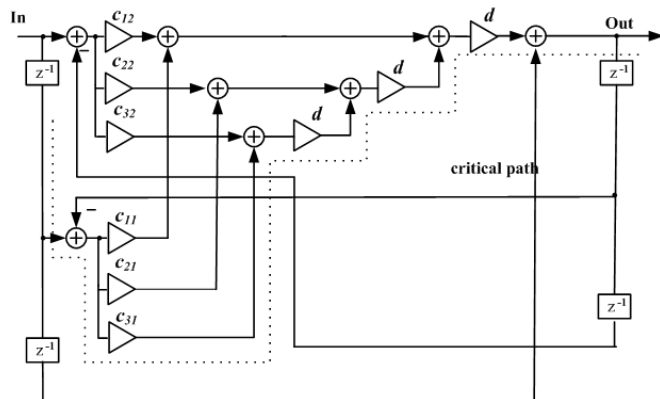


Fig. 13. Cho-Parhi structure realizing a second-order variable FD allpass filter ($P = 3$).

our method) circuit-diagrams so obtained are given in Figs. 12, 13 and 14. For our method, the IS-section (Fig. 6) with composite multipliers was used. The values of the coefficients of the three realizations are given in Table 4, Table 5, and Eq. (24), correspondingly.

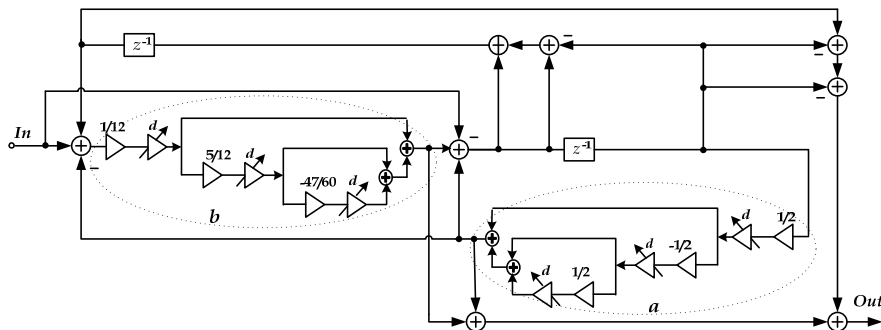


Fig. 14. IS structure realizing a second-order variable FD allpass filter with $T = 3$.

\hat{a}_k		$g(d)$
$\hat{e}_{11} = -8$	$\hat{e}_{12} = 1$	$g_0 = 0.083333$
$\hat{e}_{21} = -2$	$\hat{e}_{22} = 1$	$g_1 = -0.048611$
		$g_2 = 0.021412$
		$g_3 = -0.0084394$

Table 4. TF coefficients of gathering structure.

$c_{11} = -0.666667$	$c_{21} = 0.222222$	$c_{31} = -0.074074$
$c_{12} = 0.083333$	$c_{22} = 0.034722$	$c_{32} = -0.027199$

Table 5. TF coefficients of Cho-Parhi-structure.

In Fig. 15 the worst-case phase sensitivities of the three realizations for several values of the fractional part d of the phase-delay time are given. It is seen that our approach and the Cho-Parhi method are decreasing considerably the sensitivity, compared to that of the gathering structure, for $d = \pm 0.5$ (our structure is behaving better than that of Cho-Parhi for positive values of d and it is opposite for the negative values). For small values of d our structure is the best, but generally the IS and the Cho-Parhi structures are having similar sensitivities. The possible explanation for this is that the Cho-Parhi approach, when reducing the range of values of the multiplier coefficients, compared to those of the gathering structure, is decreasing the largest values. It is well known, that when the values of the multiplier coefficients are decreased, the sensitivities to these coefficients are decreased too.

In Table 6 the complexities of the three variable realizations are compared. The Cho-Parhi- and the IS- variable structures are having an equal number of multipliers (three of the multiplier coefficients in IS are machine representable and will be realized by using only adds and shifts), but the IS-structure has only two delays, it is not having a critical path and it will be shown in the Experiments that it is behaving better in a limited wordlength environment.

The RDI-method is not considered here, as it is not connected to some specific realization. Its accuracy is investigated in the Experiments (Sect. 7).

	Variable IS structure	Gathering structure	Cho-Parhi structure
	$T = 3$	$I = 3$	$P = 3$
Multipliers	12 (9)	13	9
Adders	14	9	8
Delay elements	2	4	4

Table 6. Comparison of the complexity of the structures.

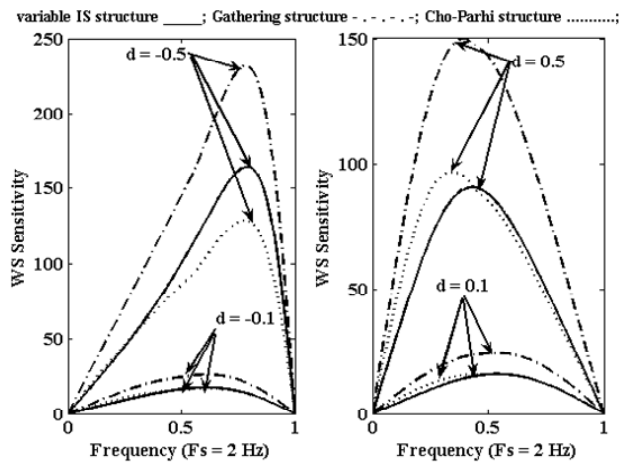


Fig. 15. Worst-case phase-sensitivities of second-order allpass based FD filter ($I = 3, P = 3, T = 3$).

7. Experiments

In order to verify the proposed low-sensitivity design procedure and to investigate how the FD time accuracy is maintained after coefficient quantization, we have designed and simulated all the five realizations considered in Sect. 5 (11th order TF realizing $D=11.2$). The phase delay responses of the quantized TFs are given in Fig. 16 (without these of 2GM2+3IS+SC, almost fully coinciding with 2GM2+3IS+MH1, as it might be anticipated from Fig. 7). The higher overall sensitivity of the 2KW2A+2KW2B+IS+SV-structure ($WS_{max}=669$ in Fig. 9) is the reason for its poor performance in a limited wordlength environment – the phase delay error for low frequencies is considerable even after a mild quantization down to 4 bits in CSD code (11.235 instead of 11.2 in Fig. 16a) and this response is almost totally destroyed for 2 bits wordlength. For the best structure (2MH2A+3IS+MH1) this error is almost negligible – 11.195 instead of 11.2 (Fig. 16d) and is quite acceptable even for wordlength of only 2 bit. The other sets from Sect. 5 are behaving as it could be predicted from Fig. 7. The main conclusion from these

experiments is that our approach is working very successfully and is ensuring a considerable improvement of the accuracy in a limited wordlength environment.

In order to observe and compare the tuning accuracy of the three methods and variable structures from Sect. 6 (gathering structure, Cho-Parhi-structure and IS-structure), we have designed three second order allpass FD filters with third order TF-coefficients approximation ($I=3, P=3, T=3$) and a given fractional delay parameter value $d=0.3$. The results after the coefficient quantization are given in Fig. 17. Because of the lower sensitivity of the IS structure the tuning accuracy is higher than that of the gathering structure and Cho-Parhi structure even when the TF coefficients are quantized to 2 significant bits (in CSD code). The deviations from the desired phase delay (0.3 samples) of variable IS FD filter near DC for 4, 3 and 2 bits are correspondingly smaller than 10^{-5} , -0.002 and -0.0179 , while these of the gathering structure are -0.0029 , -0.009 and -0.041 and of the Cho-Parhi-structure -0.0018 , -0.0086 and -0.041 .

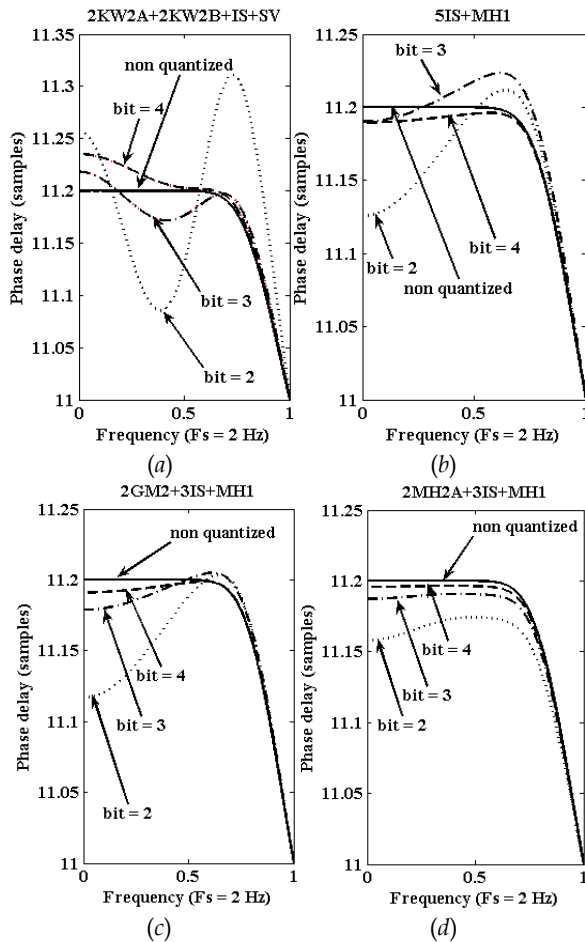


Fig. 16. Phase delay responses of the quantized structures from Sect. 5 designed for $D=11.2$.

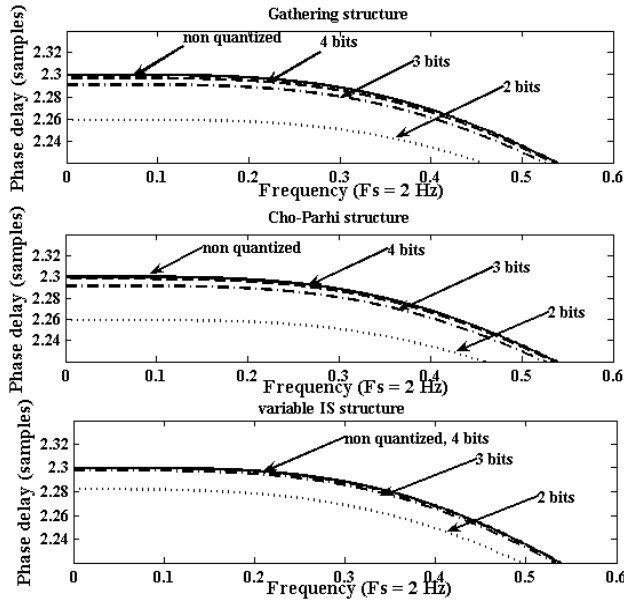


Fig. 17. Wordlength dependence of the accuracy of tuning of the phase delay of second order allpass FD filters realized as gathering-, Cho-Parhi- and IS-structures for $d = 0.3$ in a case of $I = 3, P = 3, T = 3$.

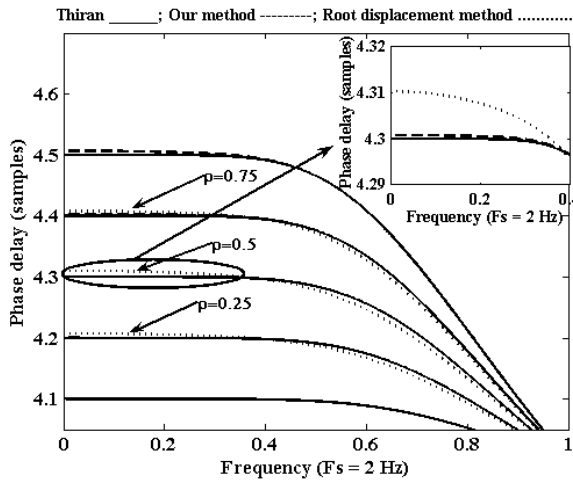


Fig. 18. Tuning accuracy comparison of the root-displacement method and our method for 4th order allpass FD filters for different values of D .

As the RDI-method is not connected to a specific structure, we have compared its accuracy to our method by simulating the tuning of the FD from 4.1 to 4.5 of the TFs with $N = 4$. For our method a direct-form structure was used and the coefficients have been approximated

by third-order Taylor polynomials. It is seen from Fig. 18 that the phase-delay of the RDI TF is having a higher error compared to that of our method and is losing its maximally-flat behavior for all intermediate values of D (note that for $D = 4.5$ there is no tuning in the case of RDI-method and thus no error will appear). It was found, additionally, that there is no direct connection between the desired value of the phase-delay D and the value of the tuning factor ρ (26) and this uncertainty in tuning cannot be avoided.

8. Conclusions and Future Work

In this chapter, a new approach to achieve a high accuracy of implementation and tuning of fixed and variable allpass-based fractional delay filters through sensitivity minimizations have been proposed. The method is based on a phase-sensitivity minimization of each individual first- and second-order allpass section in the filter cascade realization. It was shown that the poles of the FD TFs are taking positions not typical for the conventional filters. Then, after studying the possible combinations of real and complex-conjugate poles for different values of the FD parameter D and of the TF order N , it was proposed to divide the unit-circle to 11 zones and it was shown that FD TF poles (obtained using Thiran approximation) of most practical cases are located only in four of them and very often – in only one (zone 2). The behavior of the most popular allpass sections when having poles in these zones was investigated and it was shown that the proper selection of the sections is very important when trying to minimize the overall sensitivity. A new second-order allpass section, providing low sensitivity for zone 2 (and thus very suitable for high accuracy FD realizations) was developed by the authors. This section was turned also to tunable and high tuning accuracy was achieved. A new approach to obtain tunable allpass FD filters was developed and it was compared with the other known methods. It was shown also that the low sensitivity so achieved permits a very short coefficient wordlength, i.e. efficient multiplierless implementations, higher processing speed and lower power consumption.

The proposed approach to design low-sensitivity allpass-based FD filters could be easily applied to further improve the performance of different allpass-based FD filters, obtained using most of the design methods overviewed in Sect. 1.3. provided that the allpass TFs of the filters and sub-filters in these realizations are clearly identifiable.

It is well known that all IIR digital filters are producing different types of parasitic noises, especially when a fixed-point arithmetic is employed. These noises have not been investigated in the present chapter. It is also well known, however, that low sensitivity and low noises usually go together and as only allpass sections with very low sensitivities are considered here and they are selected and used in frequency ranges and TF pole-positions zones where they would exhibit their lowest sensitivities, it might be expected that they will have very low level of the noises. These noises are expected to be low also because of the specific pole-positions of the FD filters – their TF poles are usually situated in the central part of the unit circle (as shown in Sect. 3.2), while noises are dangerously growing when the poles are approaching the unit-circle (typical for highly selective amplitude filters). All this should be verified, however, and it would be done in the future work.

Next problem that should be addressed in the future is that of the transients, typical for all recursive realizations and affecting especially strongly all tunable IIR structures. These transients may compromise the proper work of the system for quite considerable time-intervals, following the moments of trimming of some multipliers, and more efficient than

the presently known methods to decrease these effects should be developed and investigated.

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The new technology advances provide that a great number of system signals can be easily measured with a low cost. The main problem is that usually only a fraction of the signal is useful for different purposes, for example maintenance, DVD-recorders, computers, electric/electronic circuits, econometric, optimization, etc. Digital filters are the most versatile, practical and effective methods for extracting the information necessary from the signal. They can be dynamic, so they can be automatically or manually adjusted to the external and internal conditions. Presented in this book are the most advanced digital filters including different case studies and the most relevant literature.

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