Design of Two-Dimensional Digital Filters Having Variable Monotonic Amplitude-Frequency Responses Using Darlington-type Gyrator Networks

Muhammad Tariqus Salam and Venkat Ramachandran, Fellow, IEEE
Department of Electrical and Computer Engineering
Concordia University
Montreal, Canada

Abstract
This paper develops a design of two-dimensional (2D) digital filter with monotonic amplitude-frequency responses using Darlington-type gyrator networks by the application of Generalized Bilinear Transformation (GBT). The proposed design provides the stable monotonic amplitude-frequency responses and the desired cutoff frequency of the 2D digital filters. This 2D recursive digital filter design includes 2D digital low-pass, high-pass, band-pass and band-elimination filters. Design examples are given to illustrate the usefulness of the proposed technique.

Index Terms – Stability, monotonic response, GBT, gyrator network.

1. Introduction
Because of recent growth in the 2D signal processing activities, a significant amount of research work has been done on the 2D filter design [1] and it is seen that monotonic characteristics in frequency response of a filter is getting more popular. The filters with the monotonic characteristics are one of the best filters for the digital image, video and audio (enhancement and restoration) [2]. The filters are widely accepted in the applications of medical science, geographical science and environment, space and robotic engineering [1]. For example, medical applications are concerned with processing of chest X-Ray, cine angiogram, projection of frame axial tomography and other medical images that occurs in radiology, nuclear magnetic resonance (NMR), ultrasonic scanning and magnetic resonance imaging (MRI) etc. and the restoration and enhancement of these images are done by the 2D digital filters [3].

The design of 2D recursive filters is difficult due to the non-existence of the fundamental theorem of algebra in that the factorization of 2D polynomials into lower order polynomials and the testing for stability of a 2D transfer function (recursive) requires a large number of
computations. But, the major drawbacks of the recursive filters are their lower-order realizations and computational intensive design techniques. Several design techniques of 2D recursive filter have been reported in the literature [2], [4] – [9] and most of these designs have problems of computational complexity, stability and unable to provide variable magnitude monotonic characteristic. A design technique of 2D recursive filters have been shown which met simultaneously magnitude and group delay specifications [4], although the technique has the advantage of always ensuring the filter stability, the difficulties to be encountered are computational complexity and convergence [5]. In [6], 2D filter design as linear programming problem has been proposed, but, this tends to require relatively long computation time. In [7], a filter design has been shown using the two specifications as the problem of minimizing the total length of modified complex errors and minimized it by an iterative procedure. Difficulties of the design obtain for two-dimensional stability testing at each iteration during the minimization procedure.

One way to ensure a 2D transfer function is stable is if the denominator of the transfer function is satisfied to be a Very Strict Hurwitz Polynomial (VSHP) [8] and that can ensure a transfer function that there is no singularity in the right half of the biplane, which can make a system unstable. In [9]-[11], stable 2D recursive filters have been designed by generation of Very Strict Hurwitz Polynomial (VSHP), but it is not guaranteed to provide the stable monotonic amplitude-frequency responses. Several filter designs with monotonic amplitude frequency response has been reported [12] – [16], but to the best of our knowledge, filter design with variable monotonic amplitude frequency response is not proposed yet.

In this paper, 2-D digital filters with variable monotonic amplitude frequency responses are designed starting from Darlington-type networks containing gyrators and doubly-terminated RLC-networks. The extension of Darlington-synthesis to two-variable positive real functions is given in [17], [18]; but they do not contain gyrators. From the 2-D stable transfer functions so obtained, the GBT [19] is applied to obtain 2-D digital functions and their properties are studied. The designed filters are used in the image processing application.

2. THE TWO BASIC STRUCTURES CONSIDERED

Two filter structures are considered for 2D digital recursive filters design and both structures are taken from Darlington-synthesis [20]. Figures 1(a) and (b) show the two structures considered in this paper.

The impedances of the filters are replaced by doubly-terminated RLC networks and the overall transfer function will be of the form

\[
H(s_1, s_2, g) = \sum_{\rho=0}^{M_x} \sum_{\nu=0}^{N_x} N_{\rho \nu}(g) s_1^\rho s_2^\nu \\
\sum_{\kappa=0}^{M_y} \sum_{\mu=0}^{N_y} D_{\kappa \mu}(g) s_1^\kappa s_2^\mu
\]

(1)

where the coefficients of \(H(s_1, s_2, g)\) are functions of \(g\).
In this paper, second-order Butterworth and Gargour & Ramachandran filters [19] are considered as doubly terminated RLC networks. For simplicity, each gyrator network is classified into three cases, such as the impedances of gyrator network are replaced by the second-order Butterworth filter and Gargour & Ramachandran filter are called case-I and case-II respectively. The impedances of gyrator network are replaced by second-order Butterworth and Gargour & Ramachandran filters is called case-III.

3. Filter 1

Transfer functions of case-I, case-II and case-III of Filter 1 (Figure 1(a)) provide stable functions, when denominators of the cases are VSHPs. This can be verified easily by the method of Inners [21]. The impedances of the cases are modified by first applying the GBT given by

\[ s_i = k_i \frac{z_i + a_i}{z_i + b_i}, \quad i = 1, 2 \]

To ensure stability, the conditions to be satisfied are:

\[ k_i > 0, \quad |a_i| \leq 1, \quad |b_i| \leq 1, \quad a_i b_i < 0 \]  

and then applying the inverse bilinear transformation [22]. In such a case, the inductor impedance becomes

\[ s_i L \rightarrow k_i L \frac{(1 - a_i) + s_i (1 + a_i)}{(1 + b_i) + s_i (1 - b_i)} \]
and the impedance of a capacitor becomes
\[
\frac{1}{s_i C} \rightarrow \frac{1}{k_i C} \left(1 - a_i\right) + s_i \left(1 + a_i\right)
\] (4b)

For example, the transfer function of the case-I represents as
\[
H_G(s_1, s_2, g) = \frac{S_1 R_1 S_2^T}{S_1 R_2 S_2^T}
\] (5)

where,
\[
S_1 = \begin{bmatrix} 1 & s_1 & s_1^2 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 & s_2 & s_2^2 \end{bmatrix},
\]
\[
R_1 = \begin{bmatrix}
2(1 + g + g^2) & 0.7 + 0.7g + 4.2g^2 & 1.5g^2 \\
0.7 + 4.2(g + g^2) & 0.23 + 1.5g + 9.1g^2 & 3g^2 \\
1.5g + 1.5g^2 & 3.1g + 0.5g^2 & g^2
\end{bmatrix},
\]
\[
R_2 = \begin{bmatrix}
3(1 + g^2) & 1 + 4.4g^2 & 1.4g^2 \\
2.8 + 6.4g^2 & 0.92 + 9.6g^2 & 3g^2 \\
0.72 + 2.1g^2 & 0.24 + 3.2g^2 & g^2
\end{bmatrix}
\]
The coefficients are dependent on the value and sign of \(g\).

The GBT [19] is applied to the transfer function (5) and it is shown that the 2D digital low-pass filters are obtained for the lower values of \(g\) and the 2D digital high-pass filters are obtained for the higher values of \(g\). But the amplitude-frequency response of the Filter 1 is constant for \(g = 1\).

If monotonicity in the magnitude response is desired, the values of \(a_i, b_i\) and \(k_i\) have to be adjusted and these are given in Table 1. Figure 2 shows the 3-D magnitude plot of such a low-pass filter.

<table>
<thead>
<tr>
<th>(g)</th>
<th>(a_i)</th>
<th>(b_i)</th>
<th>Case-I</th>
<th>Case-II</th>
<th>Case-III</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>-0.9</td>
<td>0.9</td>
<td>0.09 &gt; (k_i) &gt; 0</td>
<td>82 &gt; (k_i) &gt; 0</td>
<td>0.1 &gt; (k_i) &gt; 0</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.9</td>
<td>0.5</td>
<td>0.4 &gt; (k_i) &gt; 0</td>
<td>1.5 &gt; (k_i) &gt; 0</td>
<td>0.9 &gt; (k_i) &gt; 0</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.5</td>
<td>0.9</td>
<td>205 &gt; (k_i) &gt; 0</td>
<td>95 &gt; (k_i) &gt; 0</td>
<td>100 &gt; (k_i) &gt; 0</td>
</tr>
</tbody>
</table>

Table 1. The ranges of \(k_i\) satisfy the monotonic characteristics in the amplitude-frequency response of 2D Low-pass Filter (Filter 1).
Fig. 2. 3D magnitude plot and contour plot of the 2D digital low-pass filter (Filter 1) when \( g = 0.01 \).

### 4. Filter 2

The impedances \( Z_1, Z_2 \) and \( Z_3 \) of Filter 2 (Fig.1(b)) are replaced by impedances of the second-order RLC filters. The resultant transfer function is unstable, because, the denominator is indeterminate [8].

In order to generate a stable analog transfer function \( H_{MB2}(s_1, s_2, g) \), the impedances \( Z_1 \) and \( Z_2 \) of Filter 2 (Figure 1(b)) are replaced by the impedances of the second-order RLC filters and the third impedance \( (Z_3) \) is replaced by a resistive element. As a result, the denominator of the case-I, case-II and case-III of Filter 2 are VSHPs.

Transfer function of the case-I (Filter 2) is represented as

\[
H_{MB2}(s_1, s_2, g) = \frac{S_1 R_3 S_T}{S_1 R_4 S_T} \tag{6}
\]

where,

\[
R_3 = \begin{bmatrix}
2 + 6g & 0.68 + 8.8g & 2.8g \\
0.68 + 8.8g & 0.22 + 12g & 3.4g \\
2.8g & 3.4g & g
\end{bmatrix},
\]

\[
R_4 = \begin{bmatrix}
1.6 + 6g^2 & 16 + 8.8g^2 & 4.4 + 2.8g^2 \\
16 + 8.8g^2 & 15 + 12g^2 & 3.9 + 3.4g^2 \\
4.4 + 2.8g^2 & 3.4 + 3.9g^2 & 1 + g^2
\end{bmatrix}.
\]

The coefficients of numerator are dependent on the value and sign of ‘\( g \)’, but the coefficients of denominator are dependent only the value of ‘\( g \)’.
The GBT [19] is applied to (6) and it is shown that the 2D digital low-pass filters are obtained for the lower values of $g$, the 2D digital high-pass filters are obtained for the higher values of $g$ and inverse filter responses are obtained for the opposite sign of $g$.

If monotonicity in the magnitude response is desired, the values of $g$, $a_i$, $b_i$ and $k_i$ have to be adjusted and these are given in Table 2 and Table 3. Figure 3 shows the 3-D magnitude plot of such a high-pass filter.

<table>
<thead>
<tr>
<th>$g$</th>
<th>$a_i$</th>
<th>$b_i$</th>
<th>Case-I</th>
<th>Case-II</th>
<th>Case-III</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-0.9</td>
<td>0.9</td>
<td>$0.2 &gt; k_i &gt; 0$</td>
<td>$0.2 &gt; k_i &gt; 0$</td>
<td>$0.2 &gt; k_i &gt; 0$</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.9</td>
<td>0.5</td>
<td>$0.7 &gt; k_i &gt; 0$</td>
<td>$0.6 &gt; k_i &gt; 0$</td>
<td>$0.5 &gt; k_i &gt; 0$</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.5</td>
<td>0.9</td>
<td>$4 &gt; k_i &gt; 0$</td>
<td>$3 &gt; k_i &gt; 0$</td>
<td>$3.2 &gt; k_i &gt; 0$</td>
</tr>
</tbody>
</table>

Table 2. The ranges of $k_i$ satisfy the monotonic characteristics in the amplitude-frequency response of 2D Low-passFilter (Filter2).

<table>
<thead>
<tr>
<th>$a_i$</th>
<th>$b_i$</th>
<th>$k_i$</th>
<th>Case-I (Filter 1)</th>
<th>Case-I (Filter 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.1</td>
<td>1</td>
<td>$0.3 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 0$, $0.4 &gt; g \geq -0.1$</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.1</td>
<td>5</td>
<td>$0.1 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 9$, $0.2 &gt; g \geq -0.01$</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.1</td>
<td>10</td>
<td>$0.05 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 13$, $0.08 &gt; g \geq -0.005$</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>1</td>
<td>$0.7 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 3.2$, $0.5 &gt; g \geq -0.1$</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>5</td>
<td>$0.4 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 4.8$, $0.3 &gt; g \geq -0.04$</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>10</td>
<td>$0.18 &gt; g &gt; 0$</td>
<td>$\infty &gt; g \geq 7$, $0.2 &gt; g \geq -0.04$</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.9</td>
<td>1</td>
<td>$\infty &gt; g \geq 0$</td>
<td>$\infty &gt;</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.9</td>
<td>5</td>
<td>$4.6 &gt; g \geq -1.5$</td>
<td>$\infty &gt; g \geq 3.2$, $0.5 &gt; g \geq -0.1$</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.9</td>
<td>10</td>
<td>$1 &gt; g \geq -0.67$</td>
<td>$\infty &gt; g \geq 3.4$, $0.41 &gt; g \geq -0.09$</td>
</tr>
</tbody>
</table>

Table 3. The ranges of $g$ for the various parameter-values of the GBT, where the 2D digital high-pass filter contains the monotonic characteristics.

Fig. 3. 3D magnitude plot and contour plot of the 2D digital high-pass filter (Filter 2) when $g = -0.7$. 
5. Band-pass and band-elimination filters

In order to design the 2D digital band-pass and band-elimination filter, the following GBT [23] is applied to a stable analog transfer function.

\[ s_i = k_{1i} \frac{(z_i + a_{1i})}{(z_i + b_{1i})} + k_{2i} \frac{(z_i + a_{2i})}{(z_i + b_{2i})} \]  
(7)

To ensure stability, the conditions to be satisfied are:

\[ k_{1i} > 0, \ k_{2i} > 0, \ |a_{1i}| \leq 1, \ |a_{2i}| \leq 1, \ |b_{1i}| \leq 1, \ |b_{2i}| \leq 1, \ a_{1i}b_{1i} < 0, \ a_{2i}b_{2i} < 0 \]  
(8)

Fig. 4. 3D magnitude plot 2D digital band-pass filter (\(g = -0.001\)).

Fig. 5. 3D magnitude plot of the 2D digital band-elimination filter (\(g = -0.5\)).
The 2D digital band-pass filters and the 2D digital band-elimination filters are obtained depending on the values and sign of $g$ which is shown in Table 4. Figures 4 and 5 show the 3D magnitude plots of the digital band-pass and band-elimination filter respectively, which are obtained from Case-I (Filter1) and case-I (Filter2).

6. Digital filter Transformation

The proposed digital filter transformation provides the low-pass to high-pass filter (Table 5) or the band-pass to band-elimination filter (Table 6) or vice-versa transformation by regulating the value or sign of $g$. However, the low-pass to band-pass or the high-pass to band-elimination filter or vice versa transformation is obtained by regulating the value or sign of $g$ and the parameters of the GBT as shown in Figure 6. In Filter 1, the digital filter transformations are obtained by regulating the value of $g$. However, in Filter 2, the digital filter transformations are obtained by regulating the value or sign of $g$.

![Fig. 6. Block diagram of the digital filter transformation](image)

Table 4. The ranges of $g$ of the case-I To obtain the 2D digital band-pass and band-elimination filters.
Design of Two-Dimensional Digital Filters Having Variable Monotonic Amplitude-Frequency Responses Using Darlington-type Gyrator Networks

The 2D digital band-pass filters and the 2D digital band-elimination filters are obtained depending on the values and sign of \( g \) which is shown in Table 4. Figures 4 and 5 show the 3D magnitude plots of the digital band-pass and band-elimination filter respectively, which are obtained from Case-I (Filter1) and Case-I (Filter2).

6. Digital Filter Transformation

The proposed digital filter transformation provides the low-pass to high-pass filter (Table 5) or the band-pass to band-elimination filter (Table 6) or vice-versa transformation by regulating the value or sign of \( g \). However, the low-pass to band-pass or the high-pass to band-elimination filter or vice versa transformation is obtained by regulating the value or sign of \( g \) and the parameters of the GBT as shown in Figure 6. In Filter 1, the digital filter transformations are obtained by regulating the value of \( g \). However, in Filter 2, the digital filter transformations are obtained by regulating the value or sign of \( g \).

![Fig. 6. Block diagram of the digital filter transformation](image)

<table>
<thead>
<tr>
<th>Filter</th>
<th>Low-pass Filter</th>
<th>High-Pass Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-I (Filter 1)</td>
<td>( g = 0.01 )</td>
<td>( g = 50 )</td>
</tr>
<tr>
<td>Case-II (Filter 1)</td>
<td>( g = 0.03 )</td>
<td>( g = 100 )</td>
</tr>
<tr>
<td>Case-III (Filter 1)</td>
<td>( g = 0.05 )</td>
<td>( g = 115 )</td>
</tr>
<tr>
<td>Case-I (Filter 2)</td>
<td>( g = 10 )</td>
<td>( g = -10 )</td>
</tr>
<tr>
<td>Case-II (Filter 2)</td>
<td>( g = 8 )</td>
<td>( g = -8 )</td>
</tr>
<tr>
<td>Case-III (Filter 2)</td>
<td>( g = 9 )</td>
<td>( g = -9 )</td>
</tr>
</tbody>
</table>

Table 5. Digital filter transformation from 2D low-pass filter to high-pass filter.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Band-pass Filter</th>
<th>Band-stop Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-I (Filter 1)</td>
<td>( g = 0.01 )</td>
<td>( g = 100 )</td>
</tr>
<tr>
<td>Case-II (Filter 1)</td>
<td>( g = 0.03 )</td>
<td>( g = 150 )</td>
</tr>
<tr>
<td>Case-III (Filter 1)</td>
<td>( g = 0.05 )</td>
<td>( g = 50 )</td>
</tr>
<tr>
<td>Case-I (Filter 2)</td>
<td>( g = 5 )</td>
<td>( g = -5 )</td>
</tr>
<tr>
<td>Case-II (Filter 2)</td>
<td>( g = 25 )</td>
<td>( g = -25 )</td>
</tr>
<tr>
<td>Case-III (Filter 2)</td>
<td>( g = 100 )</td>
<td>( g = -100 )</td>
</tr>
</tbody>
</table>

Table 6. Digital filter transformation from 2D band-pass filter to band-elimination filter.

7. Applications

The designed 2D digital filters can use in the various image processing applications, such as image restoration, image enhancement. The band-width of the designed digital filter can be controlled by the magnitude of \( g \) and the parameters of the GBT. As a result, the 2D digital filter provides facilities as required in the image processing applications.

For illustration, a standard image (Lena) (Figure 7 (a)) [1] is corrupted by gaussian noises and the degraded image (Figure 7 (b)) is passed through the 2D digital low-pass filters for de-noising purposes. Table 7 shows the quality of the reconstructed images is measured in term of mean squared error (MSE) [24] and peak signal-to-noise ratio (PSNR) [24] in decibels (dB) for the most common gray image [3]. Average PSNR of the reconstructed images are obtained by Filter2 is higher than Filter1, but, some cases, Filter1 provides better performance than Filter2. Overall, it is seen that the significant amount of noise is reduced from a degraded image by the both filters.

<table>
<thead>
<tr>
<th>Filter</th>
<th>( g )</th>
<th>MSEns</th>
<th>PSNRns(dB)</th>
<th>MSEout</th>
<th>PSNRout(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-I (Filter1)</td>
<td>0.001</td>
<td>629.9926</td>
<td>20.1374</td>
<td>257.3906</td>
<td>24.0249</td>
</tr>
<tr>
<td>Case-II (Filter1)</td>
<td>0.001</td>
<td>636.2678</td>
<td>20.0944</td>
<td>257.7424</td>
<td>24.0189</td>
</tr>
<tr>
<td>Case-III (Filter1)</td>
<td>0.001</td>
<td>636.3893</td>
<td>20.0936</td>
<td>273.4251</td>
<td>23.7624</td>
</tr>
<tr>
<td>Case-I (Filter2)</td>
<td>0.001</td>
<td>630.9419</td>
<td>20.1309</td>
<td>256.4292</td>
<td>24.0411</td>
</tr>
<tr>
<td>Case-II (Filter2)</td>
<td>0.001</td>
<td>634.0169</td>
<td>20.1098</td>
<td>244.2690</td>
<td>24.2521</td>
</tr>
<tr>
<td>Case-III (Filter2)</td>
<td>0.001</td>
<td>639.1828</td>
<td>20.0746</td>
<td>253.6035</td>
<td>24.0893</td>
</tr>
</tbody>
</table>

Table 7. Denoising experiment on Lena image (gaussian noise with mean = 0, variance = 0.01 is added into the image)
Fig. 7. (a) The original image of Lena, (b) the noisy image with Gaussian noise (variance = 0.01), (c) the reconstructed image by case I (Filter 1) when $g = 0.001$ ($PSNR_{out} = 24.3337$ dB), (f) the reconstructed image by case I (Filter 2) when $g = 0.001$ ($PSNR_{out} = 24.2287$ dB)

8. Conclusion

A new design of 2-D recursive digital filters has been proposed and it includes low-pass, high-pass, band-pass and band-elimination filters using Darlington-type gyrator network. It is seen that the behavior of the gyrator filter is changed not only for the values of resistance, capacitance and inductance of the filter, but also the value and sign of $g$. The coefficients of the transfer functions of Filter 1 and Filter 2 are function of $g$. The ranges of $g$ are defined for attaining stable monotonic characteristics in the pass-band region, because $g$ has control over the frequency responses of the filters.

9. References


The new technology advances provide that a great number of system signals can be easily measured with a low cost. The main problem is that usually only a fraction of the signal is useful for different purposes, for example maintenance, DVD-recorders, computers, electric/electronic circuits, econometric, optimization, etc. Digital filters are the most versatile, practical and effective methods for extracting the information necessary from the signal. They can be dynamic, so they can be automatically or manually adjusted to the external and internal conditions. Presented in this book are the most advanced digital filters including different case studies and the most relevant literature.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
