Noise in Electronic and Photonic Devices

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1. Introduction

Modern state-of-art in the solid state technology has advanced at an almost unbelievable pace since the advent of extremely sophisticated IC fabrication technology. In the present state of microelectronic and nanoelectronic fabrication process, number of transistors embedded in a small chip area is soaring aggressively high. Any further continuance of Moore’s law on the increase of transistor packing in a small chip area is now being questioned. Limitations in the increase of packing density owes as one of the reasons to the generation of electrical noise. Not only in the functioning of microchip but also in any type of electronic devices whether in discrete form or in an integrated circuit noise comes out inherently whatever be its strength. Noise is generated in circuits and devices as well. Nowadays, solid state devices include a wide variety of electronic and optoelectronic/photonic devices. All these devices are prone in some way or other to noise in one form or another, which in small signal applications appears to be a detrimental factor to limit the performance fidelity of the device. In the present chapter, attention would be paid on noise in devices with particular focus on avalanche diodes followed by a brief mathematical formality to analyze the noise. Though, tremendous amount of research work in investigating the origin of noises in devices has been made and subsequent remedial measures have been proposed to reduce it yet it is a challenging issue to the device engineers to realize a device absolutely free from any type of noise. A general theory of noise based upon the properties of random pulse trains and impulse processes is forwarded. A variety of noises arising in different devices under different physical conditions are classified under (i) thermal noise (ii) shot noise (iii) 1/f noise (iv) g-r noise (v) burst noise (vi) avalanche noise and (vii) non-equilibrium Johnson noise. In micro MOSFETs embedded in small chips the tunneling through different electrodes also give rise to noise. Sophisticated technological demands of avalanche photodiodes in optical networks has fueled the interest of the designers in the fabrication of low noise and high bandwidth in such diodes. Reduction of the avalanche noise therefore poses a great challenge to the designers. The present article will cover a short discussion on the theory of noise followed by a survey of works on noise in avalanche photodiodes.

1.1 Mathematical formalities of noise calculation

Noise is spontaneous and natural phenomena exhibited almost in every device and circuit. It is also found in the biological systems as well. However, the article in this chapter is

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limited to the device noise only. Any random variation of a physical quantity resulting in the unpredictability of its instantaneous measure in the time domain is termed as noise. Though time instant character of noisy variable is not deterministic yet an average or statistical measure may be obtained by use of probability calculation over a finite time period which agrees well with its macroscopic character. In this sense, a noise process is a stochastic process. Such a process may be stationary or non-stationary. In stationary stochastic, the statistical properties are independent of the epoch (time window) in which the noisy quantity is measured; otherwise it is non-stationary. The noise in devices, for all practical purposes, is considered to be stochastic stationary. The measure of noise of any physical quantity, say \( x(t) \), is given by the probability density function of occurrence of the random events comprising of the noisy quantity in a finite time domain, say \( (T) \). This probability function may be first order or second order. While first order probability measure is independent of the position and width of the time-window, the second order probability measure depends. Further, the averaging procedure underlying the probability calculation may be of two types: time average and ensemble average. The time averaging is made on observations of a single event in a span of time while the ensemble averaging is made on all the individual events at fixed times throughout the observation time. In steady state situation, the time average is equivalent to the ensemble average and the system is then said to be an ergodic system. As \( x(t) \) is a real process and vanishes at \( t \to -\infty \) and \( +\infty \) one may Fourier transform the time domain function into its equivalent frequency domain function \( X(j\omega) \), \( \omega \) being the component frequency in the noise. Noise at a frequency component \( \omega \) is measured by the average value of the spectral density of the noise signal energy per unit time and per unit frequency interval centered around \( \omega \). This is the power spectral density (PSD) of the noise signal \( S_{x} \) of the quantity \( x \). The PSD of any stationary process (here it is considered to be the noise) is uniquely connected to the autocorrelation function \( C(t) \) of the process through Wiener- Khintchine theorem (Wiener, 1930 & Khintchine, 1934). The theorem is stated as

\[
S_{x}(\omega) = 4 \int_{0}^{\infty} C(t) \cos \omega t \, dt, \quad \tau \text{ being the correlation time.}
\]

Noise can also be conceptualized as a random pulse train consisting of a sequence of similarly shaped pulses randomly, in the microscopic scale, distributed with Poisson probability density function in time. Each pulse \( p(t) \) is originated from single and independent events which by superposition give rise to the noise signal \( x(t) \), the random pulse train. The PSD of such noises is given by the Carson theorem which is

\[
S_{x}(\omega) = 2a^2 | F(j\omega) |^2, \quad F \text{ being the Fourier transform}
\]

of the time domain noise signal \( x(t) \) and ‘\( a^2 \)’ being the mean square value of all the component pulse amplitudes or heights. Shot noise, thermal noise and burst noise are treated in this formalism. The time averaging is more realistically connected with the noise calculations of actual physical processes.

To model noise in devices, the physical sources of the noise are to be first figured out. A detailed discussion is made by J.P. Nougier (Nougier, 1981) to formulate the noise in one dimensional devices. The method was subsequently used by several workers (Shockley et.al., 1966; McGill et.al., 1974; van Vliet et.al., 1975) for calculation of noise. In a more
general approach by J.P.Nougier et.al.(Nougier et.al.,1985) derived the noise formula taking into account space correlation of the different noise sources. Perhaps the two most common types of noises encountered in devices are thermal noise and shot noise.

1.1.1 Noise calculation for submicron devices

Conventional noise modeling in one dimensional devices is done by any of the three processes viz. impedance field method (IFM), Langevin method and transfer impedance method. In fact, the last two methods are, in some way or other, derived form of the IFM. The noise sources at two neighbouring points are considered to be correlated over short distances, of the order of a few mean free path lengths. Let $V_{1,2}$ be the voltage between two electrodes 1 and 2. In order to relate a local noise voltage source at a point $r$ (say) to a noise voltage produced between two intermediate electrodes $1$ and $2$ a small ac current $\delta I \exp (j\omega t)$ is superimposed on the dc current $j_0 (r)$ at the point $r$. The ac voltage produced between $1$ and $2$ is given by

$$\delta V (r - dr, f) = Z (r - dr, f). \delta I ;$$

$Z$ being the impedance between the point $r$ and the electrode $2$ (the electrode $1$ is taken as reference point).

Thus, the overall voltage produced between the electrodes $1$ and $2$ is given by $\delta I. \text{Grad} Z (r,f). dr$. Grad $Z$ is the impedance field. With this definition of the impedance field, the noise voltage between $1$ and $2$ can be formulated as

$$S_V (f) = \int \int \text{Grad} Z (r, f) S_j (r, r', f). \text{Grad} Z' (r', f) d^3 (r) d^3 (r')$$

This is the three dimensional impedance formula taking into account of the space correlation of the two neighbouring sources (Nougier et.al., 1985).

2. Thermal noise

Thermal noise is present in resistive materials that are in thermal equilibrium with the surroundings. Random thermal velocity of cold carriers gives rise to thermal noise while such motion executed by hot electrons under the condition of non-equilibrium produces the Johnson noise. However, the characteristic features are not differing much and as such, in the work of noise, thermal and Johnson noises are treated equivalently under the condition of thermal equilibrium. It is the noise found in all electrical conductors. Electrons in a conductor are in random thermal motion experiencing a large number of collisions with the host atoms. Macroscopically, the system of electrons and the host atoms are in a state of thermodynamic equilibrium. Departure from the thermodynamic equilibrium and relaxation back to that equilibrium state calls into play all the time during the collision processes. This is conceptualized microscopically as a statistical fluctuation of electrical charge and results in a random variation of voltage or current pulse at the terminals of a conductor (Johnson,1928). Superposition of all such pulses is the thermal noise fluctuation. In this model, the thermal noise is treated as a random pulse train. One primary reason of noise in junction diodes is the thermal fluctuation of the minority carrier flow across the junction. The underlying process is the departure from the unperturbed hole distribution in the event of the thermal motion of the minority carriers in the n-region. This leads to
relaxation hole current across the junction and also within the bulk material. This tends to restore the hole distribution in its original shape. This series of departure from and restoration of the equilibrium state cause the thermal noise in junction diode. Nyquist calculated the electromotive force due to the thermal agitation of the electrons by means of principles in thermodynamics and statistical mechanics (Nyquist, 1928). Application of Carson’s theorem (Rice, 1945) on the voltage pulse appearing at the terminals due to the mutual collisions between the electrons and the atoms leads to the expressions of power spectral densities (PSDs) of the open circuit voltage and current fluctuations as:

\[ S_V(\omega) = \frac{4 k T R}{(1 + \omega^2 \tau^2)} \]

and

\[ S_I(\omega) = \frac{4 k T / R}{(1 + \omega^2 \tau^2)} \]

respectively, where \( k \) is the Boltzmann constant, \( T \) is the absolute temperature, \( R \) the resistive element, \( \omega \) the Fourier frequency and \( \tau \) being the dielectric relaxation time. In practice, the frequencies of interest are such that \( \omega^2 \tau^2 << 1 \).

3. Shot noise

Shot noise, on the other hand, is associated with the passage of carriers crossing a potential barrier. It is, as such, very often encountered in solid state devices where junctions of various types are formed. For example, in p-n junction diodes the depletion barrier and in Schottky diodes the Schottky barrier. These are the sources of shot noises in p-n junction devices and metal-semiconductor junction devices. Shot noise results from the probabilistic nature of the barrier penetration by carriers. Thus in the event of the current contributing carriers passing through a barrier, the resulting current fluctuates randomly about a mean level. The fluctuations reflect the random and discrete nature of the carriers. A series of identically shaped decaying pulses distributed in time domain by Poisson distribution law may be a model representation of such shot noise. The spectral density of the noise power (PSD) of such Poisson distributed of the random pulse train in time domain is given by Carson’s theorem (Rice, 1945)

\[ \overline{S_{\text{shot}}}(\omega) = 2 \nu a^2 = 2 q I \]

assuming impulse shape function of the noise; \( \nu \) and \( a^2 \) being the frequency and mean square amplitude of the pulse.

But \( \nu = I/q \) and as all the pulse amplitudes are same being equal to \( q \) so

\[ \overline{S_{\text{shot}}}(\omega) = 2 \nu a^2 = 2 q I \]

\( q \) and \( I \) being the electron charge and magnitude of the mean current. The spectral structure of shot noise is thus frequency independent and is a white noise.

In recent years, shot noise suppression in mesoscopic devices has drawn a lot of interest because of the potential use of these devices and because the noise contains important
information of the inherent physical processes as well. Gonzalez et.al.(Gonzalez et.al.1998) found, on the basis of the electrons’ elastic scatterings, a universal shot noise suppression factor of 1/3 in non-degenerate diffusive conductors. Strong shot noise suppression has been observed in ballistic quantum point contacts, due to temporally correlated electrons, possibly a consequence of space charge effect due to Coulomb interaction (Reznikov et.al.1995). Phase coherent transport may also be a cause of shot noise suppression. Resonant tunneling of electrons through the GaAs well embedded in between two barriers of AlGaAs sets another example of suppression of shot noise (Davies et.al.,1992). Shot noise can be directly calculated from the temporal autocorrelation function of current.

4. Burst noise

Burst noise manifests itself as a bistable, step waveform of same amplitude distributed randomly in a time domain of observation. In early literatures, it is sometimes called “random telegraph signal” because of its close resemblance with telegraph signal. The burst noise appears in junction devices e.g. diodes, transistors etc., in tunnel diodes and also in carbon resistors as well. Burst noise is not much observed in devices and is seen not so common as for other types of noises. It appears that such a noise is not universally present in any devices. A typical burst noise waveform is sketched in fig.1. It consists of a random, step waveform which is superimposed with a white noise. It is believed that, the burst noise in forward junctions is due to the crystallographic defects present in the vicinity of the junction while in reversed junctions it is due to an irregular on-off switching of a surface conduction path as a result of random thermal fluctuations. Hsu and Whittier (Hsu & Whittier, 1969) dealt with an issue of determining whether the burst noise in forward junctions is a surface effect or volume effect. Extensive research has suggested that the burst noise in forward biased junctions is more a surface effect than a volume effect. Updated conclusion of the origin of the burst noise to be a surface effect has received much support. This conclusion is arrived at on the basis of noise observed as a step waveform generated by microplasmas (Champlin, 1959).

![Fig. 1. Typical waveform of current burst noise (a) as observed with white noise superimposed and (b) after clipping.](www.intechopen.com)
finally results in such a noise: an avalanche effect is initiated by a carrier either generated within or diffusing in the high field region. With building up of the current, the voltage drop along the high internal series resistance also increases until the voltage drop across the high field region falls below the breakdown value at which point the secondary emission of carriers stops. Some of the carriers released in the process may be trapped in the immediate vicinity of the microplasma. Subsequent to the end of the secondary emission, some of the carriers that are re-emitted from the traps trigger the action again. The process repeats by itself resulting in a series of short avalanche current bursts until by any probability there is no further re-emission of secondary carriers to trigger fresh avalanche. A number of theoretical predictions (McIntyre, 1961, 1966, 1999; Haitz, 1964; ) were made to explain the noise in reverse biased diodes. The main suggestion came out of these theories was to consider the diode noise in two regimes e.g. avalanche and microplasma. Marinov et.al. (Marinov et.al., 2002) investigated the low frequency noise in rectifier diodes in its avalanche mode of working region and showed conclusively that in the breakdown region of the avalanche diode two competitive processes e.g. impact ionization and microplasma switching and conducting balance each other. The correlation of these two processes gives rise to a statistically fluctuating current wave of low frequency in the diode.

5. Low frequency noise

Electrical current through semiconductor devices are seen to exhibit low frequency fluctuations (generally below $10^5$ Hz.) with 1/f spectrum. The ubiquitous 1/f fluctuations i.e. noise is still a question as to its unique origin. An enormous pool of data is there on 1/f noise and different theories as opposed to other are tried to explain this noise. The 1/f noise, also known as low frequency or Flicker noise, is an intriguing type of fluctuations seen not only in the electron devices but also found in natural phenomena like earthquakes, thunderstorms and in biological systems like heart beats, blood pressure etc. Physical origin of 1/f noise is still a debatable issue. This type of noise is the limiting factor for devices like high electron mobility transistors (HEMTs) and MOS transistors and, in fact, unlike in JFETs this is very dominant MOSFETs. A number of theoretical models on LF noise in MOS transistors are based on surface related effects. There is no universally accepted unique theory or physical model of 1/f noise. Yet, in general, it is suggested that the fluctuating mechanism is a two state physical process with a characteristic time constant $\tau$. Each fluctuator produces a spectral density of Lorentzian spectrum with a specific characteristic time. If these characteristic times of the fluctuators vary exponentially with some parameter e.g. energy or distance, and if, in addition, there is a uniform distribution of the fluctuators in $\tau$ then a 1/f spectrum results. Further, there is some support for this noise in semiconductors to be linked with phonons, although no specific and unique mechanism has yet been proposed convincingly. The most complete model of noise caused by phonon fluctuation has been given by Jindal and van der Ziel (Jindal & van der Ziel, 1981). The conductance depends on the product of mobility $\mu$ and carrier density $n$. There has been considerable discussion about which of these two quantities fluctuate? Is it the mobility fluctuation $\Delta \mu$ or carrier density fluctuation $\Delta n$ or both simultaneously to fluctuate the conductance? Accordingly, there are two competing models that are invoked to figure out the reason of 1/f noise: the mobility fluctuation model devised by F.N.Hooge (Hooge,1982) and the carrier density fluctuation model by A.L McWhorter (McWhorter,1955). In McWhorter model, carrier trapping resulting in immobilization and de-trapping resulting in remobilization of carriers produce the carrier number fluctuations in the current. It is
believed that the number fluctuations of the carriers in the MOS channel due to tunneling between the surface states and traps in the oxide layer is the reason of LF noise in such devices. Assumption of electron-phonon scattering mechanism is also supposed to contribute to the resistance fluctuations and, in turn, to the generation of 1/f noise. A large number papers covering the works on 1/f flicker noise have been published by a number of authors. Recent interest in GaN-related compound materials have led to investigating the noise behavior in these materials. For example, there have been reported values of the Hooge parameter in GaN/AlGaN/sapphire HFET devices to be higher than 10^{-2}.

6. **Generation – recombination noise**

This is the noise generated as a consequence of random trapping and detrapping of the carriers contributing to the current conduction through a device. These trapping centers are the Shockley-Reed- Hall (SRH) centres of single energy states found in the band gap or in depletion region or in partially ionized acceptor/ donor level in a semiconductor. The statistics of generation –recombination (g-r) through single energy level centers in the forbidden gap of the semiconductor were formulated independently by Hall (Hall, 1952) and jointly by Shockley and Reed [Shockley & Read Jr.,1952]. The g-r noise is apparent mainly in junction devices. During a carrier diffusing from one or other of the bulk regions into the depletion region it may fall into the SRH energy trap center where it will stay for a time that is characteristic of the trap itself. This produces a recombination current pulse. Superposition of all such pulses constitutes a recombination noise current in the external circuit. Similarly, when a generation event occurs at a center, the generated carrier is swept through the depletion region by the electric field towards the bulk region. This produces a generation noise current pulse. Several authors (van der Ziel, 1950; du Pre,1950; Surdin,1951; Burgess,1955) explained the low frequency 1/f noise as a superposition of many such g-r noises and assuming the 1/\tau distribution in a very wide variation of relaxation times \tau.

7. **Noise in photonic devices**

With an exception of high frequency photonic devices, important noises are 1/f noise and shot noise. A very short report on the different types of noise in different photonic devices are given here. Mainly the devices are optical fibers, light emitting diodes (LEDs), laser diodes (LDs), avalanche photodetectors (APDs) etc.

Noise in semiconductor waveguides working on the principle of total internal reflection can be studied by considering the variation of the bandgap with temperature. This is because of the fact that the bandgap itself depends upon the refractive index of the material (Herve & Vandamme, 1995) by

$$n^2 = 1 + \left[ \frac{13.6}{E_g + 3.4} \right]^2$$

and for the relative temperature coefficient of refractive index it was proposed in ref. (Harve & Vandamme, 1995) as

$$\frac{1}{n} \frac{dn}{dt} = \frac{(n^2 - 1)^{3/2}}{13.6n^2} \left[ \frac{dE_g}{dT} + 2.5 \times 10^{-5} \right]$$
Any index difference between the core and cladding materials affects the Rayleigh scattering loss (Ohashi et al., 1992) in the fiber. Further, variation in the index with temperature causes variation in the scattering loss. The resulting fluctuation in the fiber loss shows the character of 1/f noise (van Kemenade et al., 1994). The 1/f fluctuations in optical systems had been studied by Kiss (Kiss, 1986).

8. Avalanche noise

At sufficiently high electric field, the accelerated free carriers (electrons and holes) by their drift motion in the semiconductor may attain so high kinetic energy as to promote electrons from the valence band to the conduction band by transfer of kinetic energy to the target electrons of the valence band by collision. In effect, this is the ionization of the atoms of the host lattice. The process of this ionization by impact is known as impact ionization. Many such individual primary impacts initiating the ionization process turn into repeated secondary impacts. These secondary impacts depend on the existing energy plus fresh gain in their kinetic energies from the electric field. Anyway, such multitude of uncontrollable and consecutive ionizing events result in the generation of a large multiplication of free carriers. This is what is known as “avalanche multiplication”. A huge multiplication in the number of both types of carriers, in the form of electron-hole pairs (EHPs) takes place by the process of such avalanche multiplication. The strength of ionization of a carrier is measured by its ionization coefficient and is defined by the number of ionizing collisions the carrier suffers in unit distance of its free travel. In other words, it is the ionization rate per unit path length. The minimum energy needed to ensure an impact ionization is called the ionization threshold energy. The ionization rates (also known as ionization coefficients) of electrons and holes are, in general, different and are designated by \( \alpha \) and \( \beta \) respectively. The rates are strongly dependent on the impact threshold.

There exists a probability by which the EHPs may be generated also a little bit below the threshold by highly energetic primary carriers that bombard against the valence electrons and help them to tunnel through and pass on to the conduction band. This is the tunneling-impact ionization that effectively reduces the ionization threshold (Brennan et al., 1988). Avalanche multiplication occurs in large number of electronic devices viz. p-n junction operated in reverse breakdown voltage, JFET channel under high gate voltage, reverse biased photodiode etc. In almost a majority of devices such carrier multiplication degrades the normal operation and is the limiting factor to be cared in order to save the devices from damage. On the other hand, in case of the photo-devices e.g. photodiodes, phototransistors etc. the carrier multiplication plays the key role in operating the device. Photodiodes using the principle of avalanche multiplication of carriers are known as avalanche photodiodes (APDs). These APDs are used in optical communication systems as receivers of the weak optical signals and to convert it into a strong electrical signal by the process of carrier multiplication by avalanche impact ionization. Wide bandwidth APDs are now one of the interesting areas of research work in the field of digital communication systems, transmission of high gigabit-frequency optical signal etc. However, the ionizing collisions, the key factor in the working of such APDs, are highly stochastic by nature. This results in the creation of random number of EHPs for each photo-generated carrier undergoing random transport. Moreover, the randomness in the incoming photon flux adds to the randomness in the carrier multiplication both in temporal as well as in spatial scale. This results in what is known as multiplication or avalanche noise. In some literatures it is also
termed as excess noise. The original signal is masked by this excess noise and the signal purity is obliterated.

A detailed analysis of the multiplication noise was done by Tager (Tager, 1965) considering the two ionizing coefficients to be equal while in McIntyre’s (McIntyre, 1966, 1973) work the analysis was made considering the two coefficients to be different. In the approaches of these papers continuous ionization rates were considered for both the carrier types, on the assumption that the multiplication region to be longer compared to the mean free path for an ionizing impact to occur. The noise current per unit bandwidth following McIntyre (McIntyre, 1966) is given by

\[ \overline{I}^2 = 2q I_0 M^2 F \]

where \( I_0 \) is the primary photocurrent, \( M \) is the current multiplication and \( F \) is the excess noise factor.

The validity of the continuous ionization rates for both the carriers is reasoned because of extremely large number of ionizing collisions per carrier transit. In all these conventional analyses a local field model is visualized wherein the coefficients were regarded to be the functions only of the local electric fields. It could explain the noise behavior well for long multiplication i.e. long avalanche regions.

For short regions, however, the analyses could not work and for that reason the validity of the local field effect was questionable. For short avalanche region, Lukaszek et.al. (Lukaszek et al., 1976) reported for the first time that the continuous multiplication description of avalanche process is not proper for the analysis of short region diode because here very few ionizing collisions take place per carrier transit. A very important effect, “dead space effect”, may be overlooked in case of long regions but in no way for short regions. This assertion is justified if the dead space (or, “dead length”) definition in relation to ionizing collision is understood. Dead space, for impact ionization, to take place is the minimum distance to be covered by an ionizing carrier from its zero or almost zero kinetic energy to attain a threshold energy to ensure an ionizing impact. Conflicting descriptions of the impact ionizations found in literatures raised confusions as to the exact nature of the dead length. It is reported through an investigation (Okuto & Crowell, 1974) that the average value of the dead space would effectively be increased for two possible reasons: one for the scattering of the carriers and consequently resulting in a longer path length to attain the threshold and secondly, because the nascent carriers at the point of just attaining the threshold are not so probabilistic (Marsland, 1987) to induce impact ionization but instead becomes more probabilistic with energy increasing non-linearly over the threshold. Based on these ideas, a parameter “\( p \)” signifying the degree of softness or hardness of the threshold is considered in subsequent works on avalanche ionization. Ideally, for no scattering the average dead length is smallest and is equal to \( l_0 = \varepsilon_{th} / qE_{th} \) and \( E \) being considered to be a hard threshold and electric field respectively, \( q \) the charge of the carrier. As the number scatterings are increased the dead length increases and the degree of hardness of the threshold softens. Early workers used conventionally the hard threshold which resulted in some errors. Several publications (van Vliet et al., 1979; Marsland et al., 1992; Chandramouli & Maziar, 1993; Dunn et al., 1997; Ong et al., 1998) were made to investigate the nonlocal nature of impact ionization. In another approach, Ridley (Ridley, 1983) for the first time introduced completely a different model based on lucky-drift mechanism for impact ionization. Subsequently, some other workers (Burt, 1985; Marsland, 1987) used the model in
a little modified form of the original model of Ridley (Ridley, 1983) and verified with existing experimental results. In the original model or in its derivatives, the carrier motion is divided into two parts viz. the ballistic part and the lucky drift part. In the ballistic part, carriers suffer no collisions whereas in the lucky drift part carriers undergo collisions. In an attempt to thermalize the dynamic process of the carriers’ motion with the crystal, energy relaxation or momentum relaxation is taken help of. Hayat et al. (Hayat et al., 1992) formulated a recurrence method to estimate the excess noise factor. Ong et al. (Ong et al., 1998) devised a very simple model to study the multiplication noise in avalanche photodiode by incorporating randomly generated ionization path lengths and the hard threshold concept. The model is shown to be in excellent agreement with the results derived by Monte Carlo model.

A more accurate analysis for avalanche effect especially for short regions was suggested by McIntyre (McIntyre, 1999) considering the road map of the carriers’ which includes the history of all the ionizations within the avalanche region. This reflects the fact that the impact ionization rate at a point depends simultaneously on three factors viz. (i) the local value of the electric field at that point (ii) the location of generation of the carrier and (iii) the gradient of the electric field i.e. the field profile in between the generation location and the ionizing location. Considering non-local effects and the carriers’ transport history McIntyre (McIntyre, 1999) presented approximate analytical expressions for the position dependent ionization coefficients. The results shown are in close agreement with those obtained from experimental measurement of noise in GaAs PIN photodiode. An exact calculation of the ionization probabilities with much more flexibilities in modeling the APDs may be achieved only with full band Monte Carlo technique (Bufler et al., 2000). The Monte Carlo (MC) simulation method has widely been accepted to be a reliable tool of investigating successfully a great variety of transport phenomena in semiconductor devices and materials (Kosina et al., 2000; Reggiani et al., 1997; Kim & Hess, 1986). The MC simulation offers a direct reproduction of microdynamics of the physical processes of statistical nature on the computer. The traditional drift-diffusion models rely on the assumption of equilibrium transport. They are therefore open to the question of their applicability in studying the non-equilibrium transport of hot electrons taking part in impact ionization events. Further, with the downscaling of electron devices, including the APDs as well, number of scatterings are reduced; this leads to quasiballistic and nonlocal transport; as a result the distribution function no longer remains in equilibrium.

Among the other methods, (Ridley, 1987; Herbert, 1993; Chandramouli et al. 1994) to study the impact ionization in submicron devices the MC technique of simulation has proved to be a most reliable tool as it does not suffer from any disadvantages of averaging procedures inherent in other methods. The method is recognized as the most rigorous one for carrier noise extraction as it allows the appropriate correlation functions to be calculated in a natural way from time averaging over a multi-particle history simulated during a sufficiently long time interval. At any point of time during the computer run the simulation can be stopped so that the positions of all the carriers in the real as well as in the k- space may be recorded. The frequency response of the noise is then calculated. Checked if there is sufficient accuracy, the simulation is ended; otherwise it is repeated until the desired accuracy is arrived at. Although a full band Monte Carlo (FBMC) technique (Chandramouli et al. 1994) gives a more precise and accurate result, yet the simple analytic band Monte Carlo (ABMC) method is capable of reproducing all the important high field features (Dunn et al. 1997; Di Carlo et al., 1998). An extremely large number of ionizing collisions (≈ 5x10^5)
are needed to yield an adequate statistics for the simulation purpose. This makes the FBMC method an impractical one because of the requirement of huge memory and very long run time of the computer. Recently, Ghosh et al. (Ghosh & Ghosh, 2008) used the ABMC method to study and calculate the excess noise in heterojunction APDs. The ABMC simulation is based on the hard threshold dead space effect in the displaced exponential model of distribution of random ionizing path lengths.

In the present article, the author puts forward a report of their study (Ghosh & Ghosh, 2008) of excess noise in heterojunction avalanche photodetector by Monte Carlo simulation. The MC simulation attracts much attention as it can investigate a device operation mechanism through carrier distribution dynamics and potential distribution profile. The simulation is based upon the hard threshold dead space consideration in the displaced exponential model of the distribution of ionization path lengths. As example, a material system InP / InGaAs is taken for the purpose. This heterostructure photodiode has been developed for an APD in the 1 – 1.6 µm. wavelength region for optic fiber communication system (Susa et al., 1980; Stillman et al., 1982) and for a switching photodiode in an optoelectronic switch (Hara et al., 1981). Noise in devices may be minimized by either of the two processes viz. tailoring the bandgap profile (Capasso et al., 1983) and engineering the electric field profile (Hu et al., 1996). Introduction of a heterojunction may help the less energetic carriers flowing through the large bandgap material to gain sufficient energy to ionize the low bandgap material. This results in a lower ionizing path length of electrons and longer ionizing path length of holes in the low bandgap material. In consequence, an appreciably different ionization coefficients of the electrons and holes are obtained at the band edge discontinuities. Excess noise may thus be reduced. As for the second process, it may be noted that with increasing field strength, the dead length becomes comparable to the mean ionization path length and thereby the dead length effect on the avalanche process appears to be quite significant. In the process, the carriers enter the multiplication region with high kinetic energy derived from the strong electric field existing at the sharply peaked band shape at the heterojunction. Such initial energy serves to reduce the dead space followed by the avalanche-inducing carrier.

The dead space effect on the excess noise is considered using a simplified model of Hayat et al. (Hayat et al., 2002). Here, the carriers are assumed to be injected with fixed energies in an electric field E, say. Ionisation probability of such injected projectile is set to zero within the limit of the dead length \( l_0 \). The probability distribution function (PDF) of the ionization path lengths \( x \) of an electron after each collision in the dead space model is described by the following piecewise function as

\[
P(x) = \begin{cases} 
0 & \text{for } x \leq l_0 \\
\alpha^* \exp \left[ - \alpha^* \left( l_0 - x \right) \right] & \text{for } x > l_0 
\end{cases}
\]

(1)

where \( \alpha^* \) is the ionization coefficient of electrons in the hard threshold dead space model; the ionization path lengths \( x \) are measured from the point of generation of the carriers at the instant of ionization. The multiplication in heterostructure APDs can well be studied by exploiting the eqn.(1) in conjunction with the random path length model proposed by Ong et al. (Ong et al., 1998).

Monte Carlo description of motion of electrons: - Transport dynamics of the hot electrons in the strong electric field is simulated by Monte Carlo method considering two dimensional carrier scattering of intervalley optic type. For InP a spherical and non-parabolic band
model is used while for InGaAs spherical and parabolic band model is considered. Furthermore, the composition dependent band parameters are introduced into the carrier density expressions (Yokoyama et. al., 1984). Also, it is to be noted that as the scattering of carriers in small devices does not occur instantaneously either in space or in time scale so the only compromise in dealing with such small devices is to use non-stationary carrier transport mechanism. In the MC formalism, the carrier dynamics is described in the phase space taking sample of a flux of 10,000 real particles (in this case, the real electrons) for simulation. It is to mentioned here that the entire MC algorithm (Hockney & Eastwood, Computer Simulation Using Particles, NY: McGraw-Hill, 1981) consists of two sub-sections e.g. the MC-particle dynamics where the particles are treated as real particles and in the other section the particles are treated as super-particles for particle-mesh force calculation required to set up equation of motion. For estimation of time evolution of the potential and field the potential grid is taken sufficiently dense so as to consider the k-points in the first Brillouin zone extremely close to each other. This consideration is very important in the sense that in short devices the field changes so rapidly that one may miss a significant change of track of the hot electron during its motion and in consequence some information of the carrier transport may be lost. The impact ionization rate $\lambda_{ii}$ is taken from Keldysh (Keldysh, 1964) model:

$$\lambda_{ii} = C \lambda_{ph}(e_{th}) \left[ \frac{e - e_{th}}{e_{th}} \right] \eta$$

$\lambda_{ph}$ $(e_{th})$ being the phonon scattering rate at the ionization threshold, $e$ and $e_g$ being the carrier energy and bandgap energy respectively and $C$, $\eta$, $\gamma$ are the constants. The Keldysh approximation is exploited by using the threshold and softness coefficients to fit (Spinelli & Lacaita) the measured value of the electron ionization coefficient obtained from the experiment of Bulman et.al. (Bulman et.al., 1985). In the MC formalism, the non-steady state of the ionization process is taken into account by considering the ionization probability to be a function of energy of the primary carriers; this is based on the consideration that the energy is not instantly responsive to the very fast change in the electric field at the heterojunction. For simplicity, the distribution of excess energy $(e - e_g)$ of the ionizing carriers after each impact ionization is assumed to be shared equally by the three carriers e.g. one primary electron and two secondary carriers in each of the resulting EHPs. Further, the carrier multiplication is simulated in the MC formalism by random pick up of one primary electron with zero initial speed starting at one end of the multiplication region. The transformation equation to generate random path lengths from the displaced distribution function (1) is given by:

$$l = l_0 - \frac{\ln(r)}{\alpha^*}$$

$r$ being the random number distributed uniformly in $\{0, 1\}$. The hard threshold $\alpha^*$ is obtained from the probability expression (1) as

$$\alpha^* = \frac{1}{1/\alpha - l_0}$$
\[ \alpha = \frac{g_{e-h}}{v_d} \]

where \( g_{e-h} \) is the EHP generation rate and \( v_d \) is the drift velocity of the electrons. The spatial distribution of points where the ionization events occur are recorded. At the next step, the motion of the generated EHPs from each of these points of ionizations by the primary are studied and noted also where, if any, further ionization has occurred within the avalanche region. If the total number of impact ionizations counted until all the secondary pairs and the original shooting electron leave the avalanche region be \( N \) then the multiplication \( M \) is given by \( N + 1 \) for this first trial. A large number of such trials are made and corresponding \( M \)'s are noted. Finally, the multiplication noise factor is determined by

\[ F = \frac{\langle M^2 \rangle}{\langle M \rangle^2} \]

A plot of the ionization coefficients of InP and InGaAs comprising the heterostructure versus the electric field is shown in fig.1. In the simulation, a nominal value of 0.4 \( \text{\mu m} \) is taken as the avalanche width. It is observed that the hard threshold ionization coefficients calculated in the dead length model decrease with decreasing electric field strength as is the case with the MC calculated ionization coefficient in the continuous model of McIntyre. The nature of variation of the ionization coefficient and its independence on the field orientation agree well with the results of Chandramouli et.al. (Chandramouli, 1994) where a complex band structure in FBMCs taken into account. It is apparent from the figure that the dead length effect is quite significant in strong electric field. A comparative study on the carrier multiplication individually in component materials of the heterostructure and in the heterostructure itself is made through MC simulation. Interestingly, it is observed from the graphical analysis that the multiplication and hence, in effect, the multiplication noise decreases substantially in heterojunction APDs. This means that the noise in heterojunction

Fig. 2. Ionization coefficient vs. inverse electric field. Solid line is for InP and the dotted line is for the InGaAs
Fig. 3. Multiplication as a function of electric field. The solid line is for InP while the dotted one is for InGaAs and the circled dashed line is for the heterojunction system of InP / InGaAs.

APDs is much less in comparison that in component materials. This simulated result has already been predicted in our theoretical discussion. The dead space effect is quite obvious from the shift of the multiplication curves to the right of the origin. It is also clear that the avalanche field is to be higher in heterojunction to obtain a given magnitude of carrier multiplication. Thus, we arrive at a very important conclusion that the excess noise in APDs can definitely be minimized by using heterojunction at the avalanche region. By inspection of fig.4 it is also found that the noise is more likely to depend on the ionization probability function than on multiplication. Spatial distribution of the ionizing events is suggested to be the reason behind. The same observation is supported in the works of Chandramouli et.al. (Chandramouli et.al. 1994) and of Hayat et.al. (Hayat et.al.2002).

Fig. 4. Excess noise factor varying with the multiplication. Solid line is indicative of the material InP; the dotted line is for the InGaAs while the circled dashed line represents data for the heterojunction.

The probability density function (pdf) of ionization path lengths $P(l)$ of electrons is shown in the fig.4. It shows a rounded off at the peak of the distribution curve while a sharp spike is seen in the distribution obtained in the displaced exponential model. This sharp peak at the
top of the pdf plot indicates that the impact ionization in short heterojunction APDs is more deterministic compared to that in long devices. It also points to the fact that a larger multiplication takes place at the lengths corresponding to the peaks in the P(l) plot. Thus, the dead space effect is validated by calculation of the ionization path length distribution in the displaced exponential model.

Fig. 5. Plot of P(l) versus l. The upper curve shows the effect in short heterojunction APD and the lower shows that in short one. The ionizing field is set at $65 \times 10^6$ V/m.

9. Conclusion

Device technology continues to evolve in response to demand from a myriad of applications that impact our daily lives. Inherent and irresistible noise sources, whatever be its strength or weakness, pose a problem to the high level of operational fidelity of the device. Realisation of absolutely noiseless device is far from reality. What best can be achieved is to fabricate a device with minimum possible noise. Modeling noise sources is important to characterize the noises. A plethora of such models exist in the literature and further new models continue to be introduced. Unfortunately, yet today a single unified theory of noise in devices is not available. The intricacy of 1/f noise still remains a challenge to the area of device research. Puzzling conclusions as to the cause of origin of such noise are being drawn and open up debatable issue. Modification, approximation and etc. are being used time to time to overcome the impasse. Phonon fluctuation or carrier fluctuation or mobility fluctuation – none of them is unique to explain the 1/f noise. In case of the avalanche mode photodiode the conventional continuum theory is not directly applicable to short avalanche photodiode. Lucky drift model and dead space model has improved our understanding of the excess noise in the avalanche diode. The performance of heterojunction APD in the face of noise is substantially improved compared to the homojunction diode. Aggressive downscaling of the electronic and photonic chips embedded with ultra-low dimension devices are much prone to unmanageable noise and poses a threat to the nano-device technology. Future activity in the noise modeling should be dealt with modeling of effects with specific focus related to the device dimensions.
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