Fuzzy Logic Deadzone Compensation for a Mobile Robot

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1. Introduction

Mobile manipulators have been introduced as a way of expanding the effective workspace of robot manipulators. Robots with moving vehicle such as macro-micro manipulators, space manipulators, and underwater robotic vehicles can be used for extending the workspace in repair and maintenance, inspection, welding, cleaning, and machining operation. Mobile manipulators possess strongly coupled dynamics of mobile vehicles and manipulators. With the assumption of known dynamics, much research has been carried out. Yamamoto & Yun (1996) addressed the coordination of locomotion and manipulator motion between the base and the arm, and the problem of following a moving surface. Khatib (1999) proposed the coordination and control of the mobile manipulator with two basic task-oriented controls: end-effector task control and platform self posture control. In (Bayle et al., 2003), the concept of manipulability was generalized to the case of mobile manipulators and the optimization criteria in terms of manipulability were given to generate the controls of the system.

Most approaches require the precise knowledge of dynamics of the mobile manipulator, or, they simplify the dynamical model by ignoring dynamics issues, such as vehicle dynamics, payload dynamics, dynamics interactions between the vehicle and the arm, and unknown disturbances such as the dynamic effect caused by terrain irregularity. To handle unknown dynamics of mechanical systems, robust, and adaptive controls have been extensive investigated for robot manipulators and dynamic nonholonomic systems. Dixon et al. (2000) developed a robust tracking and regulation controller for mobile robots. In (Li et al., 2008), adaptive robust output feedback motion/force control strategies were proposed for mobile manipulators under both holonomic and nonholonomic constraints in the presence of uncertainties and disturbances. Impedance control of flexible base mobile manipulator using singular perturbation method and sliding mode control law was presented in (Salehi & Vossoughi, 2008). Because of the difficulty in dynamic modeling, adaptive neural network control has been studied for different classes of systems, such as robotic manipulators (Lewis el al., 1996) and mobile robots (Jang & Chung, 2009). In (Lin & Goldenberg, 2001), adaptive neural network controls have been developed for the motion control of mobile manipulators subject to kinematic constraint. In (Mbede et al., 2005), intelligent navigation is presented for mobile manipulator using adaptive neuro-fuzzy systems. In these schemes, the controls are designed at kinematic level with velocity as input or dynamic level with torque as input, but the actuator dynamics are ignored. Therefore, the actuator nonlinearity deteriorates the system.
performance. The actuator nonlinearity compensation techniques are published in (Jang, 2009) for saturation, in (Jang, 2005) for deadzone, and in (Jang & Jeon, 2006) for backlash. In this paper, we present the deadzone compensation method for a mobile manipulator using fuzzy logic. A rigorous design procedure with proofs is given that results in a kinematic tracking loop with an adaptive FL system in the feed forward loop for deadzone compensation. We derive a practical bound on tracking error from the analysis of the tracking error dynamics and investigate the performance of the FL deadzone compensator in a mobile manipulator through the computer simulations. This paper is as follows. Section 2 provides the mobile manipulator. The FL deadzone compensation is derived in Section 3. The proposed FL deadzone compensation scheme is developed in Section 4. Simulation results of the FL deadzone compensation scheme are given in Section 5. Finally, conclusions are included in Section 6.

2. Mobile manipulator

Consider a mobile manipulator mounted on nonholonomic mobile platform, as shown in Fig. 1. The dynamics of a mobile manipulator subject to kinematics can be obtained using Lagrangian approach in the form (Yamamoto & Yun, 1996)

\[
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda
\]

where kinematic constraints are described by

\[A(q)\dot{q} = 0. \tag{2}\]

and \(q \in \mathbb{R}^p\) is the generalized coordinates, \(M(q) \in \mathbb{R}^{p \times p}\) is a symmetric and positive definite inertia matrix, \(C(q,\dot{q}) \in \mathbb{R}^{p \times p}\) is the centripetal and Coriolis matrix, \(F(\dot{q}) \in \mathbb{R}^p\) denotes the surface friction, \(G(q) \in \mathbb{R}^p\) is the gravitational vector, \(\tau_d\) denotes the bounded unknown disturbances including unstructured unmodeled dynamics, \(B(q) \in \mathbb{R}^{p \times (p-r)}\) is the input transformation matrix, \(\tau \in \mathbb{R}^{r \times r}\) is the input vector, \(A(q) \in \mathbb{R}^{p \times p}\) is the matrix associated with the constraints, and \(\lambda \in \mathbb{R}^r\) is the vector of constraint forces.

In (1), the following properties hold (Lewis et al., 1999).

**Property 1 (Skew Symmetry)**

\[
\dot{M} - 2C = -(\dot{M} - 2C)^T
\]

\[
\dot{M} = C + C^T. \tag{3}\]

The generalized coordinates \(q\) may be separated into two sets \(q = [q_v, q_r]^T\) with \(q_v \in \mathbb{R}^m\) describes the generalized coordinates appearing in the constraint equations (2), and \(q_r \in \mathbb{R}^n\) are the free generalized coordinates; \(p = m + n\). Therefore, (2) can be simplified to

\[
A_v(q_v)\dot{q}_v = 0 \tag{4}
\]

with \(A(q_v) \in \mathbb{R}^{r \times m}\). Assume that the robot is fully actuated, then (1) can be further rewritten as

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\ddot{\tilde{q}}_v \\
\ddot{\tilde{q}}_r
\end{bmatrix}
+
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{\tilde{q}}_v \\
\dot{\tilde{q}}_r
\end{bmatrix}
+
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} +
\begin{bmatrix}
G_1 \\
G_2
\end{bmatrix}
+
\begin{bmatrix}
\tau_{\dot{a}_1} \\
\tau_{\dot{a}_2}
\end{bmatrix}
=
\begin{bmatrix}
B_v\tau_v \\
0
\end{bmatrix} -
\begin{bmatrix}
A_v^T(q_v)\lambda \\
0
\end{bmatrix} \tag{5}
\]
where $\tau_c \in \mathbb{R}^{n-r}$ represents the actual torque vector of the constrained coordinates, those related to the constrained motion of the wheels, the joints, and the end effector. For simplicity in the theoretical derivation, hereafter we consider only the case where the vehicle motion is constrained. However, the proposed theory can be easily extended to include joint and/or end-effector constraints. $B_{\tau} \in \mathbb{R}^{m(n-r)}$ represents the input transformation matrix; $\tau_r \in \mathbb{R}^r$ the actuating torque vector of the free coordinates; $\tau_{d1}$ and $\tau_{d2}$ are disturbance torques bounded by $|\tau_{d1}| < \tau_{1N}$ and $|\tau_{d2}| < \tau_{2N}$, with $\tau_{1N}$ and $\tau_{2N}$ some positive constants.

It is straightforward to show that the following properties hold.

**Property 2**: 

$$
\dot{M}_{21} = C_{21} + C_{12}^T \\
M_{12} = M_{21}^T.
$$

(6)
Let $S(q_v) \in \mathbb{R}^{m \times m}$ be a full rank matrix formed by a set of smooth and linearly independent vector fields in the null space of $A_v(q_v)$, i.e.,

$$S^T(q_v)A_v^T(q_v) = 0.$$  

(7)

According to (7), it is possible to find an auxiliary vector time function $v(t) \in \mathbb{R}^{m \times m}$ such that, for all $t$

$$\dot{q}_v = S(q_v)v(t)$$  

(8)

and its derivative is

$$\ddot{q}_v = S(q_v)v + \dot{S}(q_v)v.$$  

(9)

Equation (8) is called the steering system. $v(t)$ can be regarded as a velocity input vector steering the state vector $q$ in state space.

Let us consider the first $m$ equations of (5)

$$M_{11} \ddot{q}_v + M_{12} \dot{q}_v + C_{11} \dot{q}_v + C_{12} \dot{q}_v + F_i + G_i + \tau_{d1} = B_v \tau_v - A_v^T \lambda.$$  

(10)

Multiplying both sides of (10) by $S^T$ and using (7) to eliminate the constraint force we obtain

$$S^T M_{11} \ddot{q}_v + S^T M_{12} \dot{q}_v + S^T C_{11} \dot{q}_v + S^T C_{12} \dot{q}_v + S^T F_i + S^T G_i + S^T \tau_{d1} = S^T B_v \tau_v.$$  

(11)

Substituting (8) and (9) into (11) yields

$$S^T M_{11} \ddot{v} + S^T M_{12} \dot{v} + S^T C_{11} \dot{v} + S^T C_{12} \dot{v} + S^T F_i + S^T G_i + S^T \tau_{d1} = S^T B_v \tau_v.$$  

(12)

Let us rewrite (12) in a compact form as

$$\bar{M}_{11} \ddot{v} + \bar{C}_{11} \dot{v} + f_i + \bar{\tau}_{d1} = \bar{\tau}_v$$  

(13)

where $\bar{M}_{11} = S^T M_{11} S$, $\bar{C}_{11} = S^T C_{11} S + S^T M_{12} \dot{S}$, $\bar{\tau}_{d1} = S^T \tau_{d1} i | \bar{\tau}_{d1} | \leq \bar{\tau}_{1N}$ with $\bar{\tau}_{1N}$ some positive constant, and

$$\bar{\tau}_v = S^T B_v \tau_v = \tilde{B}_v \tau_v$$  

(14)

$$f_i = S^T (M_{12} \dot{q}_v + C_{12} \dot{q}_v + F_i + G_i).$$  

(15)

$f_i$ consists of the gravitational and friction force, the disturbances on the vehicle base, and the dynamic interaction with the mounted manipulator arm which has been shown to have significant effect on the base motion, thus it needs to be compensated for (Yamamoto, 1994)

**Property 3:** $\bar{M} - 2\bar{C}_{11}$ is skew-symmetric.

Proof:

$$\bar{M} - 2\bar{C}_{11} = 2S^T M_{11} \dot{S} + S^T \dot{M}_{11} S - 2S^T M_{11} \dot{S} - 2S^T C_{11} S.$$

$$= S^T (M_{11} - 2C_{11}) S$$

(16)
Since $\dot{M} - 2C_{11}$ is skew-symmetric, therefore, $\ddot{M} - 2\dddot{C}_{11}$ is also skew-symmetric.
Let us consider the last $n$-equations of (5)

$$
M_{21}\ddot{q}_r + M_{22}\dddot{q}_r + C_{21}\ddot{q}_r + C_{22}\dddot{q}_r + F_2 + G_2 + \tau_{d2} = \tau_r .
$$

(17)

Rearrange (17) as follows:

$$
M_{22}\ddot{q}_r + C_{22}\dddot{q}_r + (M_{21}\ddot{q}_r + C_{21}\dddot{q}_r + F_2 + G_2) + \tau_{d2} = \tau_r .
$$

(18)

Equation (18) represents the dynamic equation of the mounted manipulator arm. The terms in the brackets consist of the dynamic interaction term($M_{21}\ddot{q}_r + C_{21}\dddot{q}_r$), the gravitational and friction force vector, and the disturbance on the manipulator. Equation (8), (13), and (18) form the complete dynamic model of the mobile manipulator subject to kinematic constraints.

The Lagrange formulism is used to derived the dynamic equation of the mobile manipulator. The dynamical equations of the mobile manipulator in Fig. 2 can be expressed in the matrix form where

$$
q_v = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \quad q_r = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}
$$

$$
M_{11} = \begin{bmatrix}
 m_{p12} + \frac{2I_w \sin^2 \theta}{r^2} & -\frac{2I_w \sin \theta \cos \theta}{r^2} & m_{12} d \sin \theta \\
 -\frac{2I_w \sin \theta \cos \theta}{r^2} & m_{p12} + \frac{2I_w \cos^2 \theta}{r^2} & -m_{12} d \cos \theta \\
 m_{12} d \sin \theta & -m_{12} d \cos \theta & I_p + m_{12} d^2 + \frac{2I_w d^2}{r^2}
\end{bmatrix}
$$

$$
M_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I_{12} & 0 & 0 \end{bmatrix}, \quad M_{21} = \begin{bmatrix} 0 & 0 & I_{12} \\ 0 & 0 & 0 \\ 0 & I_{12} & 0 \end{bmatrix}, \quad M_{22} = \begin{bmatrix} I_{12} & 0 \\ 0 & I_{12} \end{bmatrix}
$$

$m_{p12} = m_p + m_{12}, \quad m_{12} = m_1 + m_2, \quad I_{12} = I_1 + I_2$

$$
C_{11} = \begin{bmatrix}
 \frac{2I_w \dot{\theta} \sin \theta \cos \theta}{r^2} & \frac{2I_w \dot{\theta} \sin^2 \theta}{r^2} & m_{12} d \dot{\theta} \cos \theta \\
 \frac{2I_w \dot{\theta} \cos^2 \theta}{r^2} & \frac{2I_w \dot{\theta} \sin \theta \cos \theta}{r^2} & m_{12} d \dot{\theta} \sin \theta \\
 0 & 0 & 0
\end{bmatrix}
$$

$$
C_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$
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\[
G_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0 & 0 \\ m_2 g \ell_2 \sin \theta_2 \end{bmatrix}, \quad \tau_v = \begin{bmatrix} \tau_k \\ \tau_l \end{bmatrix}, \quad \tau_r = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad A_v = \begin{bmatrix} -\sin \theta \\ \cos \theta \\ -d \end{bmatrix}
\]

\[
B_v = \frac{1}{r} \begin{bmatrix} \cos \theta & \cos \theta \\ \sin \theta & \sin \theta \\ -l & -l \end{bmatrix}, \quad \lambda = -m_{p12} (\dot{x} \cos \theta + \dot{y} \sin \theta) \dot{\theta}.
\]  \hspace{1cm} (19)

Similar dynamical models have been reported in the literature, for instance in (Yamamoto, 1996) the mass and inertia of the driving wheels and manipulator are considered explicitly.

3. Fuzzy logic deadzone compensation

In this section a FL precompensator is designed for the non-symmetric deadzone nonlinearity. It is shown that the FL approach includes and subsumes approaches based on switching logic and indicator functions (Recker et al., 1991). This brings these references very close to fuzzy logic work in (Kim et al., 1994), and potentially allows for more exotic compensation schemes for actuator nonlinearities using more complex decision (e.g. membership) functions. This section provides a rigorous framework for FL applications in deadzone compensation for a broad class of mobile robot.

\[ N_d(u) \]

Fig. 3. Deadzone nonlinearity.

If \( u, \tau \) are scalars, the nonsymmetric deadzone nonlinearity, shown in Fig. 3, is given by

\[
\tau = N_d(u) = \begin{cases} 
0, & d_- \leq u < d_+ \\
u - d_+, & u < d_-
\end{cases}
\]

\hspace{1cm} (20)

The parameter vector \( d = [d_-, d_+]^T \) characterizes the width of the system deadband. In practical control systems the width of the deadzone is unknown, so that compensation is difficult. Most compensation schemes cover only the case of symmetric deadzones where \( d_- = d_+ \).

The nonsymmetric deadzone may be written as
\[
\overline{\tau} = N_d(u) = u - sat_d(u)
\]

where the nonsymmetric saturation function is defined as

\[
sat_d(u) = \begin{cases} 
  d_r, & u < d_-
  
  u, & d_\leq u < d_+
  
  d_r, & d_\leq u
\end{cases}
\]

To offset the deleterious effects of deadzone, one may place a precompensator as illustrated in Fig. 4. There, the desired function of the precompensator is to cause the composite throughput from \( w \) to \( \tau \) to be unity. The power of fuzzy logic systems is to that they allow one to use intuition based on experience to design control systems, then provide the mathematical machinery for rigorous analysis and modification of the intuitive knowledge, for example through learning or adaptation, to give guaranteed performance, as will be shown in Section 4. Due to the fuzzy logic classification property, they are particularly powerful when the nonlinearity depends on the region in which the argument \( u \) of the nonlinearity is located, as in the non-symmetric deadzone.

![Fig. 4. Fuzzy logic deadzone compensation of a mobile manipulator.](https://www.intechopen.com)

A deadzone precompensator using engineering experience would be discontinuous and depend on the region within which \( w \) occurs. It would be naturally described using the rules

\[
\text{If (} w \text{ is positive) then (} u = w + \hat{d}_+ \text{)}
\]

\[
\text{If (} w \text{ is negative) then (} u = w + \hat{d}_- \text{)}
\]

where \( \hat{d} = [\hat{d}_+, \hat{d}_-]^T \) is an estimate of the deadzone width parameter vector \( d \).
To make this intuitive notion mathematically precise for analysis define the membership function’s

\[
X_+(w) = \begin{cases} 
0, & w < 0 \\
1, & 0 \leq w
\end{cases},
\]

\[
X_-(w) = \begin{cases} 
1, & w < 0 \\
0, & 0 \leq w
\end{cases}.
\] (24)

One may write the precompensator as

\[
u = w + w_f
\] (25)

where \(w_f\) is given by the rule base

If ( \(w \in X_+(w)\) ) then ( \(w_f = \hat{d}_+\))

If ( \(w \in X_-(w)\) ) then ( \(w_f = \hat{d}_-\)). (26)

The output of the fuzzy logic system with this rule base is given by

\[
w_f = \frac{\hat{d}_+X_+(w) + \hat{d}_-X_-(w)}{X_+(w) + X_-(w)}. \] (27)

The estimates \(\hat{d}_+, \hat{d}_-\) are, respectively, the control representative value of \(X_+(w)\) and \(X_-(w)\). This may be written (note \(X_+(w) + X_-(w) = 1\)) as

\[
w_f = \hat{d}^rX(w)
\] (28)

where the fuzzy logic basis function vector given by

\[
X(w) = \begin{bmatrix} X_+(w) \\ X_-(w) \end{bmatrix}
\] (29)

is easily computed given any value of \(w\).

It should be noted that the membership functions (24) are the indicator functions and \(X(w)\) is similar to the regressor (Tao & Kokotovic, 1992). The composite through from \(w\) to \(\bar{r}\) of the FL compensator plus the deadzone is

\[
\bar{r} = N_d(u) = N_d(w + w_f) = w + [w_f - \text{sat}_d(w + w_f)].
\] (30)

The FL compensator may be expressed as follows

\[
u = w + w_f = w + \hat{d}^rX(w)
\] (31)

where \(\hat{d}\) is estimated deadzone widths.
Given the FL compensator with rulebase (26), the throughput of the compensator plus deadzone is given by

$$\tau = w + \tilde{d}^T X(w) \tilde{d}$$

where the deadzone width estimation error is given by

$$\tilde{d} = d - \hat{d}$$

and the modeling mismatch term $\delta$ is bounded so that $|\delta| < \delta_M$ for some scalar $\delta_M$.

4. FL deadzone compensation of a mobile manipulator

In this section, FL deadzone compensation and tuning laws will be derived for the stable joint space tracking of a mobile manipulator described by (8), (13), and (18). The mobile manipulator dynamics is redefined as an error dynamics based on a set of carefully chosen Lyapunov functions. FL deadzone compensators are constructed and new learning laws are proposed. A proof on the tracking stability of the overall closed loop system and the boundedness on FL deadzone estimation errors are provided. The proposed control structure is shown in Fig. 4.

Consider the vehicle dynamics represented by (8) and (13). Tracking control of the steering system (8) has been extensively addressed in the literature (Dixon et al., 2000). For example, for a wheeled mobile robot with two independent actuated wheels, the kinematic parameters in (8) are defined as

$$S(q_c) = \begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \\ 0 & 1 \end{bmatrix}, \quad v = \begin{bmatrix} v \\ \omega \end{bmatrix} \quad \text{and} \quad q_c = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

where $(x, y)$ represents the Cartesian coordinates of the cart, $\theta$ its orientation, $v$ and $\omega$ its linear and angular velocities, respectively. Let the reference motion of the vehicle be prescribed as

$$\begin{bmatrix} x_r \\ y_r \\ \theta_r \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix}$$

where $v_r > 0$ and $\omega_r$ are reference linear and angular velocities, respectively. Stable linear and nonlinear velocity feedback laws for (34) can be found in (Kanayama et al., 1990) to achieve the asymptotic tracking. For instance, the following feedback velocity input guarantees that the position tracking of (35) is asymptotically stable [14]:

$$v_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix}$$

where positive constant $k_1$, $k_2$, and $k_3$ are control gains, and the position tracking errors are defined as

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\[ e = \Gamma_e (q_{wl} - q_e) \]

\[
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta & 0 \\
  -\sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_e - x \\
  y_e - y \\
  \theta_e - \theta
\end{bmatrix}.
\]

(37)

Choosing the following Lyapunov function can prove the stability tracking system

\[ V_1 = k_1 (e_1^2 + e_2^2) + 2k_3 v_e (1 - \cos e_3) . \]

(38)

Differentiating yields

\[ \dot{V}_1 = 2k_1 (e_1 \dot{e}_1 + e_2 \dot{e}_2) + 2k_3 v_e \dot{e}_3 \sin e_3 . \]

(39)

Given the desired velocity \( v_c(t) \), define now the auxiliary velocity tracking error as

\[ e_v = v_c - v . \]

(40)

The velocity tracking error is

\[
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix} =
\begin{bmatrix}
  v_c - v \\
  w_c - w \\
  \omega_c + k_2 v_c e_2 + k_3 v_c \sin e_3 - \omega_3
\end{bmatrix}
\]

(41)

where \( k_1, k_2, k_3 \) are positive constants.

Substituting the derivative of the position error in (39), we obtain

\[ \dot{V}_1 = 2k_1 e_1 (v_c e_2 - v_e + v \cos e_3) + 2k_1 e_2 (-v_c e_1 + v \sin e_3) + 2k_3 v_e (\omega_c - v_2) \sin e_3 . \]

(42)

Using (41) and defining \( k_2 = (k_1 / k_3 v_c) \) yields

\[ \dot{V}_1 = -k_1^2 e_1^2 - k_3^2 v_c^2 \sin^2 e_3 - (k_1 e_1 - e_4)^2 - (k_3 v_c \sin e_3 - \omega_3)^2 . \]

(43)

Differentiating (40), multiplying both sides by \( M_{11} \) and substituting (13) into it yields

\[ \ddot{M}_{11} e_c = -\ddot{C}_{11} e_c + f_1 + \ddot{p}_{d1} + \ddot{M}_{11} \dot{v}_c + \ddot{C}_{11} v_c - \ddot{r}_v . \]

(44)

Equation (44) represents the vehicle dynamics in terms of tracking errors.

Let us choose the Lyapunov function as

\[ V_2 = \frac{1}{2} e_c^T \ddot{M}_{11} e_c . \]

(45)

Differentiating (45) yields

\[ \ddot{V}_2 = e_c^T \dddot{M}_{11} e_c + \frac{1}{2} e_c^T \dddot{M}_{11} e_c . \]

(46)
Substituting (44) into (46) we obtain
\[
\dot{V}_2 = e_c^T (f_1 + \tau_{d1} + \bar{M}_{11}\dot{v}_c + \bar{C}_{11}v_c - \tau_3) + \frac{1}{2} e_c^T (\dot{\bar{M}}_{11} - 2\bar{C}_{11}) e_c .
\]  
(47)

Now consider the arm dynamics (18). Let us define the arm error as
\[ e_r = q_{id} - q_r \]
(48)
and the tracking error as
\[ r = \dot{e}_r + \Lambda \dot{e}_r \]
(49)
where \( k = k^T > 0 \). In (49), tracking error \( r \) can be regarded as an input to a linear dynamics system with state variable \( e_r \). Therefore, when \( r \to 0 \), it can guarantee that \( e_r \to 0 \) (Lewis et al., 1999).
Differentiating (49) yields
\[
\dot{r} = \dot{e}_r + \Lambda \dot{e}_r = \ddot{q}_{id} - \ddot{q}_r + \Lambda \dot{e}_r .
\]
(50)

Therefore, we have
\[
\dot{q}_r = \ddot{q}_{id} - (r - \Lambda e_r) \]
(51)
\[
\ddot{q}_r = \ddot{q}_{id} - \dot{r} + \Lambda (r - \Lambda e_r) .
\]
(52)
The manipulator dynamics (18) can be formulated in terms of the tracking error as
\[
\dot{r} = M_{22} \dot{r} = -C_{22} r + f_2 + \tau_{d2} - \tau_r ,
\]
where the nonlinear manipulator function is
\[
f_2 = M_{22} (\ddot{q}_{id} + \Lambda \dot{e}_r) + C_{22} (\dot{q}_{id} + \Lambda e_r) + M_{22} \ddot{q}_c + C_{22} \dot{q}_c + F_2 + G_2 .
\]
(54)
The nonlinear manipulator function \( f_2 \) consists of the manipulator dynamics \( M_{22} (\ddot{q}_{id} + \Lambda \dot{e}_r) + C_{22} (\dot{q}_{id} + \Lambda e_r) \) and the dynamics of interaction with the vehicle base \( (M_{22} \ddot{q}_c + C_{22} \dot{q}_c) \).
To design the manipulator torque input, we choose the Lyapunov function as
\[
V_3 = \frac{1}{2} r^T M_{22} r .
\]
(55)
Notice that \( M_{22} \) is a symmetric positive definite matrix. Differentiating (55) yields
\[
\dot{V}_3 = r^T M_{22} \dot{r} + \frac{1}{2} r^T M_{22} r
\]
\[= r^T (-C_{22} r + f_2 + \tau_{d2}) + \frac{1}{2} r^T \dot{M}_{22} r .
\]
(56)
\[= r^T (-\tau_r + f_2 + \tau_{d2}) + \frac{1}{2} r^T (\dot{M}_{22} - 2C_{22}) r
\]
\[= r^T (-\tau_r + f_2 + \tau_{d2})
\]
Let us consider the overall dynamics (5) that combines both the arm and vehicle dynamics. Consider the Lyapunov function as

\[
V_4 = V_1 + \frac{1}{2} \left( \begin{array}{c} S e_r \\ r \end{array} \right)^T \left( \begin{array}{cc} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{array} \right) \left( \begin{array}{c} S e_r \\ r \end{array} \right). \tag{57}
\]

In the proposed Lyapunov function \( V_4 \), \( V_1 \) is used to account for the nonholonomic steering system (8), and the second term accounts for the vehicle base and manipulator arm dynamics, as well as the dynamic couplings between two.

From (57) we have

\[
V_4 = V_1 + \frac{1}{2} \left( S e_r \right)^T M_{11} (S e_r) + \frac{1}{2} r^T M_{12}^T S e_r + \frac{1}{2} \left( S e_r \right)^T M_{12} r + \frac{1}{2} r^T M_{22} r \\
= V_1 + \frac{1}{2} \left( S^T M_{11} S \right) r + r^T M_{12} (S e_r) + \frac{1}{2} r^T M_{22} r \\
= V_1 + \frac{1}{2} \left( S^T M_{11} S \right) e_r + r^T M_{12} (S e_r) + \frac{1}{2} r^T M_{22} r \\
= V_1 + V_2 + V_3 + r^T M_{12} (S e_r) \\
\]

Differentiating (58) yields

\[
\dot{V}_4 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \frac{d}{dt} \left( r^T M_{21} (S e_r) \right). \tag{59}
\]

Substituting (43), (47), and (56) into (59) yields

\[
\dot{V}_4 \leq e_r^T \left( -f_1 + \overline{M}_{11} \dot{e}_r + \overline{C}_{11} \dot{e}_r + \overline{F}_1 \right) + r^T \left( -f_2 + \tau_1 + \tau_2 \right) + \frac{d}{dt} \left( r^T M_{21} (S e_r) \right) \tag{60}
\]

where the four terms in \( (43) \) are negative.

From the definition of \( f_1 \) in (15) and (51), (52) we have

\[
f_1 = S^T \left( M_{12} \dot{q}_{id} + C_{12} \dot{q}_{ir} + F_1 + G_1 \right) \\
= S^T \{ M_{12} (\dot{q}_{id} - \dot{r} + \Lambda (r - \Lambda e_r)) + C_{12} (\dot{q}_{id} - (r - \Lambda e_r)) + F_1 + G_1 \} \\
= -S^T \{ M_{12} \ddot{r} + (C_{12} - M_{12} \Lambda) (r - \Lambda e_r) \} + \overline{f}_1 
\]

where \( \overline{f}_1 = S^T \left( M_{12} \dot{q}_{id} + C_{12} \dot{q}_{ir} + F_1 + G_1 \right) \).

From the definition of \( f_2 \) in (9) and (54) we have

\[
f_2 = M_{21} (\dot{S} v + S \dot{v}) + C_{21} S v + \{ M_{22} (\dot{q}_{id} + \Lambda e_r) + C_{22} (\dot{q}_{id} + \Lambda e_r) + F_2 + G_2 \} \\
= (M_{21} S) \dot{v} + (M_{21} \dot{S} + C_{21} S) v + \overline{f}_2 \\
= (M_{21} S) (\dot{v}_c - \dot{e}_c) + (M_{21} \dot{S} + C_{21} S) (v_c - e_c) + \overline{f}_2 
\]

where \( \overline{f}_2 = M_{22} (\dot{q}_{id} + \Lambda e_r) + C_{22} (\dot{q}_{id} + \Lambda e_r) + F_2 + G_2 \).
Substituting (61) and (62) into (60) and after some collections of them we have

\[ \dot{V}_4 \leq e_t^T (\tau_a - M_{11} \dot{v}_e + C_{11} v_e) - r^T \tau_r + e_r^T f_1 + r^T f_2 + \frac{d}{dt} \{ r^T M_2 (S e_e) \} + e_t^T \ddot{\tau}_d + r^T \tau_{d2} \]  

(63)

First of all, we carry out the following derivation

\[ e_r^T f_1 + r^T f_2 + \frac{d}{dt} \{ r^T M_2 (S e_e) \} \]

\[ = e_t^T f_1 + r^T f_2 - (S e_e)^T \{ M_{12} \dot{r} + (C_{12} - M_{12} \Lambda) (r - \Lambda e_e) \} + r^T (M_{21} \dot{S} e_e + M_{21} \dot{S} e_e + M_{21} \dot{S} e_e + C_{21} \dot{S} e_e - C_{21} S e_e) + r^T M_2 S e_e + r^T M_2 \dot{S} e_e + r^T M_2 S e_e \]  

(64)

where Properties 2 and 3 have been used in the previous derivations. Substituting (64) into (63) we obtain

\[ \dot{V}_4 \leq e_t^T (\tau_a - M_{11} \dot{v}_e + C_{11} v_e) - r^T \tau_r + e_r^T f_1 + r^T f_2 \]

\[ + (S e_e)^T \{ C_{12} \Lambda e_e + M_{12} \Lambda (r - \Lambda e_e) \} \]

\[ + r^T (M_{21} \dot{S} e_e + M_{21} \dot{S} e_e + C_{21} \dot{S} e_e + C_{21} S e_e) + e_t^T \ddot{\tau}_d + r^T \tau_{d2} \]  

(65)

Therefore

\[ \dot{V}_4 \leq e_t^T (\tau_a + \Psi_1) + r^T (\tau_a + \Psi_2) + e_t^T \ddot{\tau}_d + r^T \tau_{d2} \]  

(66)

with unknown nonlinear terms

\[ \Psi_1 = M_{11} \dot{v}_e + C_{11} v_e + f_1 + S^T \{ C_{12} \Lambda e_e + M_{12} \Lambda (r - \Lambda e_e) \} \]  

(67a)

\[ \Psi_2 = f_2 + M_{21} \dot{S} e_e + M_{21} \dot{S} e_e + C_{21} S e_e \]  

(67b)

In applications the nonlinear robot function \( \Psi_1 \) and \( \Psi_2 \) is at least partially unknown. In standard fashion [Jang & Chung, 2009; Lewis et al., 1999], the estimate \( \hat{\Psi}_1 \), \( \hat{\Psi}_2 \) may be provided by any means desired. The functional estimation error are defined as \( \hat{\Psi}_1 = \Psi_1 - \Psi_1 \) and \( \hat{\Psi}_2 = \Psi_2 - \Psi_2 \). It is assumed that the functional estimation error satisfies

\[ |\hat{\Psi}_1| \leq \Psi_{1M}(x) \]  

(68a)

\[ |\hat{\Psi}_2| \leq \Psi_{2M}(x) \]  

(68b)
for some unknown bounded function $\Psi_{1M}(x)$ and $\Psi_{2M}(x)$.
Therefore, a suitable control input for velocity following is given by the computed torque
like control

$$w_v = \dot{\Psi}_1 + k_4 e_v - \gamma_v$$  \hspace{1cm} (69a)$$

$$w_r = \dot{\Psi}_2 + k_5 r - \gamma_r$$  \hspace{1cm} (69b)$$

with $k_4, k_5$ are the diagonal positive definite gain matrix. The robustifying signals $\gamma_v(t), \gamma_r(t)$ are required to compensate the unmodeled unstructured disturbances.
Deadzone compensation is provided using

$$u_v = w_v + \hat{d}_v^T X(w_v)$$  \hspace{1cm} (70a)$$

$$u_r = w_r + \hat{d}_r^T X(w_r)$$  \hspace{1cm} (70b)$$

with $X(w_v)$ and $X(w_r)$ given by (29), which gives the overall feedforward throughout (32).
Substituting (69) and (32) into (66)

$$\dot{V}_4 \leq e_v^T (-w_v + \hat{d}_v^T X(w_v) - \hat{d}_v^T \delta_v + \Psi_1) + r^T (-w_r + \hat{d}_r^T X(w_r) - \hat{d}_r^T \delta_r + \Psi_2)$$

\[ + e_v^T T_{d1} + r^T T_{d2} \]

\[ \leq e_v^T (-\dot{\Psi}_1 - K_4 e_v + \gamma_v + \hat{d}_v^T X(w_v) - \hat{d}_v^T \delta_v + \Psi_1) \]

\[ + r^T (-\dot{\Psi}_2 - K_5 r + \gamma_r + \hat{d}_r^T X(w_r) - \hat{d}_r^T \delta_r + \Psi_2) + e_v^T T_{d1} + r^T T_{d2} \]

Let us define

$$\tau_D = \begin{bmatrix} T_{d1} \\ T_{d2} \end{bmatrix} \hspace{1cm} d = \begin{bmatrix} d_v \\ d_r \end{bmatrix}$$  \hspace{1cm} (72)$$

Based on the bounds of every element of the vectors and matrices defined above, we may show the following properties hold:

$$| \tau_D | \leq | T_{d1} | + | T_{d2} | \leq T_{1N} + T_{2N} \equiv T_M$$

$$| d | \leq | d_v | + | d_r | \leq d_{vM} + d_{rM} = d_M$$  \hspace{1cm} (73)$$

The next theorem provides an algorithm for tuning the deadzone precompensator.

**Theorem 1**: Consider the nonholonomic system (13) and (18). Select the tracking control (69) plus deadzone compensator (70), where $X(w)$ is given by (29). Choose the robustifying signal

$$\gamma_v(t) = -(\Psi_{1M} + T_M) \frac{e_v}{|e_v|}$$  \hspace{1cm} (74a)$$

$$\gamma_r(t) = -(\Psi_{2M} + T_M) \frac{r}{|r|}$$  \hspace{1cm} (74b)$$

Let the estimated deadzone widths be provided by the FL system tuning algorithm
\[ \dot{a}_c = X(w_c)e_c - k_a \dot{a}_c | e_c | \]  
(75a)

\[ \dot{a}_r = X(w_r)r - k_a \dot{a}_r | r | \]  
(75b)

where \( k_a \) is positive definite design parameter. By properly choosing the control gain and design parameter, tracking errors of error dynamics described by (8), (44), (53) and the FL deadzone estimation error \( \ddot{d} = (\ddot{d}_c, \ddot{d}_r) \) evolves practical bounds by the right hand sides of (83) and (84).

Proof) Select the Lyapunov function candidate as

\[ V = V_4 + \frac{1}{2} (\ddot{d}_c \ddot{d}_c) + \frac{1}{2} (\ddot{d}_r \ddot{d}_r) . \]  
(76)

Differentiating yields

\[ \dot{V} = \dot{V}_4 + (\ddot{d}_c \ddot{d}_c) + (\ddot{d}_r \ddot{d}_r) . \]  
(77)

From (71), (73) and robustifying term (74) it follows that

\[ \dot{V}_4 \leq -e^T_k k_c e_c + \ddot{d}_c^T (X(w_c)e_c - \delta_M e_c) + e^T_k (\Psi_1 - \dot{\Psi}_1 + \gamma_1 + \tau_{11}) + r^T k_s r + \ddot{d}_r^T (X(w_r)r - \delta_M r) \]

\[ - r^T k_s r + \ddot{d}_r^T (X(w_r)r - \delta_M r) + r^T (\Psi_2 - \dot{\Psi}_2 + \gamma_2 + \tau_{12}) . \]  
(78)

where \( | \delta_c | < \delta_{mL} \) \& \( | \delta_r | < \delta_{Mt} \) for some known positive constants \( \delta_{mL} \) and \( \delta_{Mt} \).

Using (77), we obtain

\[ \dot{V} \leq -e^T_k k_c e_c + \ddot{d}_c^T (X(w_c)e_c - \delta_{mL} e_c) - r^T k_s r + \ddot{d}_r^T (X(w_r)r - \delta_{mL} r) \]

\[ + (\ddot{d}_c \ddot{d}_c) + (\ddot{d}_r \ddot{d}_r) . \]  
(79)

Since \( \ddot{d} = -\dot{d} \), applying the tuning algorithm (75) yields

\[ \dot{V} \leq -e^T_k k_c e_c + \ddot{d}_c^T (X(w_c)e_c - \delta_{mL} e_c - X(w_c)e_c + k_s \dot{d}_c | e_c |) \]

\[ - r^T k_s r + \ddot{d}_c^T (X(w_c)r - \delta_{mL} r - X(w_c)r + k_s \dot{d}_c | r |) \]

\[ \leq -e^T_k k_c e_c + \ddot{d}_c^T (-\delta_{mL} e_c + k_s \dot{d}_c | e_c |) - r^T k_s r + \ddot{d}_c^T (-\delta_{mL} r + k_s \dot{d}_c | r |) \]

\[ \leq -e^T_k k_c e_c + \ddot{d}_c^T (\delta_{mL} e_c + k_c (d_c - \dot{d}_c) | e_c |) - r^T k_s r + \ddot{d}_c^T (\delta_{mL} r + k_c (d_r - \dot{d}_r) | r |) \]  
(80)

there results

\[ \dot{V} \leq -e^T_k k_c e_c + \ddot{d}_c^T (c_v e_c - k_v \dot{d}_c | e_c |) - r^T k_s r + \ddot{d}_c^T (c_r r - k_r \dot{d}_r | r |) \]  
(81)

with \( c_v = \delta_{mL} + k_s d_{mL} \) and \( c_r = \delta_{mL} + k_s d_{mL} \). Let \( k = \min(k_4, k_5) \) \& \( E = (e^T, r^T) \) and \( c = (c_v, c_r) \), we obtain
\[ V \leq -E^T \bar{K} E + E^T c \lvert \tilde{d} \rvert - E^T k_6 \lvert \tilde{d} \rvert^2 \leq -\lvert E \rvert [K \lvert E \rvert - c \lvert \tilde{d} \rvert + k_6 \lvert \tilde{d} \rvert^2]. \] (82)

This is negative as long as the quantity in the brace is positive. To determine conditions for this, complete the square to see that \( V \) is negative as long as either

\[ |E| > \frac{c^2}{4 \cdot \bar{k} \cdot k_6} \] (83)

or

\[ |\tilde{d}| > \frac{c}{k_6}. \] (84)

According to the Lyapunov theorem, the tracking error decreases as long as the error is bigger than the right-hand side of Eq. (83). This implies Eq. (85) gives a practical bound on the tracking error

\[ |E| \leq \frac{c^2}{4 \cdot \bar{k} \cdot k_6}. \] (85)

Also, Lyapunov extension shows that the deadzone width bound, \( |\tilde{d}| \), is bounded to a neighborhood of the right hand side of Eq. (84). Since a tracking controller, \( \bar{k} \), is determined according to the design of a tracking controller, \( \bar{k} \) cannot be increased arbitrarily. However, large \( k \) may decrease the tracking error bound as long as the kinematic controller and the robust term maintain the stability of a control system.

5. Simulation results

In this section, we illustrates the effectiveness of a proposed FL deadzone compensation method for a mobile manipulator. For computer simulations, we took the vehicle and arm parameters as \( m_p = 10[Kg] \), \( m_1 = 1[Kg] \), \( m_2 = 1[Kg] \), \( l_1 = l_2 = l_w = 1[Kg \cdot m^2] \), \( l_p = 5[Kg \cdot m^2] \), \( l_1 = l_2 = 0.05[m] \), \( 2l = 0.35[m] \), and \( r = 0.05[m] \), \( d = 0.001[m] \). The controller gains were chosen so that the closed loop system exhibits a critical damping behavior \( k_1 = 10 \), \( k_2 = 5 \), \( k_3 = 4 \), \( k_4 = diag(40,40) \), \( k_5 = diag(10,10) \), \( k_6 = 1 \), and \( \Lambda = diag(5,5) \). The reference points are constructed by using the kinematic model (35) and the following velocities, as follows:

\[ \omega_r = 1.0[m \ / \ sec] \]

\[ \omega_r = -1 + 6 \sin(0.0139t)[deg \ / \ sec]. \] (86)

The reference trajectories for the arm are \( \theta_{rj}(t) = \sin(0.0698t) \) and \( \theta_{rj}(t) = \cos(0.0698t) \). The departure posture vector is \((-5,-5,0)\) and the goal is trajectory following. Fig. 5 shows the reference trajectory response of a mobile manipulator. Since the deadzone nonlinearity is included in the mobile robot, the performance degraded by the deadzone effects in Fig. 6. The deadzone nonlinearity for mobile platform are \( d_+ = 0.33 \) and \( d_- = -0.3 \) for right wheel and \( d_+ = 0.31 \) and \( d_- = -0.3 \) for left wheel. The deadzone nonlinearity for manipulator are \( d_+ = 0.2 \) and \( d_- = -0.21 \) for arm 1 and \( d_+ = 0.19 \) and \( d_- = -0.2 \) for arm 2. In, Fig. 7, the
The proposed FL deadzone compensation shows an improvement in trajectory response compared with the dynamic controller. The velocity error, angular velocity error for vehicle, and the estimates of deadzone widths are shown in Fig 7(c)-(e).

Fig. 5. Response without deadzone nonlinearity of a mobile manipulator (a) vehicle trajectory and (b) arm position.
Fig. 6. Response with deadzone nonlinearity of a mobile manipulator (a) vehicle trajectory and (b) arm position.
Fuzzy Logic Deadzone Compensation for a Mobile Robot

(a) xy

with FL compensation → reference

(b) arm

\[ \theta_1 \rightarrow \theta_1 \]

\[ \theta_2 \rightarrow \theta_2 \]
(c)
estimates of deadzone widths for left wheel

\[ \hat{d}_+ \]

\[ \hat{d}_- \]

Time [sec]

estimates of deadzone widths for right wheel

\[ \hat{d}_+ \]

\[ \hat{d}_- \]

Time [sec]
Fig. 7.(Continued).
6. Conclusions

The FL deadzone compensation with a linear controller for tracking of a mobile manipulators has been developed. In fact, perfect knowledge of the mobile manipulator parameters is unattainable, e.g., the deadzone nonlinearity is very difficult to model by conventional techniques. To confront this, an FL deadzone compensation with guaranteed performance has been derived. The proposed control scheme is shown to be asymptotically stable through theoretical proof and simulation with a mobile manipulator.

7. References


Trying to meet the requirements in the field, present book treats different fuzzy control architectures both in terms of the theoretical design and in terms of comparative validation studies in various applications, numerically simulated or experimentally developed. Through the subject matter and through the inter and multidisciplinary content, this book is addressed mainly to the researchers, doctoral students and students interested in developing new applications of intelligent control, but also to the people who want to become familiar with the control concepts based on fuzzy techniques. Bibliographic resources used to perform the work includes books and articles of present interest in the field, published in prestigious journals and publishing houses, and websites dedicated to various applications of fuzzy control. Its structure and the presented studies include the book in the category of those who make a direct connection between theoretical developments and practical applications, thereby constituting a real support for the specialists in artificial intelligence, modelling and control fields.

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