Tracking Control of Spacecraft by Dynamic Output Feedback - Passivity- Based Approach -

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1. Introduction

In this study, we investigate the possibility of capturing an inoperative spacecraft using an orbital servicing vehicle or a space robot in future space infrastructure. These missions involve problems related to the tracking control of a target spacecraft; therefore, a control system design that takes into account the interference with the nonlinear motion of the spacecraft is required because the equations of motion of such a spacecraft are nonlinear system in which the six-degree-of-freedom (six-dof) translational motion and the rotational motion are coupled.

They have been many studies on the six-dof tracking control problem related to spacecrafts (Ahmed et al., 1998; Terui, 1998; Dalsmo & Egeland, 1999; Bošković et al., 2004; Ikeda et al., 2008; Seo & Akella, 2008). The control methods proposed by these researches are state feedback control methods and involve measurements of the linear and angular velocities of the spacecraft. It is necessary to develop an output feedback control method, which does not require velocity measurements in cases where a velocity sensor cannot be mounted on the spacecraft because of the limitations on the cost and weight of the spacecraft, or as a backup controller to ensure spacecraft stability when the velocity sensor breaks down.

For the output feedback tracking control problem, a control method that eliminate the velocity measurement via the filtering of the position and attitude information (Costic et al., 2000; Costic et al., 2001; Pan et al., 2004) or the estimation of the velocity by the observer (McDuffie & Shtessel, 1997; Seo & Akella, 2007) has previously been proposed. However, these methods cannot be used for tracking a spacecraft with an arbitrary trajectory since the attitude controller has a singular point at which the control input diverges; another instance where the method cannot be used is when the initial state of the control system is restricted.

In this paper, we propose a new passivity-based control method that involves the use of output feedback for solving the tracking control problem. Although the proposed method has a filter as well as (Costic et al., 2000), (Costic et al., 2001), and (Pan et al., 2004), and is implemented by using the conventional methods, it can track a spacecraft with an arbitrary trajectory because the controller does not have a singular point. Thus, the proposed method has characteristics that are better than those of conventional methods.

This paper is organized as follows: Section 2 describes the tracking control problem and the derivation of the relative equation of motion; the equation is then used for transforming the tracking control problem to a regulation problem. In section 3, we construct the dynamic output feedback controller that is based on passivity. Concretely speaking, the relative equation of motion is transformed into a passive system by a coordinate and feedback transformation, and a controller based on the passive system is designed. In addition, the controller obtained can be considered to be an observer. In section 4, we provide the guidelines for obtaining the controller parameters and show that the controller can be made to be similar to a proportional-derivative (PD) controller by appropriately setting the parameters. The effectiveness of the control methods is verified by performing numerical simulations in section 5. Finally, the conclusion is given in section 6.

2. Relative equation of motion of spacecraft

In this paper, we consider the tracking control problem in which the chaser spacecraft tracks to the target spacecraft that has a broken down actuator and moves in space freely. The definition of the coordinate systems and the position vectors are shown in Fig. 1. $\{i\}$, $\{c\}$, and $\{t\}$ represent the inertial, chaser, and target frame, respectively. Here, the position of the chaser conforms to a constant vector $p_t \in R^3$ fixed at $\{t\}$. In addition, the attitude of the chaser and target represent the quaternion (Hughes, 1986).

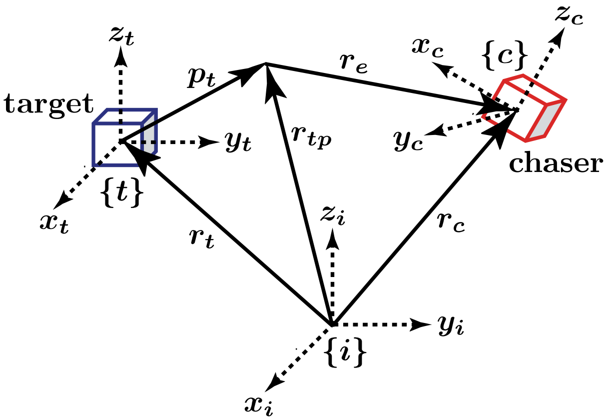


Fig. 1. Definition of the coordinate system and the position vector.

The equation of motion of the target and the chaser can be described as follows (Terui, 1998):
Target:

$$\dot{r}_t = v_t - \omega_t^{\times} r_t, \tag{1}$$

$$\dot{q}_t = \frac{1}{2}\begin{bmatrix} \varepsilon_t^{\times} + \eta_t I_3 \\ -\varepsilon_t^T \end{bmatrix} \omega_t = E(q_t)\omega_t, \ \|q_t\| = 1, \tag{2}$$

$$m_t \dot{v}_t = -m_t \omega_t^{\times} v_t, \tag{3}$$

$$J_t \dot{\omega}_t = -\omega_t^{\times} J_t \omega_t. \tag{4}$$

Chaser:

$$\dot{r}_c = v_c - \omega_c^{\times} r_c, \tag{5}$$

$$\dot{q}_c = \frac{1}{2}\begin{bmatrix} \varepsilon_c^{\times} + \eta_c I_3 \\ -\varepsilon_c^{T} \end{bmatrix}\omega_c = E(q_c)\omega_c,\ \|q_c\| = 1, \tag{6}$$

$$m_c \dot{v}_c = -m_c \omega_c^{\times} v_c + f_c, \tag{7}$$

$$J_c \dot{\omega}_c = -\omega_c^{\times} J_c \omega_c + \tau_c + \rho_c^{\times} f_c, \tag{8}$$

where $r_i \in R^3\,(i = t, c)$ is the position from the origin of the inertial frame $\{i\}$ to the center of mass of each frame, $v_i \in R^3$ is the linear velocity of the body-fixed frame with respect to $\{i\}$, $\omega_i \in R^3$ is the angular velocity of the body-fixed frame with respect to $\{i\}$, $q_i = \begin{bmatrix} \varepsilon_i^T & \eta_i \end{bmatrix}^T \in S^3$ is the quaternion, $f_c \in R^3$ is the control force, $\tau_c \in R^3$ is the control torque, $m_i \in R$ is the mass, $J_i \in R^{3\times 3}$ is the inertia matrix, $\rho_c \in R^3$ is the vector of the point of application of control force, I_n is an $n \times n$ identity matrix, and $a^{\times}$ is the skew symmetric matrix,

$$a^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{9}$$

which is induced from vector $a = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^T$. In addition, S^3 is the hypersphere of dimension three and is defined as follows:

$$S^3 = \left\{ q_i \in R^4 \mid \|q_i\| = 1 \right\}\ (i = t, c).$$

Our tracking control problem is to find a controller such that

$$r_c = r_{tp}, \varepsilon_c = \varepsilon_t, \eta_c = \eta_t, v_c = v_{tp}, \omega_c = \omega_t$$

when $t \to \infty$. The position and the velocity of the tip of vector p_t fixed at $\{t\}$ are given by

$$r_{tp} = r_t + p_t,\ v_{tp} = v_t + \omega_t^{\times} p_t. \tag{10}$$

To this end, an error system in $\{c\}$ is described as follows: Let the direction cosine matrix from $\{t\}$ to $\{c\}$ be

$$C = \left(\eta_e^2 - \varepsilon_e^T \varepsilon_e\right) I_3 + 2\varepsilon_e \varepsilon_e^T - 2\eta_e \varepsilon_e^{\times} \tag{11}$$

using the quaternion of relative attitude $q_e = \begin{bmatrix} \varepsilon_e^T & \eta_e \end{bmatrix}^T$, where ε_e and η_e are defined as

$$\varepsilon_e = \eta_t \varepsilon_c - \eta_c \varepsilon_t + \varepsilon_c^{\times} \varepsilon_t,\ \eta_e = \eta_c \eta_t + \varepsilon_c^T \varepsilon_t. \tag{12}$$

The relative position, linear velocity, and angular velocity are given in the same $\{c\}$ frame as

$$r_e = r_c - Cr_{tp},\ v_e = v_c - Cv_{tp},\ \omega_e = \omega_c - C\omega_t. \tag{13}$$

From (12) and (13), using the identity $\dot{C} = -\omega_e^{\times} C$, we obtain the relative equations of motion as

$$\dot{r}_e = v_e - \left(\omega_e + C\omega_t\right)^{\times} r_e, \tag{14}$$

$$\dot{q}_e = \frac{1}{2}\begin{bmatrix} \varepsilon_e^{\times} + \eta_e I_e \\ -\varepsilon_e^{T} \end{bmatrix}\omega_e = E(q_e)\omega_e,\ \|q_e\| = 1, \tag{15}$$

$$m_c\dot{v}_e = -m_c\left[\left(\omega_e + C\omega_t\right)^{\times} v_e + C\dot{v}_{tp} + \left(C\omega_t\right)^{\times} Cv_{tp}\right] + f_c, \tag{16}$$

$$J_c\dot{\omega}_e = -\left(\omega_e + C\omega_t\right)^{\times} J_c\left(\omega_e + C\omega_t\right) - J_c\left(C\dot{\omega}_t - \omega_e^{\times}C\omega_t\right) + \tau_c + \rho_c^{\times} f_c. \tag{17}$$

After the transform, the tracking control problem is reduced to a regulation problem to design a controller such that

$$r_e = 0, \varepsilon_e = 0, v_e = 0, \omega_e = 0$$

when $t \to \infty$ according to (14)-(17).
Hereafter, in order to simplify the derivation of the controller, the control force and torque are as follows:

$$\hat{f}_c = f_c, \hat{\tau}_c = \tau_c + \rho_c^{\times} f_c. \tag{18}$$

The controller is derived using (18) in the sequel. Since the inverse transform from $\hat{f}_c$, $\hat{\tau}_c$ to f_c, τ_c obviously exists, f_c, τ_c can be uniquely determined after $\hat{f}_c$, $\hat{\tau}_c$ is derived.
Remark 1: η_e at $\varepsilon_e = 0$ exists as $\eta_e \pm 1$ because of the constraint of the quaternion $\|q_e\| = 1$. In this paper, η_e, which should be asymptotically stabilized, is set to $\eta_e = 1$.

3. Dynamic output feedback control

3.1 Passivation of relative equation of motion

Since the relative equation of motion (14)-(17) is a complicated nonlinear time-varying system, it is difficult to design a controller based on (14)-(17). Therefore, in order to facilitate a controller, the relative equation of motion (14)-(17) is transformed into a passive system by a coordinate and feedback transformation, and a controller design based on the passive system is designed. Further, in this paper, we consider the output feedback control problem - the linear velocity v_c and the angular velocity ω_c of the chaser, in other words, the relative linear velocity v_e and the angular velocity ω_e, cannot be measured. We suppose that the states r_t, q_t, v_t, ω_t of the target can be measured in some way, for example, the target

motion estimation method using image information (Lichter & Dubowsky, 2004; Tanaka et al., 2007).
Let us consider the following coordinate and feedback transformation.

$$\bar{v}_e = v_e - (C\omega_t)^{\times} r_e, \tag{19}$$

$$\hat{f}_c = \bar{f}_c + m_c\delta_r,\ \hat{\tau}_c = \bar{\tau}_c + \delta_q, \tag{20}$$

where $\bar{f}_c$, $\bar{\tau}_c \in R^3$ are the new control inputs, and

$$\delta_r = (C\dot{\omega}_t)^{\times} r_e + (C\omega_t)^{\times}(C\omega_t)^{\times} r_e + C\dot{v}_{tp} + (C\omega_t)^{\times} C v_{tp},$$

$$\delta_q = (C\omega_t)^{\times} J_c (C\omega_t) + J_c C\dot{\omega}_t.$$

From (19) and (20), the relative equation of motion (14)-(17) is transformed into the following system:

$$\dot{r}_e = \bar{v}_e - \omega_e^{\times} r_e, \tag{21}$$

$$\dot{q}_e = E(q_e)\omega_e, \tag{22}$$

$$m_c\dot{\bar{v}}_e = -m_c(\omega_e + 2C\omega_t)^{\times}\bar{v}_e + \bar{f}_c, \tag{23}$$

$$J_c\dot{\omega}_e = -\omega_e^{\times} J_c(\omega_e + C\omega_t) - H\omega_e + \bar{\tau}_c, \tag{24}$$

where $H = (C\omega_t)^{\times} J_c + J_c (C\omega_t)^{\times}$ and H is a skew-symmetric matrix. If we can find a controller such that

$$r_e = 0, \varepsilon_e = 0, \bar{v}_e = 0, \omega_e = 0$$

when $t \to \infty$ according to (21)-(24), then the tracking control is achieved since $\bar{v}_e = 0$ implies $v_e = 0$ from (19). Therefore, the tracking control problem is reduced to a regulation problem of $(r_e, \varepsilon_e, \bar{v}_e, \omega_e)$.
At the end of this subsection, it is shown that the system (21)-(24) is passive. Let us consider the following storage function:

$$E = \frac{1}{2} m_c \bar{v}_e^T \bar{v}_e + \frac{1}{2}\omega_e^T J_c \omega_e. \tag{25}$$

By using the skew symmetric matrix properties $a^T b^{\times} a = 0$, $a^T a^{\times} = 0$, $\forall a, b \in R^3$, we can express the time derivative of (25) along with the trajectories as

$$\begin{aligned}\dot{E} &= \overline{v}_e{}^T\left[-m_c\left(\omega_e+2C\omega_t\right)^{\times}\overline{v}_e+\overline{f}_c\right]+\omega_e{}^T\left[-\omega_e{}^{\times}J_c\left(\omega_e+C\omega_t\right)-H\omega_e+\overline{\tau}_c\right]\\ &= \overline{v}_e{}^T\overline{f}_c+\omega_e{}^T\overline{\tau}_c \\ &= \overline{y}^T\overline{u},\ \overline{y}=\left[\overline{v}_e{}^T \quad \omega_e{}^T\right]^T,\ \overline{u}=\left[\overline{f}_c{}^T \quad \overline{\tau}_c{}^T\right].\end{aligned} \tag{26}$$

Therefore, the system (21)-(24) is passive with respect to input $\overline{u}$ and output $\overline{y}$.

Remark 2: In feedback transformation (20), although the acceleration $\dot{v}_{tp}$ and $\dot{\omega}_t$ are needed, this information can be calculated algebraically from (3), (4), and (10) if v_t and ω_t can be measured. In addition, we suppose that the inertia matrix J_t is kwon hereafter.

3.2 Controller design

In this subsection, the dynamic output feedback controller that asymptotically stabilizes the relative position and attitude is designed on the basis of the passivity of the system (21)-(24). With respect to the target states, the following assumption is made.

Assumption 1: The target states r_t, q_t, v_t, ω_t, $\dot{v}_t$, and ω_t are uniformly continuous and bounded.

Then, the following theorem can be obtained.

Theorem 1: Consider the following dynamic output controller

$$\begin{cases}\dot{z}_1 = A_1 z_1 + B_1 r_e \\ y_1 = C_1\dot{z}_1 = C_1\left(A_1 z_1 + B_1 r_e\right), \\ \overline{f}_c = -k_{p1} r_e - k_1 y_1\end{cases}$$
$$A_1\in R^{n_1\times n_1},\ B_1\in R^{n_1\times 3},\ C_1\in R^{3\times n_1},\ n_1\geq 3, \tag{27}$$

$$\begin{cases}\dot{z}_2 = A_2 z_2 + B_2 q_e \\ y_2 = C_2\dot{z}_2 = C_2\left(A_2 z_2 + B_2 q_e\right) \\ \overline{\tau}_c = -K\left(q_e\right)\varepsilon_e + k_1 r_e{}^{\times} y_1 - k_2 E\left(q_e\right)^T y_2\end{cases},$$
$$K\left(q_e\right)=T\left(q_e\right)^T K_{p2}-k_{p3}\left(\eta_e-1\right)I_3,\ T\left(q_e\right)=\eta_e I_3+\varepsilon_e{}^{\times}, \tag{28}$$
$$A_2\in R^{n_2\times n_2},\ B_2\in R^{n_2\times 4},\ C_2\in R^{4\times n_2},\ n_2\geq 4,$$

where k_{p1}, k_{p3}, k_1, $k_2>0$ are scalar feedback gains; $K_{p2}=K_{p2}{}^T>0$, $K_{p2}\in R^{3\times 3}$ is the matrix feedback gain; A_i, B_i, and C_i are design parameters (A_i is stable, and B_i is a full column rank matrix). Furthermore, A_i, B_i, and C_i must be designed such that there exists a matrix $P_i=P_i^T>0$ that satisfies the following matrix algebraic equations (a strictly positive real condition):

$$A_i{}^T P_i + P_i A_i = -Q_i,\ P_i B_i = C_i{}^T \tag{29}$$

for an arbitrary matrix $Q_i=Q_i{}^T>0$. Then, the state variable of the closed-loop system of (21)-(24) with (27) and (28) becomes

$$\left(r_e, \varepsilon_e, \eta_e, \bar{v}_e, \omega_e, z_1, z_2\right) \to \left(0, 0, 1, 0, 0, 0, z_2^*\right)$$
$$z_2^* = -A_2^{-1} B_2 q_e^*, \; q_e^* = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{30}$$

when $t \to \infty$ for an arbitrary initial state.

Proof: Consider the following candidate of a Lyapunov function:

$$\begin{aligned} V(x) &= E + \frac{k_{p1}}{2} r_e^T r_e + \varepsilon_e^T K_{p2} \varepsilon_e + k_{p3} (\eta_e - 1)^2 + \frac{k_1}{2} (A_1 z_1 + B_1 r_e)^T P_1 (A_1 z_1 + B_1 r_e) \\ &+ \frac{k_2}{2} (A_2 z_2 + B_2 q_e)^T P_2 (A_2 z_2 + B_2 r_e), \\ x &= \begin{bmatrix} r_e^T & \varepsilon_e^T & \eta_e & \bar{v}_e^T & \omega_e^T & z_1^T & z_2^T \end{bmatrix}. \end{aligned} \tag{31}$$

In (31), V equals to zero only when x is (30), $V > 0$ with the exception of (30). By using the skew symmetric matrix properties $a^T b^\times a = 0$, $a^T a^\times = 0$, $\left(a^\times\right)^T = -a^\times$, $\forall a, b \in R^3$ and (29), we can express the time derivative of (31) along with the trajectories as

$$\begin{aligned} \dot{V} &= \bar{v}_e^T \bar{f}_c + \omega_e^T \bar{\tau}_c + k_{p1} \bar{v}_e^T r_e + \omega_e^T T(q_e)^T K_{p2} \varepsilon_e - k_{p3} (\eta_e - 1) \omega_e^T \varepsilon_e + \sum_{i=1}^{2} \frac{k_i}{2} \dot{z}_i^T \left(A_i^T P_i + P_i A_i\right) \dot{z}_i \\ &\quad + k_1 \dot{r}_e^T B_1^T P_1 \dot{z}_1 + k_2 \dot{q}_e^T B_2^T P_2 \dot{z}_2 \\ &= -\sum_{i=1}^{2} \frac{k_i}{2} \dot{z}_i^T Q_i \dot{z}_i + \bar{v}_e^T \left(\bar{f}_c + k_{p1} r_e + C_1 \dot{z}_1\right) + \omega_e^T \left[\bar{\tau}_c + K(q_e) \varepsilon_e - k_1 r_e^\times C_1 \dot{z}_1 + k_2 E(q_e)^T C_2 \dot{z}_2\right] \\ &= -\sum_{i=1}^{2} \frac{k_i}{2} \dot{z}_i^T Q_i \dot{z}_i \le 0. \end{aligned} \tag{32}$$

Therefore, x is bounded since

$$V(x(t)) \le V(x(0)), \; \forall t \ge 0 \tag{33}$$

and V is radially unbounded in the state space $\Omega := R^{(9+n_1+n_2)} \times S^3$. Then, $\dot{x}$ is also bounded because the control inputs $\bar{f}_c$, $\bar{\tau}_c$ are bounded by Assumption 1. It follows that

$$\ddot{V} = -\sum_{i=1}^{2} k_i \dot{z}_i^T Q_i \ddot{z}_i$$

is bounded, and $\dot{V}$ is uniformly continuous with respect to t. Therefore, it is shown that

$$\dot{V} \to 0 \;\Rightarrow\; \dot{z}_i \to 0$$

when $t \to \infty$ from the Lyapunov-like lemma (Slotine & Li, 1991), and then

$$\dot{z}_i = 0, z_i = const. \;\Rightarrow\; r_e = q_e = const. \;\Rightarrow\; \dot{r}_e = 0, \dot{q}_e = 0$$

when $t \to \infty$ from (27) and (28) since B_i is a full column rank matrix, and

$$\bar{v}_e = 0, \omega_e = 0$$

from (21) and (22). Furthermore, the closed-loop system becomes

$$k_{p1} I_3 r_e = 0,\ K(q_e)\varepsilon_e = 0,\ A_1 z_1 + B_1 r_e,\ A_2 z_2 + B_2 q_e = 0. \tag{34}$$

From (34), $r_e = 0$, $\varepsilon_e = 0$ since $k_{p1} > 0$ and $\det K(q_e) \neq 0, \forall q_e$, and $\eta_e = 1$ from $V = 0$. In addition, since A_i is stable and B_i is a full column rank matrix, it follows that

$$z_1 = 0,\ z_2 = -A_2^{-1} B_2 q_e^* = 0.$$

It is known that a controller, as (27) and (28), based on the strictly positive real condition (29) is a type of observer. The controllers (27) and (28) are the observers, and the estimate errors are

$$\bar{z}_1 = z_1 + A_1^{-1} B_1 r_e,\ \bar{z}_2 = z_2 + A_2^{-1} B_2 q_e. \tag{35}$$

Then, the following corollary can be obtained.
Corollary 1: Dynamic compensators of dynamic output feedback controllers (27) and (28) are the observers; the estimate errors are (35), and

$$z_1 \to -A_1^{-1} B_1 r_e,\ z_2 \to -A_2^{-1} B_2 q_e$$

when $t \to \infty$.
Proof: By using the estimate error (35), we can represent the dynamic output feedback controllers (27) and (28) as

$$\begin{cases} \dot{\bar{z}}_1 = A_1 \bar{z}_1 + A_1^{-1} B_1 \dot{r}_e \\ y_1 = C_1 A_1 \bar{z}_1 \\ \bar{f}_c = -k_{p1} r_e - k_1 y_1 \end{cases}, \tag{36}$$

$$\begin{cases} \dot{\bar{z}}_2 = A_2 \bar{z}_2 + A_2^{-1} B_2 \dot{q}_e \\ y_2 = C_2 A_2 \bar{z}_2 \\ \bar{\tau}_c = -K(q_e)\varepsilon_e + k_1 r_e^{\times} y_1 - k_2 E(q_e)^T y_2 \end{cases}. \tag{37}$$

Consider the following candidate of a Lyapunov function:

$$V(x) = E + \frac{k_{p1}}{2} r_e^T r_e + \varepsilon_e^T K_{p2} \varepsilon_e + k_{p3} (\eta_e - 1)^2 + \sum_{i=1}^{2} \frac{k_1}{2} \bar{z}_i^T A_i^T P_i A_i \bar{z}_i. \tag{38}$$

From the calculations (32), we obtain the time derivative of (38) along with the trajectories as

$$\begin{aligned}\dot{V} &= \overline{v}_e{}^T \overline{f}_c + \omega_e{}^T \overline{\tau}_c + k_{p1} \overline{v}_e{}^T r_e + \omega_e{}^T T(q_e)^T K_{p2} \varepsilon_e - k_{p3}(\eta_e - 1)\omega_e{}^T \varepsilon_e \\ &+ \sum_{i=1}^{2} \frac{k_i}{2} z_i{}^T A_i{}^T \left(A_i{}^T P_i + P_i A_i\right) A_i z_i + k_1 \overline{z}_1{}^T A_1{}^T P_1 B_1 \left(\overline{v}_e - \omega_e{}^\times r_e\right) + k_2 \overline{z}_2{}^T A_2{}^T P_2 B_2 E(q_e)\omega_e \\ &= -\sum_{i=1}^{2} \frac{k_i}{2} \overline{z}_i{}^T A_i{}^T Q_i A_i \overline{z}_i + \overline{v}_e{}^T \left(\overline{f}_c + k_{p1} r_e + k_1 C_1 A_1 \overline{z}_1\right) \\ &+ \omega_e{}^T \left[\overline{\tau}_c + K(q_e)\varepsilon_e - k_1 r_e{}^\times C_1 A_1 \overline{z}_1 + k_2 E(q_e)^T C_2 A_1 \overline{z}_2\right] \\ &= -\sum_{i=1}^{2} \frac{k_i}{2} \overline{z}_i{}^T A_i{}^T Q_i A_i \overline{z}_i.\end{aligned} \tag{39}$$

Since $A_i{}^T P_i A_i > 0$, $A_i{}^T Q_i A_i > 0$ from P_i and Q_i are positive definite matrices and A_i is a stable matrix, $V > 0$ and $\dot{V} \le 0$ hold. Hereafter, in the same way as in the case of Theorem 1, the state variable of the closed-loop system of (21)-(24) with (36) and (37) becomes

$$\left(r_e, \varepsilon_e, \eta_e, \overline{v}_e, \omega_e, \overline{z}_1, \overline{z}_2\right) \to (0, 0, 1, 0, 0, 0, 0) \tag{40}$$

when $t \to \infty$ for an arbitrary initial state in the state space Ω.

Remark 3: In the conventional methods (Costic et al., 2000; Costic et al., 2001; Pan et al., 2004), the relative equation of motion with respect to the attitude is transformed into an Euler-Lagrange form by $(1/2)\left(\varepsilon_e{}^\times + \eta_e I_3\right) := S(q_e)$ of (15) as the coordinate transform matrix, and a controller based on the Euler-Lagrange form is designed. However, $\eta_e = 0$ is a singular point because $\det S(q_e) = 0$ when $\eta_e = 0$. In contrast, the proposed method does not exsist a singular point since a controller based on the relative equation of motion is designed.

4. Guidelines of controller parameter setting

It is difficult for dynamic output feedback controllers (27) and (28) to find a clear meaning for the design parameters A_i, B_i, and C_i (or Q_i) as the state feedback control (e.g., PD control). Therefore, the control performance deteriorates according to the value of the design parameters as the convergence of the relative error is slow or the response of the relative error vibrates. In this section, we discuss a guideline for the design parameters.

In order to simply the argument, the design parameters A_i, B_i, and $Q_i \, (i = 1,2)$ are set as follows:

$$\begin{aligned}A_1 &= -a_1 I_3,\ B_1 = -A_1 = a_1 I_3,\ Q_1 = q_1 I_3, \\ A_2 &= -a_2 I_4,\ B_2 = -A_2 = a_2 I_4,\ Q_2 = q_2 I_4,\end{aligned}$$

where $a_i, q_i > 0$. In addition, P_i and C_i are

$$P_1 = \frac{q_1}{2a_1} I_3,\ C_1 = B_1{}^T P_1 = \frac{q_1}{2} I_3,$$
$$P_2 = \frac{q_2}{2a_2} I_4,\ C_2 = B_2{}^T P_2 = \frac{q_2}{2} I_4$$

from (29). Then, the output $y_i\,(i = 1,2)$ of the dynamic compensator of (27) and (28) becomes

$$y_1 = -\frac{a_1 q_1}{2} e^{-a_1 t} I_3 \cdot z_1(0) + \frac{q_1}{2} L^{-1}\left[\left\{\frac{s}{(1/a_1)s+1}\right\} I_3 \cdot r_e(s)\right], \tag{41}$$

$$y_2 = -\frac{a_2 q_2}{2} e^{-a_2 t} I_4 \cdot z_2(0) + \frac{q_2}{2} L^{-1}\left[\left\{\frac{s}{(1/a_2)s+1}\right\} I_4 \cdot q_e(s)\right], \tag{42}$$

where $z_i(0)$ is the initial value of the dynamic compensator, $r_e(s)$ and $q_e(s)$ are the Laplace transformation of $r_e(t)$ and $q_e(t)$, and $L^{-1}[(\bullet)]$ is the inverse Laplace transformation. Moreover, the first term of (41) and (42) reveals the effect of the initial value of the dynamic compensator, and the second term of (41) and (42) reveals the effect of the input ($r_e(t), q_e(t)$) to the dynamic compensator.
From (41) and (42), we can conclude that the transfer function of the second term is an approximation differentiator. Therefore, when the value of a_i is large, the output y_i can be approximated as

$$y_1 \approx \frac{q_1}{2} L^{-1}\left[\left\{\frac{s}{(1/a_1)s+1}\right\} I_3 \cdot r_e(s)\right], \tag{42}$$

$$y_2 \approx \frac{q_2}{2} L^{-1}\left[\left\{\frac{s}{(1/a_2)s+1}\right\} I_4 \cdot q_e(s)\right], \tag{43}$$

and the terms of $-k_1 y_1$ and $-k_2 E(q_2)^T y_2$ of control law approximately become the velocity feedback with respect to $\dot{r}_e$ and $\dot{q}_e$ (Note that $\dot{r}_e \neq \overline{v}_e$, $\dot{q}_e \neq \omega_e$ from (21) and (22)). Further, parameter q_i is considered to be the feedback gain. Therefore, by setting a_i to a large value, the dynamic output feedback controllers (27) and (28) approximately become the PD controllers and $k_j, q_j\,(j = 1,2)$ become the derivative gain. However, the value of k_1 must be determined carefully because the control torque $\overline{\tau}_c$ may become excessive at a certain value of k_1 since the control law of (28) includes the term $k_1 r_e^{\times} y_1$. Moreover, although the control inputs $\overline{f}_c$ and $\overline{\tau}_c$ become large when a_i becomes large since $\overline{f}_c$ and $\overline{\tau}_c$ are represented as

$$\overline{f}_c = -\left(k_{p1} + \frac{k_1 a_1 q_1}{2}\right) r_e + \frac{k_1 a_1 q_1}{2} z_1, \tag{44}$$

$$\bar{\tau}_c = -K(q_e)\varepsilon_e + \frac{k_2 a_2 q_2}{2} E(q_e)^T z_2 - \frac{k_1 a_1 q_1}{2} r_e^{\times} z_1, \tag{45}$$

by setting the initial state $z_i(0)$ as

$$z_1(0) = r_e(0), z_2(0) = q_e(0), \tag{46}$$

the amplitude of the control inputs at an early stage can be reduced without changing the feedback gains. This reason can be expounded as follows: Since the controllers (27) and (28) are the observers and the estimate errors are (35) from Corollary 1, by setting the initial state $z_i(0)$ as (46), we find that $z_i(t)$ becomes

$$z_1(t) = r_e(t), z_2(t) = q_e(t), \ \forall t \geq 0. \tag{47}$$

Therefore, since control inputs $\bar{f}_c$ and $\bar{\tau}_c$ become

$$\bar{f}_c = -k_{p1} r_e, \ \bar{\tau}_c = -K(q_e)\varepsilon_e \tag{48}$$

by using the skew symmetric matrix property $a^{\times}a = 0, \forall a \in R^3$, the amplitude of the control inputs at an early stage can be reduced. The aforementioned results lead to the numerical simulation in the next section.

5. Numerical simulation

The simulation conditions are given in Table 1. The results of the numerical simulation are shown in Figs. 2 and 3. Fig. 2 is the result of Case1, and Fig. 3 is the result of Case2; further θ_e is the relative attitude Euler angle that transforms the relative quaternion q_e into 3-2-1 Euler angle.

From the simulation results, we conclude that the chaser tracks the target and state variables z_1 and z_2 correspond with r_e and q_e as shown by Corollary 1, that is, the dynamic compensator of the controllers (27) and (28) are the observers and the estimate errors are (35). Although the response of Euler angle θ_e deteriorates by approximately 50 [s], this is assumed to be due to the effect of the position feedback term ($k_1 r_e^{\times} y_1$) of the attitude control law. In addition, the maximum values of the control input f_c and τ_c of Case1 and Case2 are

$$Case1 : f_{c,\max} = [-44.9 \quad -14.3 \quad -45.9]^T [N], \ \tau_{c,\max} = [70.9 \quad 65.1 \quad -66.7]^T [Nm],$$
$$Case2 : f_{c,\max} = [-0.969 \quad -0.241 \quad -1.30]^T \times 10^4 [N], \ \tau_{c,\max} = [53.7 \quad 51.2 \quad -53.1]^T [Nm],$$

and the control force f_c of Case2 is relatively large at an early stage as compared to that of Case1 (Figs. 2(f) and 3(e)-(f) have an expanded vertical axis in order to show the change in input). Therefore, by setting the initial state $z_i(0)$ as (46) as described in the previous section, the amplitude of the control inputs at an early stage can be reduced without changing the feedback gains. On the other hand, the amplitude of the control torque τ_c

does not change. This could be attributed to the fact that the effect of the position feedback term (the third term) of (45) is cancelled out by the attitude feedback term (the first and the second terms). As a result, the performance of the attitude control deteriorates, and the undershoot of θ_{e1} becomes large. Therefore, it can be concluded that the effect of the position feedback term in the attitude control is suppressed by setting the initial state $z_i(0)$ as (46).

Then, we compare the proposed method to a conventional method (Costic et al., 2000; Costic et al., 2001); we consider the case where the relative attitude η_e falls into the singular point $\eta_e = 0$. The output feedback controller of the conventional method is given as follows:

$$\begin{cases} \dot{z} = -(k+1)z + k^2\varepsilon_e + \dfrac{\varepsilon_e}{(1-{\varepsilon_e}^T\varepsilon_e)^2}, \; z(0) = k\varepsilon_e(0) \\ \tau_c = S(q_e)^T\left(ke_f - W_d - \dfrac{\varepsilon_e}{(1-{\varepsilon_e}^T\varepsilon_e)^2}\right) \end{cases}, \tag{49}$$
$$e_f = -k\varepsilon_e + z,\; W_d = -J_c\dot{\omega}_t - \frac{1}{2}\omega_t^{\times}J_c\omega_t,$$

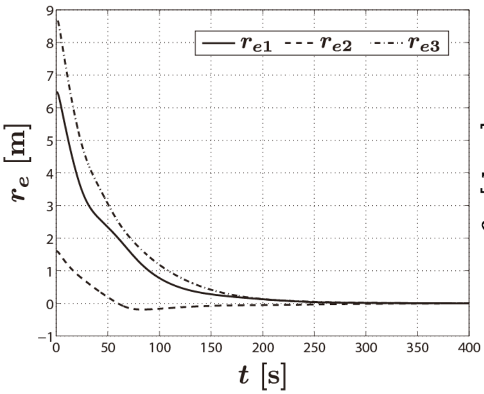
where $k > 0$ is a design parameter. The simulation results of the proposed method are shown in Fig. 4, and those of the conventional method are shown in Fig. 5. In this simulation, the initial state $\eta_e(0)$ sets the singular point (that is, $\eta_e(0) = 0$), and we consider the attitude control only because the conventional method addresses the attitude control problem. Further, from (49) and the norm constraint of the quaternion $\|q_e\| = 1$, since division by zero accrues and the control input cannot be calculated in the conventional method when $\eta_e = 0$, $\eta_e(0)$ sets $\eta_e = 8.73\times10^{-3}$ in Fig. 5. From the simulation results, it is apparent that in the conventional method, the relative quaternion fluctuates in the neighborhood of the singular point (when $\eta_e(0) \to 0$, the relative quaternion fluctuates more) and the control torque is considerably large, while in the proposed method, the tracking control is achieved even if the relative quaternion fall into the singular point. These results show that the proposed method can track a spacecraft with an arbitrary trajectory.

6. Conclusion

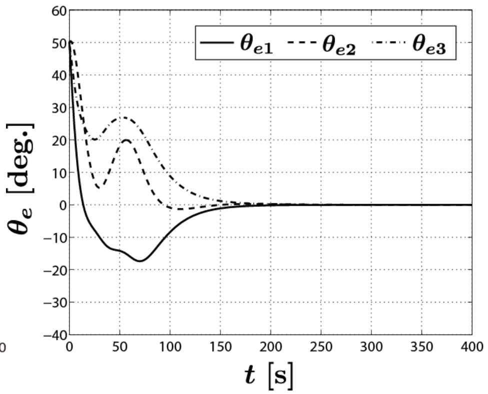
In this paper, we propose a new passivity-based control method that involves the use of output feedback for solving the tracking control problem. The proposed method has an advantage that it can track a spacecraft with an arbitrary trajectory because the controller does not have a singular point as compared to a conventional method. Furthermore, we show that the controller can be made to be similar to a PD controller by appropriately setting the controller parameters. The effectiveness of the proposed methods is verified by performing numerical simulations. Future works, include an extension to the case in which the physical parameter error exists and the robustness against a disturbance can be achieved.

Physical parameters
$m_t = 300[kg],\ J_t = diag\{50,\ 275,\ 275\}[kgm^2],\ m_c = 200[kg],\ J_c = \begin{bmatrix} 75 & -28 & -28 \\ -28 & 75 & -28 \\ -28 & -28 & 75 \end{bmatrix}[kgm^2]$ $p_t = \begin{bmatrix} 0 & 5 & 0 \end{bmatrix}^T [m],\ \rho_c = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T [m]$
Initial state of the target
$r_t(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T [m],\ v_t(0) = \begin{bmatrix} 0.005 & 0.005 & 0.005 \end{bmatrix}^T [m/s],$ $q_t(0) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T [-],\ \omega_t(0) = \begin{bmatrix} 0.005 & 0.005 & 0.005 \end{bmatrix}^T [rad/s]$
Initial state of the chaser
$r_c(0) = \begin{bmatrix} 8 & 9 & 10 \end{bmatrix}^T [m],\ v_c(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T [m/s],$ $q_c(0) = \begin{bmatrix} 0.19 & 0.51 & 0.19 & 0.82 \end{bmatrix}^T [-],\ \omega_c(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T [rad/s]$
Initial state of the dynamic compensator
$Case1\ :\ z_1(0) = r_e(0),\ z_2(0) = q_e(0),$ $Case2\ :\ z_1(0) = 0.5r_e(0),\ z_2(0) = 0.5q_e(0),$ $r_e(0) = \begin{bmatrix} 6.44 & 1.60 & 8.66 \end{bmatrix}^T [m],\ q_t(0) = \begin{bmatrix} 0.19 & 0.51 & 0.19 & 0.82 \end{bmatrix}^T [-]$
Feedback gains and design parameters
$k_{p1} = 3,\ k_1 = 150,\ K_{p2} = 30I_3,\ k_{p3} = 12,\ k_2 = 1400,$ $A_1 = -20I_3,\ B_1 = -A_1 = 20I_3,\ C_1 = I_3,$ $A_2 = -20I_3,\ B_2 = -A_2 = 20I_3,\ C_2 = I_3$

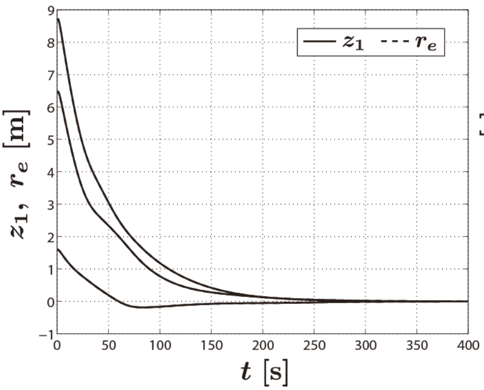
Table 1. Simulation conditions.



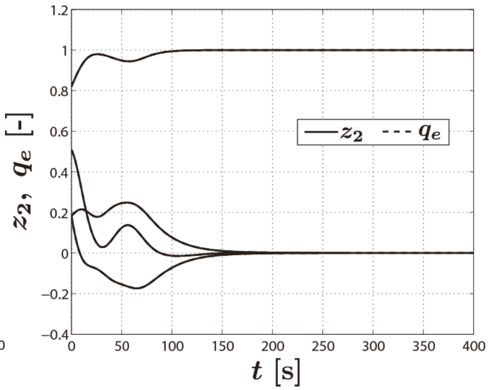
(a) r_e



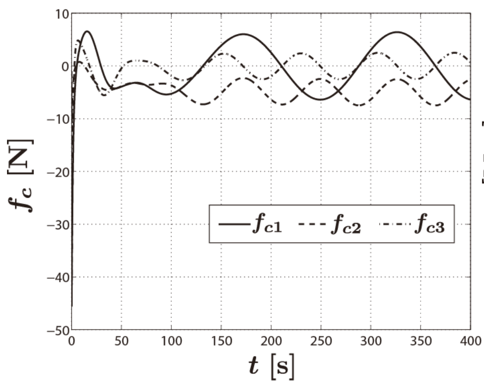
(b) θ_e



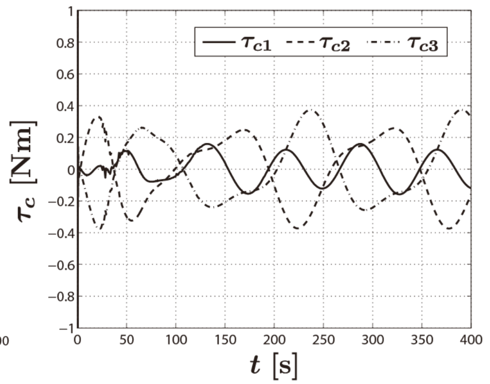
(c) z_1



(d) z_2

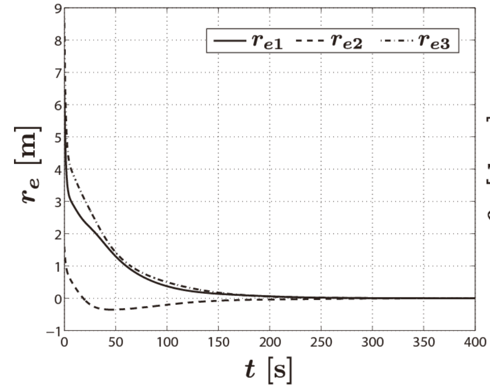


(e) f_c

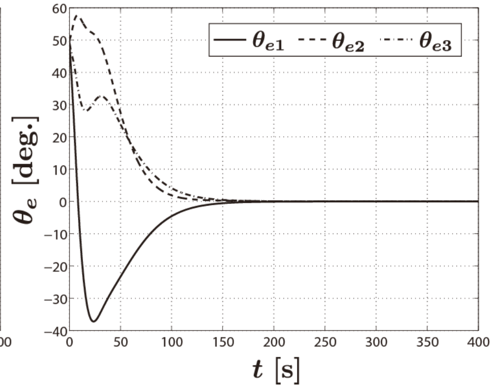


(f) τ_c

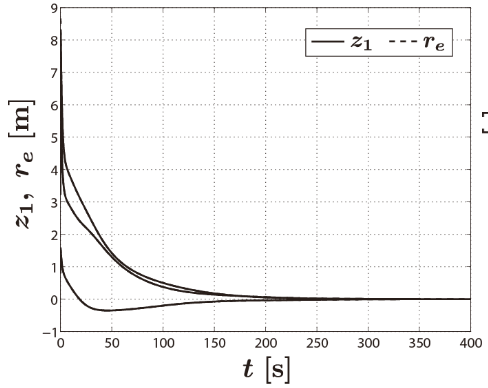
Fig. 2. Simulation results (Case1).



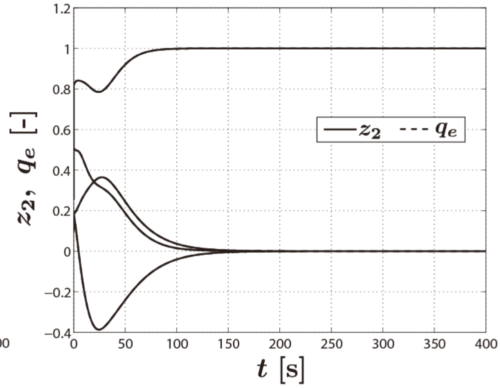
(a) r_e



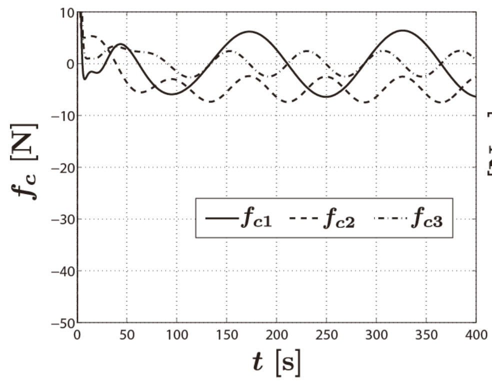
(b) θ_e



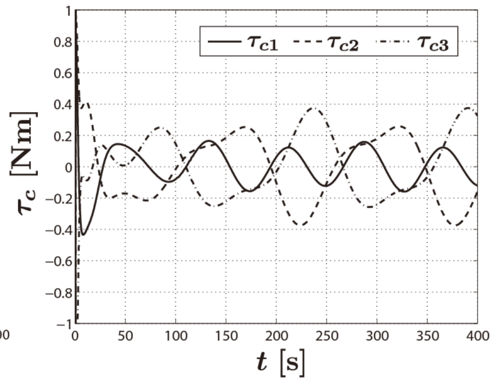
(c) z_1



(d) z_2

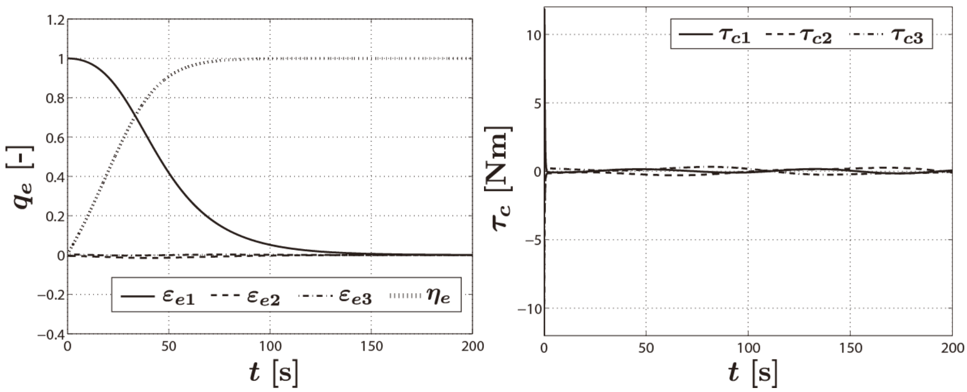


(e) f_c



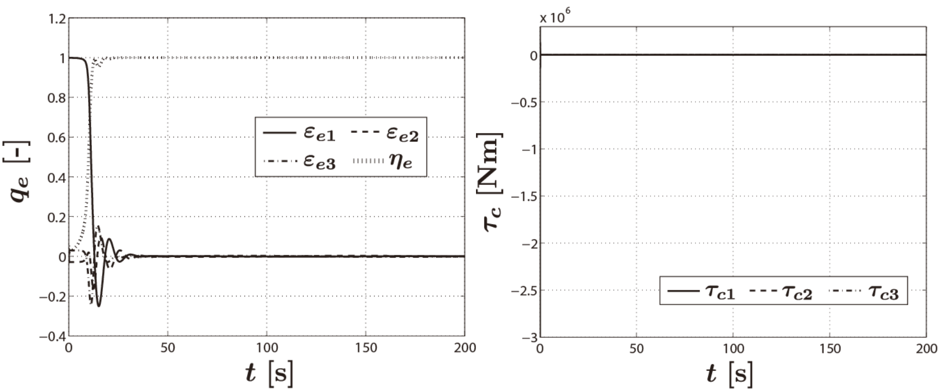
(f) τ_c

Fig. 3. Simulation results (Case2).



(a) q_e (b) τ_c

Fig. 4. Simulation results at singular point (proposed method).



(a) q_e (b) τ_c

Fig. 5. Simulation results at singular point (Costic's method).

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The development and launch of the first artificial satellite Sputnik more than five decades ago propelled both the scientific and engineering communities to new heights as they worked together to develop novel solutions to the challenges of spacecraft system design. This symbiotic relationship has brought significant technological advances that have enabled the design of systems that can withstand the rigors of space while providing valuable space-based services. With its 26 chapters divided into three sections, this book brings together critical contributions from renowned international researchers to provide an outstanding survey of recent advances in spacecraft technologies. The first section includes nine chapters that focus on innovative hardware technologies while the next section is comprised of seven chapters that center on cutting-edge state estimation techniques. The final section contains eleven chapters that present a series of novel control methods for spacecraft orbit and attitude control.

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