Transform Domain based Channel Estimation for 3GPP/LTE Systems

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1. Introduction

Orthogonal frequency division multiplexing (OFDM) is now well known as a powerful modulation scheme for high data rate wireless communications owing to its many advantages, notably its high spectral efficiency, mitigation of intersymbol interference (ISI), robustness to frequency selective fading environment, as well as the feasibility of low cost transceivers [1].

On the other hand multiple input multiple output (MIMO) systems can also be efficiently used in order to increase diversity and improve performance of wireless systems [2] [3] [4]. Moreover, as OFDM allows a frequency selective channel to be considered as flat on each subcarrier, MIMO and OFDM techniques can be well combined. Therefore, MIMO-OFDM systems are now largely considered in the new generation of standards for wireless transmissions, such as 3GPP/LTE [5] [6].

In most MIMO-OFDM systems, channel estimation is required at the receiver side for all sub-carriers between each antenna link. Moreover, since radio channels are frequency selective and time-dependent channels, a dynamic channel estimation becomes necessary. For coherent MIMO-OFDM systems, channel estimation relies on training sequences adapted to the MIMO configuration and the channel characteristics [7] and based on OFDM channel estimation with pilot insertion, for which different techniques can be applied: preamble method and comb-type pilot method.

In order to estimate the channel of an OFDM systems, one’s first apply least square (LS) algorithm to estimate the channel on the pilot tones in the frequency domain. A second step can be performed to improve the quality of the estimation and provide interpolation to find estimates on all subcarriers. In a classical way, this second step is performed in the frequency domain. An alternative is to perform this second step by applying treatment in a transform domain, that can be reached after a discrete Fourier transform (DFT) or a discrete cosine transform (DCT), and called transform domain channel estimation (TD-CE). The DFT based method is considered as a promising method because it can provide very good results by significantly reducing the noise on the estimated channel coefficients [8]. However, some performance degradations may occur when the number of OFDM inverse fast fourier transform (IFFT) size is different from the number of modulated subcarriers [8]. This problem called “border effect” phenomenon is due to the insertion of null carriers at the spectrum extremities (virtual carriers) to limit interference with the adjacent channels, and can be encountered in most of multicarrier systems.

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To cope for this problem, DCT has been proposed instead of DFT, for its capacity to reduce the high frequency components in the transform domain [9]. Its improvements are however not sufficient in systems designed with a great amount of virtual subcarriers, which suffer from a huge border effect [10]. This is the case of a 3GPP/LTE system.

The aim of the paper is to study, for a 3GPP/LTE system, two improved DCT based channel estimations, designed to correctly solve the problem of null carriers at the border of the spectrum. These two TD-CE will also be compared in terms of performance and complexity. In the first approach, a truncated singular value decomposition (TSVD) of pilots matrix is used to mitigate the impact of the “border effect”. The second approach is based on the division of the whole DCT window into 2 overlapping blocks.

The paper is organized as follows. Section II introduces the mobile wireless channel and briefly describes the MIMO-OFDM system with channel estimation component. Section III is dedicated to transform domain channel estimations (TD-CE), with description of the classical Least Square algorithm in III-A, and presents the conventional DFT and DCT based channel estimation in III-B and III-C, respectively. Next, the two proposed DCT based channel estimation are described in the sections IV and V. Finally, a performance evaluation and comparison is shown in section VI.

2. MIMO-OFDM system description

In this paper we consider a coherent MIMO-OFDM system, with $N_t$ transmit antennas and $N_r$ receive antennas. As shown in Fig.1, the MIMO scheme is first applied on data modulation symbols (e.g. PSK or QAM), then an OFDM modulation is performed per transmit antenna. Channel estimation is then required at receive side for both the one tap per sub-carrier equalization and the MIMO detection.

The OFDM signal transmitted from the $i$-th antenna after performing IFFT (OFDM modulation) to the frequency domain signal $X_i \in \mathbb{C}^{N \times 1}$ can be given by:

$$x_i(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_i(k) e^{j \frac{2 \pi i k n}{N}}, \quad 0 \leq (n,k) \leq N \tag{1}$$

where $N$ is the number of FFT points.

The time domain channel response between the transmitting antenna $i$ and the receiving antenna $j$ under the multipath fading environments can be expressed by the following equation:

$$h_{ij}(n) = \sum_{l=0}^{L-1} h_{ij,l} \delta(n - \tau_{ij,l}) \tag{2}$$

Fig. 1. MIMO-OFDM block diagram.
with \( L \) the number of paths, \( h_{ij,l} \) and \( \tau_{ij,l} \) the complex time varying channel coefficient and delay of the \( l \)-th path.

The use of a guard interval allows both the preservation of the orthogonality between the tones and the elimination of the inter symbol interference (ISI) between consecutive OFDM symbols. Thus by using (1) and (2), the received frequency domain signal is given by:

\[
R_j(k) = \sum_{i=0}^{N_t-1} X_i(k) H_{ij}(k) + \Xi(k)
\]

where \( H_{ij}(k) \) is the discrete response of the channel on subcarrier \( k \) between the \( i \)-th transmit antenna and the \( j \)-th receive antenna and \( \Xi_k \) the zero-mean complex Gaussian noise after the FFT (OFDM demodulation) process.

3. Transform Domain Channel Estimation (TD-CE)

In a classical coherent SISO-OFDM system, channel estimation is required for OFDM demodulation. When no knowledge of the statistics on the channel is available, a least square (LS) algorithm can be used in order to estimate the frequency response on the known pilots that had been inserted in the transmit frame. An interpolation process allows then the estimation of the frequency response of the channel, i.e. for each sub-carrier. In a MIMO-OFDM system, since the received signal is a superposition of the transmitted signals, orthogonally between pilots is mandatory to get the channel estimation without co-antenna interference (CAI).

We choose to apply TD-CE to a 3GPP/LTE system where the orthogonality between training sequences is based on the simultaneous transmission on each subcarrier of pilot symbols on one antenna and null symbols on the other antennas as depicted in Fig.2.

A. Least Square channel estimation (LS)

Assuming orthogonality between pilots dedicated to each transmit antenna, the LS estimates can be expressed as follows:

\[
H_{ij,LS} = H_{ij} + (\text{diag}(X))^{-1} \Xi.
\]

Therefore LS estimates can be only calculated for \( \frac{M}{N_t} \) subcarriers where \( M \) is the number of modulated subcarriers. Then interpolation has to be performed to obtain an estimation for all the subcarriers.

B. DFT based channel estimation

From (4), it can be observed that LS estimates can be strongly affected by a noise component. To improve the accuracy of the channel estimation, the DFT-based method has been proposed in order to reduce the noise component in the time domain [8]. Fig.3 illustrates the transform domain channel estimation process using DFT. After removing the unused subcarriers, the LS estimates are first converted into the time domain by the IDFT algorithm and a smoothing filter (as described in Fig.3) is applied in the time domain assuming that the maximum multi-path delay is within the cyclic prefix of the OFDM symbols. After the smoothing, the DFT is applied to return in the frequency domain.
The time domain channel response of the LS estimated channel can be expressed by (5). From (4), it is possible to divide $h_{n,LS}^{IDFT}$ into two parts.

$$h_{n,LS}^{IDFT} = \frac{1}{M-1} \sum_{k=0}^{M} H_{k,LS} e^{\frac{2\pi nk}{M}} \quad (5)$$

where $\xi_n^{IDFT}$ is the noise component in the time domain and $h_{n}^{IDFT}$ is the IDFT of the LS estimated channel without noise which is developed as:

$$h_{n}^{IDFT} = \sqrt{\frac{1}{M}} \sum_{l=0}^{L-1} h_{l} e^{-j\pi l (1-\frac{M}{N})} \sum_{k=0}^{M-1} e^{-j \frac{2\pi k}{M} \frac{M}{N} (\tau_i-l)} \quad (6)$$

It can be easily seen from (6) that if the number of FFT points $N$ is equal to the number of modulated subcarriers $M$, the impulse response $h_{n}^{IDFT}$ will exist only from $n = 0$ to $L - 1$, with the same form as (2), i.e the true channel.
Nevertheless when \( N > M \), the last term of (6) \( \sum_{k=0}^{M-1} e^{-j\frac{2\pi k}{M} \left(\frac{M}{N} \tau_l - n\right)} \) can be expressed as:

\[
\begin{cases}
M / \text{hcf}(M, N) \\
1 - e^{-j\frac{2\pi (M/\text{hcf}(M, N)) \tau_l}{N/\text{hcf}(M, N)}} \quad \text{if } \tau_l : \leq L \text{ and } \in \mathbb{N} \\
1 - e^{-j\frac{2\pi M \tau_l}{N}} \quad \text{otherwise}
\end{cases}
\]

(7)

where \( \text{hcf} \) is the highest common factor and \( \mathbb{N} \) natural integer.

From (7) it is important to note that:

- On the one hand, the channel taps are not all completely retrieved in the first \( CP \) samples of the channel impulse response.
- On the other hand, the impulse channel response obviously exceeds the Guard Interval (CP). This phenomenon is called Inter-Taps Interference (ITI). Removing the ITI by the smoothing filter generates the “border effect” phenomenon.

**C. DCT based channel estimation**

The DCT based channel estimator can be realized by replacing IDFT and DFT (as shown in Fig.3) by DCT and IDCT, respectively. DCT conceptually extends the original \( M \) points sequence to \( 2M \) points sequence by a mirror extension of the \( M \) points sequence [12]. As illustrated by Fig. 4, the waveform will be smoother and more continuous in the boundary between consecutive periods.

![DCT and DFT principle](www.intechopen.com)
The channel impulse response in the transform domain is given by:

\[ h_{n,LS}^{DCT} = \sum_{k=0}^{M-1} V_n^M H_{k,LS} \cos \frac{\pi(2k+1)n}{2M} \]  

(8)

where \( V_n^M \) is the coefficient of DCT which can take two different values, depending on the value of \( n \).

\[ V_n^M = \begin{cases} \sqrt{1/M} & n = 0 \\ \sqrt{2/M} & n \neq 0 \end{cases} \]  

(9)

From the DCT calculation and the multi-path channel characteristics, the impulse response given by (8) is concentrated at lower order components in the transform domain. It is important to note that the level of impulse response at the order higher than \( N_{\text{max}} \) is not null, but can be considered as negligible; this constitutes the great interest of the DCT. The channel response in the transform domain can be expressed by:

\[ h_{n}^{DCT} = \begin{cases} h_{n,LS}^{DCT} & 0 \leq n \leq N_{\text{max}} - 1 \\ 0 & N_{\text{max}} \leq n \leq M - 1 \end{cases} \]  

(10)

The frequency channel response is then given by:

\[ H_k^{DCT} = \sum_{n=0}^{M-1} V_n^M h_n^{DCT} \cos \frac{\pi(2k+1)n}{2M} \]  

(11)

As a summary of this conventional DCT based estimation, it is important to note this following remark:

In the conventional DCT based method, the ITI is less important than in DFT one but a residual “border effect” is still present.

4. DCT with TSVD based channel estimation

In the classical DCT approach, it is shown that all the channel paths are retrieved. Nevertheless, the residual ITI will cause the “border effect”. The following approach is a mixture of Zero Forcing (ZF) and a truncated singular value decomposition in order to reduce the impact of null subcarriers in the spectrum [13]. The DCT transfer matrix \( C \) of size \( N \times N \) can be defined with the following expression:

\[ C = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & D_N & \ldots & D_N(2N+1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & D_N(N-1) & \ldots & D_N((2N+1)(N-1)) \end{bmatrix} \]  

(12)

where \( D_N(kn) = V_n^N \cos \left( \frac{\pi}{2N} (2k+1)(n) \right) \).

To accommodate the non-modulated carriers, it is necessary to remove the rows of the matrix \( C \) corresponding to the position of null subcarriers (see Fig. 2). From (10), we can just
use the first $N_{\text{max}}$ columns of $C$. Hence the DCT transfer matrix becomes:

$$\tilde{C}'_i = C\left(\frac{N-M}{2} + i : N_i = \frac{N+M}{2} - 1, 1 : N_{\text{max}}\right)$$

where $0 \leq i \leq N_i$ is the transmit antenna index.

Let us rewrite (8) in a matrix form:

$$h_{LS}^{\text{DCT}} = \tilde{C}'.H_{LS}$$

(13)

To mitigate the ITI, the first step of this new approach is to apply the ZF criterion [14]:

$$h_{LS}^{\text{DCT-ZF}} = (\tilde{C}^tH\tilde{C}^t)^{-1}\tilde{C}^tH_{LS} = \tilde{C}^tH_{LS}$$

(14)

The main problem arises when the condition number (CN) of $\tilde{C}^tH\tilde{C}^t$, defined by the ratio between the greater and the lower singular value, becomes high. Fig.5 shows the behavior of the singular value of $\tilde{C}^tH\tilde{C}^t$ whether null carriers are placed at the edge of the spectrum or not. When all the subcarriers are modulated, the singular values are all the same and the CN is equal to 1. However, when null carriers are placed at the edges of the spectrum, the CN becomes very high. For instance, as we can see in Fig.5, if $N = 1024$, $N_{\text{max}} = 84$ and $M = 600$ as in 3GPP, the CN is $2.66 \times 10^{16}$.

To reduce the “border effect”, i.e. the impact of ITI, it is necessary to have a small condition number. The second step of this new approach is to consider the truncated singular value decomposition (TSVD) of the matrix $\tilde{C}^tH\tilde{C}^t$ of rank $N_{\text{max}}$.

Fig.6 shows the block diagram of the DCT based channel estimation and the proposed scheme. In the proposed scheme (Fig.6(b)), after performing the SVD of the matrix $\tilde{C}^tH\tilde{C}^t$, we propose to only consider the $T_h$ most important singular values among the $N_{\text{max}}$ in order to reduce the CN. The TSVD solution is defined by:

![Graph showing singular value distribution](image-url)
Fig. 6. Block diagram of channel estimation using DCT and the proposed scheme.

\[
H_{n,LS}^{DCT-ZF-TSVD} = \sum_{s=1}^{S_t} u_s^H H_{n,LS} \sigma_s
\]  

(15)

where \( T_h \) is the threshold, \( u_s, v_s \) and \( \sigma_s \) are the left singular vector, the right singular vector and the singular values of \( \tilde{C}' \).

An IDCT (\( \tilde{C}' H \)) is then used to get back to the frequency domain.

\[
H_k^{DCT-ZF-TSVD} = C_{global} = \tilde{C}' H \tilde{C}^\dagger
\]  

(16)

\( T_h (\in 1, 2, \ldots, N_{max}) \) can be viewed as a compromise between the accuracy on pseudo-inverse calculation and the CN reduction. The adjustment of \( T_h \) is primarily to enhance the channel estimation quality. Its value depends only on the system parameters (position of the null carriers), which is predefined and known at the receiver side. \( T_h \) can be in consequence calculated in advance for any MIMO-OFDM system. To find a good value of \( T_h \), it is important to master its effect on the channel estimation i.e on the matrix \( C_{global} \in \mathbb{C}^{M/N_t \times M} \). As an example, Fig. 7 shows the behavior of the \( M/N_t \) singular values of \( C_{global} \) for different \( T_h \) where CP = 72, N = 1024, M = 600 and \( N_t = 4 \).
For \( T_h = 51, 52, 53 \) the singular values of \( C_{\text{global}} \) are the same on the first \( T_h \) samples and almost zero for others samples. We can consider that the rank of the matrix \( C_{\text{global}} \) becomes \( T_h \) instead of \( N_{\text{max}} \). Therefore the noise effect is minimized and \( CN \) is equal to 1.

However, all the singular values become null when \( T_h = 50 \) due to a very large loss of energy. As illustrated by the Fig.8 which is a zoom of Fig.7 on the first singular values, their behavior can not be considered as a constant for \( T_h = 60, 70, 84 \) and then the \( CN \) becomes higher.

5. DCT with 2 overlapping blocks

The principle of this approach is to divide the whole DCT window into \( R \) blocks as proposed in [18]. In this paper we consider \( R = 2 \), that was demonstrated to reach same bit error rate (BER) performance that higher \( R \) values.

As illustrated in Fig.9, the concatenation of the 2 overlapping blocks cannot exceed \( N \).

The classical DCT smoothing process described in the section III-C is applied to each 2 blocks of size \( N/2 \) by keeping only the energy of the channel in the first \( N_{\text{max}}/2 \) samples. However, the residual ITI causes “border effect” on the edge of each block. Then, to recover the channel coefficients, we average the values in the overlapping windows between the different blocks except some subcarriers at the right and the left edge of block 1 and block 2 respectively as described in Fig.10.

The noise power is averaged on \( N \) samples instead of \( M \) in this approach. Thereby it presents a gain \( (10\log_{10}\left(\frac{N}{M}\right)) \) in comparison to the classical DCT based channel estimation.
Fig. 8. Singular values of $\tilde{C}^\dagger H \tilde{C}^\dagger$ with $CP = 72$, $N = 1024$, $M = 600$ and $N_t = 4$ for different values of $T_h$

Fig. 9. Principle of the DCT with 2 overlapping blocks
For instance, the gain is $2.31 \text{dB}$ for the studied 3GPP/LTE system ($N = 1024, M = 600$).

6. Simulations results

The different channel estimation techniques, LS estimation, classical DFT and DCT estimations and the two proposed DCT estimations (DCT with TSVD and DCT with 2 overlapping blocks) are applied to a $4 \times 2$ MIMO-OFDM system with a double-Alamouti scheme. After the description of the system parameters, the performance and complexity of the channel estimation techniques will be analysis. Note that DCT-TSVD method is named on the figures by the used threshold (DCT-TSVD with $T_h = 53$ is named $T_h = 53$), while DCT with 2 overlapping blocks is called $DCT_2$.

A. System parameters

Performance are provided over frequency and time selective MIMO SCME typical to urban macro channel model (C) without any spatial correlation between transmit antennas [15]. Double-Alamouti space-time coding consists in simultaneously transmitting two Alamouti codes on two blocks of two transmit antennas [16]. The system parameters are issued and close to those defined in 3GPP/LTE framework [6]. The detailed parameters of the system simulations are listed in Table I.

B. Performances analysis

Fig. 11 shows mean square error (MSE) on different subcarriers for the proposed DCT-TSVD based channel estimation with the optimized threshold $T_h = 53$, the proposed DCT with 2 overlapping blocks and the conventional DFT and DCT ones in 3GPP/LTE system. We can
first see that DCT based channel estimation reduces significantly the “border effect” in comparison to the conventional DFT one. The two proposed optimized DCT methods allow MSE to be improved on all subcarriers even at the edges of the spectrum compared to the conventional DCT one. For DCT with 2 overlapping blocks, this can be explained by the noise reduction obtained thanks to the averaging which is performed on the overlapped portion of the spectrum. For DCT-TSVD method, improvement is due to the minimization of the noise effect and the reduction of the CN obtained by using TSVD calculation. The MSE performance, averaged over all subcarriers, can be observed in Fig.12 which shows MSE versus $E_b/N_0$ for the different channel estimation techniques. Note that the two optimized techniques, DCT-TSVD and DCT with 2 overlapping blocks, present very similar performance.

This can also be observed in Fig.13, which represents the performance results in terms of BER versus $E_b/N_0$ for perfect, least square (LS), classical DFT and DCT, the proposed DCT-TSVD channel estimation with $T_h = 84, 70, 60, 53, 52, 51$ and the proposed DCT with 2 overlapping blocks. The classical DFT based method presents poor results due to the
Fig. 12. MSE against $E_b/N_0$ for 3GPP/LTE.

Fig. 13. BER against $E_b/N_0$ for 3GPP/LTE.
“border effect” whereas the DCT one improves the accuracy of the channel estimation by significantly reducing the noise component compared to the LS estimate (1.5 dB).

The DCT-TSVD estimator with $T_h = 84, 70, 60$ presents an error floor due to the high CN. The optimization of $T_h$ ($T_h = 53, 52, 51$) allows the residual “border effect” to be mitigated and then the noise component to be reduced. The performance is then improved compared to the classical DCT. The proposed DCT with 2 overlapping blocks also improves the performance compared to the classical DCT due to the large noise reduction.

It is important to note that, the noise reduction gain obtained by using DCT with 2 overlapping blocks depends on the system parameters ($10\log_{10}(\frac{N}{M})$). In 3GPP/LTE context ($N = 1024$ and $M = 600$), the gain is very important (2.3dB), which allows DCT with 2 overlapping blocks to have the same performance than the optimized DCT-TSVD.

C. Complexity analysis

Considering a MIMO-OFDM system with $N_t$ transmit antennas and $N_r$ received ones, the channel estimation module is used $N_t \times N_r$ times to estimate the MIMO channel between all the antenna links. Thus, the complexity of the channel estimation can be very high in term of reel multiplications and reel additions.

- In DCT-TSVD, the global matrix $C^{global}$ will be used $N_t \times N_r$ times to estimate the MIMO channel. The number of real multiplications and real additions are $(2M^2 / N_t) N_t N_r$ and $(2M(M/N_t - 1))N_t N_r$, respectively.

- In order to use the inverse fast fourier transform (IFFT) and fast fourier transform (FFT) algorithms for OFDM modulation and demodulation, the number of subcarriers ($N$) is chosen as a power of 2 in all multicarriers systems. For instance, $N = 2^6 = 64$ for IEEE802.11n, $2^{10}$ for 3GPP/LTE and $2^{11}$ for digital video broadcasting terrestrial DVBT 2k. The proposed DCT with 2 overlapping blocks uses two blocks of size $N/2$ which is a power of 2. Therefore, this channel estimation technique can be performed by using fast DCT algorithms [17]. Then, the number of real multiplications and real additions are $(\frac{N}{4})\log_2(\frac{N}{2}) \times 4N_t N_r$ and $(\frac{3N}{4})\log_2(\frac{N}{2}) - \frac{N}{2} + 1) \times 4N_t N_r$, respectively.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Multiplications</th>
<th>Additions</th>
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<tr>
<td>DCT with TSVD</td>
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<td>1431040</td>
</tr>
<tr>
<td>DCT with 2 overlapping blocks</td>
<td>73728</td>
<td>204832</td>
</tr>
</tbody>
</table>

Table II. Number of required operations to estimate the $N_t \times N_r$ subchannels ($N_t = 4, N_r = 2$)

Table II contains the number of required operations to estimate the $4 \times 2$ subchannels, for a 3GPP-LTE system. We can clearly see in this table that the complexity reduction with 2 overlapping blocks, compared to DCT-TSVD, is very important. As in the context of the study (3GPP/LTE parameters), the performance results of the two techniques are very close, the DCT channel estimation with 2 overlapping blocks becomes a very interesting and promising solution.

7. Conclusion

In this paper, two improved DCT based channel estimations are proposed and evaluated in a 3GPP/LTE system context. The first technique is based on truncated singular value
decomposition (TSVD) of the transfer matrix, which allows the reduction of the condition number (CN). The second technique is based on the division of the whole DCT window into 2 overlapping blocks. The noise reduction gain obtained by using DCT with 2 overlapping blocks, which depends on the system parameters, is very important. The simulation results in 3GPP/LTE context show that the performance results of the two techniques are very close but the DCT channel estimation with 2 overlapping blocks becomes a very interesting and promising solution due to its low complexity. It can be noted that this complexity could be further reduced by considering more than two blocks.

8. References

[15] D.S.Baum, J.Hansen, G.Del Galdo, M. Milojevic, J. Salo and P. Ky¨0sti. An interim channel model for beyond-3g systems: extending the 3gpp spatial channel model


This book "Communications and Networking" focuses on the issues at the lowest two layers of communications and networking and provides recent research results on some of these issues. In particular, it first introduces recent research results on many important issues at the physical layer and data link layer of communications and networking and then briefly shows some results on some other important topics such as security and the application of wireless networks. In summary, this book covers a wide range of interesting topics of communications and networking. The introductions, data, and references in this book will help the readers know more about this topic and help them explore this exciting and fast-evolving field.

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