Simulation of numerical distance relays

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1. Introduction

Utility engineers and consultants use relay models to select the relay types suited for a particular application, and to analyze the performance of relays that appear to either operate incorrectly or fail to operate on the occurrence of a fault. Instead of using actual prototypes, manufacturers use relay model designing to expedite and economize the process of developing new relays. Electric power utilities use computer-based relay models to confirm how the relay would perform during systems disturbances and normal operating conditions and to make the necessary corrective adjustment on the relay settings. The software models could be used for training young and inexperienced engineers and technicians. Researchers use relay model to investigate and improve protection design and algorithms. However, simulating numerical relays to choose appropriate settings for the steady state operation of over current relays and distance relays is presently the most familiar use of relay models (McLaren et al., 2001)

Numerical relay models can be divided into two categories. The models of the first category consider only the fundamental frequency components of voltages and currents. Phasor-based models were the first to be widely used to design and apply relays. The models of the second category take into consideration the high frequency and decaying DC components of voltages and currents in addition to the fundamental frequency components. These models are called transient models of relays. (McLaren et al., 2001)

The goal of this chapter is to explain the building process of MATLAB model of a distance relay and validating the relay behavior when the input data that describes the voltage and current signals at the relay location is generated by simulation of the power network using EMTP-ATP. Voltage and current signals during faults are severely distorted; this is why EMTP is used as a power simulator during faults. EMTP would present voltage and current signals during fault with their dc decaying components and high frequency oscillations. However the model was validated by a similar input data generated by the simulation of the power network using MATLAB. The validation process extended to include the cases where the measured impedance is changed due to a change of fault location, due to an existence of resistive faults or due to an existence of more than one in-feed.

The chapter began by introducing the principle of operation of distance relays and reviews the functionality of each of the internal modules of numerical relay such as, analog anti-aliasing filtering module, analog-to-digital conversion module, and phasor estimation algorithm.
2. Protective Relays

Fault current is the expression given to the current that flow in the circuit when load is shorted i.e. flow in a path other than the load. This current is usually very high and may exceed ten times the rated current of a piece of plant. Faults on power system are inevitable due to external or internal causes, lightning may struck the overhead lines causes insulation damage. Internal overvoltage due switching or other power system phenomenon may also cause an over voltage leads to deterioration of the insulation and faults. Power networks are usually protected by means of two main components, relays that sense the abnormal current or voltage and a circuit breaker that put a piece of plant out of tension. Power System Protection is the art and science of the application of devices that monitor the power line currents and voltages (relays) and generate signals to deenergize faulted sections of the power network by circuit breakers. Goal is to minimize damage to equipment and property that would be caused by system faults, if residues, and maintain the delivery of electrical energy to the consumers. Many types of protective relays are used to protect power system equipments, they are classified according to their operating principles; over current relay senses the extra (more than set) current considered dangerous to a given equipment, differential relays compare in and out currents of a protected equipment, while impedance relays measure the impedance of the protected piece of planet. For a good performance of a relay in a power system it must have the following characteristics; dependability, security, selectivity, sensitivity and speed.

Traditionally, power systems problems and applications have been solved by means of purely analog circuits, However the scenario have changed and power system area was one of the most benefited areas from the booming in area of digital and signal processing. Numerical relays are the result of the application of microprocessor technology in relay industry, they convert the measured voltages and currents from analog to digital values and calculates from these samples the relay protection criterium i.e. impedance (Ziegler, 1999). Due to processing capacity of numerical relays many protection criteria can be implemented. Protection relays, such as other monitoring and control equipments have taken the advantage from the increasing improvement of the semiconductor industry and the enormous number of digital signal processing and control algorithms. The latest generations of protective relays be provided with a large capacity of processing capabilities become more efficient and can perform a numerous number of functions such as fault locators, integrated monitoring and control functions.

Designing and modeling of numerical relay require establishing a generalized numerical relay structure, which is composed the more relevant and common internal modules employed by typical numerical relays.

3. Distance Relays

Distance relays, as the name sounds, should measure distance. In fact this is true, as in case of transmission line, distance relay measures the impedance between the relay point and the fault location. This impedance is proportional to the length of the conductor, and hence to the distance between the relaying point and the fault.
3.1 Principle of operation

The basic principle as illustrated in figure 1, involves the division of the voltage at the relaying point by the measured current. The apparent impedance is compared with the reach point impedance. If the measured impedance is less than the reach point impedance, it is assumed that a fault exists on the line between the relay and the reach point. The reach point of the relay is the point along the line impedance locus that is intersected by the boundary characteristics of the relay. Distance relay is the broader name of the different types of impedance relay.

![Fig. 1. Principle of operation of distance relay](image)

The relay is connected at position, R and receives a secondary current, iF, equivalent to a primary fault current, IF. The secondary voltage, VF, is equivalent to the product of the fault current “IF” and impedance of the line up to the point of fault, ZF. The operating torque of this relay is proportional to the fault current “IF”, and its restraining torque is proportional to the voltage “VF”. Taking into account the number of turns of each coil, there will be a definite ratio of V/I at which the torque will be equal. This is the reach point setting of the relay. The relay will operate when the operating torque is greater than the restraining torque. Thus any increase in current coil ampere-turns, without a corresponding increase in the voltage coil ampere-turns, will unbalance the relay. This means the V/I ratio has fallen below the reach point. Alternatively if the restrain torque is greater than the operating torque, the relay will restrain and its contacts will remain open. In this case the V/I ratio is above the reach point. The reach of a relay is the distance from the relaying point to the point of fault. Voltage on the primary of voltage transformer, VT, is:

\[ V = \frac{E Z_F}{(Z_s + Z_F)} \]  

(1)

The fault current, IF

\[ I_F = \frac{E}{(Z_s + Z_F)} \]  

(2)

The relay compare the secondary values of V and I, as to measure their ratio which is an impedance Zm,
For earth elements, the relevant phase voltage is supplied e.g. \( V_a \), but the corresponding current is residually compensated. The earth faults compensation factor may be calculated considering the sequence-networks connection for the phase A-to-ground fault on a transmission line. Table (1) indicates calculation formula for phase and line to line faults. In order for the relay to be correctly operated, residual factor shall be introduced as shown in the following equations:

\[
Z_m = \frac{V_{/V \cdot T \text{ Ratio}}}{I_{/C \cdot T \text{ Ratio}}}
\]

\[
Z_m = ZF \cdot C \cdot T \text{ Ratio} / V \cdot T \text{ Ratio}
\]

Zm is the measured impedance called secondary impedance. (GEC, 1990)

### 3.2 Zones of protection

Basic distance protection will comprise instantaneous directional Zone 1 protection and one or more time delayed zones. Numerical distance relays may have up to five zones, some set to measure in the reverse direction. Numerical relays usually have a reach setting of up to 85\% of the protected line impedance for instantaneous Zone 1 protection. The resulting 15\% safety margin ensures that there is no risk of the Zone 1 protection over-reaching the protected line due to errors in the current and voltage transformers, inaccuracies in line impedance data provided for setting purposes and errors of relay setting and measurement. of the distance protection must cover the remaining 15\% of the line.

The reach setting of the Zone 2 protection should be at least 120\% of the protected line impedance. In many applications it is common practice to set the Zone 2 reach to be equal to the protected line section +50\% of the shortest adjacent line. Zone 3 reach should be set to at least 1.2 times the impedance presented to the relay for a fault at the remote end of the second line section (GEC, 1990). Typical reach for a 3-zone distance protection are shown in Figure 2.

![Fig. 2. Typical 3 zones distance protection](image)

#### 3.3 Residual factor

The measuring element of the distance relay is principally laid out such that for each fault type the line impedance of the fault loop is determined. In three phase system the zone-1 of the relay will have six elements responsible for detecting both phase and earth faults (Ziegler, 2006). For phase faults elements, the difference between the two relevant phase signals are used, e.g. a-b...
elements is supplied with samples of \( V_a - V_b \) voltage and \( I_a - I_b \) current. For earth elements, the relevant phase voltage is supplied e.g. \( V_b \), but the corresponding current is residually compensated. The earth faults compensation factor may be calculated considering the sequence-networks connection for the phase A-to-ground fault on a transmission line. Table (1) indicates calculation formula for phase and line to line faults. In order for the relay to be correctly operated, residual factor shall be introduced as shown in the following equations

\[
R_K = \frac{V_K}{I_K + 3 \cdot \text{Re}(K_0 \cdot I_{0K})} \quad \text{and} \quad X_K = \frac{V_K}{I_K + 3 \cdot \text{Im}(K_0 \cdot I_{0K})}
\]

Where:
- \( K_0 \) is the compensation factor
- \( I_0 \) is zero sequence current
- \( V_k, I_k \) are the sampled voltage and current respectively

<table>
<thead>
<tr>
<th>Distance Element</th>
<th>Voltage signal</th>
<th>Current signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase A</td>
<td>( V_a )</td>
<td>( I_a + 3K_0I_0 )</td>
</tr>
<tr>
<td>Phase A - Phase B</td>
<td>( V_a - V_b )</td>
<td>( I_a - I_b )</td>
</tr>
</tbody>
</table>

Table 1. Calculation formula for phase and line to line faults

### 3.4 Effect of fault resistance on relay coverage

The earth fault resistance reduces the effective earth-fault reach of a mho Zone 1 element to such an extent that the majority of faults are detected in Zone 2 time. Figure 3 illustrates the effect of arc resistance on the relay reach. The effect of fault resistance on the reach of distance relays is better discussed with the simulation results.

![Fig. 3. Effect of arc resistance on relay coverage](image-url)
4. Numerical Relay Structure

Since their introduction on 1920, Classic distance relays based on electro-mechanical and then on static technology are still in wide use. However due to the booming in digital techniques, microprocessor-based relays were introduced. It is quite common to use term digital relay instead of numerical relay as the distinction between both rests on fine technical details. Others see numerical relays as natural developments of digital relays as a result of advances in technology. However, in US the term (digital distance protection) has always been used in the meaning of (numerical distance protection) (Ziegler, 1999). A general view of the typical digital relay is shown in figure 4.

![Block diagram of relay analogue to digital conversion arrangement](image)

The generalized numerical relay concept is directly derived from open system relaying (different relay functions can be obtained from the same hardware just by modifying microprocessor programming) (Sandro, 2006). The following hardware modules and functions constitute the generalized numerical relay.

4.1 Isolation and analog signal scaling

Current and voltage waveforms from instrument transformers are acquired and scaled down to convenient voltage levels for use in the digital and numerical relays.

4.2 Analog anti-aliasing filtering

Low-pass filters are used to avoid the phenomena of aliasing in which the high frequency components of the inputs appear to be parts of the fundamental frequency components. The analog inputs must be applied to low-pass filters and their outputs should be sampled and quantized. The use of low-pass filter is necessary to limit the effects of noise and unwanted components of frequencies. The filter is designed to remove any frequencies existing on the input signal which are greater than half the sampling frequency. The nature of the relaying task dictates the total amount of filtering required. Distance protection based on impedance measurements uses information contained in the sinusoidal steady state components of 50-60 Hz. Therefore, filtering must preserve the steady state components and reject other components. Common analog low-pass filters used in these relays are of third to fifth order with cutoff frequency of about 90 Hz. The cutoff frequency of 90 Hz implies that a sampling rate of at least three samples per cycle (180 Hz) must be used in order that the information needed to perform the distance relay functions is retained and errors due to aliasing are avoided. In practice, the sampling rate must be at least four samples per cycle (240 Hz) (Sandro, 2006).

4.3 Analog-to-digital conversion (ADC)

Because digital processors can process numerical or logical data only, the waveforms of inputs must be sampled at discrete times. To achieve this, each analog signal is passed through a sample- and-hold module, and conveyed, one at a time, to an Analog-to-Digital Converter (ADC) by a multiplexer (Mux), as shown in figure 5.
The basic function of a sample-and-hold in an analog input system is to capture an input signal and hold it constant during the subsequent ADC conversion cycle. An analog-to-digital converter (A/D converter or ADC) takes the instantaneous value of an analog voltage and converts it into an n-bit binary number that can be easily manipulated by a microprocessor. A distance relay having a minimum set impedance of 4Ω, would have a highest current level for a voltage transformer of 110 V, equal to 110/4 = 27.5 A. Allowing for offset during faults, 100%, this current could reach 55A. Suppose that the relay must operate for a minimum current level of 25 mA and this can be represented by 1 digital level. Hence the dynamic range for one polarity of the current is 55/0.025 =2200. Hence for bipolar signal the dynamic range is 4400. The ADC closest to this figure is 12 bit. In general, most high performance numeric relays use 12, 14 or 16 bit ADCs. (IEE, 1995). The n-bit number is a binary fraction representing the ratio between the input voltage and the full-scale voltage of the converter. A number of techniques can be used to achieve this conversion. The full-input voltage ranges for an ADC are typically 0 to +5 or 0 to +10 volts for unipolar operations, and −5 to +5 or −10 to +10 volts for bipolar operation (Sandro, 2006).

4.4 Quantizer

The Quantizer block passes its input signal through a stair-step function so that many neighboring points on the input axis are mapped to one point on the output axis. The effect is to quantize a smooth signal into a stair-step output. The output is computed using the round-to-nearest method, which produces an output that is symmetric about zero. The output $y$ of the quantizer is given by:

$$y = q \ast \text{round}\left(\frac{u}{q}\right)$$

where $u$ is the input, and $q$ the Quantization interval.
4.5 Digital filter

Digital filters have, for many years, been the most common application of digital signal processors. There are two basic forms of digital filters, the Finite Impulse Response (FIR) filter and Infinite Impulse Response (IIR) filter. The main drawback to the use of IIR filters in digital protection relays is that the group delay cannot be specified in the design process. This makes their use in protection somewhat onerous, in general, FIR filters are usually the preferred type. (IEE, 1995). As this the case FIR filter will be briefly explained. As seen in the block diagram of figure 6, and the second order Finite Impulse (FIR) filter shown in figure 7, the input signal $x(n)$ is a series of discrete values obtained by sampling an analogue signal. $X(0)$ correspond to the input value at $t=0$, $x(1)$ at $t=Ts$, $x(2)$ at $t=2 Ts$ and so on, where $Ts$ is sampling period $=1/fs$. The three main blocks of FIR filters are:

(a) Unit delay
Its purpose is to hold the input for a unit of time (physically equal to the sampling interval $Ts$) before it is delivered to the output. Mathematically, it performs the following operation.

$$y(n) = x(n - a)$$

Unit delay is depicted schematically in Figure 6(a). The letter D, indicating delay, sometimes is replaced by $z^{-1}$, which is the delay operator in the z domain.
Unit delay can be implemented in software in a storage variable, which changes its value when instructed by the program.

(b) Adder

(c) Multiplier

Fig. 6. Basic elements of Finite Impulse (FIR) digital filter.

Fig. 7. Second order Finite Impulse (FIR) filter.
(b) **Adder**

The purpose of the adder is to add two or more signals appearing at the input at a specific time. Mathematically, it performs the operations like the one shown in the following equation.

\[ y(n) = x_1(n) + x_2(n) + x_3(n) + ... \]  

(7)

An adder is depicted schematically in Figure 6(b).

(c) **Multiplier**

The purpose of this element is to multiply a signal (a varying quantity) by a constant number, which takes the form;

\[ y(n) = ax(n) \]  

(8)

A multiplier is depicted schematically in Figure 6(c). There is no specific symbol for the multiplier, but to show its operation, a constant factor is placed above or besides the signal line.

### 4.6 Phasor estimation algorithm

A software algorithm implemented in a microprocessor estimates the amplitude and phase of the waveforms provided to the relay. More details are given in section 8, Impedance Estimation Algorithms.

### 4.7 Relay algorithm and trip logic

After microprocessor calculates the phasors representing the inputs, acquires the status of the switches, performs protective relay calculations, and finally provides outputs for controlling the circuit breakers, the result of the algorithm transported to the control part of the relay where the results is compared with the settings of the relay and trip signal may be generated. Trip signal has to be secured and it should not be released unless the fault is stable within the tripping zone. Since impedance measurement falling within the relay characteristic is not a reliable indication of fault, counter may be used to establish a decision scheme that decides the trip signal generation. One of the employed counters techniques increases when the impedance is in the tripping zone and decreases when outside the tripping zone, other, remaining the impedance values in the characteristic for a certain period of time before fault is reliably evaluated, i.e. a number of successive samples are in tripping zone. The processor may also support communications, self-testing, target display, time clocks, and other tasks (McLaren et al., 2001).

### 5. Numerical Relays Operating Principles

When the distance relays receive discrete voltage and current signal, it converts it to a phasor. However faults on transmission lines cause the voltage and current signals to be severely distorted. These signals may contain decaying dc components, subsystem frequency transients, high frequency oscillation quantities, and etc. The higher frequency components can be eliminated using low pass anti-aliasing filters with appropriate cut-off frequency, but the anti-aliasing filters cannot remove decaying dc components and reject
low frequency components. This makes the phasors very difficult to be quickly estimated and affects the performance of digital relaying. Therefore, the Discrete Fourier transform is usually used to remove the dc-offset components. DFT is a digital filtering algorithm that computes the magnitude and phase at discrete frequencies of a discrete time sequence.

6. Current and Voltage Signal During Faults

The voltage and current signals in resistance-inductance behavior of power network are as usual sinusoid with exponentially decaying offsets. The offsets can severely affect the currents but seldom affect the voltage. Figure 8, shows the shape of the fault current at the terminal of a synchronous machine (Nasser, 2008)

![Fig. 8. Three-phase short-circuit fault at a synchronous machine terminals](image)

Non-linear loads, power transformers and instrument transformers can produce harmonics. Figure 9 shows a composite harmonic waveform. (Barry, 2000). In addition to that, capacitive series compensation introduces subsystem frequency transients. This transient depends on the percentage of capacitive compensation. Attention has to be given to filters, no matter how they are built, they should have the following characteristics:-
- Band pass response, about the system frequency, because all other components are of no interest.
- Dc rejection to guarantee decaying exponential are filtered out.
- Harmonic attenuation or rejection to limit effects of nonlinear loads.
- Reasonable bandwidth for fast response.

![Fig. 9. Composite harmonic waveform.](image)
7. Relay Models

A successful relay model must produce the same output for the same inputs as its real counterpart. However, numerical relay models can be divided into two categories. First, the models that consider only the fundamental frequency components of voltages and currents. Phasor-based models were the first to be widely used to design and apply relays. The second category models take into consideration the high frequency and decaying DC components of voltages and currents in addition to the fundamental frequency components (McLaren et al., 2001).

7.1 Transient relay models

Transient relay models mimic the behavior of numerical relays including their performance in the transient state and the impact of the transient components in the input signal. The availability of detailed information of the internal functioning of relays is critical in the process of producing a close-to-real transient relay model. According to the available information, transient models can be categorized in generic and detailed models (Sandro, 2006).

Generic models give considerable insight into the operation of the relay type but may not be suitable for marginal cases and precise timing. They may not have detailed logic provided in specific implementation of the generic principle in a specific relay. This logic is often applied to make specific functions interact with other functions to make a protection system. Because of this limitation generic model determine the best use for checking specific functions, rather than complete systems that are made up of numerous interacting functions. Detailed models preserve all the advantages of being able to examine the internal operation of any function. Detailed models are more useful than generic models for checking the performance of complete systems since all logic is represented. Unfortunately, detailed models are not as readily available as the generic models because they may include trade secrets of the manufacturers.

Manufacturers are in position to design accurate transient models, particularly for new digital relays, for the reason that, in the designing process, the software model may precede the hardware design. Where algorithms and hardware are known in detail, very precise performance can be achieved in the modeling.

8. Impedance Estimation Algorithms

The estimated phasors of voltages and currents are used in the implementation of protection algorithms in numerical relays. A relay algorithm is a set of equations whose evaluation and comparison with certain predetermined levels determines the operation of the relay. A number of algorithms can be regarded as impedance calculations in that the fundamental frequency component of both voltages and currents are obtained from the samples. The ratio of appropriate voltages and currents then provide the impedance to the fault. The performance of all of these algorithms is dependent on obtaining accurate estimate of the fundamental frequency component of a signal from a few samples. The algorithm based on series R&L model has the apparent advantage of allowing all signals that satisfy the differential equations to be used in estimating the R and L of the model (Phadke & Thorp, 1990). The equations and parameters that represent the relay algorithm of distance relays are simplified hereinafter. The algorithms are classified according to the approach used to calculate the impedance based on the voltage and current measurements.
8.1 Transmission Line Model

By assuming that the transmission line to which the relay is connected is composed of a series resistance and inductance, the fundamental equation is:

\[ v(t) = Ri(t) + L \frac{di(t)}{dt} \]  

(9)

where, R and L are the resistance and reactance of the fault loop (up to the fault point respectively).

Any sampled voltage and current signals taken at any time is considered to obey above equation.

To solve the equation (9) and calculate R and L, two equations are required. This can be achieved by measuring \( v(t) \), \( i(t) \) and \( \frac{di}{dt} \) at two different instants of time

\[ v(t_n) = Ri(t_n) + L \frac{di(t_n)}{dt} \]  

(10)

\[ v(t_{n-1}) = Ri(t_{n-1}) + L \frac{di(t_{n-1})}{dt} \]  

(11)

By solving equations (10) & (11), R and L may obtained from the following matrix

\[
\begin{bmatrix}
  v(t_n) \\
  v(t_{n-1})
\end{bmatrix}
= 
\begin{bmatrix}
  i(t_n) & \frac{di(t_n)}{dt} \\
  i(t_{n-1}) & \frac{di(t_{n-1})}{dt}
\end{bmatrix}
\begin{bmatrix}
  R \\
  L
\end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
  R \\
  L
\end{bmatrix} = \frac{1}{D} 
\begin{bmatrix}
  \frac{di(t_{n-1})}{dt} & -\frac{di(t_n)}{dt} \\
  -i(t_{n-1}) & i(t_n)
\end{bmatrix} \begin{bmatrix}
  v(t_n) \\
  v(t_{n-1})
\end{bmatrix}
\]

where D is matrix determinant. The derivative of the current may be calculated from difference formula,

\[
\frac{di(t_n)}{dt} = \frac{i(t_n) - i(t_{n-1})}{T}
\]

(13)

It is obvious how sampled voltage and current signals can be combined to form the resistance and inductance of the fault loop.
8.2 Discrete Fourier Transform (DFT)

In this approach the estimation is based on equation $Z = \frac{v}{i}$. The sampled current and voltage signals are initially transformed into phasor quantities (both direct and quadrature components). The estimation approach includes estimation of the first harmonic; calculation from equation $Z = \frac{v}{i}$ the impedance as a quotient of voltage and current phasors. Based on fault type (using residual factors as explained in section 3.3), the resistance and reactance up to the relay point is calculated.

Mathematical Background

Signal at any given time may be described by a phasor. Phasor actually is a vector rotating in the complex plane with a speed $\omega$ radian/sec, a snapshot in time, the signal at that time, $x(t)$ is given in rectangular form by; (Marven & Gillian, 1993)

$$x(t)|_{t=T} = (\text{realcoordinate}) + j(\text{imaginarycoordinate})$$

And in polar form by

$$x(t) = Ae^{j\omega t}$$

(14)

Considering the initial value at $t=0$,

$$x(0) = Ae^{j\alpha}$$

The general form of $x(t)$ is;

$$x(t) = Ae^{j(\omega t + \alpha)}$$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$\cos \omega t = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

$$\sin \omega t = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$$

Therefore, sine or cosine signal can be represented by two phasors form a conjugate pair, i.e. if $x(t) = A \cos \omega t$, then $x(t)$ may be written as;

$$x(t) = \frac{A}{2}(e^{j(\omega t + \alpha)} + e^{-j(\omega t + \alpha)})$$

(18)

The above discussion is related to a simple cosine or sine functions of a single frequency, most signals are composed of many cosine and sine waves. Therefore any complex periodic signal can be described as sum of many phasors. Fourier series assumes that a set of phasors have frequencies which are multiples of some fundamental frequency, $f_0$, i.e.
The individual frequency components are known as harmonics. If the complex signal is not periodic the phasor frequencies are not related, thus the Fourier general form may be written as:

\[ x(t) = \sum_{k=-N}^{N} A_k e^{j(k\omega_0 t)} \]  

(19)

In digital domain (discrete time), replace the continuous function, \( t \), with a function progresses in jumps of \( \omega_0 T_s \), thus phasor description of single frequency signal would be:

\[ x(n) = A e^{j(n\omega_0 T_s + \alpha)} \]

\[ e^{j(n\omega_0 T_s)} = \cos(n\omega T_s) + j \sin(n\omega T_s) \]  

(21)

where, \( T_s \) is the sampling interval

A real signal can be described using Fourier in discrete domain called (Discrete Fourier Series) as,

\[ x(n) = \sum_{k=-N}^{N} A_k e^{j(k\omega_0 T_s n)} \]  

(22)

which is a simple phasor model that describes a general discrete signal.

The discrete Fourier transform (DFT) is a digital filtering algorithm that computes the magnitude and phase at discrete frequencies of a discrete time sequence. Fast Fourier transforms are computationally efficient algorithms for computing DFTs. FFTs are useful if we need to know the magnitude and/or phase of a number individual or band of frequencies. The DFT is ideal method of detecting the fundamental frequency component in a fault signal. However, DFT, Least Error Square LES and Walsh Function algorithms are among the most popular phasor estimation techniques employed in numerical relays (Phadke & Thorp, 1990). As we are dealing with a 50-60 Hz signal that is sampled synchronously. This means that the sample interval is the inverse of an integer multiple of 50 or 60. We need to compute the DFT for the fundamental using equation (1), where, \( k \) equal to one for the fundamental and \( n \) is the coefficient subscript. Two digital filters are required, one to get the real part and one for the imaginary part.
Fig. 10. Block diagram of the developed distance relay model

9. Developing Procedures of Distance Relay Model Using MATLAB

MATLAB development environment, is a set of tools to help the use of MATLAB functions and files (Matlab, 2006), where Simulink is an interactive tool for modeling, simulating and analyzing dynamic systems, including control and many complex systems. (Simulink, 2001). MATLAB and Simulink were used to model the relay components such as ADC and digital filters. (Abdlmnam, 2007). Figure 10, shows block diagram for developed distance relay model. The voltage and current data are derived using the power simulator EMTP-ATP. It is possible to derive these values from any power system simulator such as MATLAB, EMTP, NEPLAN....etc. and converted to a MATLAB format. Simulation of electric power systems has been a common practice for more than thirty years. Computer models of major power system components have been used in software packages such as short circuit programs, load flow, stability programs, and electromagnetic transient programs. In most of the cases the power system is represented by a single line diagram which is representing either a three or single phase system. This may include three phase source, three phase transmission line (lines may be represented using n model), current transformers, voltage transformers and voltage and current measurements. The voltage and current input signals are inserted in a MATLAB window which is designed to set the distance relay parameters. As these signals generated by applying faults they may include a dc offset and a high frequency traveling waves which, if not suppressed, may lead to misjudgment to the fault location.
Figure 11 a&b shows a sample of a current and voltage waveform before, during and after fault, while Figure 12 a@b shows the Matlab window that contain the input signals as appeared after low pass filter. Thus data is passed through low pass filter to remove the effects, on the voltage and current signals, of the traveling waves instigated by the fault.

The input filtered signals then passed through A/D convertor. Figure 13 shows the output signal of A/D convertor. The output signal becomes ready to be used by the Discrete Fourier Transform. Figure 14 shows the input voltage and current signals amplitude as determined by Discreet Fourier Transform model. Data applied to the developed relay model, is then analyzed to evaluate the relay response i.e. whether the impedance trajectory of the relay during fault denotes to the proper zone. MATLAB program then used to plot the characteristic of mho distance relay, the behavior of Z during the sampled period. The results are presented in graphical form using an R-X diagram.
10. Simulation Results

The developed distance relay model is evaluated using data generated from power simulator. The output signals as resulted from faults set over a power network using EMTP are input to the MATLAB relay model. Evaluation extended to include different power networks at different fault locations. The faults were also set over the power network when fault resistances at different values were assumed, and when the power network consist of more than one in-feed. This is to evaluate performance of the developed model at different operating conditions and to check the effect of system conditions, fault resistance and load conditions on the performance of the developed distance relay model.

A Single line diagram representing a single 220 kV 50 Hz over head line connected to a single power source is shown in figures 15, where the overhead line is modeled as a lumped \( \pi \) model. The positive and zero sequence impedance of the source are:
10.1 Case one: Single line to ground faults at different distances from the relay location

Single line to ground faults were set on EMTP model of the power system shown in figure 13 at a distance of 10 Km, 20 Km and 35 Km from the location of bus-A. The distances representing 10% to 80% of line A-B length. Similarly few more Single line to ground faults were set at 5 Km, 10Km and 25 Km from the location of bus-B and bus-C. The selected distances are to check the relay behavior at faults that covers the different zones of protection of the relay. The voltage and current signal before and during fault were fed to the relay model. Figures 16, show the impedance trajectory for few samples of these cases. In all cases the output results which are the impedance trajectory of the digital distance relay model had the expected behavior where the impedance trajectory calculations start the trajectory from the load area, before fault, and end up at the proper zone.

Fig. 16. Impedance trajectory for faults at different locations, case 1.
10.2 Case two: Single line to ground faults with fault resistance

Single line to ground faults with different fault resistances were set on EMTP model of the power system shown in Figure 15 at different fault locations. Figure 17 show the impedance trajectory for two of these cases. The shown cases illustrate the behavior of the relay when fault resistance is 2Ω and 10Ω. (Abdlmnam & Sherwali, 2009) In first case the relay detects the fault in zone 1 as the resistance value were not enough to change the reach of the relay, while in the second case the value of the resistance was enough to make the impedance presented to the relay lies in zone two, even though the fault were set in zone one. However, in all cases the output results which are the impedance trajectory of the digital distance relay model had the expected behavior where the effect of the arc resistance reflected on the value of the impedance seen by the relay. Impedance trajectory calculations start the trajectory from the load area, but due to the existence of fault resistance the relay judges the location of the fault considering the effect of arc resistance, as expected.

![Impedance trajectory for faults](http://www.intechopen.com)

(a) Fault resistance of 2 Ω (b) Fault resistance of 10 Ω

Fig. 17. Impedance trajectory of the relay for faults accompanied by fault resistances.

10.3 Case three: Double circuit fed from more than one in-feed

A distance relay is said to under-reach when the impedance presented to it is apparently greater than the impedance to the fault. The main cause of underreaching is the effect of fault current in-feed at remote busbars. High voltage power system usually interconnected and run in double circuits for a reliable system. This usually implies an existence of more than one in-feed point which may cause the distance relay to under. EMTP-ATP is used to simulate the power system network shown in figure 18, to evaluate the developed model under this circumstance. Fault location is shown on the single line diagram and system data is as shown below:

![Single line diagram of the simulated multi in-feed power network](http://www.intechopen.com)

Fig. 18. Single line diagram of the simulated multi in-feed power network.
The positive and zero sequence impedance of the sources are
Zs1 of G1=0.819+6.76j     Zs1 of G2= 4.5+12.8j     Zs1 of G3= 1.4+8.8j
Zs0 of G1=3.48+22.51j      Zs0 of G2=10.6+38.8j      Zs0 of G3= 6.6+27.8j

The positive and zero sequence impedance of the transmission lines are
Z1= 0.09683 + 0.9034j Ω /km.,  Z0= 0.01777 + 0.4082j Ω /km.

Where the network voltage is 400kV, the current transformer ratio is 400/1A, the voltage transformer ratio is 400kV/110V and the setting of relay A is as follows:
Zone one = 1.79 ohm-secondary (80 % of the protected line).
Zone two = 3.55ohm-secondary (100% of the protected line + 50% of the shortest adjacent line).
Zone three = 7.64 ohm-secondary (120% of the impedance presented to the relay for a fault at the remote end of the longest adjacent line).

As seen in figure 19, the impedance trajectory moves into zone three not in zone two. If the fault impedance is calculated assuming a single in-feed the relay, A would see the fault within its zone two, however due to the under-reach caused by the in-feed from the parallel line, relay A sees the fault in zone three.

![Impedance trajectory for faults on multi in-feed power network](image)

**Fig. 19. Impedance trajectory for faults on multi in-feed power network**

**11. Future Research**

Since the developed model was not provided with a decision scheme, work may be extended to include a comprehensive relay model including trip scheme. Work may be extended to incorporate algorithms used to improve the relay behavior when overhead lines are compensated by series capacitors or/and when the model is to be used to protect power systems incorporate power cables having a considerable capacitance.

**12. Conclusions**

As modern numerical relays are widely employed in protection systems nowadays and modeling of these types of relays is important to adjust and settle protection equipment in electrical facilities and to train protection personnel, the simulation of distance relays using MATLAB offers a good opportunity to perform these activities efficiently and with minimum cost. Another advantages is that, as MATLAB is a powerful tool rich with component models, any shape of relay characteristic (Impedance, mho, quadrilateral,...) can
be employed. The simulation of numerical distance relay using MATLAB/SIMULINK was explained in details and the behavior of the developed relay model was tested under different onerous conditions. From impedance calculation point of view, the relay model was able to identify the proper zone of operation. In all of the cases presented to test the model, the model judged the fault location as expected including the cases were the measured impedance was changed due to a change of fault location, due to an existence of resistive faults and/or due to the change in apparent impedance as a result of an existence of more than one in-feed. The impedance trajectory that reflects the behavior of the developed model under different fault locations and at different arc resistances, for few of the cases tested, was presented and discussed. However, trip signal was not generated since the model was not provided with a decision scheme that decides when to generate the trip signal.

13. References
Abdlmenam A. Abdllaheh, Modeling of distance relays for power system protection, M.Sc. dissertation, EE&E Dept., Faculty of Engineering, Al-Fatah University, Fall 2007.
Sandro Gianny Aquiles Perez, “Modeling Relays for Power System Protection Studies”, Thesis Submitted to the College of Graduate Studies and Research, Department of Electrical Engineering University of Saskatchewan, Saskatchewan, Canada, July 2006

www.intechopen.com
This book is a collection of 19 excellent works presenting different applications of several MATLAB tools that can be used for educational, scientific and engineering purposes. Chapters include tips and tricks for programming and developing Graphical User Interfaces (GUIs), power system analysis, control systems design, system modelling and simulations, parallel processing, optimization, signal and image processing, finite different solutions, geosciences and portfolio insurance. Thus, readers from a range of professional fields will benefit from its content.

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