# Modelling of DC-DC converters

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## 1. Introduction

The DC-DC converters are electrical circuits that transfer the energy from a DC voltage source to a load and regulate the output voltage. The energy is transferred via electronic switches, made with transistors and diodes, to an output filter and than is transferred to the load.

DC-DC converters are used to convert unregulated dc voltage to regulated or variable dc voltage at the output. They are widely used in switch-mode dc power supplies and in dc motor drive applications. In dc motor control applications, they are called *chopper-controlled drives.* The input voltage source is usually a battery or derived from an ac power supply using a diode bridge rectifier. These converters are generally either hard-switched PWM types or soft-switched resonant-link types.

These converters employ square-wave pulse width modulation to achieve voltage regulation. The output voltage is regulated varying the duty cycle of the power semiconductor switch driving signal. The voltage waveform across the switch and at the input of the filter is square wave in nature and they generally result in higher switching losses when the switching frequency is increased. Also, the switching stresses are high with the generation of large electromagnetic interference (EMI), which is difficult to filter. However, these converters are easy to control, well understood, and have wide load control range.

These converters operate with a fixed-frequency, variable duty cycle. This type of signal is called Pulse Width Modulated signal (PWM). Depending on the duty cycle, they can operate in either continuous current mode (CCM) or discontinuous current mode (DCM). If the current through the output inductor never reaches zero then the converter operates in CCM; otherwise DCM occurs.

The output voltage will be equal with the average value on the switching cycle of the voltage applied at the output filter. Due to the losses on the ON or OFF state of the ideal transistor are zero, the theoretical efficiency of the switching mode converters is up to 100%. But, considering the real switches, with parasitic elements, the efficiency will be a little bit lower, but higher than linear regulators.

Another advantage of switching mode converters consist in the possibility to use the same components but in other topology in order to obtain different values of the output voltages: positive or negative, lower or higher than input voltage.

There are various analysis methods of DC-DC converters. Throughout the chapter an extended analysis and modelling for DC-DC power converters is proposed.  In this approach, the differential equations that describe the inductor current and capacitor voltage are determined and are solved according with the boundary conditions of the switching periods. The values of currents and voltages at the end of a period become initial conditions for the next switching period. This method is very accurate and produces a set of equations that require extensive computation.

In addition, for specified values of converter parameters (inductance and capacitor) we can calculate the maximum value of transistor and diode current and reverse voltage, in order to help user to choose the appropriate type of transistor and diode.

## 2. Buck converter

The buck (or step-down converter), shown in the figure 1, contain a capacitor and an inductor with role of energy storing, and two complementary switches: when one switch is closed, the other is open and vice-versa.
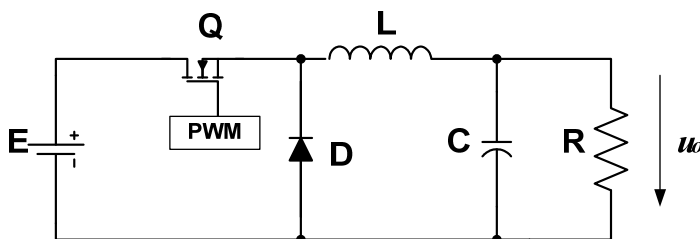
Fig. 1. The buck converter diagram

The switches are alternately opened and closed with at a rate of PWM switching frequency. The output that results is a regulated voltage of smaller magnitude than input voltage. The converter operation will be analyzed function of switches states.

**The first time interval: The transistor is in ON state and diode is OFF.**

During this time period, corresponding with duty cycle of PWM driving signal, the equivalent diagram of the circuit is presented bellow:
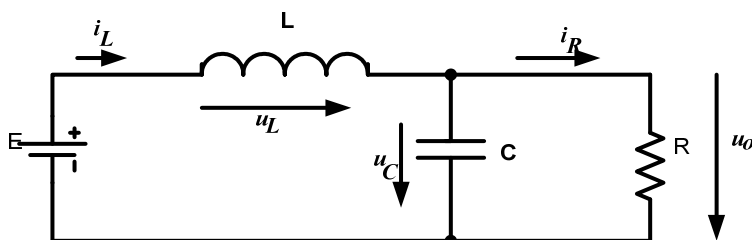
Fig. 2. The equivalent circuit during the ON state of transistor and OFF state of diode

For this equivalent circuit will write the equations that describe the converter operation:

$$\begin{cases} \dfrac{du_o}{dt} = (i_L - \dfrac{u_o}{R})\dfrac{1}{C}; \\ \dfrac{di_L}{dt} = \dfrac{E - u_o}{L}; \end{cases} \tag{1}$$

**The second time period: the transistor is OFF and diode is ON**

In the moment when the transistor switch in OFF state, the voltage across the inductor will change the polarity and the diode will switch in ON state. The equivalent diagram of converter during this period is shown in the bellow figure:
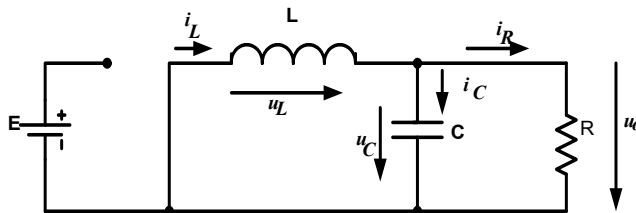


Fig. 3. The equivalent circuit for OFF state of transistor and ON state of diode

For this operation period, the output voltage $u_0$ and the current through the inductor $i_L$ satisfy the following equations:

$$\begin{cases} \dfrac{du_o}{dt} = \left(i_L - \dfrac{u_o}{R}\right)\dfrac{1}{C}; \\ \dfrac{di_L}{dt} = -\dfrac{u_o}{L}; \end{cases} \tag{2}$$

**The third operation mode: The both transistor and diode are OFF**

If the inductor current becomes zero before ending the diode ON period, both transistor and diode will naturally closed. This operation regime is called *discontinuous current mode*. The equivalent diagram of this operation regime is shown bellow.
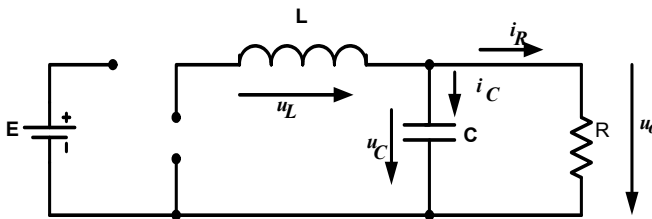


Fig. 4. The equivalent circuit with transistor and diode in OFF state

For this operation period, the output voltage $u_o$ and the current through the inductor $i_L$ satisfy the following equations:

$$\begin{cases} \dfrac{du_o}{dt} = -\dfrac{u_o}{R} \cdot \dfrac{1}{C}; \\ \dfrac{di_L}{dt} = 0; \end{cases} \tag{3}$$

## 2.1 CCM inductance

The minimum value of inductance for continuous current mode (CCM) operation is calculated from output voltage and inductor current equations.

Thus, the output voltage $u_o$ and the current through the inductor $i_L$ satisfies the following equations:

$$\begin{cases} u_L = E - Uo........t \in (0, D \cdot T) \\ u_L = -Uo............t \in (D \cdot T, T) \end{cases} \tag{4}$$

$$\begin{cases} i_L = \dfrac{E - Uo}{L} \cdot t...........t \in (0, D \cdot T) \\ i_L = -\dfrac{Uo}{L} \cdot t...............t \in (D \cdot T, T) \end{cases} \tag{5}$$

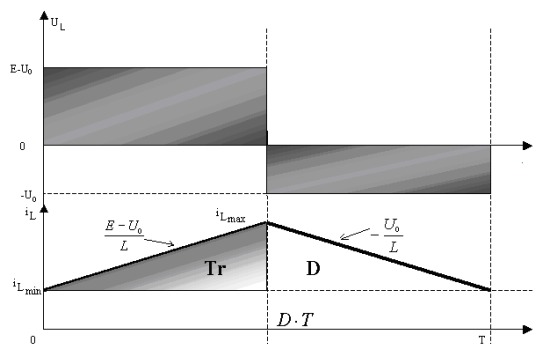The waveforms of inductor voltage and current on a switching period are shown in the figure 5:



Fig. 5. The waveforms of output voltage and inductor current

In the steady state regime, the average value of voltage across the inductor is zero. Thus,

$$(E - Uo) \cdot D \cdot T = Uo \cdot T (1 - D) \Rightarrow Uo = D \cdot E \tag{6}$$

Based on the inductor current waveform, the following equation can be write:

$$I_{L\max} = I_{L\min} + \frac{E - Uo}{L} \cdot D \cdot T \tag{7}$$

The average value of the inductor current is equal with the output current:

$$\frac{I_{L\max} + I_{L\min}}{2} = \frac{U_o}{R} = I_o R \tag{8}$$

From the equations (7) and (8) results the minimum ad the maximum values of inductor current.

$$I_{L\min} = \frac{U_o}{R} - \frac{E - U_o}{2L} \cdot D \cdot T \tag{9}$$

$$I_{L\max} = \frac{U_o}{R} + \frac{E - U_o}{2L} \cdot D \cdot T \tag{10}$$

Thus:

$$\Delta i = I_{L\max} - I_{L\min} = \frac{E - U_o}{L} \cdot D \cdot T \tag{11}$$

From the equations (6) and (11) the inductor current ripple can be calculated.

$$\Delta i = \frac{E \cdot D \cdot (1 - D)}{L} \cdot T \tag{12}$$

From the condition, $I_{L\min} \geq 0$ , it results:

$$\frac{2L}{RT} \geq 1 - D . \tag{13}$$

This relation can be used to determine the minimum value of inductance, when the switching frequency and load value are known.

$$L_{\min} = \frac{R \cdot T}{2} \cdot (1 - D) \tag{14}$$

## 2.2 The discontinuous current mode

In discontinuous current regime, the waveforms of inductor voltage and current are shown in the figure bellow:
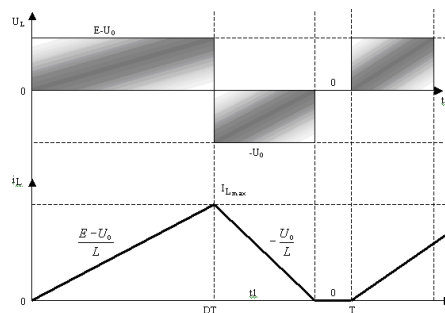


Fig. 6. The waveforms of voltage and inductor current in DCM

The average value of the input current is equal with the current through the switching transistor.

$$I_{iav} \cdot T = \frac{1}{2} \cdot D \cdot T \cdot I_{L\max} \tag{15}$$

$$I_{L\max} = \frac{E - U_o}{L} \cdot D \cdot T \tag{16}$$

From the above equations, it results:

$$I_{iav} = \frac{E - U_o}{2L} D^2 T \ . \tag{17}$$

Considering that there are no losses in the circuit, the input and the output powers are equals.

$$P_{in} = P_{out} \Leftrightarrow \frac{U_o^2}{R} = E \cdot I_{iav} \tag{18}$$

Thus,

$$\frac{U_o^2}{R} = E \frac{E - U_o}{2L} D^2 T \tag{19}$$

$$\frac{U_o^2}{E^2} = \frac{RT}{2L} \left(1 - \frac{U_o}{E}\right) D^2 \tag{20}$$

Denoting the circuit transfer ratio $\dfrac{Us}{E}$ =M:

$$M^2 + \frac{R \cdot T \cdot D^2}{2 \cdot L} \cdot M - \frac{R \cdot T \cdot D^2}{2 \cdot L} = 0 \tag{21}$$

Denoting
$$a = \frac{R \cdot D^2 \cdot T}{2 \cdot L} \ , \tag{22}$$

the solutions of the above equation are :

$$M = \frac{-a \pm \sqrt{a^2 + 4a}}{2} \ . \tag{23}$$

Analyzing those solutions, can be observed that the single valid solution is

$$M = \frac{-a + \sqrt{a^2 + 4a}}{2} \tag{24}$$

The variation of circuit transfer ratio M function of PWM signal duty-cycle D, for different values of $\dfrac{2L}{RT}$ parameters is shown in the figure:
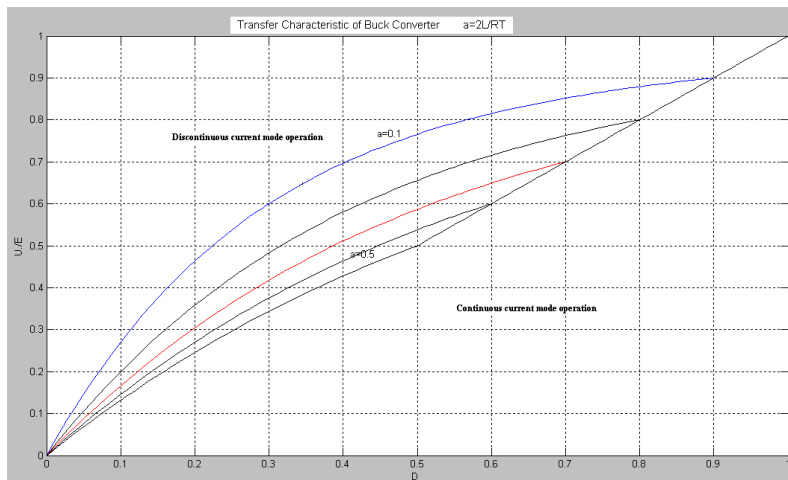
Fig. 7. Variation of circuit transfer ratio M function of PWM signal duty-cycle D

## 2.3 Filtering capacitor

Other important parameter that is important to be determined is the value of the output capacitor, in order to obtain a specific value of the output voltage ripple.

The capacitor charging current is equal with difference between the inductor current $i_L$ and the output current $i_O$. Considering a constant output current, the electric charge stored in the capacitor during a switching period is equal with the shade area from the figure bellow:
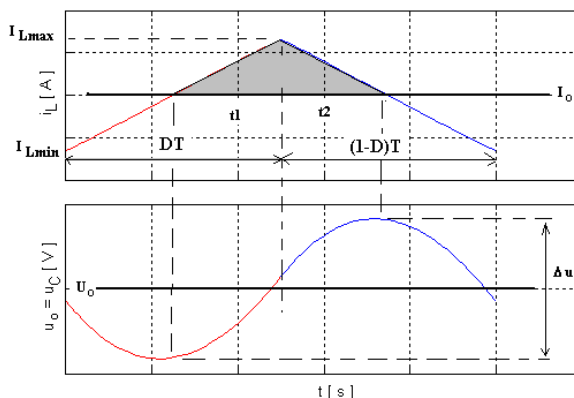


Fig. 8. The waveforms of inductor current and output voltage

$$Q = \Delta u \cdot C = \frac{1}{2}(I_{L\max} - I_O)(t_1 - t_2) \tag{25}$$

$$\frac{t_1}{DT} = \frac{t_2}{(1-D)T} = \frac{t_1 + t_2}{T} = \frac{I_{L\max} - I_O}{I_{L\max} - I_{L\min}} \tag{26}$$

Thus, the value of the output capacitor can be calculated with the following formula:

$$C = \frac{(E - U_O)DT^2}{8L\Delta u} = \frac{E(1-D)D}{8Lf^2\Delta u} \tag{27}$$

It can be seen in this formula that the capacitor value depends by the switching frequency. Increasing the switching frequency, the capacitor value will be smaller.

## 3. Boost Converter

The boost (or step-up converter), shown in the figure 9, contains, like the Buck converter, a capacitor and an inductor with role of energy storing, and two complementary switches. In the case of the boost converter, the output voltage is higher than the input voltage.
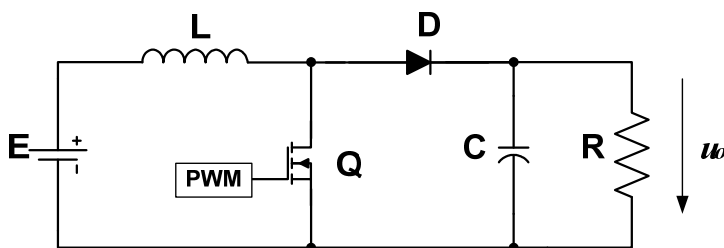
Fig. 9. The boost converter diagram

The switches are alternately opened and closed with at a rate of PWM switching frequency. As long as transistor is ON, the diode is OFF, being reversed biased. The input voltage, applied directly to inductance L, determines a linear rising current. When transistor is OFF, the load is supplied by both input source and LC filter. The output that results is a regulated voltage of higher magnitude than input voltage. The converter operation will be analyzed according with the switches states.

**The first time interval: The transistor is in ON state and diode is OFF.**

During this time period, corresponding with duty cycle of PWM driving signal, the equivalent diagram of the circuit is presented bellow. In this time period the inductance L store energy.
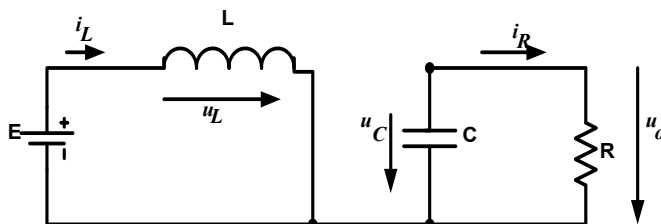
Fig. 10. The equivalent circuit during the ON state of transistor and OFF state of diode

For this operation period, the output voltage $u_o$ and the current through the inductor $i_L$ satisfies the following equations:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{E}{L}; \\ \dfrac{du_o}{dt} = -\dfrac{u_o}{R \cdot C}; \end{cases} \tag{28}$$

**The second time period: the transistor is OFF and diode is ON**

In the moment when the transistor switch in OFF state, the voltage across the inductor will change the polarity and diode will switch in ON state. The equivalent diagram of converter during this period is shown in the bellow figure:
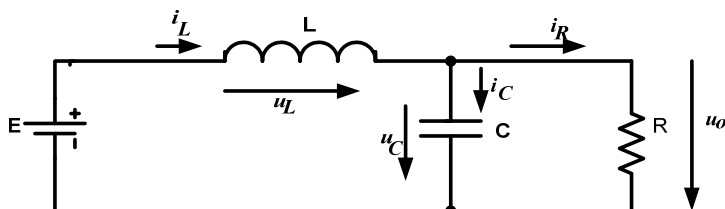


Fig. 11. The equivalent circuit for OFF state of transistor and ON state of diode

For this operation period, the output voltage $u_o$ and the current through the inductor $i_L$ satisfy the following equations:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{E - u_o}{L}; \\ \dfrac{du_o}{dt} = \dfrac{1}{C}\left( i_L - \dfrac{u_o}{R} \right); \end{cases} \tag{29}$$

**The third operation mode: The both transistor and diode are OFF**

If the inductor current becomes zero before ending the diode conduction period, both the transistor and the diode will be in OFF state. Due to the diode current becomes zero, the diode will naturally close, and the output capacitor will discharge on the load. This operation regime is called *discontinuous current mode*. The equivalent diagram of this operation regime is shown bellow.
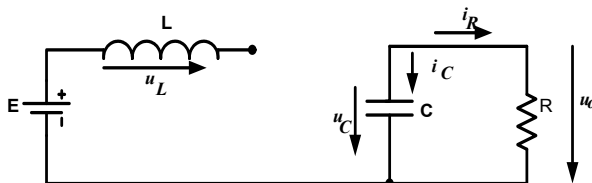


Fig. 12. The equivalent circuit with transistor and diode in OFF state

For this operation period, the output voltage $u_o$ and the current through the inductor $i_L$ can be calculated from the following equations:

$$\begin{cases} \dfrac{di_L}{dt} = 0; \\ \dfrac{du_o}{dt} = -\dfrac{u_o}{R \cdot C}; \end{cases} \tag{30}$$

### 3.1 CCM inductance

The minimum value of inductance for continuous current mode (CCM) operation is calculated from inductor current equation. In the steady state regime, the average value of voltage across the inductor is zero.
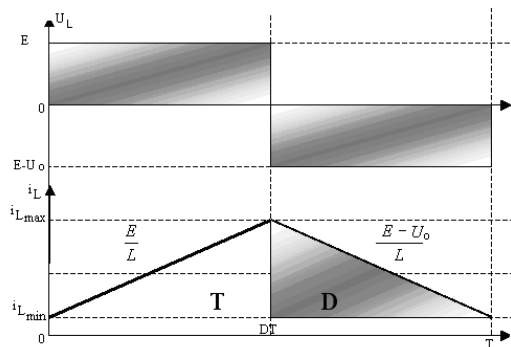


Fig. 13. The waveforms of inductor voltage and current in steady-state regime

Thus, the output voltage $u_0$ and the current through the inductor $i_L$ satisfies the following equations:

$$E \cdot D \cdot T = T \cdot (1 - D) \cdot (Uo - E); \Rightarrow Uo = \frac{E}{1 - D} \tag{31}$$

Based on the above waveforms, the maximum value of the inductor current is:

$$i_{L_{max}} = i_{L_{min}} + \frac{E}{L} \cdot D \cdot T \tag{32}$$

The output current is equal with the diode average current:

$$\frac{i_{Lmax} + i_{Lmin}}{2} \cdot T \cdot (1 - D) = \frac{U_o}{R} \cdot T \tag{33}$$

Based on the equations (32) and (33), results:

$$i_{Lmax} = \frac{U_o}{R \cdot (1 - D)} + \frac{E \cdot D \cdot T}{2 \cdot L} \tag{34}$$

$$i_{L \ \min} = \frac{U_o}{R \cdot (1-D)} - \frac{E \cdot D \cdot T}{2 \cdot L} \tag{35}$$

Based on the equations (34) and (35), can be determined the inductor current ripple:

$$\Delta i = \frac{E \cdot D \cdot T}{L} \tag{36}$$

From the condition for continuous conduction mode, $i_{L_{\min}} \geq 0$, results:

$$\frac{2L}{R \cdot T} \geq D \cdot (1-D)^2 \tag{37}$$

This condition can be used to determine the minimum inductance value, for a specific switching period T and a specific load value R.

$$L_{\min} = \frac{R \cdot T}{2} D \cdot (1-D)^2 \tag{38}$$

### 3.2 The discontinuous current mode
In discontinuous conduction mode, the waveforms of the inductor voltage and current are shown in the figure bellow:
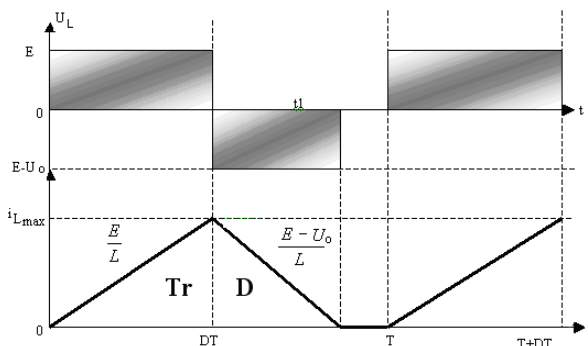


Fig. 14. The waveforms of voltage and inductor current in DCM

The average value of the input current is equal with the inductor average current.

$$I_{i_{av}} \cdot T = \frac{1}{2} \cdot I_{L_{\max}} \left( D \cdot T + t_1 \right) \tag{39}$$

where

$$I_{L_{\max}} = \frac{E \cdot D \cdot T}{L} \ . \tag{40}$$

Based on the equations (39) and (40), results:

$$I_{i_{av}} = \frac{E \cdot D}{2 \cdot L} \cdot \left( D \cdot T + t_1 \right) \tag{41}$$

The average value of the inductor voltage during a switching period is zero.

$$E \cdot D \cdot T = t_1 (Uo - E); \Rightarrow t_1 = \frac{E \cdot D \cdot T}{U_0 - E} \qquad (42)$$

Replacing equation (42) in the equation (41), and also considering the input and the output power equals,

$$P_{in} = P_{out} \Leftrightarrow \frac{Uo^2}{R} = E \cdot I_{i_{av}} \qquad (43)$$

it results:

$$\frac{Uo}{E} \left( \frac{Uo}{E} - 1 \right) = \frac{D^2 \cdot T \cdot R}{2 \cdot L} \ . \qquad (44)$$

Denoting the voltage transfer ratio with M=$\frac{Uo}{E}$, the equation (44) becomes:

$$M(M-1) = \frac{D^2 \cdot T \cdot R}{2 \cdot L} \qquad (45)$$

The solution of this equation is:

$$M = \frac{1 + \sqrt{1 + \dfrac{4 \cdot D^2 \cdot T \cdot R}{2L}}}{2} \qquad (46)$$

The variation of circuit transfer ratio M function of PWM signal duty-cycle D, for different values of $\frac{2L}{RT}$ parameters is shown in the figure:
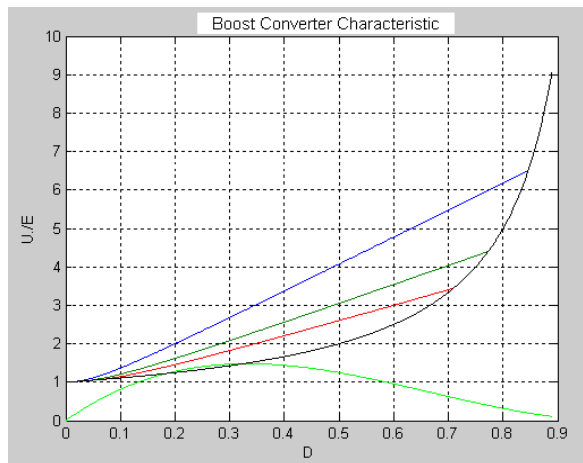


Fig. 15. Variation of circuit transfer ratio M function of PWM signal duty-cycle D

### 3.3 Filtering capacitor

Other important parameter that is important to be determined is the value of the output capacitor, in order to obtain a specific value of the output voltage ripple.

The capacitor charging current is equal with the difference between the diode current $i_D$ and the output current $i_o$. Considering a constant output current, the electric charge stored in the capacitor during a switching period is equal with the shade area from the figure bellow:
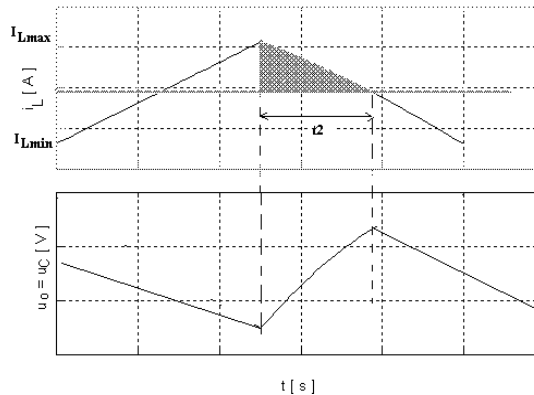
Fig. 16. The waveforms of inductor current and output voltage

$$Q = C \cdot \Delta u = \frac{1}{2}(I_{L\max} - I_o)t_2 \tag{47}$$

$$\frac{t_2}{T(1-D)} = \frac{I_{L\max} - I_o}{I_{L\max} - I_{L\min}} \tag{48}$$

Thus, the value of the output capacitor can be calculated with the following formula:

$$C = \frac{\left(I_{L\max} - I_o\right)^2 T(1-D)}{2\left(I_{L\max} - I_{L\min}\right)} \tag{49}$$

## 4. Buck-Boost converter

The buck-boost converter (polarity inverter) is shown in figure 17.
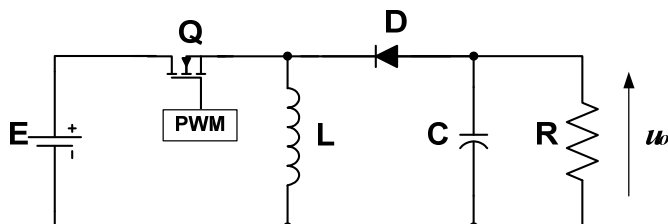
Fig. 17. Buck-Boost converter diagram

The switches are alternately opened and closed with at a rate of PWM switching frequency. As long as the transistor is ON, the diode is OFF, being reversed biased. The input voltage, applied directly to inductance L, determines a linear rising current. The capacitor is discharged on the load circuit. When the transistor is OFF, the load is supplied by LC filter. The output that results is a regulated voltage of smaller or higher magnitude than input voltage, depending on the value of duty cycle, but it has a reverse polarity. The converter operation will be analyzed according with the ON or OFF state of switches.

**The first time interval: The transistor is in ON state and diode is OFF.**

During this time period, corresponding with duty cycle of PWM driving signal, the equivalent diagram of the circuit is presented bellow. In this time period the inductance L stores energy. The load current is assured by the output capacitor.
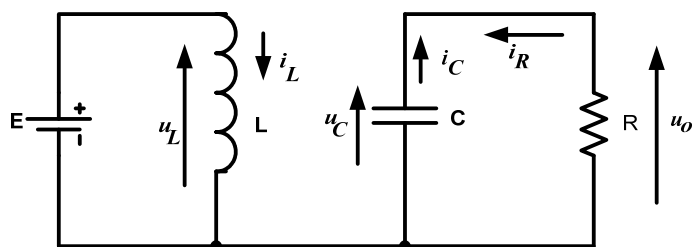


Fig. 18. The equivalent circuit during the ON state of transistor and OFF state of diode

For this operation period, the output voltage $u_o$ and the current through the inductor $i_L$ are given by the following equations system:

$$\begin{cases} \dfrac{di_L}{dt} = E \\ \dfrac{du_o}{dt} = -\dfrac{u_o}{R \cdot C} \end{cases} \tag{50}$$

**The second time period: the transistor is OFF and diode is ON**

In the moment when the transistor switch in OFF state, the voltage across the inductor will change the polarity and diode will switch in ON state. The energy stored in the inductor will supply the load. The equivalent diagram of converter during this period is shown in the figure bellow:
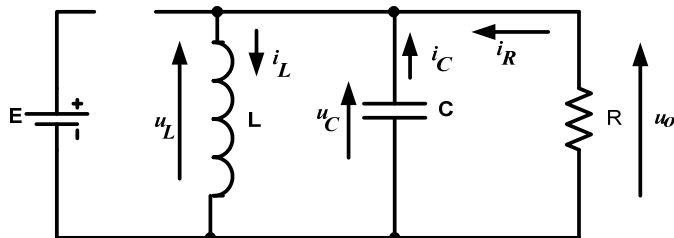


Fig. 19. The equivalent circuit during the OFF state of transistor and ON state of diode

For this operation period, the following equations for the output voltage $u_o$ and the current through the inductor $i_L$ can be written:

$$\begin{cases} \dfrac{di_L}{dt} = -\dfrac{u_o}{L} \\ \dfrac{du_o}{dt} = \left( i_L - \dfrac{u_o}{R} \right) \cdot \dfrac{1}{C} \end{cases} \tag{51}$$

**The third operation mode : The both transistor and diode are OFF**

If the inductor current becomes zero before ending the diode ON period, both the transistor and the diode will be OFF. Due to the diode current becomes zero, the diode will naturally close, and the output capacitor will discharge on the load. This operation regime is called *discontinuous current mode*. The equivalent diagram of this operation regime is shown bellow.
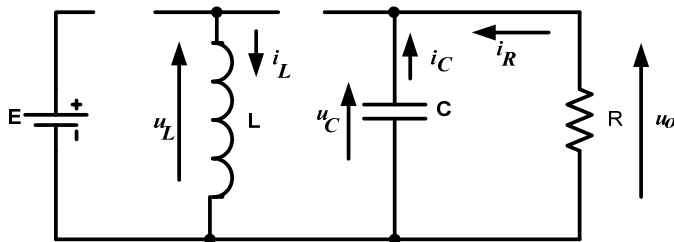


Fig. 20. The equivalent diagram for discontinuous conduction mode operation

For this operation mode, the output voltage $u_o$ and the current through the inductor $i_L$ can be calculated from the following differential equations:

$$\begin{cases} \dfrac{di_L}{dt} = 0 \\ \dfrac{du_o}{dt} = -\dfrac{u_o}{R \cdot C} \end{cases} \tag{52}$$

**4.1 CCM inductance**
The minimum value of the inductance for continuous current mode (CCM) operation is calculated from the the inductor current equations.
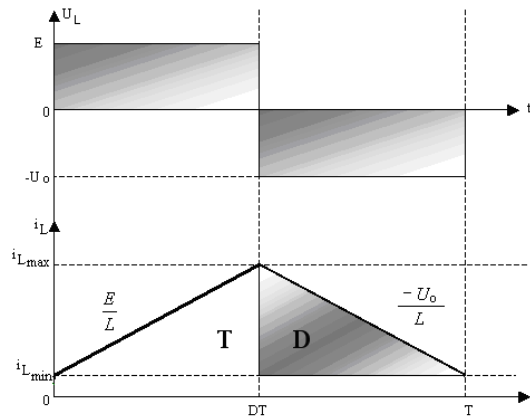
Fig. 21. The inductor voltage and current waveforms in steady-state regime

In the steady state regime, the average value of the voltage across the inductor is zero. From this condition, the output voltage *Uo* can be determined:

$$E \cdot D \cdot T = Uo \cdot T(1-D) \Rightarrow E \cdot D = Uo(1-D)$$
$$\Rightarrow Uo = \frac{E \cdot D}{1-D} \tag{53}$$

Based on the above waveforms:

$$I_{L\max} = I_{L\min} + \frac{E}{L} \cdot D \cdot T \tag{54}$$

Also, the average diode current is equal with the output current.

$$\frac{I_{L\max} + I_{L\min}}{2} \cdot T \cdot (1-D) = \frac{U_o}{R} \cdot T \tag{55}$$

Based on the equations (54) and (55), the maximum and the minimum value of the inductor current will be:

$$I_{L_{\max}} = \frac{U_o}{R \cdot (1-D)} + \frac{E \cdot D \cdot T}{2 \cdot L} \tag{56}$$

$$I_{L_{\min}} = \frac{U_o}{R \cdot (1-D)} - \frac{E \cdot D \cdot T}{2 \cdot L} \tag{57}$$

Thus, the inductor current ripple is:

$$\Delta i = \frac{E \cdot D \cdot T}{L} \tag{58}$$

From the condition for continuous conduction mode, $i_{L_{\min}} \geq 0$ , results:

$$\frac{2L}{RT} \geq (1-D)^2 \tag{59}$$

This condition can be used to determine the minimum inductance value, for a specific switching period T and a specific load value R.

$$L_{\min} = \frac{R \cdot T \cdot (1-D)^2}{2} \tag{60}$$

## 4.2 The discontinuous current mode

In discontinuous conduction mode, the waveforms of the inductor voltage and current are shown in the figure bellow:
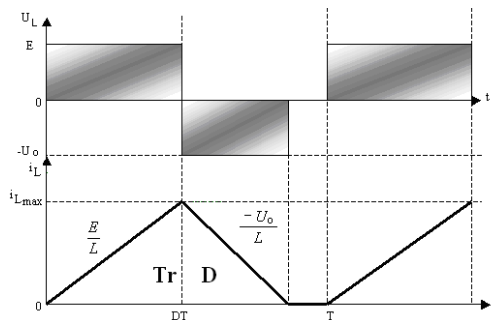


Fig. 22. The waveforms of voltage and inductor current in DCM

The average value of the input current is equal with the transistor average current.

$$I_{i_{av}} \cdot T = \frac{1}{2} \cdot I_{L\max} D \cdot T \tag{61}$$

where,

$$I_{L\max} = \frac{E \cdot D \cdot T}{L} \tag{62}$$

From the above equations results:

$$I_{i_{av}} = \frac{E \cdot D^2 T}{2 \cdot L} \tag{63}$$

Neglecting the losses in the circuit, the input power is equal with the output power.

$$P_{in} = P_{out} \Leftrightarrow \frac{U_o^2}{R} = E \cdot I_{i_{av}} \tag{64}$$

$$\frac{E^2 D^2 T}{2L} = \frac{U_o^2}{R} \tag{65}$$

Denoting the voltage transfer ratio $\dfrac{Uo}{E}$ =M , it results:

$$M = D\sqrt{\dfrac{R\,T}{2L}} \tag{66}$$

The variation of circuit transfer ratio M function of PWM signal duty-cycle D, for different values of $\dfrac{2L}{RT}$ parameters is shown in the figure:
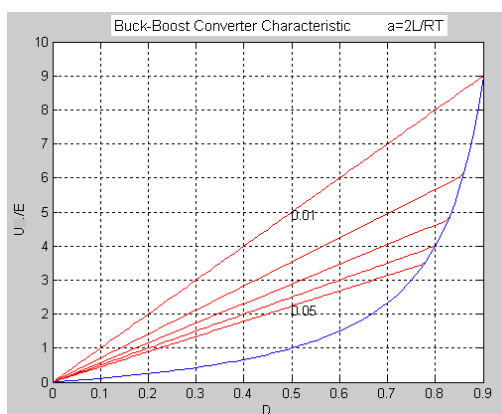


Fig. 23. Variation of circuit transfer ratio M function of PWM signal duty-cycle D

### 4.3 Filtering capacitor

Other important parameter that must be determined is the value of the output capacitor, in order to obtain a specific value of the output voltage ripple.

The capacitor charging current is equal with difference between the diode current $i_D$ and the output current $io$. Considering a constant output current, the electric charge stored in the capacitor during a switching period is equal with the shade area from the bellow figure:



Fig. 24. The waveforms of the inductor current and output voltage

$$Q = C \cdot \Delta u = \frac{1}{2}(I_{L\max} - I_o)t_2 \tag{67}$$
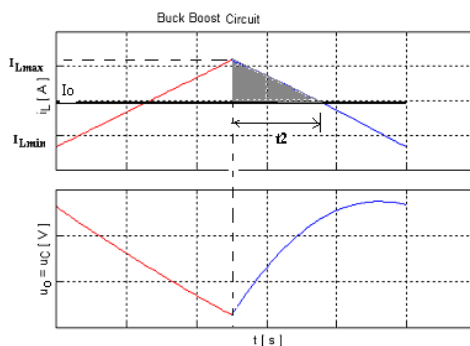
$$\frac{t_2}{T(1-D)} = \frac{I_{L\max} - I_o}{I_{L\max} - I_{L\min}} \tag{68}$$

The value of the output capacitor can be calculated with the following formula:

$$C = \frac{\left(I_{L\max} - I_o\right)^2 T(1-D)}{2\left(I_{L\max} - I_{L\min}\right)} \tag{69}$$

## 5. Matlab Modeling of DC-DC Converters

In order to simulate the converters, the equations that describe the converter operation on each of the three possible operating stages are implemented in Matlab, and solved using Matlab facilities.

The program structure consists in two files. The first file initializes the default values of converter parameters: the input voltage E, the inductance value L, the capacitor value C, the load value R, the switching period T, the duty-cycle D and the number of periods to be displayed. All the parameters can be changed during the converters simulation. The structure of this file is:

Listing for initialization of default parameters values

```
clear all;
close;
E=10;      %input voltage value
L=1e-4;    %inductor value
C=10e-6;   %capacitor value
R=10;      %load value
%--------------------
Q=sqrt(L./C)./R;
T0=2.*pi.*sqrt(L.*C);
%------------
T=50e-6;   %switching period
D=0.5;     %duty-cycle
N=20;      %numbers of periods to be displayed
%--------------
p=1;       % default plotting regime (transient)
%--------------------
type=1;    % default analyzed converter -Buck
%type=2;  % Boost
%type=3;   % Buck-Boost
%----------------
ed_converter(E,L,C,R,T,D,N,p,type);   % function for converters simulation
```

As it can be seen at the end of the file, the *ed_converter(E,L,C,R,T,D,N,p,type)* function is called. This function is implemented in a file with the same name, and had as arguments the converter parameters. In the first part of the file are created the buttons that allow to change the values of the converter parameters. Than, are implemented the functions that solve the differential equations that describe the converter operation and are calculated the critical values of inductor for continuous conduction mode operation and value of output voltage. Also, are defined the plots for output voltage and input current.

The structure of this file is presented bellow:

Listing of function file

```
function ed_converter(E,L,C,R,T,D,N,p,type);
%create a new figure;
Fig=figure('Name','  DC-DC Converters',...
        'Numbertitle','off', 'color', [1, 1, 1]);

% creating 7 text buttons B_T;

txt=['E [V]  L[ H]  C[ F] R[ohm]  T[ s]  D     N   '];
  for k=1:7
   B_T(k)=uicontrol('Style','text', ...
            'Units','normalized', ...
            'backgroundcolor',[1, 1, 1],...
            'Position',[0.91 0.95-0.1.*(k-1) 0.10 0.04], ...
            'String',txt((7.*(k-1))+1:7.*k), ...
            'Callback','close; ');
   end

% Creating 7 Edit buttons B_E ;
var=['E';'L';'C';'R';'T';'D';'N'];
val=[E;L;C;R;T;D;N];

 xc= '=str2num(get(gco,"String"));close;ed_converter(E,L,C,R,T,D,N,p,type)';
for i=1:7
 B_E= uicontrol('Style','edit',...
            'Units','normalized',...
            'backgroundcolor',[1, 1, 0],...
            'Position',[0.91 0.90-0.1*(i-1) 0.10 0.04],...
            'String',val(i),...
            'Callback',cat(2,var(i),xc));
    end

%Creating the control buttons for selection of converter type: Buck, Boost or Buck-Boost
            Buck=uicontrol('Style','pushbutton',...
             'Units','normalized',...
             'Position',[0.05 0.01 0.17 0.04],...
             'String','Buck',...
```

```
      'backgroundcolor',[0, 1, 0.5],...
      'Callback','type=1,close;ed_converter(E,L,C,R,T,D,N,p,type)');
   Boost=uicontrol('Style','pushbutton',...
      'Units','normalized',...
      'Position',[0.25 0.01 0.17 0.04],...
      'String','Boost',...
      'backgroundcolor',[0, 1, 0.5],...
      'Callback','type=2,close;ed_converter(E,L,C,R,T,D,N,p,type)');
 Buck_Boost=uicontrol('Style','pushbutton',...
      'Units','normalized',...
      'Position',[0.45 0.01 0.17 0.04],...
      'String','BuckBoost',...
      'backgroundcolor',[0, 1, 0.5],...
      'Callback','type=3,close;ed_converter(E,L,C,R,T,D,N,p,type)');

%---------------
   Bp=uicontrol('Style','pushbutton',...
      'Units','normalized',...
      'Position',[0.01 0.92 0.19 0.04],...
      'String','Steady-State Regime',...
      'backgroundcolor',[0, 1, 1],...
      'Callback','p=0;close;ed_converter(E,L,C,R,T,D,N,p,type)');
   if p==0
      set(Bp,'String','Transient Regime');
      set(Bp,'Callback','p=1,close;ed_converter(E,L,C,R,T,D,N,p,type)');
   end
% When a button is pushed, the callback will call again the function file with the
newer parameters
%Routine for solving the function ec_conv, that describes the converters operation
t=0;
y=[0 0];
for k=1:N
nt=length(t);
t0=(k-1).*T;
tf=t0+D.*T;
ci=y(nt,:);
interval=1;
[t,y]=ode45(@ec_conv,[t0,tf],[ci],[],E,R,L,C,type,interval);
nt=length(t);
%Setting the plots
subplot('Position',[0.10 0.55 0.80 0.35]);
plot(t,y(:,1),'r');grid on;hold on;
subplot('Position',[0.10 0.15 0.80 0.35]);
plot(t,y(:,2),'r');grid on;hold on;
%--------------------interval=2;
t0=(k-1).*T+D.*T;
```

```
        tf=k.*T;
        ci=y(nt,:);
        interval=2;
        options=odeset('Events',@conv_ev);
        [t,y,te,ye,ie]=ode45(@ec_conv,[t0,tf],[ci],[options],E,R,L,C,type,interval);
        nt=length(t);
        %----------------------
         subplot('Position',[0.10 0.55 0.80 0.35]);
        plot(t,y(:,1),'b');grid on;hold on;
        subplot('Position',[0.10 0.15 0.80 0.35]);
        plot(t,y(:,2),'b');grid on;hold on;
        %---------------------------interval=3;
        if te>0;
        t0=t(nt);
        tf=k.*T;
        ci=y(nt,:);
        interval=3;
        [t,y]=ode45(@ec_conv,[t0,tf],[ci],[],E,R,L,C,type,interval);
        %----------------------
        subplot('Position',[0.10 0.55 0.80 0.35]);
        plot(t,y(:,1),'g');grid on;hold on;
        subplot('Position',[0.10 0.15 0.80 0.35]);
        plot(t,y(:,2),'g');grid on;hold on;
        %--------------------
        end
            if (p==0)&(j<N-1)
                    subplot('Position',[0.10 0.55 0.80 0.35]);
                  hold off;
              end

            if (p==0)&(j<N-1)
                    subplot('Position',[0.10 0.15 0.80 0.35]);
                  hold off;
            end

        end
        %====================================
        subplot('Position',[0.10 0.55 0.80 0.35]);
        ylabel(['iL [ A ]']);
         switch type;
            case 1
    Lm=R.*T.*(1-D)./2;    %Calculating the minimum value of inductance for Buck Converter
            if 2.*L./(R.*T)>=1-D
    Uo=E.*D;              %Calculating the output voltage in Continuous conduction mode

            else
```

```
            z=0.5.*R.*D.^2.*T./L;
            v=0.5.*(sqrt(z.^2+4.*z)-z);
        Uo=v.*E;          %Calculating the output voltage in Discontinuous conduction mode
            end
        title(['Buck Converter',' Uo = ',num2str(Uo),' [ V ]',' Lm = ',num2str(Lm),' [ H ]']);


        case 2
      Lm=R.*T.*D.*(1-D).^2./2;      %Calculating the minimum value of inductance for
      Boost Converter
            di=E.*D.*T./L;


          if 2.*L./(R.*T)>=D*(1-D).^2
      Uo=E./(1-D);          %Calculating the output voltage in Continuous conduction mode


            else
      v=0.5.*(1+sqrt(1+2.*D.^2.*T.*R./L));
        Uo=v.*E;          %Calculating the output voltage in Discontinuous conduction mode
            end
        title(['Boost Converter',' Uo = ',num2str(Uo),' [ V ]',' Lm = ',num2str(Lm),' [ H ]']);


        case 3
              Lm=R.*T.*(1-D).^2./2, %Calculating the minimum value of inductance
      for Buck-Boost Converter
            if 2.*L./(R.*T)>=(1-D).^2
      Uo=E.*D./(1-D);   %Calculating the output voltage in Continuous conduction mode


            else
            v=D.*sqrt(0.5.*T.*R./L);
      Uo=v.*E;          %Calculating the output voltage in Discontinuous conduction mode
            end
title(['Buck-Boost Converter',' Uo = ',num2str(Uo),' [ V ]',' Lm = ',num2str(Lm),' [ H ]']);


        end


        subplot('Position',[0.10 0.15 0.80 0.35]);
        ylabel(['Uo = uC [ V ]']);
        xlabel(['t [ s ]']);


        %Function that describes the converters operation
        function dy=ec_conv(t,y,E,R,L,C,type,interval);
        dy=zeros(2,1);
        switch type;


          case 1
            if interval==1
                a=1;b=1;c=1;
```

```
        elseif interval==2
          a=0;b=1;c=1;
        else
          a=0;b=0;c=0;
        end

    case 2
      if interval==1
        a=1;b=0;c=0;
      elseif interval==2
        a=1;b=1;c=1;
      else
        a=0;b=0;c=0;
      end

    case 3
      if interval==1
        a=1;b=0;c=0;
      elseif interval==2
        a=0;b=1;c=1;
      else
        a=0;b=0;c=0;
      end
end

        dy(1)=(a.*E-b.*y(2))./L;    % Current equation
        dy(2)=(c.*y(1)-y(2)./R)./C; % Voltage equation

%===========================================================
function [value,isterminal,direction] =conv_ev(t,y,E,R,L,C,type,interval);
value = y(1);    % detect iL = 0
isterminal = 1;   % stop the integration
direction = -1;   % negative direction
```

As it can be seen in the converters description, for all types of converters, the equation that describes the operation has the same shape. The difference consists in the value of the coefficients. From this reason, the same equations are used for the simulation of the converters operation and from each converter only the value of *a*, *b*, and *c* coefficients are set. The equations system is:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{a \cdot E - b \cdot u_o}{L} \\ \dfrac{du_o}{dt} = \left(c \cdot i_L - \dfrac{u_o}{R}\right)\dfrac{1}{C} \end{cases} \qquad (70)$$

The simulation results of the dc-dc converters are presented in the following figure:
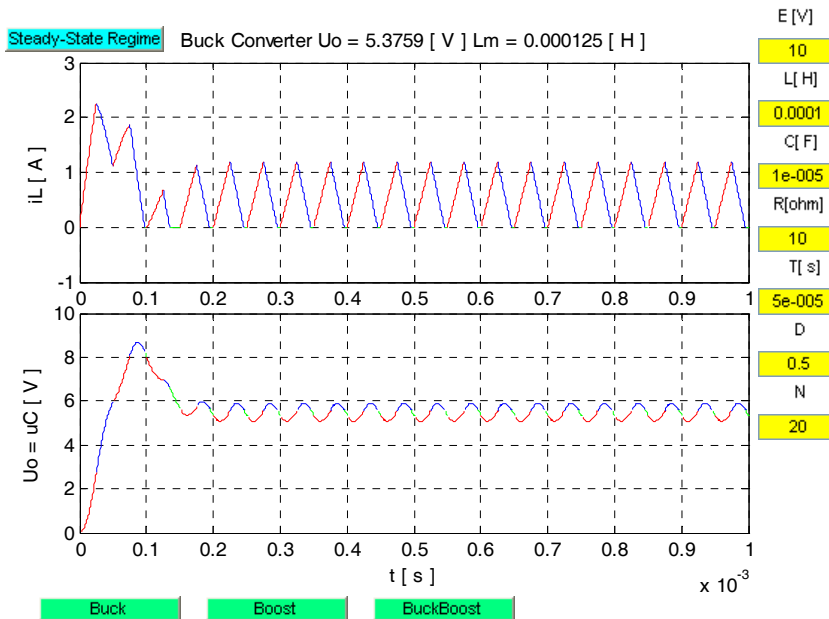


Fig. 25. The converters simulation results

As can be seen in the figure, from the upper left side button can be chosen the display mode: transient, when all the simulated periods are plotted or steady state regime when only the last simulated period is plotted.

From the right site editing buttons, all of the converter parameters can be set. From the bottom side it can be selected the desired converter: buck, boost or buck-boost. Also, for each converter type, the program displays the output voltage value and the minimum inductance value in order to obtain continuous current mode operation.

# 6. References

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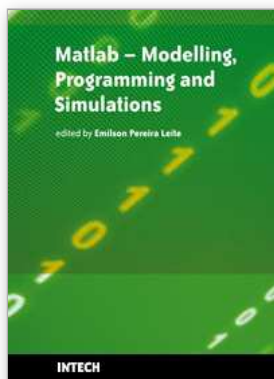
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