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# The Use of Simulation in Urban Modelling

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## 1. Introduction

The impact caused by the fast urban growth and by the occupation of ambient protection areas, demand efficient problem evaluation tools that may be capable to give support in the process of cities territorial, social and economics planning in a short and average stated period. One of the most serious problems of several cities is the disordered urban expansion, which becomes worse because of the lack of planning and specific strategies for this control.

Urban systems are becoming ever larger and increasingly complex as urban economies, social and political structures and norms, and transportation and other infrastructure systems and technologies involved.

We intent to provide a reasonable understanding of the context and objectives for urban simulation modelling, the limitations and challenges of urban simulation models, the design choices involved in developing operational models, and how such models are applied.

The process of urban growth, however, is complex and difficult to model, due to the great number of operating actors in the city and the integrated landscape in different scales. The cities size changes all the time and all of possible rules that could be established are extremely complex (Allen, 1997, and Wu, 2002).

To simulate operations and reactions of these real world processes, models of urban environment and the involved actors, are used to assist in exploration of the hypotheses, analysing the ambient processes and giving some answers about urban changes. To deal with these changes it is necessary to know the processes that caused them and identify the conditions. It is important to know how the changes of a city can occur. Cities are non linear complex systems, and their characteristics are difficult to be modelled by conventional methods (static and linear). New strategies have been developed for this class of problems.

New methods of system modelling must be used in modelling and simulating urban phenomenon. Among others, cellular automata and multi-agent systems are being used successfully in cities simulation. CA is simple and is a well established method.

Since the 1980's, the development of discrete choice modelling and the emergence of cellular automata and multi-agent simulation techniques have created a proliferation of modelling approaches. We discuss each of these approaches below, and the supporting role of Geographic Information Systems and the integration of several of these approaches in the design of urban simulation. In this work, a CA urban simulator model will be presented.

## 2. Modelling Urban System

Some isolated urban process and phenomena can be modelled and simulated by well known methods: Geographic Information System (GIS) for spatial data, virtual reality or computer graphics for architectural issues, finite elements methods for structural and mass flux problems. But a tool for an overall description of the relationship among these process and problems is not completely developed, and is a great research field.

Besides of the complexity of these process and the difficulties in understand, modelling and simulating the interaction between them, urban problems are dependent of the scale. People, for example, can be modelled as a person, with wishes, behaviour all individual thoughts or can be modelled as a mass, reacting to external forces.

Some methods have been developed to carry out the task of simulate urban process and their interactions, such as: Cellular Automata (CA), Multi-agent Systems (MAS) and Artificial Life (AL). Cellular Automata are simple, flexible and has been shown good results (Benenson & Torrens, 2005).

## 3. Simulators, Agents and Models

A simulator is chosen, following the premise of that it is possible to carry out complex tasks (simulation of urban growth, etc.), through the work carried out for a great number of agents, acting in set, but, separately they are considered very simple.

Agents are computational systems that inhabit some environment of complex dynamics. They feel and act independently in this environment, and carry through a series of objectives or tasks for which they had been assigned. Agents operate without human beings or others direct intervention, and have some control on its action and internal states, perceiving the environment and answer the changes, taking initiatives to reach an objective.

Models of physical iterations are used and are the base of agents modelling, allowing iterations as the attraction and repulsion, which are used in the simulator.

The problem is treated as being a set of agent's interaction. Each one has its own objectives (behaviours) and act on layers, representing the city characteristics in study (eg. railroads, highways, rivers). The exit map is generated as a resultant of the simulation. Figure 1 illustrates the simulator structure.

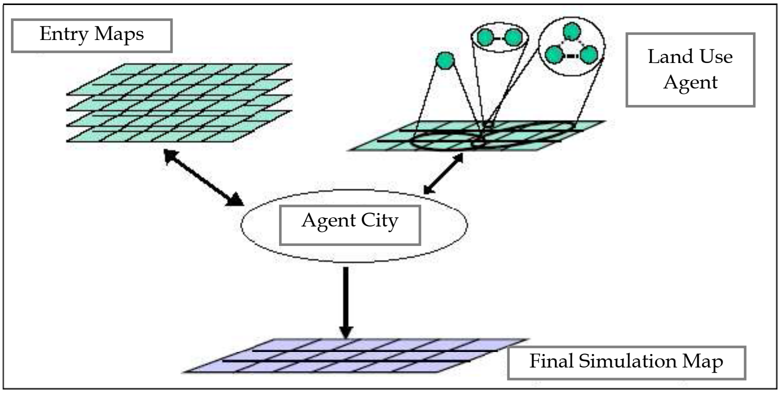

Fig. 1. Simulator structure (Bastos, 2007)

In relation to the model, for understanding the changes that occur in a city, it is necessary not only observe an isolated moment, but also a process of transformation throughout one determined period of time (Portugali, 2000).

Artificial life, multi-agent systems and cellular automata are good examples of simulators.

## 4. Artificial Life

The field of artificial life (or A-life) is concerned with human-made systems that exhibit life-like characteristics and behaviours. As well as exploring the possibility of creating artificial life, the field is focused on understanding natural life by attempting to abstract the fundamental dynamical principles that underpin biological phenomena - in short, this is a search for the rules that make life possible. A-life has close parallels with complexity studies. Life and living organisms represent some of the best examples of complex adaptive systems. Very simple genetic rules applied across many cells spawn advanced biological structures and organisms. The quest for artificial life is being pursued by replicating the complex dynamics of living systems in other physical media that make them accessible to new kinds of experimentation and testing (Sipper, 1997). In this sense, it is hoped that some of the principles that govern real life can be uncovered in the process. An obvious candidate for such a laboratory is the computer.

## 5. Multi-Agent Systems (MAS)

Agents are also automata and, thus, incorporate all of the features of basic automata that have just been discussed. However, there are some important distinctions between CA and MAS, particularly when agents are specified with mobility, which is the common interpretation in geographic models.

According to Batty (1976), CA is capable of diffusing state information to neighbouring. However, the individual cells of CA remain fixed in their simulated spaces, they cannot change location. Mobile agents transmit information by themselves, moving to another location, which can be at any distance from an agent's current position. Agents' spatial behaviour can manifest more complex forms than simple relocation. For example, landlord agents might perform spatially mediated sale and purchasing of real estate; the spatial behaviour of agents designed to represent car drivers could include the choice of links and turning opportunities at junctions.

Generally, agent automata employed in social science research (Epstein, 1999, and Kohler, 2002) are used to represent individual decision-makers (or, sometimes, groups of decision-makers). Consequently, the states that are attributed to social science agents are usually designed to represent socio-economic characteristics, and agent transition rules commonly correspond to human-like behaviours. For the most part, however, work in agent-based simulation in the social sciences outside geography is non-spatial in nature, as are the tools that are used. Many of the decisions and behaviours of geographic agents are spatial in nature, and this distinguishes agent tools for geographic applications. MAS, allow a non linear dynamic development (Portugali, 2000).

Multi-agent simulation (MAS) models are related to CA in that both draw on complex systems theory, but, MAS differ from CA emphasis on emergent system behaviour arising from interactions between agents. Research and testing of MAS models accelerated rapidly after the SWARM software environment (Swarm, 2000) was developed for implementing models of this type. The MAS approach is gaining substantial research interest across the social sciences, since it opens new avenues to analyse social behaviour from an interactive perspective. In economics, the adoption of MAS has come to be known as Agent-Based Computational Economics (Tesfatsion, 2000).

## 6. Cellular Automata (CA)

Cellular Automata simulation is a useful tool in addressing long-term environmental issues. For a long time, urban modellers have been pursuing a “scientific” way that is intuitively equivalent to “precise”, of description and prediction. However, similar to engineering context, as the complex of the system increased, the useful information provided by traditional mathematical models is declining rapidly. This is particularly true in the case of long-term environmental issues, where decision making is characterised by the combination of complexity and uncertainty (Benenson & Torrens, 2005).

In urban grow process, usually, the decision of developing a particular site is made individually by development projects, without co-ordination between projects. Land uses sometimes leads to contradictory policies related to development and preservation. All these features suggest that a simulation approach under a self-organising paradigm is much more appropriate. Through simulating different “rules” the model can generate alternative urban scenarios that may reveal the risk associated with certain development policies, thus allowing necessary precautions to be taken against disastrous consequences.

In recent years there are many researchers have shown interest to analysis and design techniques for complex systems. Cellular automata (CA) are one of the effective methods. Although Cellular Automata (CA) was proposed firstly by Von Neumann and Ulam, from theoretical point of view, in the late 1940’s, John Horton Conway’s “game of life” ensures the new idea of its application in the computing field (Toffoli & Margolus, 1987). CA are henceforth considered as powerful modelling approach for complex systems in which global behaviour arises from the collective effect of many locally interacting simple components. Subsequently several tools based on CA are proposed to provide meaningful results for real world applications.

Perceiving the city as an open, complex, far from equilibrium and thus self-organised system, we can understand how the global pattern of the city is constructed from uncoordinated local decision-making processes. Cellular Automata (CA) provide a way to simulate such a self organised process. Through development decisions being made on the basis of individual sites, a complex urban pattern can be emerging. The decision of developing a particular site is affected by the pattern in the immediate past. In other words, development is proceeding through discreet interactions during which urban space is constantly evolved (Wolfram, 1984, and Wolfram, 1986).

Interaction among developments is confined within a limit of proximity which is measured by a neighbourhood space. No pre-knowledge of a global pattern exists to guide the direction towards the city is evolved. The transition rule is simple in the sense that it is applied simultaneously to all development sites. Moreover, any rules modifications are being applied instantly to all sites (Wolfram, 1984, and Wolfram, 1986).

Simulation is particularly useful when the issue under question becomes less “predictable” due to its complexity, uncertainty, and non linear iterative natures. The value feature of CA simulation is not its “predictive” power, because the property of a self-organising system is that it is largely unpredictable and uncontrollable (Wolfram, 1986), (Toffoli & Margolus, 1987). Given identical initial conditions, each CA simulation run is unique and never fully repeats itself (Portugali & Benenson, 1995). Although the simulation is unable either to replicate or to predict exact development patterns, it can reveal some qualitative features inherent in the evolution of the system, e.g. the overall rate of land lost. This is largely because the final state of CA is controlled by a set of transition rules. Through linking the rules with their consequences, the model can provide “artificial planning experience” (Portugali & Benenson, 1995) to suggest alternative scenarios of urban growth.

Summarily, the simulation involves the following aspects (Wolfram, 1986): selection of appropriate states in the neighbourhood which are relevant to a particular transition; planning of criteria based on the concerned states to reflect stimulus or constraint to the particular transition; inference of the truth of the particular transition from criteria according to the specified decision-making process; and comparison of all possible transitions associated with a cell and to decide the transition of the cell.

## 6.1 The Evolution of CA

In the beginning, the early development of the CA framework took place in the 1950s and 1960s and is generally associated with famous names and great discoveries of the twentieth century. (Benenson & Torrens, 2005). Cellular Automata in their classic sense were invented by Ulam and Von Neumann in the mid-1940s. They were interested in exploring whether the self-reproducing features of biological systems could be reduced to purely mathematical formulations (Sipper, 1997). At that time, the two worked at Los Alamos Laboratories on the atomic and, later, hydrogen bombs and Stanislaw Ulam, together with Edward Teller, signed the patent application for the latter. Mathematical folklore attributes the CA idea to Ulam, who had exceptional mathematical imagination and avoidance of writing. Although there are doubts about the origins of the idea: “one can say that the “cellular” comes from Ulam and the “automata” comes from Von Neumann” (Rucker, 1999). By 1943, Ulam suggested the idea of cellular space, where each cell is an independent automaton, interacting with adjacent cells, and shared the idea with Von Neumann. The common view, now, is that Ulam’s idea was also secondary one, and was based on paper by Alam Turing, (1936), where he demonstrated that a simple automation, later termed a “Turing machine”, can simulate any discrete recursive function. Regardless of the origins, CA came into being amid a soup of very talented intellects.

Having being responsible for researching some of the most critical defence projects of World War II, Ulam and Von Neumann did not care too much about publishing their theoretical thoughts. Most of the papers by Von Neumann on CA were completed and published after his death, in the 1960´s (Taub, 1961), (Burks, 1966). The first paper by Von Neumann, “The General and Logical Theory of Automata”, introducing what are now know as cellular automata, was published in 1951 (Von Neumann, 1951), and discussed the problem of designing a self-reproducing machine.

The developed urban models in the end of 50’s until the half of 80’s, in a general way, did not operate on a space dimension. The urban space was disaggregated in units (generally zones of origin-destination), but, the result of these models could not be visualised in space. In fact, effective advances in urban models space representation occurred only in the end of 80’s, when models of cellular automata had started to be used on a large scale.

Stephen Wolfram, one of the most famous theoreticians defines the Cellular Automata as being a mathematical idealisation of physical systems, in which space and time are discrete, and the attributes assume a set of discrete values too. A cellular automata is a regular uniform grating or matrix field, generally infinite in its extension, with one discrete variable in each locality (cell), evolving in discrete spaces of time. The variable value in one cell is affected by the values in cells neighbouring, found in the previous time step. Each cells variables are brought up to date simultaneously, based on the neighbouring variable values in previous time step and in agreement with a set of pre-defined local rules (Wolfram, 1983).

CA models have applications in most different areas, since in physic until changes in land use and covering, engineering and traffic control, dissemination of epidemics, biology, among others. CA had been, in a implicit way, in the first generation of computational models in 60's with experiments, executed in North Carolina. In 70's, Tobler, influenced by quantitative geography, suggested cellular models for the development of Detroit. Shortly afterwards he started to explore the form through which CA could be applied to geographic systems, resulting in his famous article " Cellular Geography" (Tobler, 1979). Finally, on the end of 80's, CA had widely started to be used for urban questions, impelled for the parallel development of the graphical computation and the theory of the complexity, similar chaos and fractals (Batty et al., 1997).

The 90's had lived deeply successive improvements in CA urban models, which had started to incorporate ambient dimensions, partner-economic and politics, and had finally been successful in small and macro scales space. A example of this last case is presented by White (1998), where the demand for residential area use is esteem through a social subsystem that takes in consideration migratory flows between regions, and where the demand for economic activities (industrial, advertising, services) is obtained by means of region subsystems that evaluate the performance of different economic sectors, supplying, thus, parallel, information on job chances, that again are used to compute the residential demand.

This model esteem the demand for different kinds of land use, considering the support ambient capacity of the sites in question (natural subsystem), as well as the imposed restrictions in local level for function's, physical's, institution's and infrastructure's aspects.

Theoretical progress in the vast field of artificial intelligence, such as, neural specialists systems, artificial nets and evolutionary computation, which is anchored in the concept of genetic algorithms, recently had been included in target simulations in CA. As showed by (Almeida et al., 2002), just-incorporated methods in CA models, as tools of adjustments of neural nets (Yeh & Li, 2001) and evolutionary learning (Papini et al., 1998), have shown themselves as the most promising for CA next generation urban models.

## 6.2 CA Models – Basic Concepts

CA models consist in cells arranged in a regular grid that change state according to specific transition rules. These rules define the new state of the cells as a function of their original state and local neighbourhood (Ramos & Silva, 2002).

CA models have three important characteristics: massive parallelism, cellular interactions localisation and basic components simplicity - cells. A construction of a CA model, destined to simulate a specific problem, like the dynamics of population growth, must obey some rules. Among these, the most important are: net geometry, the size of the neighbourhood, the border initial conditions, the states classes and the transition rules (Ramos & Silva, 2002).

The net geometry consists in its form and dimension. In two dimensions there are three types of regular nets (Viher et al, 1998): triangular (Fig. 2), square (Fig. 3), and hexagonal (Fig. 4). In the majority of the cases the square shaped net is used, due to easiness of representation and visualisation.

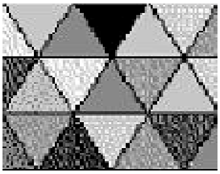
Fig. 2. Triangular net

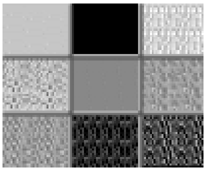
Fig. 3. Square net

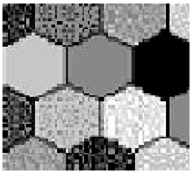
Fig. 4. Hexagonal net

After the definition of the net form, it is chosen the neighbourhood in which the cells can interact. Usually, the models are: Moore neighbourhood, with eight neighbours (Fig. 5), or the Von Neumann neighbourhood (Fig. 6), with four neighbours (Viher et al., 1998).

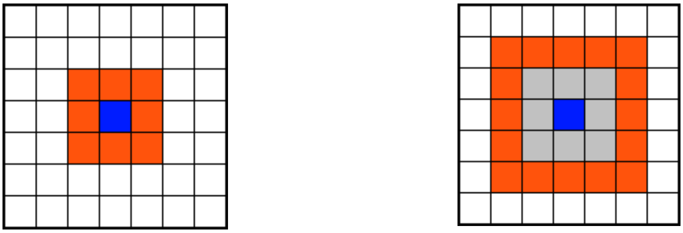
(a) (b)

Fig. 5. (a) Moore first neighbourhood , (b) Moore second neighbourhood

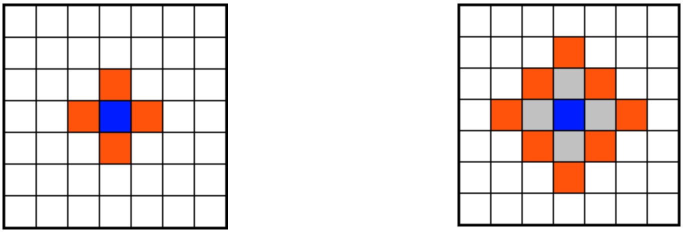
(a) (b)

Fig. 6. (a) Von Neumann first neighbourhood, (b) Von Neumann second neighbourhood

In the normal CA definition, it is demanded that the net has been defined in all the dimensions, what becomes impossible to simulate an infinite net, truly, in computer. Consequently, it can be prescribed, some border conditions. The initial condition, the classes states and the transition rule, are highly independent aspects (Batty et al., 1997).

The initial condition is the departure scene for the real problem analysis; the cell state classes can represent any characteristic of them, like land use (residential or commercial), population density, among others; and the transition rules, can be determined in other to reflect the way as the real phenomenon happens, and can be interpreted in the simulation, as algorithms. The transition rules specify the behaviour of the cells with time evolution, deciding the future conditions of these cells (Torrens, 2000). Batty et al., (1997) says that, these rules substitute the traditional mathematical functions in the models with procedures based on rules. The author argues, yet, that there are advantages in this methodology: the rules reflect as the real systems operate and allow the reduction of complicated systems in simple ones that have their directed dynamic.

It is important to detach that the GIS, and the graphical technology related to them, supply the necessary platform to increase the complexity of the cellular models, mainly in the study of urban models. Efforts toward deeply understanding about natural phenomena of time-space dimensions have been made. The objective to represent them under the form of dynamic space models by considering future events forecast, consist in promising research areas. Therefore GIS techniques already emphasise the representation of dynamic space phenomena, they are not adequate to foresee future events in changing scenarios.

To represent the relations of interaction, or space tension, between the cells, it is necessary that this structure has been converted into a graph. This is possible because each cell could be considered a vertex of the graph arcs (Granero & Polidori, 2002)(Fig. 7).

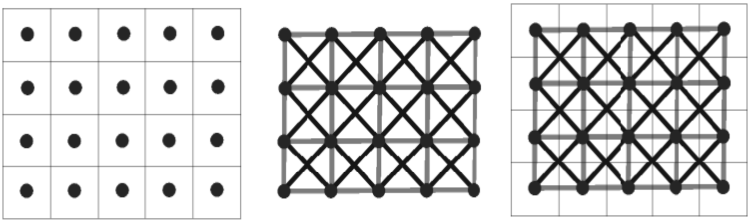

Fig. 7. Cells transformation in graphos (= cells + graphos)

### 6.3 CA – Some Application Areas in Urban Simulation

Automata systems are the basis of Urban Simulation. Automata-based modelling tools hold many advantages for simulation of urban phenomena in space. The decentralised structure of automata systems, their ability to directly handle individual spatial and non spatial elements, simplicity of formulation, thus, all of these features offer many benefits to model builders (Benenson & Torrens, 2005).

#### 6.3.1 Drainage Network Systems

Although the disadvantages of the urbanisation for the ecosystem and human well-being are known, people are always arriving at urban areas (Geiger, 1993). The growth and development of the cities, many times occurs in a disordered and irregular way, as a consequence of the lack of efficient development plans, supervise and control. This growth leads to a change in land use with greater soil extension watertight.

This increasing of the urban space watertight reflects in cities flooding increase. So, each time more, it is necessary to use tools that make possible urban drainage planners to foresee what could happen, in case of risk's scenarios (as a population increase) becomes a reality.

The numerical simulation appears as a possible tool to be used, allowing the impact evaluation as a consequence of these urbanisation, and from these results to analyse solutions that could minimise the impacts.

The possibility of analysing the impact from different developments scenarios and the combination with the use of tools to control the flooding, become the simulation, in general, a tool widely used in urban drainage managing plans. Normally, the difficulties usually related with the accomplishment of a simulation for urban areas have a relationship with necessary information (drainage network systems, impervious rates, observed runoff, etc.) and with the appropriate choice of the simulation model, and this is conditioned by the available information. However, the ideal would be the possibility of a detailed representation of the urban space, using a model compatible with this proposes. For example, the use of a hydrological model, called Schaake (Schaake, 1971) was presented for the detailed representation of surfaces in urban areas. The concept of source control using on-site detention was used during the simulations. The versatility of this model showed the possibilities for drainage planning in urban areas, mainly those that are in developing.

### 6.3.2 Application of Space Dynamic Models in the Dynamics of Land Use Change

An increasing number of models for predicting land use change in rapidly urbanising regions are being proposed and built using ideas from cellular automata (CA). Calibrating such models to real situations is highly problematic and to date, serious attention has been focused on the estimation problem. These modelling experiments synthesise various information about spatial infrastructure as the driver of urban land use change. This indicates the relevance of the approach for generating forecasts of growth for Brazilian cities in particular and for world-wide cities in general.

The results obtained with land use change simulations by using CA, have the possibility to be clearly understood by politics, planners and decision-makers, in particular, as well as, the public in general. The dynamics of urban land use models that show to be useful in the identification of the main vectors of urban growth and its general tendencies of land use. In this way, it allows that the local authority power, can command and give a direction to urban growth, as the capacity of ambient support and the infra and superstructure availability at the present and on future (Almeida et al., 2003).

The prognostics of urban expansion supplied by these models are also useful to help local managers, in the establishment of goals for social investments in infrastructure and equipment, as for example the prolongation of ways, expansion in the water and sewer net, creation of new bus lines, construction of day-care centers, schools, hospitals, etc.

Decision-makers of private side can equally benefit from these data modelling, a time that transport companies, fixes and cellular telephony, handle TV, Internet suppliers and others, will have subsidies to define priorities and what could be the intensity to invest. The organised civil society, either through not governmental organisations or quarter inhabitants associations, could use the prognostics, in a way of legitimating claim. In this way, arguments will be based on real expansion trends in short and average stated period.

Batty (1976), displays the key-ideas in relation to the applications and proposals of the urban modelling when affirming that: "…There are many reasons for the development of such models: their object in assisting scientists to understand the urban phenomena, through the analysis and experimentation, represents a traditional objective of science; however, the urban modelling has the same importance when helping planners, politicians and the community in general to foresee, to prescribe and to invent the urban future".

### 6.3.3 Cellular Automata Models of Road Traffic

Traffic cellular automata (TCA) models, are a class of computationally efficient microscopic traffic flow models. TCA models arise from the physics discipline of statistical mechanics, having the goal of reproducing the correct macroscopic behaviour based on a minimal description of microscopic interactions.

The performance in measurements on a TCA model's cells lattice, is represented in mathematical notations and these quantities are converted into real-world units and vice versa. There is an extensive account of the behaviour aspects of several TCA models encountered in literature. Already, several reviews of TCA models exist, but none of them consider all the models exclusively from the behavioural point of view.

Some TCA models are used to describe city traffic as a two-dimensional grid of cells, or as a road network with explicitly modelled intersections. Cellular Automata have the advantage of modelling the traffic flow on the microscopic scale of individual vehicles and allow the study of large systems due to a simple type of dynamics (Maerivoet & Moor, 2005).

### 6.4 Advantages and Potentialities of the Urban Models Based on Cellular Automata

CA models had become popular, because they are easy to handle and has an operational simplicity by generating dynamics that can reproduce traditional processes of changes, beyond containing complexity enough to simulate unexpected and surprising changes, as the observed ones in emergent phenomena. These phenomena are flexible. They supply a structure not overloaded with theoretical assumptions, and that it is applicable to a represented space as a regular grid. These models can be articulated with matrix data, normally used in GIS (Geographic Information Systems).

Although dynamic models have been criticised, due to limitations on a way of capturing the integral inherent complexities to the reality (Briassoulis, 2000), it can be argued in favour of their existence and continuity, because they offer an incomparable way of abstracting standards, dynamic order and trends lines of processes direction of the real world.

As displayed by Batty (1976), "… standard and order exist, in fact, and it is relatively easy to identify them… in urban and regional systems. If a person agrees or not with the description statistics of these standards, it is a question of opinion, and lately of faith in the fundamental ideas."

In the truth, urban models must be conceived, manipulated, applied and interpreted, of a wise and critical form, in a way that the planners and decision-makers, the private and politics sphere, can extract the best of its results and sensibly recognise its limits. These ideas are well synthesised by (Batty, 1976), when affirming that: " a more liberal perspective of the state of the art, of all involved ones, is necessary in the urban modelling, promoting the vision of that models, assisting the imagination inside of a bigger project process and on the solution of the problems and in decision making, in the society as a whole".

## 7. Conclusions

Urban model is a young field of development, although the beginning studies come from the 50´s. The problem is so vast and complex and we are in the beginning of understanding the complexity of the processes and the interactions between the actors, in modelling urban systems. The rapid development of computers, bringing more and more computational power at lower costs, allows that new forms of exploration can be used in world modelling. Nowadays, bigger and more detailed models are possible, allowing the researchers to improve the complete description of the city behaviour.

The Urban simulation system is being further developed to adapt to varying data availability. Different factors influencing agent choices in locations ranging from newer and rapidly growing.

Careful design at each stage of the process is needed to make the model sensitive to the policies of principal concern, to make the data and computational requirements manageable, to make the model usable by staff and other users with appropriate levels of training, and to fit into the operational practices of the relevant organisations.

To be relevant in the policy process, model design should carefully integrate the elements into a design that fits well into a specific institutional and political context, and evolve to adapt to changing conditions. Careful design at each stage of the process is needed to make the model sensitive to the policies of principal concern, to make the data and computational requirements manageable, to make the model usable by staff and other users with appropriate levels of training.

To deal with these new models, new approaches for simulation have been developed, and CA seems to be the one of the most adequate simulator. It is simple, modular, and easy to implement, and permit that the problem could be represented in almost any scale.

Cellular Automata can take full advantages by using Parallel Processing, a new and powerful computation category.

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