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Kalman Filter in Control and Modeling

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1. Introduction

Originaly, a filter is a physical device for removing unwanted components of mixtures (gas, liquid, solid). In the area of telecommunications, signals are mixtures of different frequencies, and the term of filter is used to describe the attenuation of the unwanted frequencies. Since 1940, the concept of a filter was extended to the separation of signals from noise. With Kalman filter, the meaning of filter is well beyond the notion of separation. It also includes the solution of an inversion problem, in which one knows how to represent the measurable variables as functions of variables of principle interest.
Least squares method proposed by Carl Friedrich Gauss in 1795 was the first method for forming an optimal estimate from noisy data, and it provides an important connection between the experimental and theoretical sciences.
Before Kalman, Norbert Wiener proposed his famous filter called Wiener filter which was restricted only to stationary scalar signals and noises, the solution obtained by this filter is not recursive and needs the storing of the entire pas observed data.
Kalman filter is a generalization of Wiener filter. The significance of this filter is in its ability to accommodate vector signals and noises which may be non stationary. The solution is recursive in that each update estimate of the state is computed from the previous estimate and the new input data, so, contrary to Wiener filter, only the previous estimate requires storage, so Kalman filter eliminate the need for storing the entire pas observed data.
In this chapter, we present two important applications of Kalman filter. In the first one we show how this filter can be used as an adaptive controller system (Chafaa et al., 2006). Studies proposed in this part illustrate a structure for the control of a positional system towards a mobile target in a three dimensional space (see Fig.1). In the presence of a random disturbances (white noise) or when few system parameters change, the use of an adaptive and optimal controller turns out necessary (Mudi & Nikhil, 1999; Zdzislaw, 2005). In this case we are choosing to use Kalman filter as a controller. This technique is based on the theory of Kalman's filtering (Kalman, 1960; Eubank, 2006), it transforms Kalman's filter into a Kalman controller.
In the second application we give the use of such filter in estimating the membership functions of fuzzy sets in order to obtain a fuzzy model (Chafaa et al., 2007). Fuzzy modelling is an effective tool for the approximation of nonlinear systems. Takagi-Sugeno (TS) model is widely used fuzzy modeling technique (Takagi & Sugeno, 1986; Angelov & Filev, 2004). The TS model utilizes the idea of linearization in a fuzzily defined region of the state space. Due to the fuzzy regions, the nonlinear system is decomposed into a multi-

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model structure consisting of linear models that are not necessarily independent (Johansen & Babuska, 2003). A TS fuzzy model is usually constructed in two steps: Step 1: Determine the membership functions of the antecedents; Step 2: Estimate the parameters of the consequent functions.
One of the most techniques used to release the first step is the fuzzy clustering in the Cartesian product-space of the inputs and outputs (Babuska & Verbruggen, 1995; Babuska & Verbruggen, 1997; Bezdek & Dunn, 1975). As the consequent functions are usually chosen to be linear in their parameters, the second step is done by standard linear least-squares methods (Babuska & Verbruggen, 1997; Babuska et al., 1998).
Many clustering algorithms can be found in the literature, they are based on the optimization of fuzzy C-means functional (Nascimento et al., 2003). Some of them utilize an Euclidian distance norm (Bezdek et al., 1987; Hathaway & Bezdek, 1991) in which the detected clusters have an hyperspherical shapes, i.e., clusters whose surfaces of constant membership are hyperspheres. Others extend the Euclidian distance norm to an adaptive distance norm (Bezdek & Dunn, 1975; Gustafson & Kessel, 1997; Gath & Geva, 1998) in order to detect clusters of different geometrical shapes in one data set.

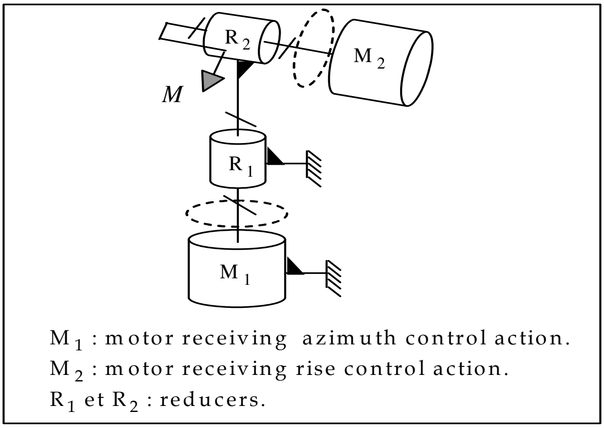

Fig. 1. Positioning system

Fuzzy clustering in the Cartesian product-space of the inputs and the outputs has been extensively used to obtain the antecedent membership functions (Babuska & Verbruggen, 1997; Babuska, 1998; Sugeno & Yasukawa, 1993). Attractive features of this approach are the simultaneous identification of the antecedent membership functions along with the consequent local linear models and the implicit regularization (Johansen& Babuska, 2003).
By clustering in the product-space, multidimensional fuzzy sets are initially obtained, which are either used in the model directly or after projection onto the individual antecedent variables (regressors). As it is generally difficult to interpret multidimensional fuzzy sets, projected one-dimensional fuzzy sets are usually preferred.
Babuska and Verbruggen (Babuska & Verbruggen, 1997) proposed a fuzzy modeling scheme based on Gustafson-Kessel clustering algorithm (GKCA) to estimate the premise membership functions and on least-squares method to estimate the parameters of the consequence functions. Abony et al (Abonyi et al., 2002) proposed to use the Gath-Geva

(GG) clustering algorithm instead of GKCA method, because with GG method, the parameters of the univariate membership functions can directly be derived from the parameters of the clusters.

In this part, a fuzzy modeling algorithm combining GKCA and Kalman filter (KF) is proposed (Chafaa et al., 2007). We use GKCA in order to detect clusters of different geometrical shapes in the data set and to obtain the point-wise membership functions of the premise. After that a Kalman filter is introduced to estimate the parameters of the premise membership functions and those of the consequence functions. In the premise part, the membership functions are triangular functions, then Kalman filter will estimate the parameters of a straight line functions by using the data corresponding to the premise membership functions defined point-wise, but in the consequence part, Kalman filter will be used as a linear regression to estimate the parameters of the TS fuzzy model using the input-output data set.

2. Kalman controller for target tracking system

2.1 Target tracking system

A target tracking system is a system for which inputs are the azimuth and rise, and outputs are the control actions for locating the motors. The target moves through azimuth-rise space. Two dc-motors adjust the platform position constantly towards the target (Chafaa et al., 2006; Brookner, 1998). The platform can be any directional system which can turn up exactly towards the target; such system can be a Laser, a video camera or an antenna. We suppose that we have a potentiometric system which can discover the direction of the platform towards the target (Ogata, 1970). The Radar sends azimuth and rise coordinates to the target tracking system in the end of every time interval, we calculate the current error and its variation in the platform position. Then, a Kalman controller determines the control actions for dc-motors, one action for azimuth motor and the other one for rise motor. These actions are going to reposition the platform as shown in Fig. 2. We can control independently the azimuth and rise positions by applying the same algorithm twice, it facilitates us calculations.

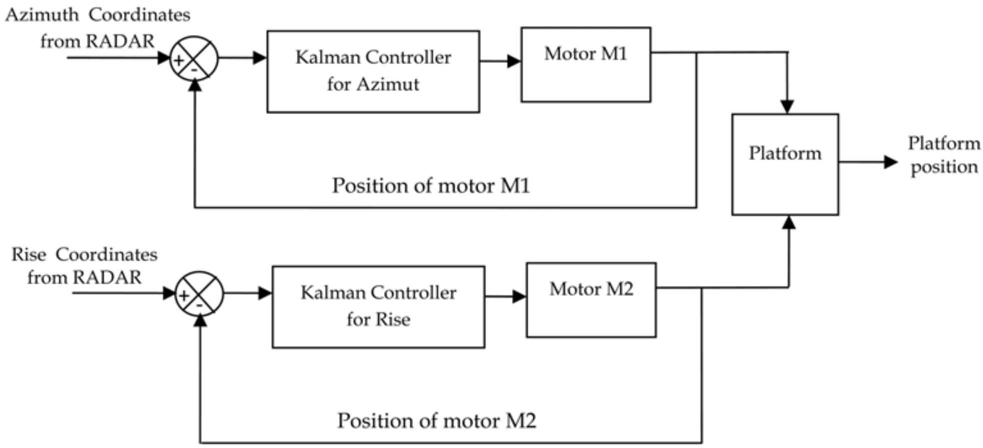

Fig. 2. Control system

2.2 Kalman controller

The Kalman filter is used for estimating or predicting the next stage of a system based on a moving average of measurements driven by white noise, which is completely unpredictable. It needs a model of the relationship between inputs and outputs to provide feedback signals but it can follow changes in noise statistics quite well. The Kalman filter is an optimum estimator that estimates the state of a linear system developing dynamically through time. An optimum estimator can be defined as an algorithm that processes all the available data to yield an estimate of the "state" of a system whilst at the same time estimating some predefined optimality criterion.

In this section we will conceive another type of controllers called "Kalman Controller" or "Kalman Filter controller". This technique consists to achieve a one-dimensional Kalman Filter acting as an alternative controller, i.e., it can provides the control actions to the dc-motor in addition to its filtering function (Kosko, 1992).

In the discrete state space formulation, the state and measurement equations for the controllers are given by:

$$\begin{aligned} x_{k+1} &= Gx_k + Hc_k + w_k \\ z_k &= Cx_k + v_k \end{aligned} \tag{1}$$

Our proposed control structure contains two Kalman controllers, one for azimuth (Azimuth controller) and another for rise (rise controller). Since the two controllers act independently, so we can assume them to have one-dimensional models such that :

$$G = H = C = 1 \tag{2}$$

Since the state is a control action, so we can take the input c_k to be :

$$c_k = e_k + \dot{e}_k \tag{3}$$

Let x_{k+1} denotes the control action necessary at the moment *k* to exactly lock onto the target at the moment *k*+1. Then, the controller output at the moment *k* will be considered equal the prediction $u_k = \hat{x}_{k+1/k}$

Let us note that:

$$\begin{aligned} e_k &= x_k - \hat{x}_{k/k-1} \\ \dot{e}_k &= e_k - e_{k-1} \end{aligned} \tag{4}$$

By substitution of 2 and 3 in 1 we obtain the new state equation:

$$x_{k+1} = x_k + e_k + \dot{e}_k + w_k \tag{5}$$

where w_k represents a white noise that models target acceleration or other unmodeled effects. The new equation of measurements is

$$\begin{aligned} z_k &= x_k + v_k \\ &= \hat{x}_{k/k-1} + e_k + v_k \\ &= \hat{x}_{k/k-1} + v'_k \end{aligned} \tag{6}$$

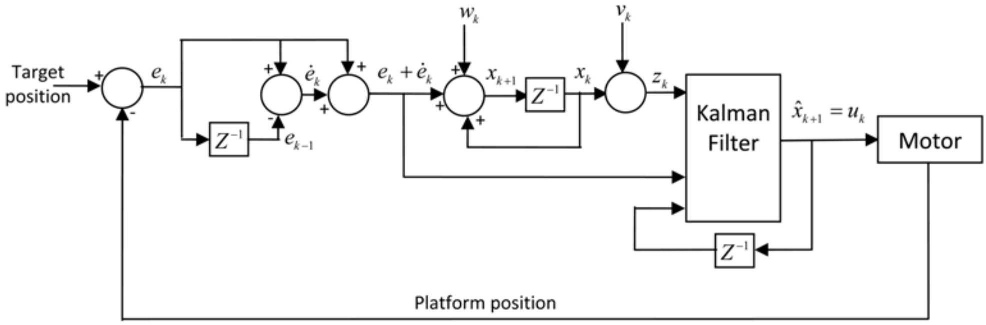

Fig. 3. Control system

Since we assume that e_k and v_k are uncorrelated, the variance of v'_k is:

$$\begin{aligned} R'_k &= E\left[v'^2_k\right] \\ &= E\left[\left(e_k + v_k\right)^2\right] \\ &= E\left[e_k^2\right] + E\left[v_k^2\right] \\ &= P_{k/k-1} + R_k \end{aligned} \tag{7}$$

The recursive equations of Kalman Filter take the following general form:

$$\begin{aligned} \hat{x}_{k+1/k} &= G\hat{x}_{k/k} + HU_k \\ P_{k/k-1} &= GP_{k-1/k-1}G^T + Q_{k-1} \\ K_k &= P_{k/k-1}C^T\left[CP_{k/k-1}C^T + R_k\right]^{-1} \\ \hat{x}_{k/k} &= \hat{x}_{k/k-1} + K_k\left[z_k - C\hat{x}_{k/k-1}\right] \\ P_{k/k} &= P_{k/k-1} - K_kCP_{k/k-1} \end{aligned} \tag{8}$$

To obtain the one–dimensional Kalman controller, we substitute 2, 3, 6 and 7 in 8, and then we obtain it under the following form:

$$\begin{aligned} u_k &= \hat{x}_{k/k} + e_k + \dot{e}_k \\ P_{k/k-1} &= P_{k-1/k-1} + Q_{k-1} \\ K_k &= \frac{P_{k/k-1}}{R'_k} \\ \hat{x}_{k/k} &= u_{k-1} + K_k v'_k \\ P_{k/k} &= \left[1 - K_k\right]P_{k/k-1} \end{aligned} \tag{9}$$

Figure 3 illustrates the detailed structure of Kalman controller. The simulation results of this controller are presented in figures 4 and 5, where we see in figure 4 that the load disturbance is rejected and in figure 5 the tracking is carried out.

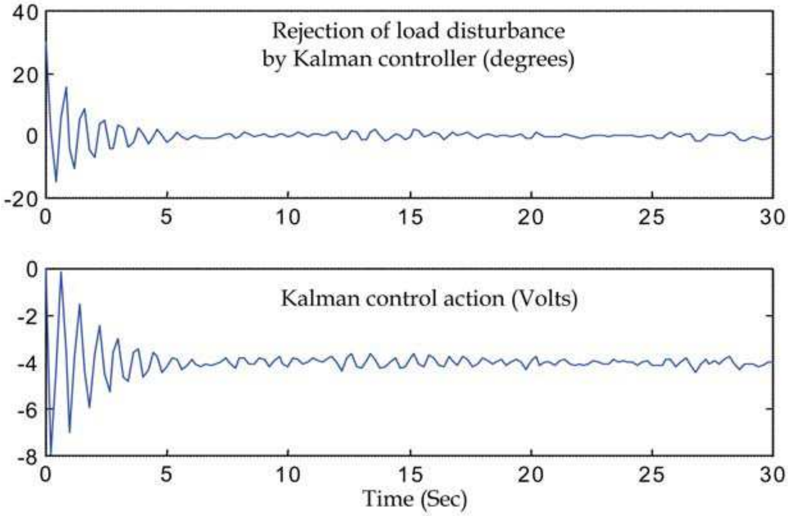

Fig. 4. System response and Kalman control action to a strong load disturbance

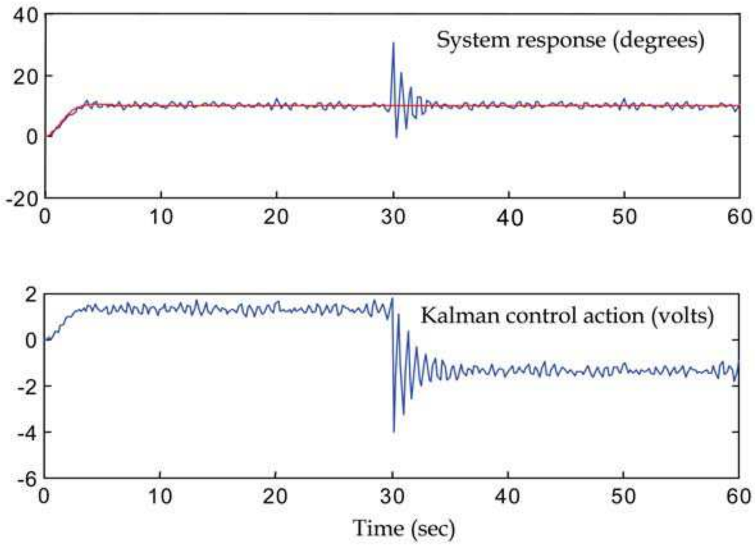

Fig. 5. System response and Kalman control action to a step input and a strong load disturbance

3. Fuzzy modeling and identification

3.1 Takagi-Sugeno fuzzy models

The TS fuzzy model can represent or model any unknown nonlinear system $y = f(\boldsymbol{x})$, based on some available input-output data $\boldsymbol{x}_k = \left[x_{1k}, x_{2k}, ..., x_{nk}\right]^T$ and y_k. The index k denotes the individual data samples and n the number of regressors.

In the TS fuzzy model, the rule consequents are crisp functions of the model inputs:

$$R_i : IF\ \boldsymbol{x}\ is\ \boldsymbol{A}_i(\boldsymbol{x})\ THEN\ y_i = \boldsymbol{a}_i^T \boldsymbol{x} + b_i \qquad i = 1,2,...c \tag{10}$$

Where x is $n \times 1$ input variable, $y_i \in R$ is the output variable. $n \times 1$ vector $\boldsymbol{a}_i$ and $b_i \in R$ are the TS parameters. R_i denotes the ith rule and c is the number of rules in the rule base. $\boldsymbol{A}_i$ is the premise multivariable membership function of the ith rule. $\boldsymbol{\theta}_i = \left[\boldsymbol{a}_i \ \ b_i\right]^T$ is the parameter vector of the ith rule.

The premise proposition " $\boldsymbol{x}\ is\ \boldsymbol{A}_i(\boldsymbol{x})$ " can be expressed as a logical combination of propositions with univariate fuzzy sets defined for the individual components of $\boldsymbol{x}$, usually in the following conjunctive form (Kukolj & Levi, 2004) :

$$R_i : IF\ x_1\ is\ A_{i1}(x_1)\ AND......AND\ x_n\ is\ A_{in}(x_n)\ THEN\ y_i = \boldsymbol{a}_i^T \boldsymbol{x} + b_i \qquad i = 1,2,...c \tag{11}$$

the degree of fulfilment of the rule is calculated as the product of the individual membership degrees :

$$\beta_i(\boldsymbol{x}) = \prod_{j=1}^{n} \mu_{A_{ij}}(\boldsymbol{x}) \tag{12}$$

where $\mu_{A_{ij}}(\boldsymbol{x})$ is the membership function of the fuzzy set A_{ij} .

The inference is reduced to the fuzzy-mean defuzzification formula (Takagi & Sugeno, 1986 ; Kukolj & Levi, 2004) :

$$y = \frac{\sum_{i=1}^{c} \beta_i(\boldsymbol{x})\left(\boldsymbol{a}_i^T \boldsymbol{x} + b_i\right)}{\sum_{i=1}^{c} \beta_i(\boldsymbol{x})} \tag{13}$$

From (11) and (13), it is noted that TS fuzzy model approximates a nonlinear system with a combination of several linear systems by decomposing fuzzily the whole input space into several partial spaces and representing each input-output space with each linear equation.

3.2 New fuzzy modeling algorithm

The structure of the proposed algorithm is presented in Fig. 6. The identification algorithm proceeds in three steps:

1. from the input-output sequences $\{(x_k, y_k\}_{k=1}^{N}$, partition the data into a set of local linear submodels by using GKCA in the product space $\boldsymbol{X} \times \boldsymbol{Y}$
2. Obtain the membership functions for the premise variables by using cluster projections and Kalman filtering.
3. Estimate the consequent parameters by Kalman filter algorithm.

The three procedures are repeated to find the appropriate number of clusters *c* as shown in Fig. 6 in which the performance index used is the mean squared error (MSE), so when $\mathrm{MSE} \le \varepsilon$ the loop is stopped and the optimal *c* is obtained.

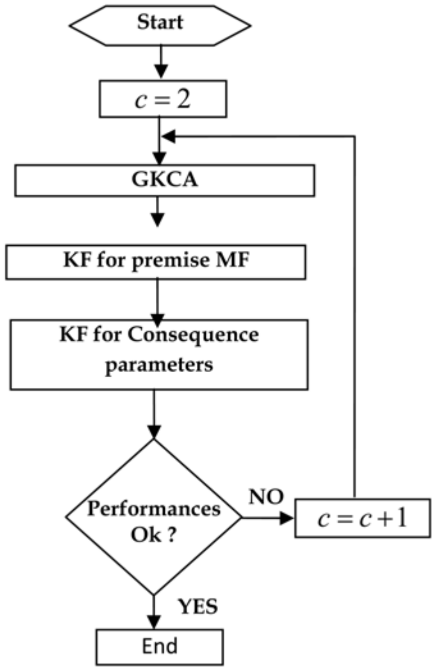

Fig. 6. Structure of the proposed fuzzy modeling

A. Fuzzy clustering

Clustering of numerical data forms the basis of many classification and system modeling algorithms (Bezdek & Dunn, 1975; Babuska et al., 1998). The purpose of clustering is to distil natural grouping of data from a large data set, producing a concise representation of a system's behavior. In particular, The GKCA has been widely studied and applied by many researchers (Bezdek et al., 1987; Hathaway & Bezdek, 1991). The GKCA is an iterative optimization algorithm that minimizes the cost function:

$$J = \sum_{i=1}^{c} \sum_{k=1}^{N} \mu_{ik}^{m} (\boldsymbol{z}_k - \boldsymbol{v}_i)^T \boldsymbol{M}_i (\boldsymbol{z}_k - \boldsymbol{v}_i) \tag{14}$$

where N is the number of data points, c is the number of clusters, $\boldsymbol{z}_k$ is the k 'th data point, $\boldsymbol{v}_i$ is the i 'th cluster center, μ_{ik} is the degree of membership of the k 'th data in the i 'th cluster, M_i is the norm-inducing matrix of the i 'th cluster and m is a weighting exponent which determines the fuzziness of the resulting clusters (typically $m = 2$).

In this work, the GKCA is applied in order to obtain the fuzzy partition matrix $\boldsymbol{U} = \left[\mu_{ik}\right]_{c \times N}$, with $\mu_{ik} \in \left[0 \quad 1\right]$ is a membership degree.

B. Premise membership functions

The premise membership functions can be obtained from the results of fuzzy clustering by projecting the fuzzy sets defined point-wise in the partition matrix onto the premise variables x_j, $1 \le j \le n$. The TS rules are then expressed in the conjunctive form as in (11).

In order to obtain membership functions for the premise fuzzy sets $A_{ij}, 1 \le i \le c, 1 \le j \le n$, the multidimensional fuzzy set defined point-wise in the ith row of the partition matrix is projected onto the regressors x_j. Note that the resulting membership functions are defined point-wise, for the identification data only and may be nonconvex, which is caused by the probabilistic constraint in most fuzzy clustering algorithms and by the noise in the data (Babuska et al., 1998).

In order to obtain a prediction model or a model suitable for control purposes, the premise membership functions must be expressed in a form that allows computation of the membership degrees, also for input data not contained in the data set. To achieve this step, we propose to use Kalman filter to approximate the point-wise defined membership functions by some suitable straight line functions (triangular functions) as depicted in Fig. 7.

The Kalman filtering process is a recursive minimum mean square estimation procedure (Kalman, 1960; Mohinder & Angus, 2001). Each update estimate of the parameter vector corresponding to a straight line equation is computed from the previous estimate and the new input data (here the input data are the point-wise values of the membership functions). In this sense we propose to use Kalman filter as a linear regression as follows : Consider $2cn$ sets, each set represent the linear part of the point-wise set of a certain premise membership function. The linear part is obtained by taking the α -cut of the considered membership function. So we obtain $2cn$ parameter vectors (one for each set). In each set we will have N_j data (samples), where j denotes the jth set.

Then each set can be modeled by the following measurement equation:

$$\begin{aligned} y_{k_j}^j &= a^j x_{k_j} + b^j + v_{k_j} && j = 1, 2, \ldots, 2cn \\ &= \begin{bmatrix} x_{k_j} & 1 \end{bmatrix} \begin{bmatrix} a^j \\ b^j \end{bmatrix} + v_{k_j} && k_j = 1, 2, \ldots, N_j \\ &= \boldsymbol{C}_{k_j}^j \boldsymbol{\theta}_{k_j}^j + v_{k_j} \end{aligned} \tag{15}$$

where $\boldsymbol{C}_{k_j}^j$ is the observation vector at the moment k_j, $\boldsymbol{\theta}_{k_j}^j = \begin{bmatrix} a^j & b^j \end{bmatrix}^T$ is the parameter vector, v_{k_j} is the measurement noise, N_j is the number of data (samples) in the jth set and the superscript j denotes the jth straight line regression. For simplicity, we will denote k_j by k. From equation (15), $\boldsymbol{\theta}_k^j$ will be considered as a state variable, so the state equation will be

$$\boldsymbol{\theta}_k^j = \boldsymbol{A}^j \boldsymbol{\theta}_{k-1}^j + \boldsymbol{w}_{k-1}^j \qquad j = 1, 2, \ldots, 2cn \tag{16}$$

where $\boldsymbol{A}^j$ is an 2×2 state transition matrix, and $\boldsymbol{w}_k^j$ is the state noise and $\boldsymbol{\theta}_k^j$ is the value of the state variable at the moment k.

The state noise and the measurement noise are assumed to be statistically independent (Haykin, 2001) and can be modeled as zero mean, white noise processes whose covariances are given as

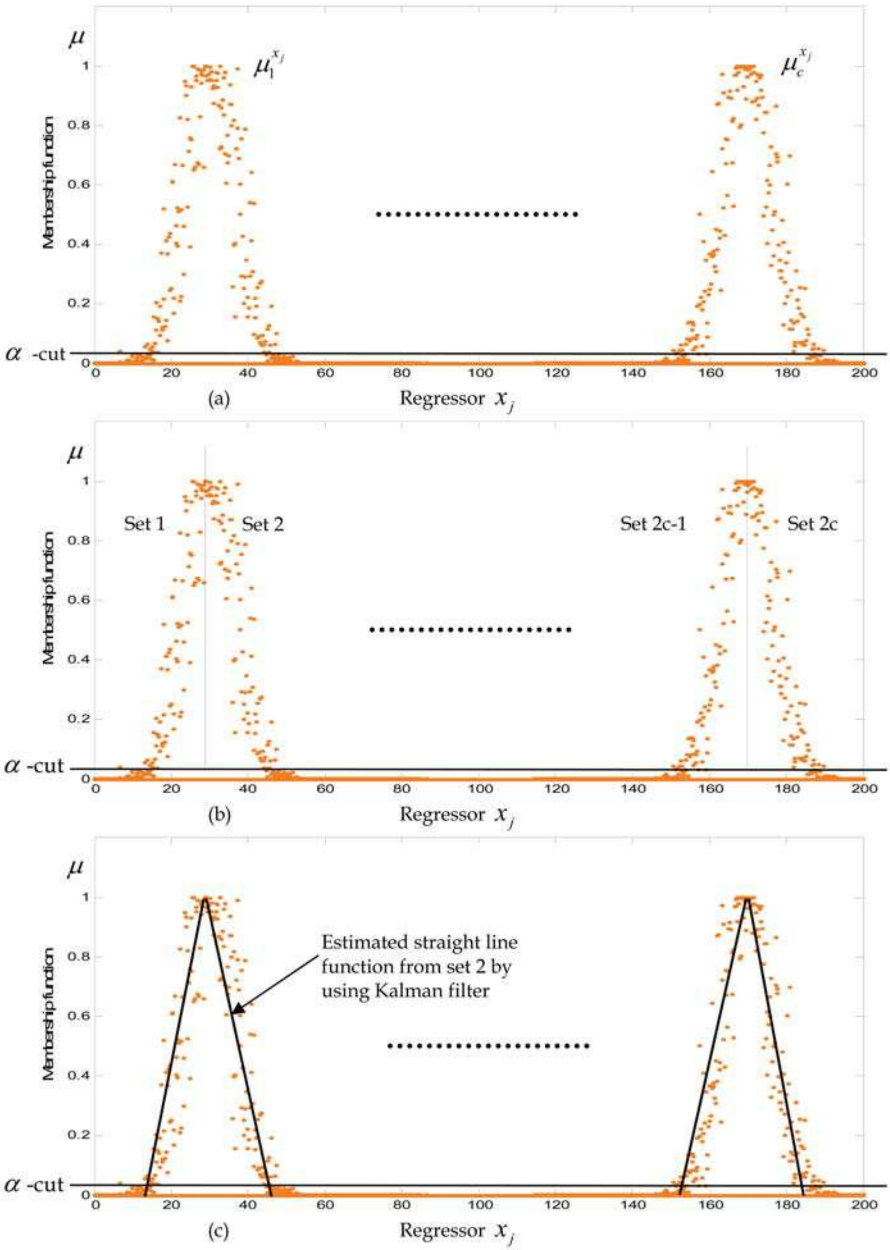

Fig. 7. Determination of premise membership functions :
(a) Premiss MF defined point-wise for the regressors x_j, j=1,2,...n
(b) Division of each membership function into 2 sets (2c sets are obtained for each x_j and 2cnsets are obtained for all regressors)
(c) Approximation of each set by a straight line function by using Kalman filter

$$E\left[\boldsymbol{w}_i^T \boldsymbol{w}_j\right] = \begin{cases} \boldsymbol{Q} & i = j \\ 0 & i \neq j \end{cases}$$
$$E\left[v_i v_j\right] = \begin{cases} r & i = j \\ 0 & i \neq j \end{cases} \tag{17}$$
$$E\left[\boldsymbol{w}_i v_j\right] = 0 \quad \forall i, j$$

By recurrence proceeding, the update state equation and the predicted measure will be given by the following equations:

$$\hat{\boldsymbol{\theta}}_{k/k-1}^j = \boldsymbol{A}\hat{\boldsymbol{\theta}}_{k-1/k-1}^j \tag{18}$$

$$\hat{y}_k^j = \boldsymbol{C}_k^j \hat{\boldsymbol{\theta}}_{k/k-1}^j \tag{19}$$

Now that the model representation of the parameter vectors is complete, the training of the parameters via Kalman filter technique is in order. The update of the parameters is according to the following recursion:

$$\hat{\boldsymbol{\theta}}_{k/k}^j = \hat{\boldsymbol{\theta}}_{k/k-1}^j + \boldsymbol{K}_k^j \left(y_k^j - \boldsymbol{C}_k^j \hat{\boldsymbol{\theta}}_{k/k-1}^j \right) \tag{20}$$

where $\boldsymbol{K}_k$ is the computed Kalman gain. The computed Kalman gain can be viewed as an adaptive learning rate (Tzeng et al., 1994) ant its computation is according to the following steps :

$$\boldsymbol{K}_k^j = \boldsymbol{P}_{k/k-1}^j \boldsymbol{C}_k^{jT} \left(\boldsymbol{C}_k^j \boldsymbol{P}_{k/k-1}^j \boldsymbol{C}_k^{jT} + r \right)^{-1} \tag{21}$$

$$\boldsymbol{P}_{k/k-1}^j = \boldsymbol{A}^j \boldsymbol{P}_{k-1/k-1}^j \boldsymbol{A}^{jT} + \boldsymbol{Q} \tag{22}$$

$$\boldsymbol{P}_{k/k}^j = \boldsymbol{P}_{k/k-1}^j - \boldsymbol{K}_k^j \boldsymbol{C}_k^j \boldsymbol{P}_{k/k-1}^j \tag{23}$$

where $\boldsymbol{P}_{k/k-1}^j = E\left[\left(\boldsymbol{\theta}_k^j - \hat{\boldsymbol{\theta}}_{k/k-1}^j\right)\left(\boldsymbol{\theta}_k^j - \hat{\boldsymbol{\theta}}_{k/k-1}^j\right)^T\right]$ and $\boldsymbol{P}_{k/k}^j = E\left[\left(\boldsymbol{\theta}_k^j - \hat{\boldsymbol{\theta}}_{k/k}^j\right)\left(\boldsymbol{\theta}_k^j - \hat{\boldsymbol{\theta}}_{k/k}^j\right)^T\right]$

are the one step predicted and filter estimate error covariance matrices, respectively.
To simplify the implementation of the Kalman filtering technique, we assume that $\boldsymbol{A}^j = \boldsymbol{I}$ where $\boldsymbol{I}$ is a unit matrix ; $\boldsymbol{Q}$ and r are an assigned variances of the process noise and measurement noise, respectively. The initial parameters values are set to be random numbers.

C. Estimating consequent parameters

There are several methods to obtain the consequent parameters (Angelov & Filev, 2004; Babuska & Verbruggen, 1997; Abonyi et al., 2002). In part we propose an algorithm based also on KF that can compute directly the consequent parameters from the data set and the estimated premise membership functions.

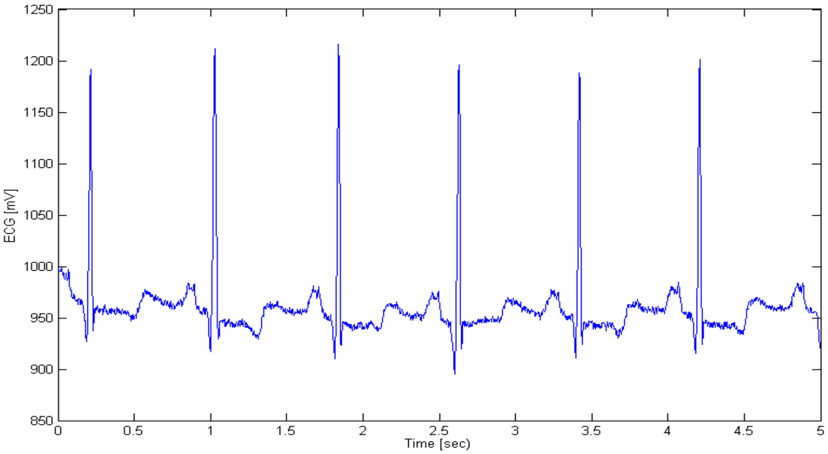

Fig. 8. ECG signal

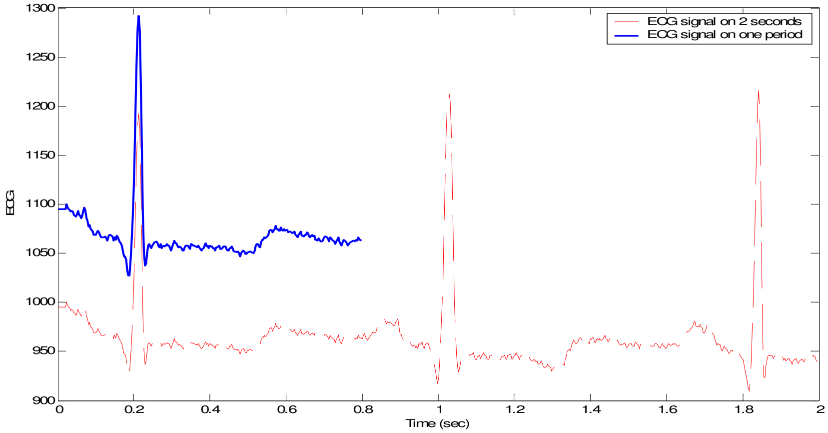

Fig. 9. Extraction of one period ECG signal

From equation (13) we have

$$y = \sum_{i=1}^{c} \varphi_i(\boldsymbol{x})\left(\boldsymbol{a}_i^T \boldsymbol{x} + b_i\right) \tag{24}$$

where $\varphi_i(\boldsymbol{x}) = \dfrac{\beta_i(\boldsymbol{x})}{\sum_{i=1}^{c} \beta_i(\boldsymbol{x})}$ is the normalized activation value of the ith rule. The development of (24) gives

$$y = \begin{bmatrix} \varphi_1(\boldsymbol{x})[\boldsymbol{x} \quad 1] & \varphi_2(\boldsymbol{x})[\boldsymbol{x} \quad 1] & .... & \varphi_c(\boldsymbol{x})[\boldsymbol{x} \quad 1] \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ \vdots \\ a_c \\ b_c \end{bmatrix} \tag{25}$$

Let $\boldsymbol{\Theta} = \begin{bmatrix} a_1 & b_1 & \dots & a_c & b_c \end{bmatrix}^T$ the $c(n+1)\times 1$ TS parameters vector and let the extended vector $\boldsymbol{x}_e = [\boldsymbol{x} \quad 1]$ with dimension $1\times(n+1)$, also if we put $\boldsymbol{C} = [\varphi_1 \boldsymbol{x}_e \quad \varphi_2 \boldsymbol{x}_e \quad .... \quad \varphi_c \, \boldsymbol{x}_e]$, then equation (25) can be rewritten as follows :

$$y = \boldsymbol{C}\boldsymbol{\Theta} \tag{26}$$

with $\boldsymbol{C}$ is an $1\times(n+1)c$ vector.
To apply Kalman filter, we must introduce the measurement noise v_k , so the measurement equation corresponding to (27) at the moment k will take the following form :

$$y_k = \boldsymbol{C}_k \boldsymbol{\Theta}_k + v_k \tag{27}$$

then, we can consider that the state variable is $\boldsymbol{\Theta}_k$, so the state equation will take the following expression :

$$\boldsymbol{\Theta}_k = \boldsymbol{A}\boldsymbol{\Theta}_{k-1} + \boldsymbol{w}_{k-1} \tag{28}$$

where $\boldsymbol{A}$ is an $c(n+1)\times c(n+1)$ transition matrix and $\boldsymbol{w}_k$ is a state noise. v_k and $\boldsymbol{w}_k$ must satisfy some conditions as cited in the previous subsection.
Now we can apply Kalman filter to estimate the TS parameter vector $\boldsymbol{\Theta}_k$ as follows:

$$\hat{\boldsymbol{\Theta}}_{k/k-1} = \boldsymbol{A}\hat{\boldsymbol{\Theta}}_{k-1/k-1} \tag{29}$$

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{A} \; \boldsymbol{P}_{k-1/k-1}\boldsymbol{A}^T + \boldsymbol{Q} \tag{30}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1}\boldsymbol{C}_k^T \left(\boldsymbol{C}_k \boldsymbol{P}_{k/k-1}\boldsymbol{C}_k^T + r\right)^{-1} \tag{31}$$

$$\hat{\boldsymbol{\Theta}}_{k/k} = \hat{\boldsymbol{\theta}}_{k/k-1} + \boldsymbol{K}_k \left(y_k - \boldsymbol{C}_k \hat{\boldsymbol{\Theta}}_{k/k-1} \right) \tag{32}$$

$$\boldsymbol{P}_{k/k} = \boldsymbol{P}_{k/k-1} - \boldsymbol{K}_k \boldsymbol{C}_k \boldsymbol{P}_{k/k-1} \tag{33}$$

where $\hat{\boldsymbol{\Theta}}_k$ is the estimated value of $\hat{\boldsymbol{\Theta}}_k$ and $\boldsymbol{K}_k$ is the computed Kalman gain, $\boldsymbol{P}_{k/k-1}$ and $\boldsymbol{P}_{k/k}$ are the one step predicted and filter estimate error covariance matrices, respectively. Also, for simplicity we will take $\boldsymbol{A} = \boldsymbol{I}$.

3.3 Application

In order to illustrate the effectiveness of the proposed method, we consider the problem of approximating the electrocardiogram (ECG) signal. The ECG is the graphical representation

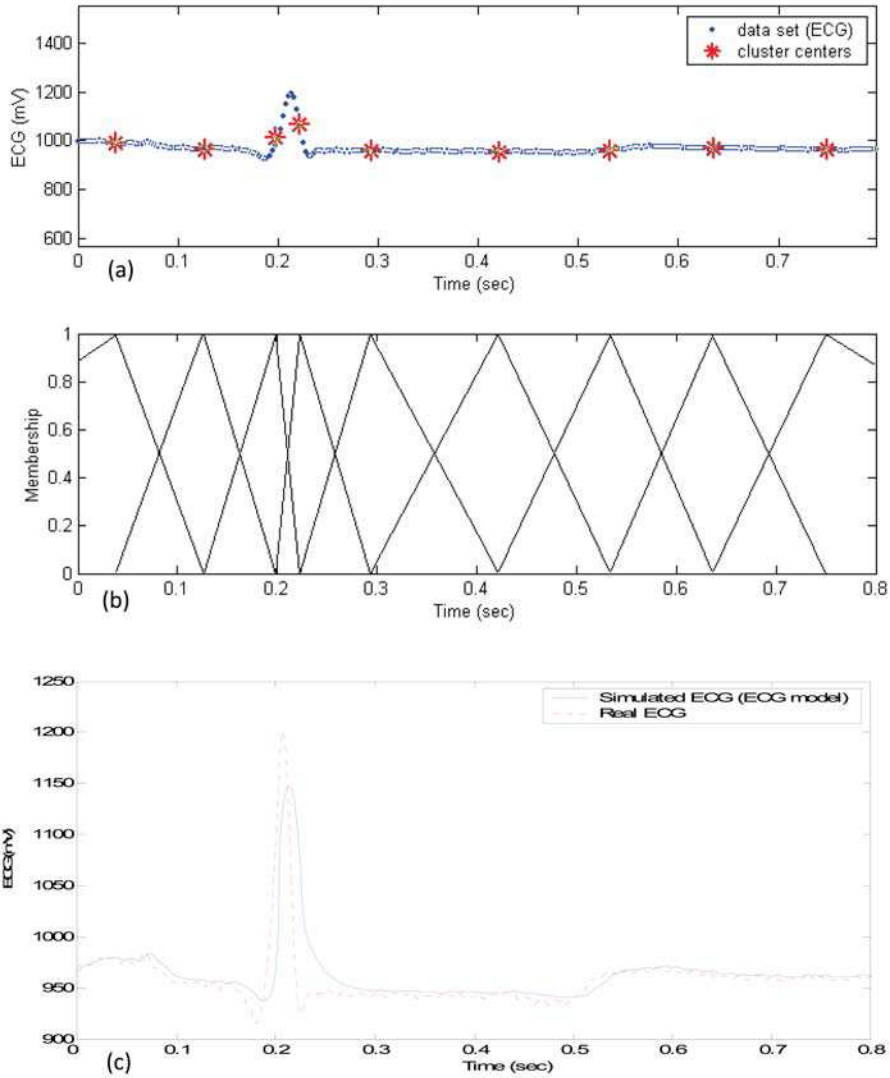

Fig. 10. (a) Clustering result for the ECG signal
(b) Obtained membership function
(c) Performance of our model for the ECG signal.

of the electrical activity generated by the heart. This activity shows dynamical behavior which is neither periodic nor deterministically chaotic.
To avoid the confusion between the two proposed Kalman filters, we will denote KF1 the filter used for premise membership functions and KF2 the filter used for the consequence parameters.
The considered ECG signal is taken from a publically available database of MIT (see figure 8). Before structure identification, extraction of one period ECG is done (see figure 9). In the

structure identification of the proposed method, 9 clusters are detected as shown in Fig. 10(a). By taking the projection of U on x, taking an $\alpha - cut = 0.1$ and by applying Kalman filter KF1, the premise membership functions are obtained (see figure 10(b)). The resulting model using our strategy is as follows:

$$R_i : If \quad x \ is \ A_i , \ Then \ \ y = a_i x + b_i \qquad i = 1, 2, ..., 9 \tag{34}$$

where A_i are the obtained premise membership functions, and a_i, b_i are the TS parameters to be estimated. After applying Kalman filter KF2, the TS parameters of the fuzzy model are obtained and presented in the antecedent membership function. The ECG signal and the simulated ECG signal (ECG model) are shown in Figure 10(c).

4. Conclusion

Investigations presented in this chapter were divided into two parts. In the first part, Kalman filter was used as an alternative controller. The main idea of this technique is to transform the Kalman filter from a state estimator to a control action estimator. We developed a Kalman controller system for real-time target tracking. According to our simulation results, we can say that this type of controller is very robust to load and stochastic disturbances.

In the second part, a fuzzy modelling algorithm is proposed and its validity is verified through computer simulations. This new algorithm has an excellent capacity to describe a given system. We have showed that Kalman filter can be used with fuzzy clustering to obtain a useful method to fuzzy modeling. The proposed algorithm is composed of three steps: 1) fuzzy clustering; 2) determination of premise membership functions; 3) estimation of the TS parameters. In the first step, the GKCA algorithm was used in order to detect clusters of different shapes. In the second step, a Kalman filter has been used in order to estimate the parameter values of the premise membership functions by considering the point-wise defined membership functions as a training sets. In the third step, Kalman filter is also used as a linear regression to efficiently choose the parameter values of the consequent part (TS parameters) of the fuzzy model from the input output data of the identified system. Consequently, the hybrid clustering and Kalman filter method can be efficiently constructed. The performances of the proposed modeling technique was demonstrated on modeling of ECG signal.

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The Kalman filter has been successfully employed in diverse areas of study over the last 50 years and the chapters in this book review its recent applications. The editors hope the selected works will be useful to readers, contributing to future developments and improvements of this filtering technique. The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and science. The book is divided into 20 chapters corresponding to recent advances in the filed.

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