1. Introduction

It is well known that multiple-input multiple-output (MIMO) systems can provide very high spectral efficiencies in a rich scattering propagation medium Telatar (1999). They are hence a promising solution for high-speed, spectrally efficient, and reliable wireless communication Raoof et al. (2008). When coherent signal detection is to be performed at the receiver, channel state information at the receiver (CSIR) is required, for which a channel estimation step is necessary. Channel estimation plays a critical role in the performance of the receiver. It is a real challenge in practical MIMO systems where the quality of data recovery is as important as attaining a high data throughput.

In order to obtain the CSIR, usually some known training (also called pilot) symbols are sent from the transmitter, based on which the receiver estimates the channel before proceeding to the detection of data symbols. The classical approach consists in time-multiplexing pilot and data symbols, usually referred to as pilot symbol-assisted modulation (PSAM) Cavers (1991). We start the chapter by introducing the PSAM channel estimation for MIMO systems (Section 2). Instead of this classical channel estimation based on pilot symbols only, we can perform semi-blind estimation that in addition to pilot symbols, makes use of data symbols in channel estimation. In this way, a considerable performance improvement can be achieved at the price of increased receiver complexity De Carvalho & Slock (1997); Giannakis et al. (2001); Sadough (2008). Usually, these semi-blind approaches are implemented in an iterative scheme when channel coding is performed. That is, channel estimation is performed iteratively together with signal detection and channel decoding Sadough, Ichir, Duhamel & Jaffrot (2009). We, hence, continue Section 2 by considering semi-blind estimation for the case of time-multiplexed pilots.

The drawback of the PSAM scheme is the encountered loss in the spectral efficiency by the periodic insertion of pilot symbols. As an alternative to this method, overlay pilots (OP) can be employed, where pilot symbols are sent in parallel with data symbols Hoeher & Tufvesson (1999). We introduce the OP approach in Section 3 and explain, in particular, pilot-only-based
and semi-blind estimation approaches. The pros and cons of OP with respect to PSAM are discussed too.
Whatever the channel estimation technique, in practice, the receiver can only obtain an imperfect estimate of the channel. Classically, for signal detection, the estimated channel is considered as the perfect estimate. This sub-optimal approach is usually called the mismatched receiver. Its sub-optimality is due to the fact that the receiver does not take into account the presence of channel estimation errors Sadough & Duhamel (2008); Sadough (2008). A more appropriate approach is to take into account channel estimation inaccuracies in the formulation of the detector. We firstly consider in Section 4 the effect of estimation errors on the receiver performance and the impact of the employed space-time coding scheme.
Next, in Section 5, we consider maximum-likelihood (ML) signal detection and show how to integrate the imperfect channel knowledge into the design of the detector. More precisely, we consider two iterative detectors based on maximum a posteriori (MAP) and soft parallel interference cancellation (soft-PIC), and propose for each case modifications to the MIMO detectors for taking into account the channel estimation errors. The implementation complexity issues are also discussed. We present some numerical results to demonstrate the performance improvement obtained via the use of the improved detectors. Finally, Section 8 concludes the chapter.

1.1 Assumptions and notations
We consider a single-user MIMO system with $M_T$ transmit and $M_R$ receive antennas, transmitting over a frequency non-selective channel and refer to it as an $(M_R \times M_T)$ MIMO channel. Unless otherwise mentioned, single carrier modulation and block fading channel model is considered where the channel is assumed to remain almost constant over the duration of a block of symbols.

Figure 1 shows the block diagram of the transmitter that employs the bit-interleaved coded modulation (BICM) scheme which is known to be a simple and efficient method for exploiting channel time-selectivity. The binary data sequence $b$ is encoded by a forward error correction (FEC) code before being interleaved by a quasi-random interleaver. The output bits $d$ are mapped to constellation symbols $s$ and then either multiplexed spatially or encoded according to a space-time scheme before being sent through the wireless channel. Let us denote by $x$ and $y$ respectively the $(M_T \times 1)$ and $(M_R \times 1)$ vectors of transmit and received symbols at a given time reference. For simplicity, we assume for now the simple spatial multiplexing scheme. We have:

$$y = Hx + z$$

where $H$ denotes the $(M_R \times M_T)$ channel matrix and $z$ is the vector of additive complex white Gaussian noise of zero mean and covariance matrix $\Sigma_z = \sigma_z^2 I_{M_R}$, where $I_n$ denotes an $(n \times n)$ Identity matrix. We assume here that $\sigma_z^2$ is perfectly known at the receiver and focus on the estimation of $H$. 

![Fig. 1. Transmitter architecture of MIMO-BICM scheme.](image-url)
2. Time-multiplexed Pilots and Data

Most current systems use a training-based channel estimation scheme in the form of time-multiplexed pilot symbols. In what follows, we present the PSAM approach for MIMO systems and explain two cases of pilot-only-based and semi-blind estimation; the latter is implemented in an iterative receiver.

2.1 Pilot-only-based PSAM channel estimation

When using the PSAM method, we have a trade-off between the channel estimation quality and the data throughput. With an increased number of pilots, a better channel estimate can be obtained, but at the same time, the spectral efficiency is sacrificed more. There is a minimum number of channel-uses that should be devoted to the transmission of pilots in order that the MIMO channel be identifiable at the receiver. As a general case, if \( L \) denotes the maximum length of the underlying subchannels’ impulse response, the number of pilot channel-uses \( N_p \) should satisfy Balakrishnan et al. (2000):

\[
(N_p - L + 1) \geq M_T L \quad (2)
\]

Under flat fading conditions where \( L = 1 \), this implies: \( N_p \geq M_T \). Several works have been done on the optimal placement of pilots in a frame of symbols, as well as on the optimal power allocation between pilot and data symbols Hassibi & Hochwald (2003); Dong & Tong (2002); Adireddy et al. (2002); Ma et al. (2003). They consider the criteria of mean-square channel estimation error, channel capacity, or the Cramér-Rao bounds. In particular, it is shown in Hassibi & Hochwald (2003) that, if power optimization over pilot and data symbols is allowed, the optimum \( N_p \) is \( N_{p,opt} = M_T \). In such a case, we should place pilot symbols with lower power at the beginning and the end of the frame, and those with higher power in the middle of the frame Dong & Tong (2002). However, if equal power has to be allocated to pilot and data symbols, then \( N_{p,opt} \) can be larger than \( M_T \) Hassibi & Hochwald (2003).

Considering the simple case of flat block-fading MIMO channel, to estimate the channel, corresponding to each fading block, we send \( N_p \) pilot symbol vectors with the same power as the data symbols. Let \( \mathbf{x}_p[k] \) denote an \((M_T \times 1)\) pilot symbol vector at the time sample \( k \). We denote the received vector corresponding to \( \mathbf{x}_p[k] \) by \( \mathbf{y}_p[k] \). We can constitute the \((M_T \times N_p)\) matrix \( \mathbf{X}_p \) by stacking in its columns the pilot vectors \( \mathbf{x}_p[k], k = 0, 1, N_p - 1, \) i.e., \( \mathbf{X}_p = [\mathbf{x}_p[0], \ldots, \mathbf{x}_p[N_p - 1]] \).

According to (1), during a given channel training interval, we have:

\[
\mathbf{Y}_p = \mathbf{H}\mathbf{X}_p + \mathbf{Z}_p. \quad (3)
\]

The definitions of \( \mathbf{Y}_p \) and \( \mathbf{Z}_p \) are similar to that of \( \mathbf{X}_p \). We denote by \( \mathbf{E}_p \) the average power of the training symbols on any subcarrier as

\[
\mathbf{E}_p \triangleq \frac{1}{N_pM_T} \text{tr} (\mathbf{X}_p^H \mathbf{X}_p^T). \quad (4)
\]

The maximum likelihood (ML) channel estimate \( \hat{\mathbf{H}} \), which is equivalent to the least-squares solution, is Balakrishnan et al. (2000):

\[
\hat{\mathbf{H}} = \left( \sum_{k=0}^{N_p-1} \mathbf{y}_p[k] \mathbf{x}_p^T[k] \right) \left( \sum_{k=0}^{N_p-1} \mathbf{x}_p[k] \mathbf{x}_p^T[k] \right)^{-1}, \quad (5)
\]
which can be written in a more compact form as:
\[
\hat{H} = Y_p X_p^H (X_p X_p^H)^{-1},
\]
where \(\cdot^\dagger\) denotes transpose-conjugate.

Let us denote by \(E\) the matrix of estimation errors, that is, \(E = \hat{H} - H\). From (3) and (6), it is easy to show that
\[
E = Z_p X_p^H (X_p X_p^H)^{-1}.
\]

It is known that the best channel estimate is obtained with mutually orthogonal training sequences, which result in uncorrelated estimation errors. In other words, we should choose \(X_p\) with orthogonal rows such that
\[
X_p X_p^H = N_p E_p I_M.
\]
Then, the \(j\)-th column \(E_j\) of \(E\) has the covariance matrix \(\Sigma\) given by
\[
\Sigma_e = E [E_j E_j^\dagger] = \sigma_e^2 I_{M_R}, \quad \text{where} \quad \sigma_e^2 = \frac{\sigma^2_z}{N_p E_p}.
\]

### 2.1.1 Statistics of the Channel Estimation Errors

We saw that the estimated channel matrix \(\hat{H}\) can be viewed as a noisy version of the perfect channel matrix \(H\). In Section 5 we show that for channel estimators having this feature, the detection performance can be improved if the statistics of the channel estimation errors are known.

Let us reconsider the pilot-based ML channel estimator of equation (7). The good feature of this estimator is that the statistics of the channel estimation error matrix \(E\) are known (see equation (9)). By using these statistics and equation (7), the conditional pdf of \(\hat{H}\) given \(H\) can be easily expressed as:
\[
p(\hat{H}|H) = CN(H, I_M \otimes \Sigma_e),
\]
where \(\otimes\) denotes the Kronecker product and \(CN\) denotes the complex Gaussian distribution.

Furthermore, we assume that the channel matrix \(H\) has a normal prior distribution as:
\[
H \sim CN(0, I_M \otimes \Sigma_H) = \frac{1}{\pi^{M_R M_T} \det \{\Sigma_H\}^{M_T}} \exp \left\{ - \text{tr} \left( H \Sigma_H^{-1} H^\dagger \right) \right\}
\]
where \(\Sigma_H\) is the \((M_R \times M_R)\) covariance matrix of the columns of \(H\), and \(\det\{\}\) denotes matrix determinant. We assume that the entries of \(H\), i.e., the fading coefficients of different subchannels, are i.i.d. Then, \(\Sigma_H\) is a diagonal matrix with equal diagonal entries \(\sigma^2_h\).

By using the prior pdf of \(H\) from (11) and the pdf of \((\hat{H}|H)\) from (10), we can derive the posterior distribution of the perfect channel matrix, conditioned on its ML estimate, as follows (see Sadough & Duhamel (2008) for the details of the derivation):
\[
p(H|\hat{H}) = CN(\Sigma_\Delta \hat{H}, I_M \otimes \Sigma_e \Sigma_e),
\]
where
\[
\Sigma_\Delta = \Sigma_H (\Sigma_e + \Sigma_H)^{-1}.
\]

Under the above-mentioned assumptions, we have
\[
\Sigma_\Delta = \delta I_{M_R}
\]
where
\[
\delta = \frac{\sigma_h^2}{\sigma_h^2 + \sigma_e^2}.
\]
In particular, when the number of pilot symbols tends to infinity, it is not difficult to see that \(\delta \to 1\) and \(\delta \sigma_e^2 \to 0\) and consequently \(p(H|\hat{H})\) tends to a Dirac delta function. The availability of the estimation error distribution is an interesting feature of pilot-only-based PSAM channel estimation that we used to derive the posterior distribution (12). This distribution constitutes a Bayesian framework which is exploited in Section 5 for the design of detectors by taking into account channel estimation inaccuracies.

2.2 Semi-blind PSAM channel estimation

In order to preserve the spectral efficiency for the transmission of data symbols, we are interested in minimizing the number of pilot symbols in a frame. However, by reducing the number of pilot symbols, the channel may be learned improperly and channel estimation errors may become important. This can result in a considerable performance degradation and in the need to data retransmission. This performance degradation can be compensated by smart signal processing at the receiver. In fact, instead of estimation methods based on pilot symbols only, we can use semi-blind approaches that in addition to pilot symbols, make use of data symbols in channel estimation. In this way, a considerable performance improvement can be achieved at the price of increased receiver complexity de Carvalho & Slock (2001); Sadough, Ichir, Duhamel & Jaffrot (2009). We present here two semi-blind channel estimation schemes that we implement in an iterative receiver. The first semi-blind method that we consider is the thresholded hard-decisions (Th-HD) method and the second one is based on the expectation maximization (EM) algorithm Dempster et al. (1977); Moon (1996). For both methods, at the first iteration, we calculate a primary channel estimate based on the pilot sequences only, which allows the semi-blind estimator, used in the succeeding iterations, to bootstrap. Before describing these methods, we present in the following details on the iterative receiver.

2.2.1 Iterative signal detection

We usually consider in this paper iterative signal detection in the case of using non-orthogonal space-time codes at the transmitter. As shown in Fig. 2, the receiver mainly consists of a combination of two sub-blocks that exchange soft information with each other. The first sub-block, referred to as soft detector or demapper, produces extrinsic soft information from the input symbols and send it to the second sub-block, the soft-input soft-output (SISO) channel decoder. Here, we consider SISO channel decoding based on the well known forward-backward algorithm Bahl et al. (1974). Soft MIMO signal detection and soft-input SISO channel decoding are performed iteratively and the estimates of the channel coefficients are updated at each iteration of the turbo-detector Sadough, Ichir, Duhamel & Jaffrot (2009); Berthet et al. (2001). The blocks \(\Pi\) and \(\Pi^{-1}\) denote bit-level interleaver and de-interleaver, respectively, corresponding to the BICM scheme used at the transmitter.

2.2.2 Th-HD semi-blind estimation

In the Th-HD method, in addition to pilot symbols, we use in channel estimation the symbols detected with high reliability at each iteration Khalighi & Boutros (2006); Sellathurai & Haykin (2002). For instance, consider the a posteriori probability \(P_i^{(m)}\) at the decoder output at iteration \(m\), corresponding to the coded bit \(c_i\). We compare it with a threshold \(0.5 < P_{TH} < 1\). If \(P_i^{(m)} > P_{TH}\), we make the hard decision \(\hat{c}_i^{(m+1)} = 1\); otherwise,
Fig. 2. Iterative channel estimation and data detection, $y_i$ denotes the received signal on the $i$th antenna.

if $P_i^{(m)} < (1 - P_{TH})$, we make the hard decision $\hat{c}_i^{(m+1)} = 0$; and if none of these conditions are verified, we give up the channel-use corresponding to $c_i$ and do not consider it in channel estimation. If a hard decision is made on all $BM_T$ constituting bits of a channel-use, we use the resulting hard-detected symbol vector in channel estimation, in the same way as pilot symbols. The resulting channel estimate is then used in the next iteration of the detector.

The performance of Th-HD depends highly on the choice of the threshold $P_{TH}$ that determines whether or not the SISO decoder soft-outputs are reliable enough. The practical limitation is that the optimum threshold value depends on the MIMO structure, i.e., the number of transmit and receive antennas, as well as on the actual SNR Khalighi & Boutros (2006). Note that, if we take $P_{TH}$ very close to 0.5, we effectively make hard decisions on all detected symbols and use them in channel estimation. This coincides with the so called decision-feedback channel estimation Visoz & Berthet (2003).

### 2.2.3 EM-based semi-blind estimation

The interest of the EM algorithm is that it is guaranteed to be stable and to converge to an ML estimate Moon & Stirling (2000). We do not present here the details on the formulation of the EM-based estimator and refer the reader to Khalighi & Boutros (2006), for instance. A simple and classical formulation is when data and pilot symbols are used in the same way in channel estimation. By this approach, the estimated channel matrix is given below:

$$\hat{H} = \overline{R}_{yx} \overline{R}_{x}^{-1},$$

(16)

where

$$\overline{R}_{yx} = \sum_{k=1}^{N_s} y[k] \hat{x}^*[k]$$

(17)

and

$$\overline{R}_{x,i,j} = \left\{ \begin{array}{ll}
N_s & ; i = j \\
\sum_{k=1}^{N_s} \hat{x}_i[k] \hat{x}_j^*[k] & ; i \neq j
\end{array} \right.$$ (18)
Here, $N_s$ denotes the number of channel-uses per frame, $y[k]$ is the received symbol vector at the time reference $k$, and $\hat{x}[k]$ is the corresponding soft-estimates of the transmitted symbol vector, calculated using the SISO channel decoder outputs at the preceding iteration. For $k = 1, \cdots, N_p$, we have $\hat{x}[k] = x_p[k]$. Also, $\mathbf{R}_{x_{i,j}}$ denotes the $(i,j)$th entry of matrix $\mathbf{R}_x$. The formulation that we provided for EM can be modified further to improve the receiver performance. Interested reader may refer to Khalighi & Boutros (2006); Khalighi et al. (2006) for details.

2.2.4 Case study

BER curves versus $E_b/N_0$ are shown in Fig. 3 for the case of $(8 \times 8)$ and $(8 \times 6)$ MIMO structures using the PSAM technique. Rayleigh independent quasi-static fading model is considered with blocks of length $N_s = 64$ channel-uses. The number of channel-uses devoted to pilot transmission is $N_p = 10$. Three cases of pilot-only-based estimation, and Th-HD and EM-based semi-blind estimations are considered. The case of perfect-CSIR is also provided as reference. The simple spatial multiplexing scheme is used at the transmitter and MIMO signal detection is based on soft parallel interference cancellation (Soft-PIC) Sellathurai & Haykin (2002); Lee et al. (2006). Results shown in Fig. 3 correspond to the eighth iteration of the receiver. We notice that the performance of the semi-blind Th-HD and EM-based estimator are very close to each other and they outperform the pilot-only-based method. Yet, their performance is about 2 dB away from the perfect-CSIR case that is due to the relatively high co-antenna interference, as we have $M_T = 8$. We can approach further the perfect-CSIR case by increasing $N_p$.

3. Superimposed Pilots and Data

The main drawback of the PSAM approach is that, for finite-length blocks, if channel estimation is to be done on each block of symbols, the periodic insertion of pilot symbols can result in a considerable reduction of the achievable data rate. This loss in the data rate becomes important, specially for large number of transmit antennas, at low SNR, and when the channel undergoes relatively fast variations Hassibi & Hochwald (2003). As an alternative, we can use overlay pilots (OP), also called superimposed or embedded pilots, for channel estimation Hoeher & Tuòvesson (1999); Zhu et al. (2003). In this approach, a pilot sequence is superimposed on the data sequence before transmission, as shown in Fig. 4; thus, no separate time slot is dedicated to pilot transmission.

3.1 Channel estimation using OP

By using OP, we prevent the loss in the data throughput but we experience degradation in the quality of the channel estimate due to the unknown data symbols Hoeher & Tuòvesson (1999). As a matter of fact, here also there is a trade-off between high quality channel estimation and the information throughput: To obtain a better channel estimate, we should increase the percentage of the power dedicated to pilot symbols; this, however, reduces the SNR for the detection of data symbols Tapio & Bohlin (2004). In general, OP may be preferred to PSAM for high SNR, not too short channel coherence times, and larger number of receive than transmit antennas Khalighi et al. (2005).

Consider again the block fading channel model. Assuming uncorrelated data and pilot sequences, we can estimate channel coefficients by calculating the cross-correlation between the received sequences on each antenna and the transmitted pilot sequences, known to the
Fig. 3. Comparison of different estimation methods based on PSAM, iterative Soft-PIC detection, 8th iteration of the receiver (almost full convergence); (8 × 8) and (8 × 6) systems, i.i.d. Rayleigh quasi-static fading, QPSK modulation, (5,7)8 NRNSC rate 1/2 channel code, orthogonal pilot sequences with $N_p = 10$, $N_s = 64$ channel-uses. $E_b/N_0$ takes into account the receiver antenna gain $M_R$.

receiver. As the pilot sequences transmitted from different antennas are orthogonal, the estimation errors arise from noise and the data sequences. Indeed, the main problem in the estimation of channel coefficients concerns the latter interference component, i.e., the unknown data symbols. In fact, although data and pilot sequences are statistically uncorrelated, the cross-correlation is calculated over a block of symbols of limited length, over which the channel coefficients are supposed to remain unchanged. The smaller is the block length (i.e., the faster the channel fading), the more important this cross-correlation is. This can result in an error floor in the receiver BER performance Jungnickel et al. (2001), especially at high SNR, and make the OP scheme lose its interest.

Fig. 4. Overlay pilot scheme

pilot symbols, $\sigma_p^2$

data symbols, $\sigma_d^2$
3.2 Iterative channel estimation for OP

Iterative data detection and channel estimation can be a solution to the problem of error floor Zhu et al. (2003); Khalighi et al. (2005); Cui & Tellambura (2005). We consider in the following, two such iterative schemes; a pilot-only-based decision-directed estimator Khalighi et al. (2005) and an EM-based semi-blind estimator Khalighi & Bourennane (2007).

Let us denote by \( \mathbf{x}_d \) and \( \mathbf{x}_p \) the vectors of data and pilot symbols, respectively, corresponding to the transmitted \((M_T \times 1)\) vector \( \mathbf{x} = \mathbf{x}_d + \mathbf{x}_p \). We denote by \( \sigma_d^2 \) and \( \sigma_p^2 \) the power allocated to the entries of \( \mathbf{x}_d \) and \( \mathbf{x}_p \), respectively.

3.2.1 Pilot-only-based decision-directed estimator

Consider the estimation of the entry \( H_{ij} \) of the channel matrix \( \mathbf{H} \). As explained above, this estimate can be obtained by calculating the cross-correlation \( \Gamma_{ij} \) between the sequence received on the antenna \#i, \( \mathbf{y}_i \), and the pilot sequence transmitted on the antenna \#j, \( \mathbf{x}_{p,j} \). By the decision-directed method that we denote by DD, we use in each iteration, the soft-estimates \( \tilde{x}_d \) of transmitted data symbols using a posteriori LLRs at the SISO decoder output, and cancel their effect in \( \Gamma_{ij} \).

3.2.2 EM-based semi-blind estimator

To obtain a better performance, similar to the case of PSAM, we can employ semi-blind estimation methods Khalighi & Bourennane (2008); Bohlin & Tapiö (2004); Meng & Tugnait (2004) at the price of increased Rx complexity. For instance, a semi-blind estimator based on the EM algorithm may be used. Its formulation is analogous to the case of PSAM and can be found in Khalighi & Bourennane (2007).

3.2.3 Data-dependent overlay pilots

A solution for getting rid of the interference from data symbols in channel estimation is to use data-dependent overlay pilot sequences such that the corresponding pilot and data sequences are orthogonal Ghogho et al. (2005). The drawback of this method is that it results in nulls in the equivalent channel impulse response (seen by data symbols), and hence, in a performance degradation. This degradation could be reduced by performing iterative channel equalization Lam et al. (2008). On the other hand, such a scheme leads to increased envelope fluctuations of the transmitted signal. We do not consider this scheme here.

3.2.4 Case study

Let us denote by \( \alpha \) the ratio of the power of pilot symbols to the total transmit power at a symbol time, i.e., \( \alpha = \sigma_p^2 / (\sigma_p^2 + \sigma_d^2) \). For a \((2 \times 4)\) MIMO system, we have presented in Fig. 5, BER curves versus SNR for DD, EM-based, and perfect channel estimation, and different values of \( \alpha \). Pilot sequences for \( M_T \) antennas can be QPSK modulated and chosen according to the Walsh-Hadamard series to ensure their orthogonality. Results correspond to the fifth iteration of the Rx where almost full convergence is attained. SNR in Fig. 5 stands for the actual average received SNR, i.e., \( M_R / (\sigma_d^2 + \sigma_p^2) / \sigma_n^2 \), in contrast to \( E_b / N_0 \) that takes into account only \( \sigma_d^2 \). In this way, we can directly see the compromise between the channel estimation quality and the data detection performance, e.g. by increasing \( \alpha \). Again, Soft-PIC MIMO detection is performed. For both estimation methods, we notice an error floor at high SNR for \( \alpha < 15\% \) which is especially visible for \( \alpha = 2\% \) and \( \alpha = 5\% \). On the other hand, by increasing \( \alpha \), better channel estimates are obtained, but at the same time, less power is dedicated to data symbols. So, increasing \( \alpha \) too much, will result in an overall performance degradation. Comparing
the performance curves of the EM-based and DD estimators, we see that the performance improvement by semi-blind estimation is quite considerable. This improvement is more important for smaller $\alpha$ values. For increased $\alpha$, more power is dedicated to pilot symbols, and hence, the performance of the pilot-only-based estimator approaches that of the semi-blind estimator.

4. Impact of Space-Time Scheme

An important aspect is the impact of channel estimation errors on the receiver performance for different ST schemes. We would like to compare the two general classes of ST schemes, i.e., orthogonal and non-orthogonal schemes. In the case of perfect channel knowledge, a comparison is made between these two categories in Khalighi et al. (2009), where iterative signal detection based on Soft-PIC is proposed for the case of non-orthogonal schemes. Note that optimal signal detection is too computationally complex for these schemes. Conditioned to the presence of sufficient (time or frequency) diversity, it is shown that a substantial gain is obtained by using the appropriate non-orthogonal schemes for moderate-to-high spectral efficiency MIMO systems, as compared to orthogonal schemes, which justifies the increased complexity of the receiver Khalighi et al. (2009).
Concerning the practical case of imperfect channel estimate, in fact, lower-rate orthogonal schemes could be more sensitive to channel estimation errors as, in general, they have to use a larger signal constellation to attain a desired spectral efficiency. Concerning non-orthogonal schemes, we need, in general, smaller constellation sizes as compared to orthogonal ones. However, the iterative detector for the non-orthogonal schemes could be more sensitive to channel estimation errors because its convergence, and hence, its performance is affected by these errors.

To study the effect of channel estimation errors, let us consider pilot-only-based channel estimation using time-multiplexed pilots. For each fading block, we devote \( N_p \) channel-uses to the transmission of power-normalized mutually orthogonal QPSK pilot sequences from \( M_T \) transmit antennas Khalighi & Boutros (2006). For a \( (2 \times 2) \) MIMO system, we have shown in Fig. 6 the average BER after four detector iterations versus \( N_p \) for a spectral efficiency of \( \eta = 2 \) bps/Hz. We have considered the Alamouti code Alamouti (1998) as the orthogonal scheme, and two cases of spatial multiplexing (denoted here by MUX) and Golden coding Belfiore et al. (2005) (denoted by GLD) as non-orthogonal schemes. The \( E_b/N_0 \) for each ST scheme is set to what results in BER \( \approx 10^{-4} \) in the case of perfect channel knowledge. From Fig. 6 we notice an almost equivalent sensitivity to the channel estimation errors for MUX and GLD schemes. This comparison makes sense as the SNRs for these schemes are close to each other. On the other hand, we see that the Alamouti scheme has the lowest sensitivity. This is due to the orthogonal structure of the code, and the fact that the SNR is higher, compared to those for MUX and GLD schemes, and as a result, the quality of channel estimate is much better.
5. Improved Signal Detection in the Presence of Channel Estimation Errors

For the case of time-multiplexed pilot and data, as we explained previously, in order to obtain a good channel estimate, we should increase $N_p$, which in turn, results in a larger loss in the spectral efficiency, specially for relatively fast time-varying communication channels Hassibi & Hochwald (2003). One solution is to use semi-blind channel estimation in order to reduce the number of channel-uses devoted to pilot transmission, as seen in Section 2.2. The disadvantage of semi-blind approaches is the increased receiver complexity. For a reduced-complexity semi-blind joint channel estimator and data detector the reader is referred to Sadough, Ichir, Duhamel & Jaffrot (2009).

An alternative to this solution is to modify the detector so as to take into account channel estimation errors. As a matter of fact, the classical approach is to use the channel estimate in the detection part in the same way as if it was a perfect estimate, what is known as mismatched signal detection. Obviously, this approach is suboptimal and can degrade considerably the receiver performance in the presence of channel estimation errors.

In this section we provide the general formulation of a detection rule that takes into account the available imperfect CSIR and refer to it as the improved detector. To this end, we consider the model (1) and denote by $J(y, x, H)$ the quantity (cost function) that would let us to decide in favor of a particular $x$ at the receiver if the channel was perfectly known. Note that depending on the detection criteria, the quantity $J(y, x, H)$ can be the posterior pdf $p(x|y, H)$, the logarithm of the likelihood function $p(y|x, H)$, the mean square error (as in Sadough, Khalighi & Duhamel (2009); Sadough & Khalighi (2007)), etc. Assume a channel estimator in which the statistics of the estimation errors are known. Such a scenario occurs for instance in pilot-only-based PSAM channel estimation studied in Subsection 2.1 where we saw that the estimation process can be characterized by the posterior pdf of the channel (12). In this case, we propose a detector based on the minimization of a new cost function defined as

$$\tilde{J}(y, x, \hat{H}) = \int_H J(y, x, H) p(H|\hat{H}) dH = E_{H|\hat{H}}[J(y, x, H)|\hat{H}]$$  \hspace{1cm} (19)

where by using the posterior distribution (12), we have averaged the cost function $J$ over all realizations of the unknown channel $H$ conditioned on its available estimate $\hat{H}$. Note that the mismatched detector is based on the minimization of the cost function $J(y, x, \hat{H})$. This latter cost function is obtained by using the estimated channel $\hat{H}$ in the same metric that would be used if the channel was perfectly known, i.e., $J(y, x, H)$. Using the metric of (19) differs from the mismatched detection on the conditional expectation $E_{H|\hat{H}}[\cdot]$ which provides a robust design by averaging the cost function $J(y, x, H)$ over all (true) channel realizations which could correspond to the available estimate.

Consider the problem of detecting symbol vector $x$ from the observation model (1) in the ML sense, i.e., so as to maximize the likelihood function $p(y|x, H)$. It is well known that under perfect channel knowledge and i.i.d. Gaussian noise, detecting $x$ by maximizing the likelihood $p(y|x, H)$ is equivalent to minimizing the Euclidean distance $D_{ML}$ as

$$\hat{x}_{ML}(H) = \arg\min_{s_0, \ldots, s_{M-1} \in C} \{ D_{ML}(x, y, H) \},$$  \hspace{1cm} (20)

with $D_{ML}(x, y, H) \triangleq - \log p(y|x, H) \propto ||y - Hx||^2$, where $\propto$ means “is proportional to” and $C$ denotes the set of constellation symbols of size $M$. Assuming $B$ bits per symbol, we have $M = 2^B$.
The detection rule (20) requires the knowledge of the perfect channel matrix \( \mathbf{H} \). The sub-optimal mismatched ML detector consists in replacing the exact channel by its estimate \( \hat{\mathbf{H}} \) in the receiver metric as

\[
\mathbf{x}_{\text{MM}}(\hat{\mathbf{H}}) = \underset{s_0, \ldots, s_M-1 \in \mathcal{C}}{\arg \min} \{ \mathcal{D}_{\text{MM}}(\mathbf{x}, \mathbf{y}, \hat{\mathbf{H}}) \} = \underset{s_0, \ldots, s_M-1 \in \mathcal{C}}{\arg \min} \{ \| \mathbf{y} - \hat{\mathbf{H}} \mathbf{x} \|^2 \},
\]

where

\[
\mathcal{D}_{\text{MM}}(\mathbf{x}, \mathbf{y}, \mathbf{H}) \triangleq \mathcal{D}_{\text{ML}}(\mathbf{x}, \mathbf{y}, \mathbf{H}) \bigg|_{H=\hat{H}},
\]

and the subscript -MM denotes mismatched. Obviously, the sub-optimality of this detection technique is due to the mismatch introduced by the channel estimation errors; while the decision metric is derived from the likelihood function \( p(\mathbf{y}|\mathbf{H}, \mathbf{x}) \) conditioned on the perfect channel \( \mathbf{H} \), the receiver uses an estimate \( \hat{\mathbf{H}} \) different from \( \mathbf{H} \) in the detection process.

As an alternative to this mismatched detection, an improved ML detection metric is proposed in Tarokh et al. (1999); Taricco & Biglieri (2005). This metric is based on modified likelihood \( p(\mathbf{y}|\hat{\mathbf{H}}, \mathbf{x}) \) which is conditioned on the imperfect channel \( \hat{\mathbf{H}} \). The pdf \( p(\mathbf{y}|\hat{\mathbf{H}}, \mathbf{x}) \) can be derived as follows:

\[
p(\mathbf{y}|\hat{\mathbf{H}}, \mathbf{x}) = \int_{\mathbf{H} \in \mathcal{C}} p(\mathbf{y}, \mathbf{H}|\hat{\mathbf{H}}, \mathbf{x}) \, d\mathbf{H} = \int_{\mathbf{H} \in \mathcal{C}} p(\mathbf{y}|\mathbf{H}, \mathbf{x}) \, p(\mathbf{H}|\hat{\mathbf{H}}) \, d\mathbf{H} = E_{\mathbf{H}|\hat{\mathbf{H}}} \left[ p(\mathbf{y}|\mathbf{H}, \mathbf{x}) | \hat{\mathbf{H}} \right],
\]

where \( p(\mathbf{H}|\hat{\mathbf{H}}) \) is the channel posterior distribution of equation (12) and \( \mathcal{C} \) denotes the set of complex matrices of size \((M_R \times M_T)\). In fact, equation (22) shows that \( p(\mathbf{y}|\hat{\mathbf{H}}, \mathbf{x}) \) can be simply derived from the general formulation in (19). It is shown in Sadough & Duhamel (2008) that the averaged likelihood in (22) is shown to be a complex Gaussian distributed vector given by

\[
p(\mathbf{y}|\hat{\mathbf{H}}, \mathbf{x}) \sim \mathcal{CN}(\mathbf{m}_M, \Sigma_M),
\]

where \( \mathbf{m}_M = \delta \hat{\mathbf{H}} \mathbf{x} \), and \( \Sigma_M = \Sigma_z + \delta \Sigma_e \| \mathbf{x} \|^2 \). Finally, the estimate of the symbol \( \mathbf{x} \) is

\[
\hat{\mathbf{x}}_M(\hat{\mathbf{H}}) = \underset{s_0, \ldots, s_M-1 \in \mathcal{C}}{\arg \min} \{ \mathcal{D}_M(\mathbf{x}, \mathbf{y}, \hat{\mathbf{H}}) \},
\]

where

\[
\mathcal{D}_M(\mathbf{x}, \mathbf{y}, \hat{\mathbf{H}}) \triangleq -\log p(\mathbf{y}|\mathbf{x}, \hat{\mathbf{H}}) = M_R \log \pi (\sigma_e^2 + \delta \sigma_e^2 \| \mathbf{x} \|^2) + \frac{\| \mathbf{y} - \delta \hat{\mathbf{H}} \mathbf{x} \|^2}{\sigma_e^2 + \delta \sigma_e^2 \| \mathbf{x} \|^2}
\]

is referred to as the improved ML decision metric under imperfect CSIR.

Note that when CSIR tends to the exact value, which is obtained when the number of pilot symbols tends to infinity, we have \( \delta \to 1, \sigma_e^2 \to 0 \), and the improved metric (25) tends to the mismatched metric:

\[
\lim_{N \to \infty} \frac{\mathcal{D}_M(\mathbf{x}, \mathbf{y}, \hat{\mathbf{H}})}{\mathcal{D}_{\text{MM}}(\mathbf{x}, \mathbf{y}, \hat{\mathbf{H}})} = 1.
\]

In the following two sections, we apply the proposed receiver design method of equation (19) for improving the performance of two usually-used MIMO receivers working under imperfect channel estimation: one based on the maximum a posteriori (MAP) criterion, and the other on Soft-PIC. For both cases, we consider the simple spatial multiplexing as the space-time scheme at the transmitter, and iterative MIMO detection and channel decoding at the receiver.
6. Reception Scheme I: Iterative MAP Detection

Here, we consider MIMO signal detection based on the MAP algorithm. In the following, we make use of the improved ML metric derived in the previous section to modify the MAP detector part for the case of imperfect CSIR Sadough et al. (2007). Let us denote by \( \mathbf{x}[k] \) and \( \mathbf{y}[k] \), the transmitted and received symbol vectors corresponding to the time slot \( k \), simply by \( \mathbf{x}_k \) and \( \mathbf{y}_k \), respectively. Also, let \( d_{kj} \) denote the \( j \)-th (\( j = 1, \ldots, BM_T \)) coded and interleaved bit corresponding to \( \mathbf{x}_k \). We denote by \( L(d_{kj}) \) the coded log-likelihood ratio (LLR) of the bit \( d_{kj} \) at the output of the detector. Conditioned on the imperfect CSIR \( \hat{\mathbf{H}}_k \), \( L(d_{kj}) \) is given by:

\[
L(d_{kj}) = \log \frac{P_{\text{dem}}(d_{kj} = 1 | \mathbf{y}_k, \hat{\mathbf{H}}_k)}{P_{\text{dem}}(d_{kj} = 0 | \mathbf{y}_k, \hat{\mathbf{H}}_k)},
\]

(27)

where \( P_{\text{dem}}(d_{kj} | \mathbf{y}_k, \hat{\mathbf{H}}_k) \) is the probability of transmission of \( d_{kj} \) at the detector output. We partition the set \( C \) that contains all possibly-transmitted symbol vectors \( \mathbf{x}_k \) into two sets \( C^m_0 \) and \( C^m_1 \), for which the \( j \)-th bit of \( \mathbf{x}_k \) equals “0” or “1”, respectively. We have:

\[
L(d_{kj}) = \log \frac{\sum_{\mathbf{x}_k \in C^m_1} e^{-D_M(\mathbf{x}_k, \mathbf{y}_k, \hat{\mathbf{H}}_k)} \prod_{i=1}^{BM_T} p^1_{\text{dec}}(d_{ki})}{\sum_{\mathbf{x}_k \in C^m_0} e^{-D_M(\mathbf{x}_k, \mathbf{y}_k, \hat{\mathbf{H}}_k)} \prod_{i=1}^{BM_T} p^0_{\text{dec}}(d_{ki})},
\]

(28)

where \( p^1_{\text{dec}}(d_{kj}) \) and \( p^0_{\text{dec}}(d_{kj}) \) are prior probabilities on the bit \( d_{kj} \) coming from the SISO decoder.

Note that using the metric \( D_M(\mathbf{x}_k, \mathbf{y}_k, \hat{\mathbf{H}}_k) \) for the evaluation of the LLRs in (28) is an alternative to using the mismatched ML metric \( D_{\text{MM}}(\mathbf{x}_k, \mathbf{y}_k, \hat{\mathbf{H}}_k) \) which replaces at each iteration, the exact channel \( \mathbf{H}_k \) by its estimate \( \hat{\mathbf{H}}_k \) in \( D_{\text{ML}}(\mathbf{x}_k, \mathbf{y}_k, \mathbf{H}_k) \). By doing so, the LLRs are adapted to the imperfect channel knowledge available at the receiver and consequently the impact of channel uncertainty on the SISO decoder performance is reduced. We refer to the latter approach as improved MAP detector Sadough et al. (2007).

The summations in (28) are taken over the product of the likelihood \( p(\mathbf{y}_k | \mathbf{x}_k, \hat{\mathbf{H}}_k) = e^{-D_M(\mathbf{x}_k, \mathbf{y}_k, \hat{\mathbf{H}}_k)} \) given a symbol \( \mathbf{x}_k \) and the estimated channel coefficient \( \hat{\mathbf{H}}_k \), and of the a priori probability on \( \mathbf{x}_k \) (the term \( \prod P_{\text{dec}}(d_{ki}) \), fed back from the SISO decoder at the previous iteration. In this latter term, the a priori probability of the bit \( d_{kj} \) itself has been excluded, so as to let the exchange of extrinsic informations between the channel decoder and the soft detector. Also, note that this term assumes independent coded bits \( d_{kj} \), which is a reasonable approximation for random interleaving of large size. At the first iteration, no a priori information is available on bits \( d_{kj} \), therefore the probabilities \( p^0_{\text{dec}}(d_{kj}) \) and \( p^1_{\text{dec}}(d_{kj}) \) are set to 1/2. The decoder accepts the LLRs of all coded bits and computes the LLRs of information bits, which are used for decision, at the last iteration.

6.1 Case study

We now present some numerical results. First, the BER performance of the improved and mismatched detectors are compared. Let us first address the case of BICM iterative decoding with 16-QAM and Gray labeling for a \( 2 \times 2 \) MIMO channel. It can be seen from Fig. 7 that for \( N_p = 2 \) (the shortest possible training sequence), the improvement in terms of required \( E_b / N_0 \)
in order to attain a BER of $10^{-5}$ is about 1 dB, compared to the mismatched solution, while staying still 3 dB away from the perfect channel knowledge case. We also notice that, logically, these quantities are reduced when increasing the length of the training sequence, that is, the performances of mismatched and improved detectors get closer to the case of perfect channel knowledge.

Similar plots are shown in Fig. 8 for the case of 16-QAM and set-partition (SP) labeling on the $(2 \times 2)$ MIMO channel. These show the behavior of the detectors with respect to the type of bit-symbol labeling. At a BER of $2 \times 10^{-4}$ with $N_p = 2$, we obtain an SNR gain of about 1.4 dB by using the improved detector. In other words, iterative decoding with SP labeling benefits more from the improved metric than the one with Gray labeling. Otherwise, similar conclusions hold between the SP-labeling curves.

![Fig. 7. BER performance improvement over $(2 \times 2)$ MIMO channel with i.i.d. Rayleigh fading for various training sequence lengths. 16-QAM modulation with Gray labeling, iterative MAP detection after four receiver iterations.](image)

**7. Reception Scheme II: Iterative Soft-PIC Detection**

MAP detection is the optimal solution under perfect CSIR in the sense of bit error rate but its complexity grows exponentially with the number of transmit antennas and the signal constellation size. For this reason, suboptimal detection techniques are usually preferred. One interesting solution is that based on Soft-PIC and linear minimum mean-square error (MMSE) filtering Wang & Poor (1999); Sellathurai & Haykin (2002); Lee et al. (2006), what we considered in Sections 2 to 4. In fact, in these parts, we considered in the iterative detector a sim-
Fig. 8. BER performance improvement over \((2 \times 2)\) MIMO channel with i.i.d. Rayleigh fading for various training sequence lengths. 16-QAM modulation with set-partition labeling, iterative MAP detection after four receiver iterations.

The simplified formulation of Soft-PIC, which assumes perfect interference cancellation after the first iteration. In this section, however, we consider the exact formulation of Soft-PIC. In order to better understand the formulation of the improved detector, we present in the following the formulation of (exact) Soft-PIC under perfect channel knowledge at the receiver. Then, we present the improved Soft-PIC detector in the presence of channel estimation errors in Subsection 7.2.

### 7.1 Soft-PIC detection under perfect channel knowledge

Consider the general block diagram of Fig. 2. Here, to detect a symbol transmitted from a given antenna, we first make use of the soft information available from the SISO channel decoder to reduce and hopefully to cancel the interfering signals arising from other transmit antennas. At the first iteration where this information is not available, we perform a classical MMSE filtering.

Let us consider the transmitted vector \( \mathbf{x}_k = [x_{k1}, \ldots, x_{kM_T}]^T \) at time \( k \) and assume that we are interested in the detection of its \( i \)-th symbol \( x_k^i \). We start by evaluating the parameters \( \hat{x}_k^i \) and
\[ \sigma^2_{x_k} \] for the interfering symbols \( x_k^j, j \neq i \), from the SISO decoder as follows:

\[
\hat{x}_k^i = \mathbb{E}[x_k^i] = \sum_{j=1}^{2^M} x_k^j P[x_k^j] \\
\sigma^2_{x_k^i} = \mathbb{E}[|x_k^i|^2] = \sum_{j=1}^{2^M} |x_k^j|^2 P[x_k^j]
\]

(29)

(30)

where \( P[x_k^i] \) is the probability of the transmission of \( x_k^i \) and is evaluated using the probabilities \( P_{\text{dec}}(d_k^n) \) at the decoder output:

\[
P[x_k^i] = K \prod_{n=1}^{B} P_{\text{dec}}(d_k^n),
\]

where \( K \) is a normalization factor. We further introduce the following definitions. \( \mathbf{H}_i \) is the \((M_R \times (M_T - 1))\) matrix constructed from \( \mathbf{H} \) by discarding its \( i\)-th column, namely \( \mathbf{h}_i \). We also define the \(((M_T - 1) \times 1)\) vectors

\[
\mathbf{x}_k^i \triangleq [x_k^1, x_k^2, ..., x_k^{i-1}, x_k^{i+1}, ..., x_k^{M_T}]^T
\]

and

\[
\hat{\mathbf{x}}_k^i \triangleq [\hat{x}_k^1, \hat{x}_k^2, ..., \hat{x}_k^{i-1}, \hat{x}_k^{i+1}, ..., \hat{x}_k^{M_T}]^T,
\]

where \( \hat{x}_k^i \) are estimated in (29).

Now, given the received signal vector \( \mathbf{y}_k \), a soft interference cancellation is performed on \( \mathbf{y}_k \) for detecting the symbol \( x_k^i \) by subtracting to \( \mathbf{y}_k \) the estimated signals of the other transmit antennas as Sadough, Khalighi & Duhamel (2009):

\[
\mathbf{y}_k^i = \mathbf{y}_k - \mathbf{H}_i \hat{\mathbf{x}}_k^i = \mathbf{h}_i x_k^i + \mathbf{H}_i \mathbf{x}_k^i - \mathbf{H}_i \hat{\mathbf{x}}_k^i + \mathbf{z}_k, \text{ for } i = 1, ..., M_T.
\]

(31)

Except under perfect prior information on the symbols which leads to \( \hat{x}_k^i = x_k^i \), there remains a residual interference in \( \mathbf{y}_k^i \). In order to reduce further this interference, an instantaneous linear MMSE filter \( \mathbf{w}_k^i \) is applied to \( \mathbf{y}_k^i \) to minimize the mean square value of the error \( e_k^i \) defined as

\[
e_k^i = x_k^i - r_k^i
\]

(32)

where the filter output \( r_k^i \) is equal to

\[
r_k^i = \mathbf{w}_k^i \mathbf{y}_k^i.
\]

(33)

Here, \( \mathbf{w}_k^i \) is obtained as

\[
\mathbf{w}_k^i = \arg \min_{\mathbf{w}_k^i \in \mathbb{C}^{M_R}} \mathbb{E}_{\mathbf{x}_k, \mathbf{z}_k} \left[ |x_k^i - \mathbf{w}_k^i \mathbf{y}_k^i|^2 \right].
\]

(34)

By invoking the orthogonality principle Scharf (1991), the coefficients of the MMSE filter \( \mathbf{w}_k^i \) are given by

\[
\mathbf{w}_k^i = \mathbf{h}_i^\dagger \left[ \mathbf{h}_i \mathbf{h}_i^\dagger + \frac{\mathbf{H}_i (\Lambda_{k,i} - \tilde{\Lambda}_{k,i}) \mathbf{H}_i^\dagger}{\sigma^2_{x_k^i} + \frac{\sigma^2_{x_k^i}}{\sigma^2_{x_k^i}}} + \frac{1}{M_R} \right]^{-1}
\]

(35)
where

\[
\Lambda_{k,i} = \mathbb{E}[x_k^i x_k^i] \approx \text{diag} \left( \mathbb{E}[|x_k^1|^2], ..., \mathbb{E}[|x_k^{i-1}|^2], \mathbb{E}[|x_k^{i+1}|^2], ..., \mathbb{E}[|x_k^{M}|^2] \right), \quad \text{and}
\]

\[
\tilde{\Lambda}_{k,i} = \tilde{x}_k^i \tilde{x}_k^i \approx \text{diag} \left( |\tilde{x}_k^1|^2, ..., |\tilde{x}_k^{i-1}|^2, |\tilde{x}_k^{i+1}|^2, ..., |\tilde{x}_k^{M}|^2 \right).
\]

Note that the off-diagonal entries in \( \Lambda_{k,i} \) and \( \tilde{\Lambda}_{k,i} \) have been neglected to reduce the complexity without causing significant performance loss Lee et al. (2006).

At the first decoding iteration, we have no prior information available on the transmitted data, i.e., \( \Lambda_{k,i} = \sigma_x^2 I_{M_T} \) and \( \tilde{\Lambda}_{k,i} = 0_{M_T} \). Consequently, (35) reduces to

\[
w_k^i = h^T \left[ HH^T + \frac{\sigma_x^2}{\sigma_{x_k^i}^2} I_{M_R} \right]^{-1}
\]

which is no more than the linear MMSE detector for \( x_k^i \).

Before passing the detected symbols \( r_k \) to the SISO decoder, we convert them to LLR. This is done assuming a Gaussian distribution for the residual interference after Soft-PIC detection (see Wang & Poor (1999) for details on the LLR conversion).

### 7.2 Improved Soft-PIC Detection Under Imperfect Channel Estimation

As we see from (31) and (35), we need the channel \( H \) for both interference canceling and MMSE filtering. As the receiver has only an imperfect channel estimate \( \tilde{H} \), the suboptimal mismatched solution consists in replacing \( H \) and \( h \) in (31) and (35) by their estimates \( \tilde{H} \) and \( \tilde{h} \), respectively. As a first step toward a realistic design, we make use of the available channel estimate \( \tilde{H} \) for interference cancellation. That is, equation (31) is rewritten as

\[
y_k^i = y_k - \tilde{H}_i \tilde{x}_k^i = h_i x_k^i + \tilde{H}_i \tilde{x}_k^i + z_k, \quad \text{for } i = 1, ..., M_T
\]

where \( \tilde{H}_i \) is the \((M_R \times (M_T - 1)) \) matrix constructed from \( \tilde{H} \) by discarding its \( i \)-th column, namely \( \tilde{h}_i \). We note that (37) naturally depends on the unknown channel matrix \( H \) of which the receiver has only an imperfect estimate available. Instead of replacing the unknown channel by its estimate (i.e., the mismatched approach), we use the posterior distribution (12) and make two modifications to the detector described in Subsection 7.1, as follows (see Sadough, Khalighi & Duhamel (2009) and Sadough & Khalighi (2007) for more details).

The first modification concerns the design of the filter \( w_k^i \) in (34). The modified filter \( \tilde{w}_k^i \) should minimize the average of the mean square error over all realizations of channel estimation errors. In other words,

\[
\tilde{w}_k^i = \arg \min_{\tilde{w}_k^i \in \mathbb{C}^{M_R}} \mathbb{E}_{H,x_k,z_k} \left[ |x_k^i - \tilde{w}_k^i y_k^i|^2 \right] = \arg \min_{\tilde{w}_k^i \in \mathbb{C}^{M_R}} \mathbb{E}_{H,x_k,z_k} \left[ |x_k^i - \tilde{w}_k^i y_k^i|^2 \right] = \arg \min_{\tilde{w}_k^i \in \mathbb{C}^{M_R}} \mathbb{E}_{H,x_k,z_k} \left[ \left| x_k^i - \tilde{w}_k^i y_k^i \right|^2 \right]
\]

where we have assumed the independence between \( H, x_k, \) and \( z_k \). After some simple algebraic manipulations Sadough, Khalighi & Duhamel (2009); Scharf (1991), we obtain:

\[
\tilde{w}_k^i = R_k x_k^i R_k^{-1}
\]
The second modification concerns the application of the derived filter different due to the inherent averaging in (38), which provides a robust design that adapts the presence of estimation errors, the proposed improved and mismatched detectors become CSIR. We observe that the gain in SNR of the improved detector to attain the BER of $10^{-5}$ is proportional to the channel knowledge is available at the receiver, i.e., $\hat{H} = H$ and $\sigma_{x_k}^2 = 0$. We note that in this case, $\delta = 1$ and the posterior pdf (12) reduces to a Dirac delta function; consequently the two filters $\tilde{w}_k^i$ and $w_k^i$ coincide. Similarly, under near-perfect CSIR, obtained either when $N_p \rightarrow \infty$, we have $\delta \rightarrow 1$, and the filter $\tilde{w}_k^i$ gives a similar expression as $w_k^i$ in (35). However, in the presence of estimation errors, the proposed improved and mismatched detectors become different due to the inherent averaging in (38), which provides a robust design that adapts itself to the channel estimate available at the receiver.

To get more insight on the proposed detector, let us consider the ideal case where perfect channel knowledge is available at the receiver, i.e., $\hat{H} = H$ and $\sigma_{x_k}^2 = 0$. We observe that the gain in SNR of the improved detector to attain the BER of $10^{-5}$ is proportional to the channel knowledge is available at the receiver, i.e., $\hat{H} = H$ and $\sigma_{x_k}^2 = 0$. We note that in this case, $\delta = 1$ and the posterior pdf (12) reduces to a Dirac delta function; consequently the two filters $\tilde{w}_k^i$ and $w_k^i$ coincide. Similarly, under near-perfect CSIR, obtained either when $N_p \rightarrow \infty$, we have $\delta \rightarrow 1$, and the filter $\tilde{w}_k^i$ gives a similar expression as $w_k^i$ in (35). However, in the presence of estimation errors, the proposed improved and mismatched detectors become different due to the inherent averaging in (38), which provides a robust design that adapts itself to the channel estimate available at the receiver.

The second modification concerns the application of the derived filter $\tilde{w}_k^i$ to the received signal $\tilde{y}_k^i$. As this latter depends on $H$ (see (37)), we average the filter output $r_k^i$ as follows:

$$\tilde{r}_k^i = E_{H,H} \left[ r_k^i \right] = \delta \mu_{k,i} \tilde{h}_i \bar{H}_k \bar{h}_k - \delta \bar{w}_k^i \bar{H}_k \bar{h}_k + \tilde{w}_k^i \bar{z}_k = \mu_{k,i} x_k^i + \eta_{k,i},$$

where $\eta_{k,i}$ contains interference and noise. From (42) it is clear that the output of the improved MMSE filter can be viewed as an equivalent AWGN channel having $x_k^i$ at its input. The parameters $\mu_{k,i}$ and $\sigma_{x_k}^2$ are calculated at each time-slot by using the symbols statistics. In order to transform the detected symbols at the output of the MMSE filter to LLRs on the corresponding bits, we approximate $\mu_{k,i}$ by a zero-mean Gaussian random variable with variance $\sigma_{\eta_{k,i}}^2$ (see Sadough, Khalighi & Duhamel (2009) for details on the calculation of this variance).

Let $d_k^i,m$ denote the $m$-th ($m = 1, \ldots, B$) bit corresponding to $x_k^i$. The LLR on $d_k^i,m$ is given by:

$$L(d_k^i,m) = \log \frac{P_{dem}(d_k^i,m = 1|r_k^i,\mu_{k,i})}{P_{dem}(d_k^i,m = 0|r_k^i,\mu_{k,i})} = \log \frac{\sum_{x_k^i \in \mathcal{S}_1^m} \exp \left\{ - \frac{|r_k^i - \mu_{k,i} x_k^i|^2}{\sigma_{\eta_{k,i}}^2} \right\}}{\sum_{x_k^i \in \mathcal{S}_0^m} \exp \left\{ - \frac{|r_k^i - \mu_{k,i} x_k^i|^2}{\sigma_{\eta_{k,i}}^2} \right\}}.$$

Note that here the cardinality of the sets $\mathcal{S}_1^m$ and $\mathcal{S}_0^m$ equals $2^B - 1$.

### 7.2.1 Case study

Figure 9 shows BER curves of the mismatched and improved receivers for the case of QPSK modulation and a $(2 \times 2)$ MIMO system. The number of channel uses for pilot transmission is $N_p \in \{2, 4, 8\}$. As a reference, we have also presented the BER curve for the case of perfect CSIR. We observe that the gain in SNR of the improved detector to attain the BER of $10^{-5}$ is about 1.4 dB, 0.5 dB, and 0.2 dB, respectively for $N_p = 2, 4, 8$. 

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The interesting point is that the two detectors have almost the same convergence trend and the major improvement is obtained after the second iteration for both detectors Sadough, Khalighi & Duhamel (2009). So, if for the reasons of complexity reduction, we only process two receiver iterations, we still have a considerable performance gain by using the improved detector.

Fig. 9. BER performance of improved and mismatched iterative Soft-PIC; (2 × 2) MIMO with MUX ST scheme, i.i.d. Rayleigh block-fading channel with 4 fades per frame, QPSK modulation, training sequence length \( N_p \in \{2, 4, 8\} \).

8. Conclusions

We studied in this chapter the interaction between iterative data detection and channel estimation in realistic wireless communication systems where the receiver disposes only of an imperfect estimate of the unknown channel parameters. To obtain the CSIR, we considered different recent and classically-used techniques. First, we presented pilot-only based channel estimation and showed that an accurate estimate of the channel through this method would require a large number of pilots per frame, which can result in a considerable loss in the system data throughput. Overlay pilots may be preferred to time-multiplexed solution from this point of view, however, the quality of channel estimate is, in general, worse, as compared to PSAM. We also presented semi-blind channel estimation methods that, in addition to pilot symbols, make use of the data symbols in the estimation process. Although iterative semi-blind channel estimation outperforms pilot-only assisted channel estimation, it has a higher complexity, which may be of critical concern for practical implementations.
Regardless of the channel estimation technique, an important point is the impact of estimation errors on the receiver performance. The usually-used approach is to consider the (imperfect) channel estimate as perfect and to use it in data detection. We called this the mismatched approach. In such case, we saw that, the impact of estimation errors is somehow similar for orthogonal and non-orthogonal space-time schemes. We then considered the improved approach by which we take into account the channel estimation inaccuracies in data detection. More precisely, by using the statistics of the channel estimation errors, we use a new detection rule instead of the sub-optimal mismatched detector. Applying this detection design rule to MAP and Soft-PIC detectors, we showed that a significant improvement can be obtained as compared to the mismatched detector. Finally, it is worth mentioning that adopting the improved reception scheme does not increase considerably the complexity. In fact, the improved detectors require just a few more matrix additions and multiplications, which does not have an important impact on the receiver complexity.

9. References


In the last decades the restless evolution of information and communication technologies (ICT) brought to a deep transformation of our habits. The growth of the Internet and the advances in hardware and software implementations modified our way to communicate and to share information. In this book, an overview of the major issues faced today by researchers in the field of radio communications is given through 35 high quality chapters written by specialists working in universities and research centers all over the world. Various aspects will be deeply discussed: channel modeling, beamforming, multiple antennas, cooperative networks, opportunistic scheduling, advanced admission control, handover management, systems performance assessment, routing issues in mobility conditions, localization, web security. Advanced techniques for the radio resource management will be discussed both in single and multiple radio technologies; either in infrastructure, mesh or ad hoc networks.

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