Kinematic calibration of articulated arm coordinate measuring machines and robot arms using passive and active self-centering probes and multipose optimization algorithm based in point and length constrains

Jorge Santolaria and Juan José Aguilar

Universidad de Zaragoza
Spain

1. Introduction

The kinematic modeling of articulated arm coordinate measuring machines (AACMM) has inherited both the previous developments in the field of robot arms and manipulators, and their calibration and parameter identification techniques, given the similarity of their mechanical characteristics. Traditional approaches to the problem of kinematic parameters identification in both cases use an objective function in terms of quadratic sum of errors of measurement or positioning, formulated as the euclidean distance between the points materialised by a gauge or measuring instrument, and the points obtained through the kinematic model. By capturing data at various positions in the workspace, those approaches follow a resolution scheme that involves indirect optimization, such as the Moore-Penrose pseudoinverse or other methods for solving systems of linear equations to obtain the set of kinematic model parameters which minimize the error in the positions considered.

This chapter first presents a review of developments and state of the art concerning the kinematic modeling of robot manipulators and AACMM, as well as aspects to consider regarding the influence of the model chosen on the subsequent parameters identification procedure. Secondly an optimization algorithm based on an objective function that considers terms related to the accuracy and repeatability is shown. This algorithm follows a pure optimization scheme from data obtained through probing several spheres of a ball-bar gauge placed at several positions in the working range for both systems. In addition to the distance errors from the nominal coordinates of the gauge, it is possible to optimize the repeatability from the captured pose values for the same sphere in several arm orientations. To capture data, a passive self-centering probe and an active self-centering probe are used to directly probe the center of each sphere for a large number of arm orientations, and also to analyze the effect of probing force in the identification process and the generalization of the error results for any arm position and orientation. Experimental results of the capture and identification technique are presented with both probes linked to a Faro AACMM, as well as
the comparison of these results with those obtained applying traditional identification techniques. Finally a description of the technique for a Kuka robot with the active self-centering probe is shown, and also the integration of both mathematical models to obtain the sphere center coordinates in the robot global reference frame from readings expressed on the reference system of the self-centering probe.

2. Evolution of kinematic modelling for robot calibration

Since the development of the well-known model proposed by Denavit and Hartenberg (D-H) (Denavit & Hartenberg, 1955), multiple variants of kinematic models based on homogenous transformations have been considered. These later approaches solve problems of indetermination derived from the use of D-H model in robot calibration procedures. In (Roth et al., 1987), three levels of calibration for robots are established: joint level, kinematic model level and dynamic model level. The work presented in this chapter is focused on the second level of calibration. Thereby, the possible influences in the final measurement error of an AACMM will be studied to determine the mathematical model parameters. Those influences can be both geometrical -due to the kinematic model- and non-geometrical -due to other factors as assembly, damage in joints or transmission errors-. The approach of models which try to separate geometrical and non-geometrical errors has been a constant in literature on robot arms, without any apparent success from the point of view of a generalizable model regarding both robots and AACMMs. Based on the mathematical separation of both influences through the consideration of additional parameters in the model, the determination of a set of parameters related to each type of modeled error is carried out following optimization techniques without maintaining a direct relation between the real physical parameters and those obtained by optimization. Thus, these parameters cannot be considered more than an adjustment to the captured data which minimizes the positioning error in the case of robots but which do not have any numerical restriction to maintain the relation between results and real physical parameters, not being justifiable in many cases additional parameters that cause redundancy and complicate unnecessarily the mathematical model. In (Goswami & Bosnik, 1993) the relationship between the mathematical results and the physical reality, along with the influence of the redundancy in model parameters on a calibration method, are studied. The detection of the error sources is carried out by the way the model is approached. However, traditionally, the second calibration level only studies the identification of geometric parameters, whereas the non-geometrical and temperature influences are considered in a different way from model to model.

Regarding the D-H model itself, Hayati and Mirmirani (Hayati, 1983; Hayati & Mirmirani, 1985) outlined the discontinuity and non-proportionality of the model proposed by D-H for robotic arms with parallel or nearly parallel joint axes. In these cases the elimination of a parameter and the inclusion of a new rotation parameter are proposed. Many other authors have tried to solve the problem of the model proposed by D-H. In (Hsu & Everett, 1985) a modification of the D-H parameterization is proposed, adding a fourth joint parameter which describes with a variable the reference frame position in the direction of the previous frame axis. Subsequently, new approximations considering the general equation of rigid body spatial movement (Mooring & Tang, 1984) to model transformations between joints appeared, solving the indetermination of the D-H model but introducing redundant
parameters. However, it is desirable, and indeed essential for many researchers, that a calibration model is free of redundancies in order to identify each error with a parameter, to establish reliable correction methods and even to improve the machine design. This assumption cannot be generalized, since to detect an error and to find its relation to a particular source depends on many other factors. Although (Everett et al., 1987) and (Everett & Suryohadiprojo, 1988) describe the maximum number of independent parameters necessary in order to define the geometry of an arm depending on the type of joints, redundancy may exist. In (Goswami et al., 1993), a series of considerations regarding the minimum number of parameters to describe the kinematics of an arm can be found, along with the need for redundancy in certain robot configurations in order to maintain the physical-mathematical link in model parameters.

The specific inclusion of non-geometrical errors effect on a robot calibration process by mean of model parameters or series approximation of them appears in (Mooring, 1983) and (Whitney et al., 1984), beyond considering the arm errors in terms of variation of geometrical parameters. In (Chen & Chao, 1986) there is an approximation to the non-geometrical errors model based on the mathematical model proposed by Sheth and Uicker (Sheth & Uicker, 1971), which, like the previous ones, allows maintaining the nominal positions of the joints reference frames, since it uses different frames to model the geometrical and non-geometrical errors. In this way, the definition of the nominal model does not require any particular modeling procedure, since the errors are modeled by way of new matrices and transformations to the nominal systems. Subsequently, the inclusion of intermediate error matrices has been given extensive consideration in the biographical resources on robots to model non-geometrical errors. In (Drouet et al., 2002), an efficient method to consider any kind of errors is shown, including an intermediate matrix which relates joint nominal frame position with a second joint frame in its real position.

Stone and others (Stone et al., 1986) proposed a model, known as S-model, which adds two parameters to the basic D-H notation in order to allow variable positioning of the reference frames. Again, a solution to the indetermination problem is obtained, resulting in a complete but non-proportional model. Hollerbach and Wampler (Hollerbach & Wampler, 1996) later proposed a model based on a mix of the D-H parameters and the Hayati notation, using model parameters depending on the type of joint in order to avoid indeterminations. In the search for a model which fulfils the three basic properties to be used for calibration (Everett et al., 1987), Zhuang et al. (Zhuang & Roth, 1992) proposed the CPC model. Based on the idea that an incomplete model can be made complete by adding parameters, from the D-H model, they considered two more parameters to model the arbitrary position of the frames. Finally, models which consider the physical reference position of the encoder zero and which determine the relation between this position and the initial nominal position of the model (Mooring & Tang, 1984; Mooring, 1983; Park & Brockett, 1994) should be considered as the last main group of kinematic models.

Apart from the modeling techniques presented, all of them with later approximations in bibliography, and those which combine parameters associated to geometrical and non-geometrical error (Caenen & Angue, 1990; Vincze et al., 1999)– given the difficulty of modeling separately –, one of the current modeling trends involves an approximation to the final error based on the simplest kinematic models (Alici & Shirinzadeh, 2005). In this way, they do not try to explicitly model each parameter based on a parametric model which includes the error. So far, the models reviewed increase in mathematical complexity when
modeling each supposedly identifiable error by way of parameters not directly measurable. Once the value of these parameters has been established by optimization, it is possible to correct the final position of the end of the arm through an error model using off-line correction tools. Using different linear and non-linear regression mathematical tools, such as polynomials, Bezier curves, Fourier series, wavelets, neural networks, etc..., it is possible to make an approximation to the arm error in line with the error characteristics observed in different positions of the work space. The non-parametric nature of these tools makes more difficult to link the geometrical and non-geometrical parameters of a model with the physical reality of the arm. Most applications which use regression as an approximation to errors try to combine traditional modeling and regression techniques to allow the error to be linked in some way to measurable arm parameters.

In this way, despite not fulfilling the conditions of proportionality and equivalence (Everett et al., 1987), the D-H model avoids redundancies and perfectly describes the kinematics of a measurement arm, not presenting indeterminations in practically all AACMMs on the market, whose dual joints define consecutive perpendicular axes. For this reason, in the presented work the AACMM has been modeled by way of D-H when dealing with the identification procedure, although it is easily generalizable to any kinematic model.

3. AACMM Kinematic model

AACMMs make up a special group within coordinate measurement due to their special characteristics and differences with regards to traditional coordinate measuring machines (CMMs). While the CMMs, whether they are bridge, gantry or horizontal arm type, have a cartesian configuration which allows the measurement of the physical displacement of each of the three linear axes, in the AACMMs the measured point is a result of a series of coordinate transformations, depending on the model. AACMMs adopt the kinematic structure and model of robotic arms, and, similarly, are made up of a series of straight sections linked by rotary joints which provide them with the dof necessary to reach the required measurement positions. The differences compared to robotic arms are great, both in terms of accuracy and functionality, paying special attention to the accuracy of the sensors, materials used and dynamic conditions. Moreover, another important difference compared to CMMs and robotic arms is their manual and portable operation, instead of having machine axes automatically controlled.

The growing use of the AACMMs has been accompanied by an absence of standards on verification and calibration procedures, both from the point of view of the user and of the manufacturer. Traditionally, each AACMM manufacturer has adopted its own evaluation procedures. Firstly, it is necessary to determine the value of the parameters of the kinematic model of the arm. To this end, each manufacturer uses its own methods depending on the model and parameters implemented in each arm. Both the mathematical model considered and the method used to identify parameters constitutes restricted information which is not available to the final user. These evaluation methods are based on the procedures set out by the predecessors of the three main standards for performance evaluation in current CMMs, UNE-EN ISO 10360, ASME B89.4.1 and VDI/VDE 2617. However, the special characteristics of the AACMMs require different verification procedures. While for a CMM an X, Y, Z point clearly defines a position of the three machine axes, for an AACMM the possible positions of its elements to achieve a fixed point defined in the measurement volume are practically
infinite, according with its inverse kinematics. Moreover, for CMMs, besides the general evaluation methods, it is possible to carry out evaluation tests in order to extract the positioning errors to finally implement correction models (Trapet & Wäldele, 1991). The CMM evaluation tests achieve a high level of maintenance of the physical-mathematical relations between the error model parameters and the error physically committed by the machine. This is not the case of AACMMs, in which the application of these models does not make sense, given the difficulty of directly relating the error committed with the model parameters, which are obtained by using optimization procedures.

ASME B89.4.22-2004 and VDI/VDE 2617-9:2008 are nowadays the only standards existing in the field of AACMM verification. They recommend the methods which should be followed for reliable performance evaluation of the measurement arms. These documents deal with complete standardization in this field. They are focused on contact measuring with active and passive probes, covering the common measurement arm applications. In order to uniformize and eliminate ambiguity in evaluation methods for AACMMs, these standards fulfil their function, without making any indications with regards to parameter identification methods, calibration or correction.

The static calibration of an AACMM establish a parametric model of its kinematic behavior in order to determine, numerically, the relationship between the joint variables and the probe position for any arm posture. A direct kinematic model takes the form of (1).

\[ y = f(\theta, q) \]  

with \( i=1, \ldots, n \) for an arm with \( n \) rotating joints. This model calculates the position and orientation of the AACMM probe \( y \), according to the value of the joint variables \( \theta_i \) and to the equations of the model defined in \( f \), which depend on the parameters vector \( q \). This parameters vector contains the geometric parameters of the model, which must be optimized in order to obtain the lowest possible measurement error. Depending on the chosen kinematic model, the way the equations are obtained in \( f \) changes, along with the number of geometric parameters necessary to be included in \( q \). The D-H basic model uses four parameters \((d_i, a_i, \theta_i \text{ and } \alpha_i)\) to model the transformation of coordinates between successive reference systems. The homogeneous transformation matrix between frame \( i \) and \( i-1 \) of equation (2) depends on those four parameters. Calculating successive transformations of coordinates, by pre-multiplying the transformation matrix between a frame and the previous one, it is possible to obtain the global transformation matrix of the arm, which obtains coordinates of the center of the probe sphere with respect to the base of the AACMM. In this manner, considering 0 as the global fixed reference system of the base and reference frame 6 moving with the rotation of the last joint, the desired homogeneous transformation can be obtained by way of equation (3). Apart from the four joint parameters indicated, it is necessary to consider an extra joint parameter \( \theta_0 \) which will define the relationship between the encoder physical reference mark position of each joint and the initial position considered in the definition of the kinematic model.
\begin{equation}
\begin{bmatrix}
\cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & \alpha_i \cos \theta_i \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & \alpha_i \sin \theta_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{equation}

\begin{equation}
{^0}T_6 = {^0}A_1 {^1}A_2 {^2}A_3 {^3}A_4 {^4}A_5 {^5}A_6 \quad \overline{X}_{AACMM} = \overline{X}_{\text{Probe}}
\end{equation}

Thus, the joint variable \( \theta_i \) of the model is related to the rotation reading provided by the encoder through equation (4), where \( \theta_0 \) must also be identified from its nominal value defined for the initial position.

\begin{equation}
\theta_i = \theta_{\text{enc}} - \theta_0
\end{equation}

The use of a reference system defined in the robotic hand responds generally to the need to have a frame which follows the characteristic directions of the tool, which will be subsequently controlled. In this case, since the aim is to obtain the sphere center coordinates of a static probe, the use of a reference system attached to the probe is omitted, assuming the link between the probe center and frame 6 to be a translation expressed by the coordinates \( X_{\text{Probe}}, Y_{\text{Probe}}, Z_{\text{Probe}} \) of the sphere center with respect to this frame. With this, the model has 4 joint parameters plus the three coordinates of the probe sphere center in frame 6, what makes a total of 27 parameters for the 6 dof AACMM considered. The initial values taken for the arm model parameters are shown in Fig. 1.

![Model definition posture of FARO AACMM with D-H convention. Initial values of parameters.](image)

**4. Parameter identification**

The AACMM used in the present work is a 6 dof Sterling series FARO arm with a typical 2-2-2 configuration and a-b-c-d-e-f deg rotation, with a nominal value of \( 2\sigma = \pm 0.102 \text{ mm} \).
obtained in a single-point articulation performance test of the arm manufacturer, without specifying the number of positions of the kinematic seat and points captured. Simple evaluation tests have been carried out to evaluate the current state of the measurement arm using the manufacturer capture software and model, in order to know the expected initial values for accuracy and repeatability with the arm in the current situation. Using a spherical-point passive probe of 6 mm in diameter, 14 spheres of a ball-bar gauge have been measured in a single gauge position, obtaining a mean deviation of the lengths measured with regards to the gauge lengths of 0.854 mm. On the other hand, a single-point articulation performance test has been carried out using a conical seat to capture 100 points in different AACMM orientations for a single kinematic mount position. The result obtained in this test was 0.347 mm for 2σ, with respect to the mean point of the captured data. Without the need for further initial evaluation tests or complete tests in accordance with ASME B89.4.22-2004, it can clearly be appreciated that the AACMM is out of calibration with the manufacturer parameters and model. Hence, it will be taken as the initial situation when dealing with the proposed calibration procedure.

4.1 Data capture setup and evaluation

Once the mathematical model is defined, the next step involves the capture of nominal coordinates in the workspace of the AACMM. All the calibration procedures, both for robotic arms and AACMMs, establish a system which acquires coordinates or nominal distances in the workspace, in order to capture points which allow the error to be evaluated and minimized.

Current identification procedures for robotic arms are based on a small set of positions where the coordinates of the robot hand are captured discretely. In (Alici & Shirinzadeh, 2005), based on the results obtained in (Driels & Pathre, 1990) and (Borm & Menq, 1991), 85 discrete robot positions were selected to cover the entire work range and extrapolate the results obtained to any arm position. The selection of capture positions to identify kinematic parameters must be preceded by tests which characterize the influence of each joint on the final error, to finally choose positions in accordance with this influence. For this reason, the application of a number of specific positions is not generalizable from one robotic arm to another, since the errors committed by each robot will depend on their configuration and assembly defects. In (Chunhe et al., 2000), the method described to identify the geometrical parameters takes 30 capture positions, without any apparent justification. It is common to consider a very different number and type of robot positions in the bibliography, constituting a very important step of the procedure for the extrapolation of the results. The referenced procedures successfully carry out their task, and hence both the capture of points and the subsequent test positions are captured discretely in robot configurations which are very similar to each other and also similar to the common robot work positions. This means that the positioning accuracy of the robot is improved globally in common work configurations. Since the optimization of the kinematic parameters is a least squares method, the adjustment of the parameters which minimize the error in the robot identification positions will mean small errors occur in positions similar to those of capture, but will cause bigger errors in very different positions.

In this work, a continuous data capture method has been developed. This technique allows the massive capture of arm positions corresponding to several points of the workspace. To this end, a ball-bar gauge of 1.5 m long was placed in 7 positions within the workspace of
the arm in order to cover the maximum number of possible AACMM positions, to subsequently extrapolate the results obtained throughout the volume. Fig. 2 shows the considered positions for the bar in a quadrant of the workspace. The ball-bar comprises a carbon fiber profile and 15 ceramic spheres of 22 mm in diameter, reaching calibrated distances between the centers with an uncertainty, in accordance with its calibration certificate, of \((1+0.001L)\mu m\), with L in mm. The ball-bar profile is made of a carbon fiber layer having a balanced pair of carbon fiber plies embedded in a resin matrix, with a nominal coefficient of thermal expansion (CTE) between \(\pm 0.5 \times 10^{-6} \text{ K}^{-1}\). The position of the fibers in the profile allows compensating this coefficient, obtaining a mean CTE near zero.

Fig. 2. Ball bar positions for each quadrant. Sample of position P6.

The capture of data both for calibration and for verification of the arms is usually performed by way of discrete contact probing of surface points of the gauge in order to obtain the center of the spheres from several surface measurements. This means that the time required for the capture of positions is high, and then, identification is generally carried out with a relatively low number of arm positions. In the present work, two specific probes, capable of directly probing the center of the spheres of the gauge without having to probe surface points, were designed. As seen in Fig. 3, one of the probes comprises three tungsten carbide spheres of 6 mm in diameter, laid out at 120° on the end of the probe. Since the ceramic spheres of the gauge have a diameter of 22 mm, it is necessary to establish the geometrical relationships in order to ensure the proper contact of the three spheres and the stability of this contact. In general, in order to maintain this stability, it is recommended a contact between the spheres of the kinematic mount and the sphere to fit between them at 45° with respect to the plane formed by the centers of the mount spheres. Thereby, the centering of the probe direction with regards to the sphere center is ensured, making this direction cross it (Fig. 3) for any orientation of the probe. Thus, in this case, it is possible to define a probe with zero probe sphere radius and with the distance from the position of the housing to the center of the probed sphere of 22 mm as length, allowing direct probing of the sphere center when the three spheres of the probe and the sphere of the gauge are in contact.

On the other hand, we have reproduced the process of data capture for all positions of the gauge with a self-centering active probe. This probe, specifically designed for probing spheres, is composed of three styli positioned to form a trihedron with their probing directions. Each individual stylus has been designed by using a linear way together with a LED+PSD sensor combination to measure its displacement. From the readings of the displacement of the three styli and its mathematical model, the probe is able to get the center of the probed sphere in its reference system. So, it is necessary to link the reference system
with the last reference system of the kinematic chain of the AACMM, to express the center of the probed sphere in the global reference system. The method followed to determine the relationship between the two frames is described in the last section of this chapter, since this probe is particularly suitable for parameter identification procedures in robots.

![Diagram](image)

**Fig. 3.** Passive and active self-centering probes used in the capture of AACMM positions for parameter identification

Besides characterizing and optimizing the behavior of the arm with regards to error in distances, its capacity to repeat measurements of a same point is also tested. Hence, an automatic arm position capture software has been developed to probe each considered sphere of the gauge and to replicate the arm behavior in the single-point articulation performance test, but in this case, to include the positions captured in the optimization from the point of view of this repeatability. The rotation angle values of the arm joints for each position, reached in the continuous probing of each sphere, are stored to obtain the coordinates of the measured point with respect to the global reference system for any set of parameters considered. In this way, it is possible to capture the maximum possible number of arm positions, thus covering a large number of arm configurations for each sphere considered. Fig. 4 shows the capture scheme followed. As a general rule, the indicated trajectories will be followed for a probed sphere. Moreover, positions causing maximum variation of the arm joints in all the possible directions at the start, end and midpoint of each trajectory will be searched. The capture will be continuous and we will try to capture data in symmetrical trajectories in the sphere, in order to minimize the effect of probing force on the gauge. Thereby, around 400 rotation angle combinations \( \theta_{i} \) (\( i=1,\ldots,6 \)) have been captured for the joints to cover the positions of the arm probing the center of the measured sphere. With this configuration, 4 spheres of the gauge in each of the 7 positions considered for each of the quadrants of the arm work volume were probed. The measuring of a sphere center with the self-centering probe from different arm orientations should result in the same point measured.
The unsuitable value of the kinematic parameters of the model will be shown by way of a probing error. This error produces different coordinates obtained for the same measured point in different arm orientations. In this manner, by probing four spheres of each position of the gauge with an approximate average of 400 arm positions per sphere for the passive self centering probe (250 for the active self centering probe), a series of 400 XYZ coordinates measured for each sphere center will be obtained. The deviations, initially due to the value of the parameters of the model between these 400 points in each sphere, will be used to characterize and optimize the arm point repeatability. In addition, in each gauge location 6 nominal distances between the four probed spheres are reached (Fig. 5a). The nominal distances of the gauge will be compared to the distances measured by the arm. Since an average of 400/250 centers per sphere are captured, the mean point of the set of points captured will be taken as the center of the sphere measured, in order to determine the distances between spheres probed by the arm (Fig. 5b). Thereby, a method for the subsequent combined optimization of the AACMM error in distances and point repeatability is defined.

In order to analyze the metrological characteristics of the AACMM for a specific set of parameters, both the error in distances of the arm and the dispersion of the points captured for each probed sphere center will be studied. As can be seen in Fig. 5, the parameters to evaluate are the six distances between the centers of the four spheres probed by bar location and the standard deviation of the points captured for each of the spheres probed. The 3D
distance between pairs of spheres, based on the mean points calculated in each of them, is shown in equation (5).

$$D_{jk} = \sqrt{\left(\overline{X}_j - \overline{X}_k\right)^2 + \left(\overline{Y}_j - \overline{Y}_k\right)^2 + \left(\overline{Z}_j - \overline{Z}_k\right)^2}$$  (5)

in which $D_{jk}$ represents the Euclidean distance between sphere j and sphere k of the gauge i location, with coordinates corresponding to the mean of the points captured for sphere j and sphere k according to equation (6).

$$\overline{X}_j = \frac{\sum_{m=1}^{n_j} X(m)_j}{n_j}$$  (6)

In equation (6), $n_j$ is the number of angle combinations captured for sphere j in identification position i of the gauge, analogously for the coordinates Y and Z. In this manner, considering $D_{0jk}$ as the nominal distance between spheres j and k obtained in the gauge calibration table, it is possible to calculate the error in distance between spheres j and k in location i in accordance with equation (7).

$$E_{jk} = \sqrt{(D_{jk} - D_{0jk})^2}$$  (7)

Since there are 4 spheres per gauge location, a total of 6 distances in each one are calculated, bringing a total of 42 distances. Considering in the previous equations $i=1,\ldots,7$, and j and k covering each one of the spheres of the gauge for each position (1, 6, 10 and 14), eliminating the terms in which $j=k$ comes about, and considering that $D_{jk} = D_{kj}$, it is possible to evaluate the errors in distances obtained for all the gauge locations. Moreover, as a second quality value of a set of parameters the maximum standard deviation of the points captured in each of the measured spheres is chosen.

$$2\sigma_{X_j} = 2\sqrt{\frac{\sum_{m=1}^{n} \left(X(m)_j - \overline{X}_j\right)^2}{n_j - 1}}$$  (8)

In equation (8), $\sigma_{X_j}$ represents the standard deviation in coordinate X of the points obtained for sphere j of position i. Analogously for the coordinates Y and Z. Following the diagram presented in Fig. 6 we obtain the values of all the possible standard deviations and distances for the data captured, considering a given set of parameters. The quality indicators for the self centering passive probe of the initial set of parameters considered in Fig. 1 are shown in Fig. 6. In the first column is shown the maximum error in distances for all the positions of the gauge, the position of identification in which it is produced and the distance at which the maximum error has been obtained. Likewise, the minimum error and its location, and the mean value of all the errors in distance observed are shown. In the case of standard
deviation, Fig. 6 also includes the coordinate in which the value has been obtained, since both parameters are calculated separately for the three point coordinates. As can be seen, the values obtained for the initial set of parameters are large, as was expected given the initial lack of adjustment of the AACMM kinematic parameters.

### 4.2 Non-linear least squares identification

Kovac and Klein present in (Kovac & Klein, 2002) an identification method based on nominal data obtained with the gauge developed in (Kovac & Frank, 2001). This method uses an objective function as used in robots, along with commercial software to identify kinematic parameters, without focusing the study on the particularities of the measurement arms. In (Furutani et al., 2004), Furutani et al. describe an identification procedure for measurement arms and make an approximation to the problem of determination of AACMM uncertainty. This study is centered on the type of gauge to be used according to the arm configuration and analyses the minimum number of necessary measurement positions for identification, as well as the possible gauge configurations to be used. Again, this work does not specify the procedure to obtain the parameters of the model, nor the type of model implemented, and does not show experimental results for the method proposed. In (Ye et al., 2002), Ye et al. develop a simple parameters identification procedure based on arm positions captured for a specific point of the space. In (Lin et al., 2006), Lin et al. perform an error propagation analysis from the definition of several error geometrical parameters. This study shows the influence of the error parameters defined by its authors in their model and, even though it is not generalized to the geometrical errors propagation from the parameters identification, it shows an effective method to elaborate a software-based error correction procedure.

As indicated in section 3, the kinematic model implemented in the measurement arm can be described, for any arm position, by way of equation (9), based on the formulation of direct kinematic problem.
in which \( p = [\mathbf{X} \, \mathbf{Y} \, \mathbf{Z} \, 1]^T \) are the coordinates of the point measured with respect to the arm global reference frame at the base, corresponding to the value of the geometrical parameters and to the joints rotation angles in the current arm position. There are many alternatives when dealing with an optimization procedure, although the most widely used in the field of robot arms and AACMMs are the formulations based on least squares fitting. Given the non-linear nature of the arm kinematic model, it is not possible to obtain an analytical solution to the problem of parameter identification. Therefore, it is necessary to use non-linear optimization iterative procedures. In this way, for the mathematical formulation of the optimization method it is common to define the objective function to minimize in terms of square error components. Based on the nominal coordinates reached by the gauge and those corresponding to the points measured, we can obtain the arm measurement error as the Euclidean distance between both points, as shown in equation (5), although applied to the difference between the measured point and the nominal point. Since the identification procedure both in robots and in AACMMs is based on the capture of discrete positions within the workspace, all the reviewed optimization procedures use equation (10) as basic objective function to minimize.

\[
\phi = \sum_{i=1}^{m} [\Delta p]^T [\Delta p]
\]

Equation (10) quantifies the error in distances between the nominal point and the point reached for all the positions captured, formulated as the quadratic sum. There are variants of this expression in those cases in which a capture procedure based on nominal distances is proposed, where the measurement error of the arm is obtained in accordance with equation (7) for each distance considered, to obtain the errors in all the distances and with the target function being the quadratic sum.

In this work, in order to choose the objective function to be minimized, consideration has been given to the error in distances presented in equation (7) for the 42 distances measured. Therefore, it is possible to evaluate all the combinations of six values of joint angles captured for each set of kinematic parameters, and to obtain the centers as the mean value of the coordinates corresponding to each sphere as shown in equation (6). Finally, we evaluate all the distances in each iteration of the optimization procedure. The objective function can be formulated as the quadratic sum of all the errors in distances calculated by way of equation (7). Hence an objective function similar to those commonly chosen in robot and AACMMs parameter identification is obtained.

Given the arm positions capture setup used, and the fact that point repeatability in any arm probe orientation is a very important parameter in order to characterize the metrological behavior, unlike traditional expressions, our objective function in equation (11) includes both the errors in distance and the deviation of the points measured in each sphere showing the influence of the volumetric accuracy and point repeatability, minimizing simultaneously the errors corresponding to both parameters.
In the objective function proposed, with the capture setup described, \( r = 7 \) positions of the ball bar and \( s = 4 \) spheres (1, 6, 10 and 14) per bar position. Again, in equation (11) it is necessary to consider the elimination of the terms in which \( j = k \), in order to avoid the inclusion of null terms or considering as duplicate the influence of the error on distances, taking into account that \( D_{j_k} = D_{k_j} \). The first term of equation (11) corresponds to the error in distances in position \( i \) of the gauge between sphere \( j \) and sphere \( k \), whereas the other terms refer to twice the standard deviation in each of the three coordinates for sphere \( j \) in position \( i \) of the gauge. Finally, again by mathematical formulation of the optimization problem, it is necessary to consider the sum of all the square errors calculated. With the objective function of equation (11), 126 quadratic error terms will be obtained to calculate the final value of the objective function after each optimization algorithm stage. This value will show the influence of the kinematic parameters as well as of the joint variables through the calculation of the points coordinates corresponding to the arm positions captured in both cases, active and passive probe. The Levenberg-Marquardt (L-M) method (Levenberg, 1944; Marquardt, 1963) has been chosen as optimization algorithm for parameter identification, given its proven efficiency in robot parameter identification procedures (Goswami et al., 1993; Alici & Shirinzadeh, 2005). The selection of a specific optimization procedure implies to avoid the influence of the mathematical method itself with regards to the data captured on the result. One of the most suitable methods to solve this problem is the L-M algorithm. Table 1 shows the AACMM kinematic model parameters finally identified, based on the initial values and for the objective function of equation (11) and the arm positions considered with the passive self-centering probe. Also, the error values obtained for the identified set of parameters for the passive self centering probe are shown in Table 1. Results of distance errors between centers have been obtained for each of the 6 distances materialized in each of the 7 ball bar positions for the two probes considered. Measured distances for each sphere in the 7 different positions were compared with the distances obtained with the ball bar gauge thus obtaining the error in distance (Fig. 7a), as well as the differences between the distance errors of the active and the passive self centering probes in all 42 positions that were considered (Fig. 7b). In Fig. 7b, a positive difference represents a smaller error in the active probe and in that case this probe is considered better than the passive one. In the case of positions 3, 4 and 7, three spheres were not measured, so a value of zero was assigned in the graphs. From Fig. 7a, we can observe that on average, the error made by the self-centering active probe was less than the one corresponding to the self-centering passive probe; the errors obtained with the active probe, when greater than those corresponding to the passive probe, can be associated to AACMM as it approaches its workspace frontier.
Kinematic calibration of articulated arm coordinate measuring machines and robot arms using passive and active self-centering probes and multipose optimization algorithm based in point and length constrains

Table 1. Identified values for the model parameters by L-M algorithm and quality indicators for these parameters over 7 ball bar locations with equation (11) as objective function. Data from passive probe.

<table>
<thead>
<tr>
<th>Joint</th>
<th>$c_i$ (mm)</th>
<th>$c'_i$ (*)</th>
<th>$c''_i$ (mm)</th>
<th>$c'''_i$ (*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.039832</td>
<td>-90.052248</td>
<td>-0.000062</td>
<td>-0.128434</td>
</tr>
<tr>
<td>2</td>
<td>0.102486</td>
<td>90.044751</td>
<td>47.891183</td>
<td>14.642165</td>
</tr>
<tr>
<td>3</td>
<td>0.097868</td>
<td>-90.020969</td>
<td>645.700223</td>
<td>-0.994088</td>
</tr>
<tr>
<td>4</td>
<td>-0.132013</td>
<td>90.068889</td>
<td>24.011312</td>
<td>-3.838998</td>
</tr>
<tr>
<td>5</td>
<td>0.057626</td>
<td>90.010114</td>
<td>8.16242900</td>
<td>83.770480</td>
</tr>
<tr>
<td>6</td>
<td>0.392776</td>
<td>-5.52698</td>
<td>0.150712</td>
<td>-0.817073</td>
</tr>
<tr>
<td>$X_{*}$ (mm)</td>
<td>$Y_{*}$ (mm)</td>
<td>$Z_{*}$ (mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.267276</td>
<td>1.3845807</td>
<td>54.65780</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance Error (mm)</th>
<th>2σ by Sphere (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max.</td>
<td>0.144</td>
</tr>
<tr>
<td>Cheming Post</td>
<td>POS2</td>
</tr>
<tr>
<td>Cheming Dist</td>
<td>D1</td>
</tr>
<tr>
<td>Min.</td>
<td>0.006</td>
</tr>
<tr>
<td>Cheming Post</td>
<td>POS1</td>
</tr>
<tr>
<td>Cheming Dist</td>
<td>D2</td>
</tr>
<tr>
<td>Med.</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. Comparison between passive and active self-centering probes with the identified parameters in each case over the identification data: (a) Error in distance of the centers measured, (b) Difference in distance errors.

The repeatability error values for all measured points are shown in Fig. 8a and 8b, for the self-centering active probe and self-centering passive probe respectively. These values represent the errors made in $X$, $Y$ and $Z$ coordinates of each one of the approximately 10000 points obtained with each probe, corresponding to the 7 positions of the ball-bar gauge with regards to the mean obtained for each sphere. The repeatability error value for each coordinate as a function of the 6 joint rotation angles is given by equation (12). This information can also be used to obtain empirical error correction functions as a function of the angles (Santolaria et al., 2008).

\[
\varepsilon_{Xijk}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = \bar{X}_{ij} - X_{ij}
\]
\[
\varepsilon_{Yijk}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = \bar{Y}_{ij} - Y_{ij}
\]
\[
\varepsilon_{Zijk}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = \bar{Z}_{ij} - Z_{ij}
\]
Fig. 8. Point repeatability errors for the optimal sets of model parameters over identification AACMM positions: (a) Active probe, (b) Passive probe.
It can be observed that the error made by the self-center active probe is a lot smaller than the error made by the self-center passive probe and that in both graphs the error shows an increment in the Z coordinate. This behavior in the Z coordinate, could be explained by the fact that, unlike what happens in the X and Y coordinates, there is no self-compensation effect in the gauge deformation due to the probing force in this coordinate.

In Fig. 9 we can observe the standard deviation corresponding to the 7 different positions in X, Y and Z for both types of probes. As expected, the standard deviation in the self-centering active probe is smaller than the one obtained with the self-centering passive probe, except as mentioned earlier, in the positions were spheres were not measured and a value of zero was assigned in the graph.

![Fig. 9. Standard deviation of the center of the spheres probed.](image)

In order to study the influence of the inclusion of the standard deviation on the objective function, we have complete optimizations taking as function only the terms corresponding to the error in distances for the 10,780 positions captured with the passive probe, as would correspond to a common objective function for parameter identification of robots.

\[
\phi = \sum_{i=1}^{r} \sum_{j=1}^{t} \left( D_{ij} - D_{ij}^0 \right)^2
\]  

(13)

Compared to the maximum and mean error obtained in Table 1, using equation (13) as objective function, a maximum error of 15 µm was obtained and a mean error of 5 µm for the same arm positions. However, for the parameters identified with the objective function of equation (13), the maximum value obtained for 2σ is 1.8932 mm compared to 0.249 mm.
obtained using equation (11), and the mean value is 1.009 mm. As can be seen in the results, an optimization equivalent to those commonly found in robots produces excellent results for errors in distance but inadequate results for range and standard deviation. Hence, to obtain a set of parameters which allows the arm to be repeatable in a point for any measurement orientation and not only in the orientation captured for optimization, it is necessary to consider the range or the standard deviation in objective function. There may exist cases of robot arms in which an optimization scheme without considering repeatability evaluation parameters is useful for work positions and orientations similar to those used in identification. However, in general for robots and always in the case of AACMM parameter identification, regarding the standard deviation results, the traditional objective functions should be completed with repeatability evaluation parameters, obtaining kinematic parameters that makes more reliable the generalization to the measurement volume of the error values obtained.

5. Generalization tests with the identified sets of parameters

The generalization of an identified set of parameters to the rest of the measurement volume involves the obtaining of deviation and error values smaller than the maximums obtained for the identification process for any arm position. For this reason, the use of at least one test position different to the identification positions is recommended. Thereby, the maximum error for the identification positions, in those cases in which a lower number of gauge or arm positions have been taken, has proven to be better than that finally considered as optimum. However, in these conditions, the evaluation of the identified parameters on positions not considered before has resulted in worse values than those obtained in identification with consideration of all the positions captured. For this reason, the use of all the positions captured as representative of the arm measurement volume was the option taken. Thus, a sufficiently representative set is obtained in order to absorb all the influences on the final error and to obtain a set of kinematic parameters which make the error obtained in identification be realistic and truly the maximum for the arm for any position in the measurement volume. As is shown in Table1, a maximum error of 144 µm and a mean error of 66 µm are obtained for all the measurement volume of quadrant 1 with the passive probe. This can be compared to maximum error in distances (0.854 mm) and to mean error (0.262 mm) obtained in one single evaluation position in the initial situation. In normal operation of the arm - probing discrete points of the center of the sphere probe - the error obtained with the identified set of parameters for the passive probe will be normally around the mean value of 66 µm, producing the maximum error in certain specific arm positions.

Once the optimization process is complete, as the final stage of the presented parameter identification procedure, it is necessary to evaluate the behavior of the arm with the optimum set of parameters on arm positions different to those used during identification. The more similar the evaluation positions subsequent to those used in identification, the better the results. Hence, it is necessary to find different measurement arm positions to evaluate the level of fulfillment of the error values obtained in other measurement volume positions.

In this case, as test bar location subsequent to identification was chosen in the upper part of quadrant 1. Based on the same orientation of position P1, the bar was rotated approximately 25° both horizontally and vertically. For this ball bar location, angle combinations
corresponding to the arm positions probing the centers of the 14 gauge spheres were captured for both probes. In this way around 6,000 arm positions were captured for the test position for each probe, which is a reliable check of the measurement arm error on positions not used. Table 2 shows the error values obtained for the 14 test position spheres.

![Image](57x437 to 424x554)

Table 2. Quality indicators for the identified sets of model parameters over 14 spheres of ball bar test location: (a) Passive probe, (b) Active probe.

As can be seen in the results obtained, the mean error is of the same order as in the identification positions and the maximum is below the maximum obtained in that case for both probes. It should be considered that the maximum values of standard deviation are obtained in the end spheres of the gauge, in more forced positions of the measurement arm. Given that we check the error values in one single test position of the gauge, better results in the arm behavior could be expected. However, it should be remembered that, for this ball bar location, over 6,000 arm positions are evaluated, both from the point of view of point repeatability and error in distances based on the calculation of the mean point probed for each sphere. For this reason, as the conclusion of the evaluation test, the importance of the data captured should be again emphasized. A high number of arm positions, different to those chosen for identification, should be searched in the way recommended in normalized evaluation test, in order to conclude with the acceptance of the identified model parameters. In this case, the number of arm positions considered for evaluation is high compared to those used in identification, obtaining values below the maximum error, meaning the arm behavior is verified in accordance with these maximum errors within the volume considered.

### 6. Application to kinematic calibration of robot arms with active self-centering probes

This section describes the application of the identification method presented to robot arms. Due to its automatic movement, it is not appropriate in this case to probe the spheres of the gauge with a self-centering passive probe. Influences of probing force or incorrect position of the robot's hand are removed by using a self-centering active probe (Fig. 10).
Both the data capture procedure and the identification are the same as the ones presented in section four for AACMMs, so it is necessary to capture points of several spheres of the gauge at various gauge positions distributed within the workspace of the robot. This makes it necessary manual probing of the first and last sphere in each gauge position, to know its center coordinates in robot reference system. Once these coordinates are known, it is possible to automatically generate the measuring program for a gauge position. Thus, from the nominal positions of the spheres of the gauge expressed in robot reference system, it is possible to generate the probing trajectories of each sphere through the inverse kinematics model (Fig. 11). As a result, the inverse model will provide the position and orientation of the robot's hand. At each point of the trajectory, by inverse kinematics, it should be captured the maximum possible robot positions for this position and orientation of the hand. This will capture all the possible influences of the joints on the position and orientation at each probing point.

After probing the selected spheres of all the positions of the gauge, we will have information related to both volumetric accuracy and point repeatability. So, with the objective function of equation (11) and the described procedure it is possible to identify the parameters of the kinematic model of the robot. This procedure will lead to a set of parameters that will
improve the accuracy of the robot throughout its workspace, considering also its ability to reach a point from many different postures, unlike the procedures that identify parameters only in some specific working positions of the robot.

6.1 Linking the mathematical model of the probe with the kinematic model of the robot

As discussed above, the self-centering active probe obtains in its reference system the coordinates of the probed sphere center from the readings of displacement of its three styli. Therefore it is necessary to obtain the homogeneous transformation matrix that relates the coordinates of a sphere expressed in the probe reference system with the coordinates of the same sphere in the last reference system of the robot arm, once mounted the probe. This will provide the coordinates of the sphere center in the global reference system of the robot. This can be achieved following several methods, all of them based on least squares.

This section presents the method for obtaining this homogeneous matrix from the probing of a single sphere. This self-calibration method will allow obtaining the matrix that relates the two reference systems without knowing the coordinates of the probed sphere in robot reference system. Assuming a robot with six joints, the equation (14) obtains the coordinates of the center of a sphere in robot reference system from the coordinates of the points probed expressed in the probe reference system.

$$
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = T_{\text{ROBOT}}^i \cdot P \cdot
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
$$

(14)

where $[X\ Y\ Z\ 1]^T_{\text{PROBE\_i}}$ are the coordinates of the probed sphere expressed in the probe reference system; $P$ is the 4x4 matrix that relates the probe frame with the last joint frame of the robot, constant for any position; $T_{\text{ROBOT}}^i$ is the 4x4 robot matrix in the probing pose $i$, that relates coordinates in the last joint frame of the robot with coordinates expressed in the base frame; and $[X\ Y\ Z\ 1]^T_{\text{ROBOT}}$ are the coordinates of the probed sphere center in robot base frame, invariants for any position and orientation of the robot.

In equation (14), both robot matrix and the coordinates of the points probed expressed in the probe frame are known for each probing posture, while $P$ matrix and the coordinates of the sphere expressed in robot reference system are unknown. Thus, it is possible to propose a least squares resolution for the overdetermined system of equation (15).

$$
Aq = b
$$

(15)

where

$$
A =
\begin{bmatrix}
- t_{1}\ t_{1}\ t_{2}\ t_{2}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ 0\ 0 \\
0\ t_{1}\ t_{1}\ t_{2}\ t_{2}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
- t_{2}\ t_{2}\ t_{2}\ t_{2}\ t_{2}\ t_{2}\ t_{2}\ t_{2}\ t_{2}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
- t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
- t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ t_{3}\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
\end{bmatrix}
$$

(16)
As a result of the resolution of the previous system, the vector
\[
q = \begin{bmatrix}
  p_{11} & p_{12} & p_{13} & p_{14} & p_{21} & p_{22} & p_{23} & p_{24} & p_{31} & p_{32} & p_{33} & p_{34} & X_g & Y_g & Z_g
\end{bmatrix}
\]
is obtained. This vector contains the searched terms of the matrix and also the coordinates of the sphere center in robot reference system. It is possible to follow the same resolution strategy but probing more than one sphere and introducing the corresponding nominal distance constrains between gauge spheres, leading to a more accurate solution in fewer iterations.

7. Conclusions

In this chapter, a comparison between two different probing systems applied to capturing data for parameter identification and verification of AACMM is presented. Besides the probing systems traditionally used in the verification of AACMM, self-centering probing systems with kinematic coupling configuration and self-center active probing systems have also been used for the presented method. Such probing systems are very suitable for use in verification procedures and capturing data for parameter identification, because they drastically reduce the capture time and the required number of positions of the gauge as compared to the usual standard and manufacturer methods. These systems are also very suitable for their capacity of capturing multiple positions of the AACMM for a single gauge position, so that the accuracy results obtained after a procedure of identification or verification are more generalizable than those obtained with the traditional probing systems.

The effect of auto compensation of the gauge deformation has been shown by properly defining the trajectories of capture or the direction of probing during the process of capturing data. Moreover, it has been demonstrated that the smallest influence of the probing force is obtained in the case of the self-centering active probe, this being the most adequate system in tasks of verification or capturing data for the identification of kinematic parameters if no configuration or application restrictions are imposed, specially for robot arms.

8. References

As a result of the resolution of the previous system, the vector verification are more generalizable position, so that the accuracy results obtained after a procedure of identification or suitable for their capacity of capturing multiple positions of the AACMM for a single gauge compared to the usual standard and manufacturer methods. These systems are also very suitable for use in probing systems traditionally used in the verification of AACMM, self-centering probing data for parameter identification and verification of AACMM is presented. Besides the In this chapter, a comparison between two different probing systems applied to capturing

7. Conclusions

The effect of auto compensation of the gauge deformation systems.

8. References


Robot manipulators are developing more in the direction of industrial robots than of human workers. Recently, the applications of robot manipulators are spreading their focus, for example Da Vinci as a medical robot, ASIMO as a humanoid robot and so on. There are many research topics within the field of robot manipulators, e.g. motion planning, cooperation with a human, and fusion with external sensors like vision, haptic and force, etc. Moreover, these include both technical problems in the industry and theoretical problems in the academic fields. This book is a collection of papers presenting the latest research issues from around the world.

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