Robust Shaping Indirect Field Oriented Control for Induction Motor

M. Boukhnifer, C. Larouci and A. Chaibet
Laboratoire Commande et Systèmes, ESTACA, 34-36 rue Victor Hugo, 92 300 Levallois-Perret, France
{mboukhnifer, clarouci, achaibet}@estaca.fr

1. Introduction

Over the past years, thanks to the systematic use of digital microprocessors in industry, we have seen a very significant development in the regulation controls of the asynchronous machine. The latter is widely used in industry for its diversity of use and its ability to withstand great variations in its nominal regime. Currently, several types of control are proposed. Nonlinear controls such as linearization input-output (Benchab & Edwards, 2000) (Chan et al., 1990) (De Luca & Ulivi, 1989) (Marina & Valigi, 1991), the controls resulting from the theory of passivity (Nicklasson et al., 1997) (Gokdere, 1996) which generally require the measurement of all system states (currents and flux). As the flux of the motor cannot be measured, a great part of the literature is devoted to the control problem coupled with a nonlinear flux observer (Kanellakopoulos et al., 1992). Other controls using only the exit returns (rotor speed and stator currents) were developed resulting in controls of the passive type (Abdel Fattah & Loparo, 2003), other controls using the technique of backstepping or the techniques derived from the orientation of the flux field (Peresada et al., 1999) (Barambones et al., 2003).

The parameters of such controls must be selected in order to ensure total stability for a given nominal running and nominal values of the parameters. Thus, different robust controls with parameter uncertainties, such as the discontinuous or adaptive controls, were developed (Marina et al., 1998). These techniques adapt the controls to the variations of resistances and the load couple. A control which is very common in industry is the indirect field oriented control (FOC) based on the orientation of the field of rotor flux. This control allows the decoupling of speed and flux, and we obtain linear differential equations similar to the D.C. machine. The regulation is carried out finally by simple controller PI (cf Fig. 1).

However, the decoupling observed is only asymptotic. The behaviour of the transient regime and the total stability of the system remain a major problem. In addition, the modifications of parameters such as rotor resistance or resistive torque deteriorate the quality of decoupling.

In this paper, we propose a diagram of $H_\infty$ regulation, linked to the field oriented control allowing a correct transient regime and good robustness against parameter variation to be
ensure. This Paper is divided into several parts: The first one describes the model of the asynchronous machine, using the assumptions of Park. In the second part, we present the field oriented control principles, as well as regulation simulations, using PI. The third part is devoted to the problems of the $H_\infty$ control and the loop shaping design procedure approach originally proposed in (McFarlane et al., 1988) and further developed in (McFarlane & Glover, 1988) and (McFarlane & Glover, 1989) which incorporates the characteristics of both loop shaping and $H_\infty$ design. Specifically, we make use of the so-called normalized coprime factor $H$, robust stabilization problem which has been solved in (Glover & McFarlane, 1988) (Glover & McFarlane, 1989) and is equivalent to the gap metric robustness optimization as in (McFarlane & Glover, 1992). The design technique has two main stages: 1) loop shaping is used to shape the nominal plant singular values to give desired open-loop properties at frequencies of high and low loop gain; 2) the normalized coprime factor $H_\infty$ problem mentioned above is used to robustly stabilize this shaped plant. Finally, the last part shows how to integrate the loop shaping design procedure into the field oriented control with the Luenberger observer and proposes simulations results.

2. Mathematical model of asynchronous machine

We use some simplifying assumptions and Park transformation. The stator currents $(I_{ds}, I_{qs})$, the rotor flux $(\phi_d, \phi_q)$ and the rotation speed $\omega_m$ are considered as state variables. The model of the asynchronous machine in the reference axes $d, q$ related to the rotating field is given in the form:

$$X = f(x) + BU$$

(1)

$X = (I_{ds}, I_{qs}, \phi_d, \phi_q, \omega_m)^T$ is the state vector.

$U = (v_{ds}, v_{qs})^T$ is the control vector.

$$\begin{align*}
\frac{dx}{dt} & = & \frac{1}{2}\begin{bmatrix}
R_d + R_m & L_d & \frac{L_m^2}{L_T} \\
L_d & \frac{L_m^2}{L_T} & \frac{L_m^2}{L_T} \\
0 & \frac{L_m^2}{L_T} & \frac{L_m^2}{L_T}
\end{bmatrix} \begin{bmatrix}
I_{ds} \\
I_{qs} \\
\phi_d
\end{bmatrix} + \begin{bmatrix}
L_d & \frac{L_m^2}{L_T} \omega_m \\
\frac{L_m^2}{L_T} & \frac{L_m^2}{L_T} \omega_m \\
0 & \frac{L_m^2}{L_T} \omega_m
\end{bmatrix} \begin{bmatrix}
\dot{I}_{ds} \\
\dot{I}_{qs} \\
\dot{\phi}_d
\end{bmatrix} + \begin{bmatrix}
\frac{L_m^2 \omega_m}{L_T} & \frac{L_m^2 \omega_m}{L_T} & \frac{L_m^2 \omega_m}{L_T}
\end{bmatrix} \begin{bmatrix}
\frac{I_{ds}}{L_T} \\
\frac{I_{qs}}{L_T} \\
\frac{\phi_d}{L_T}
\end{bmatrix}
\end{align*}$$

(2)

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oriented control with the Luenberger observer and proposes simulations results. Finally, the last part shows how to integrate the loop shaping design procedure into the field desired open-loop properties at frequencies of high and low loop gain; 2) the normalized main stages: 1) loop shaping is used to shape the nominal plant singular values to give robustness optimization as in (McFarlane & Glover, 1992). The design technique has two (Glover & McFarlane, 1988) (Glover & McFarlane, 1989) and is equivalent to the gap metric. We use some simplifying assumptions and Park transformation. The stator currents

\[ w_{qI} = w_q - w_r, \quad w_r = \frac{d\theta}{dt}, \quad w = pw_m \]

\[ C_{em} = p(I_q\phi_{dI} - I_d\phi_{qI}), \quad \phi_r = \sqrt{\phi_{dI}^2 + \phi_{qI}^2} \]

\[ \sigma = 1 - \frac{L_m}{L_s L_r}, \quad T_r = \frac{L_r}{R_r} \]

Where \( J \) is the moment of inertia, \( L_r, L_s \) and \( L_m \) are respectively rotor inductance, stator inductance and mutual inductance. \( R_r \) and \( R_s \) are respectively the resistance of the rotor and of the stator, \( P \) is the number of pole pairs of the machine, \( k_f \) is the friction coefficient and \( \sigma \) is Blondel’s dispersion coefficient.

3. **Indirect field oriented control**

The aims of this method of frequency control (Slipway Frequency Control) consist in not using the flux rotor amplitude but simply its position calculated according to the reference variables (Peresada et al., 1999) (Blaschke, 1972). This method does not use a flux sensor (physical sensor or dynamic model) but needs the rotor speed sensor. Fig. 1 shows an example of an applied indirect field control with a type PI regulation on the asynchronous machine fed by an inverter controlled by the triangulo-sinusoidal strategy with four bipolar carriers.

3.1 **Field oriented control**

The FOC (Field Oriented Control) is an arithmetic block which has two inputs (\( \phi_r^* \) and \( C_{em}^* \)) and generates the five variables of the inverter (\( v_{dI}^*, v_{dI}^*, w^*_r, v_{dI}^*, v_{dI}^* \)). It is defined by leading the static regime for which the rotor flux and the electromagnetic couple are maintained constant equal to their reference values. If we do not take into account the variations of the direct currents and the squaring component, the equations of this block are deduced in the following way:

\[
\begin{align*}
    i_{dI}^* &= \frac{\phi_r^*}{L_m} \\
    i_{qI}^* &= \frac{L_r C_{em}^*}{pL_m \phi_r^*} \\
    \omega^*_r &= \omega_m + \frac{L_m R_s i_{qI}^*}{L_r \phi_r^*} \\
    v_{dI}^* &= R_s i_{qI}^* - \omega^*_r \sigma L_s i_{qI}^* \\
    v_{qI}^* &= R_s i_{qI}^* + \omega^*_r L_s i_{qI}^* 
\end{align*}
\]

This method consists to control the direct component \( I_{dI} \) and the squaring \( I_{qI} \) stator current in order to obtain the electromagnetic couple and the flux desired in the machine.
3.2 Simulation of field oriented control

The best-known inverters up to now are the two level inverters. However, some applications such as electric traction require three-phase asynchronous variators functioning at very high power and/or speeds. These two level inverters are limited in tension (1.4kV) and power (1MVA). To increase power and tension, we use a multilevel inverter. In our work, the multilevel inverter used is controlled by the triangulo-sinusoidal strategy with four bipolar carriers (Boukhnifer, 2007).

Fig. 2 shows the results of the indirect field control of an asynchronous machine fed by this inverter. The decoupling is maintained and the speed follows the reference very well and is not affected by the application of a resistive torque. In the next section, we will explain briefly the principles of the $H_{\infty}$ control and how it can be integrated into the indirect field control.

4. Robust control

4.1 $H_{\infty}$ Control

For given $P(s)$ and $\gamma > 0$, the $H_{\infty}$ standard problem is to find $K(s)$ which:

- Stabilize the loop system in Fig. 3 internally.
- Maintain the norm $\|F_{L}(P, K)\| < \gamma$

with $F_{L}(P, K)$ defined as the transfer function of exits $Z$ according to entries $W$.

4.2 $H_{\infty}$ Coprime factorization approach

An approach was developed by McFarlane and Glover (McFarlane & Glover, 1988) (McFarlane & Glover, 1989) starting from the concept of the coprime factorization of a transfer matrix. This approach presents an interesting properties and its implementation uses traditional notions of automatics.
4.3 Robust controller design using normalized coprime factor

We define the nominal model of the system to be controlled from the coprime factors on the left: $G = \tilde{M}^{-1} \cdot \tilde{N}$. Then the uncertainties of the model are taken into consideration so that (see Fig. 4)

$$\tilde{G} = (\tilde{M} + \Delta_M)^{-1} \cdot (\tilde{N} + \Delta_N)$$

Fig. 4. Coprime factor stabilization problem
where $\tilde{G}$ is a left coprime factorization (LCF) of $G$, and $\Delta_u, \Delta_x$ are unknown and stable transfer functions representing the uncertainty. We can then define a family of models as follows:

$$\xi_\varepsilon = [\tilde{G} = (\tilde{M} + \Delta_u) \cdot (\tilde{N} + \Delta_x) : \|\Delta_u, \Delta_x\|_\infty, \varepsilon_{\text{max}}]$$

Where $\varepsilon_{\text{max}}$ represents the margin of maximum stability. The robust stability problem is thus to find the greatest value of $\varepsilon = \varepsilon_{\text{max}}$, so that all the models belonging to $\xi_\varepsilon$ can be stabilized by the same corrector $K$. The problem of robust stability $H_\infty$ amounts to finding $\gamma_{\text{min}}$ and $K(s)$ stabilizing $G(s)$ so that:

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - K \cdot G)^{-1} (I G) \right\| = \gamma_{\text{min}} = \frac{1}{\varepsilon_{\text{max}}}$$

However, McFarlane and Glover (McFarlane & Glover, 1992) showed that the minimal value of $\gamma$ is given by:

$$\gamma_{\text{min}} = \varepsilon_{\text{max}} = \sqrt{1 + \lambda_{\text{sup}}(XY)}$$

where $\lambda_{\text{sup}}$ indicates the greatest eigenvalue of $XY$, moreover for any $\varepsilon(\varepsilon_{\text{max}}$, a controller stabilizing all the models belonging to $\xi_\varepsilon$ is given by:

where $A$, $B$, and $C$ are state matrices of the system defined by the function $G$ and $X$, $Y$ are the positive definite matrices and the solution of the Ricatti equation:

$$A^T X + X A - X B^T B X + C^T C = 0$$

$$A Y + Y A^T - Y C^T C Y + B B^T = 0$$

### 4.4 Loop-shaping design procedure

Contrary to the approach of Glover-Doyle, no weight function can be introduced into the problem. The adjustment of the performances is obtained by affecting an open modelling (loop-shaping) process before calculating the corrector. The design procedure is as follows:

![Fig. 5. The loop-shaping design procedure](www.intechopen.com)
order to improve the performances of the system so that \( G_2 = W_1 GW_2 \) (see Fig.5). In the monovariable case, this step is carried out by controlling the gain and the phase of \( G_2(j\omega) \) in the Bode plan. From coprime factorizations of \( G_2(s) \), we apply the previous results to calculate \( \varepsilon_{\text{max}} \), and then synthesise a stabilizing controller \( K \) ensuring a value of slightly lower than \( \varepsilon_{\text{max}} \):

\[
\begin{bmatrix}
I \\
K
\end{bmatrix} (I - K \cdot W_2 \cdot G \cdot W_1)^{-1} (I \cdot W_2 \cdot G \cdot W_1) = \gamma = \frac{1}{\varepsilon}
\]

(10)

The final feedback controller is obtained by combining the H\(\infty\) controller \( K \) with the shaping functions \( W_1 \) and \( W_2 \) so that \( G_\alpha(s) = W_1 GW_2 \). (See Fig.5).

5. Robust control of the asynchronous machine

When the reference is directed we have \( \phi_{dr} = \phi_r \) and \( \phi_{qg} = 0 \). In this case, the expression of the electromagnetic couple can be written in the form:

\[
C_{\text{em}} = k_i_c i_{qg}, \quad k = \frac{p}{L_r}
\]

(11)

This equation simplifies the model of the asynchronous machine as follows:

\[
\begin{align*}
\frac{dl_{d}}{dt} &= \frac{1}{\alpha C_{\text{em}}} \left( - (R_s + \omega_m^2 L_s) l_{d} + \alpha t \omega_s j_{w} + \frac{L_s R_r}{L_r} \phi_r + v_s \right) \\
\frac{dl_{q}}{dt} &= \frac{1}{\alpha C_{\text{em}}} \left( - (R_s + \omega_m^2 L_s) l_{q} + \frac{L_s R_r}{L_r} \phi_r + v_q \right) \\
d\phi_r &= \frac{L_s R_r}{L_r} - \frac{R_s}{L_r} \phi_r \\
d\omega_s &= \frac{p^2 L_s}{L_r J} \omega_s - \frac{p c_f}{J} \phi_r \\
\omega_s &= \omega_s + \frac{L_s R_r}{L_r} \phi_r
\end{align*}
\]

(12)

By using the transform of Laplace, we can write that:

\[
\phi_s = \frac{i_{d}}{1 + \frac{j \omega_s}{L_s} \cdot \phi_s}, \quad C_{\text{em}} = \frac{p l_n}{l_r} \phi_r, \quad \Omega = \frac{C_{\text{em}}}{j \omega_s + k_f}
\]

(13)

The equation (13) shows that we can act independently on rotor flux and the electromagnetic couple by means of components \( I_{d1} \) and \( I_{q1} \) respectively of the stator current. The goal consists in controlling the direct component \( I_{d1} \) and in squaring component \( I_{q1} \) of the stator current in order to obtain the electromagnetic couple and the flux desired in the machine. We can represent our system by combining equations (3) and (13) in two sub-systems with the transfer functions described below, See (Boukhnifer, 2007) for details:
\[ G_{\text{flux}} = \frac{1/T_r}{s + 1/T_r} \quad G_{\text{speed}} = \frac{1/J}{s + k_f/J} \] (14)

In order to ensure a high gain in low frequencies and a low gain in high frequencies, we add the weight functions for flux and speed respectively so that:

\[ W_f = \frac{2 \cdot (s + 5)}{s} \quad W_\Omega = \frac{2.5 \cdot (s + 2)}{s} \] (15)

5.1 Loop shaping controller

The calculation of the flux controller by MATLAB® software gives:

\[ \frac{0.8140 \cdot s + 6.7347}{s + 5.4817} \quad \varepsilon_{\text{max}} = 0.7756 \] (16)

The calculation of the speed controller by MATLAB® software gives:

\[ \frac{1.0208 \cdot s + 1.9587}{s + 1.9994} \quad \varepsilon_{\text{max}} = 0.6998 \] (17)

Fig. 6. Robust control of asynchronous motor

5.2 Simulation of the robust control

To illustrate the performances of the \( H_\infty \) control, we simulated a no-load start with application of the load (nominal load \( C_r=10 \text{Nm} \)) at \( t_1 = 1.5 \text{Sec} \) to \( t_2 = 2.5 \text{Sec} \). Then the machine is subjected to an inversion of the instruction between 100 rad/sec at \( t_3=3 \text{Sec} \) (Fig.7).

The speed regulation presents better performances with respect to the pursuit and the rejection of the disturbances. We note that the current is limited to acceptable maximum values. The decoupling is maintained and the speed follows the reference well and is not affected by the application of a resistive torque.

6. Luenberger observer

We apply the Luenberger observer method for the estimation of the rotor flux components (Orlawska-Kowalska, 1989). The model of the reference machine linked to the stator field is
linear in the electromagnetic states. The two stator current components are measurable. We will consider them as outputs of the model:

\[
\dot{x} = Ax + Bu
\]
\[
y = Cx
\]

with:

\[
U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad i_s
\]

and:

\[
A = \begin{bmatrix} -\gamma & 0 & \frac{k}{T} & 0 \\ 0 & -\gamma & 0 & \frac{k}{T} \\ \frac{1}{T} & 0 & \frac{1}{T} & -\rho \Omega \\ 0 & \frac{1}{T} & \rho \Omega & -\frac{1}{T} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sigma_d} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\]

\[
\hat{z} = F\hat{z} + ky + Hu
\]

The dimensions of the vectors and matrices which appeared in this relation are:

\[
z(3,1), F(2,2), k(2,2), H(2,2).
\]
Vector $Z$ is related to the initial state vector $x$ by the transformation matrix $T$:

$$Z = Tx$$ (23)

To determine the relations between the matrices of system $A, B$ and $C$ and the matrices of the observer $F, K$ and $H$, the equation of error is calculated ($e = \dot{z} - Tx$):

$$\dot{e} = \dot{z} - Tx$$
$$\dot{e} = F\dot{z} + ky + Hu - TAx - Tu$$
$$\dot{e} = F\dot{z} + kCx + Hu - TAx - Tu$$
$$\dot{e} = F(e + Tx) + kCx + Hu - TAx - Tu$$
$$\dot{e} = Fe + (FT + kC - TA)x + (H - TB)u$$ (24)

To give the equation of error the form:

$$\dot{e} = Fe$$ (25)

We must check the relation:

$$TA - FT = KC$$
$$H = TB$$ (26)

The error dynamics (25) is described by the eigenvalue of the state matrix of observer $F$. We impose to this matrix the following form:

$$F = \text{diag}(\lambda_1, \lambda_2)$$ (27)

In order to stabilize the error dynamics, $\lambda_1$ and $\lambda_2$ must be negative. With this choice of $F$, the explicit equations of the observer are given by:

$$\dot{z}_1 = \lambda_1z_1 + k_{11}y_1 + k_{12}y_2 + h_{11}u_1 + h_{12}u_2$$
$$\dot{z}_2 = \lambda_2z_2 + k_{21}y_1 + k_{22}y_2 + h_{21}u_1 + h_{22}u_2$$ (28)

We impose to the transformation matrix $T$ the following form:

$$T = \begin{pmatrix} t_{11} & t_{12} & 1 & 0 \\ t_{21} & t_{22} & 0 & 1 \end{pmatrix}$$ (29)

The elements of the $T, K$ matrix and $H$ are obtained from the equations (25):
\[
\begin{align*}
    t_{11} &= \frac{\dot{\theta}^2 + \lambda_1 \theta + p^r \Omega^2}{k(\dot{\theta}^2 + p^r \Omega^2)} \\
    t_{12} &= -\frac{\lambda_1 p \Omega}{k(\dot{\theta}^2 + p^r \Omega^2)} \\
    t_{21} &= -\frac{\lambda_1 p \Omega}{k(\dot{\theta}^2 + p^r \Omega^2)} \\
    t_{22} &= \frac{\dot{\theta}^2 + \lambda_1 \theta + p^r \Omega^2}{k(\dot{\theta}^2 + p^r \Omega^2)}
\end{align*}
\]

Thus, rotor flux:
\[
\phi_r = \sqrt{\phi_{ar}^2 + \phi_{br}^2}
\]

6.1 Simulations Results
The results of simulations show that the Luenberger observer gives an error tends to zero and the flux observed follows very well the real flux of the machine and has a better robustness as regards parametric variations (variations of rotor resistance). The results of simulations of robust control are present in (Fig. 9) and we note clearly that decoupling is maintained and the speed follows the reference well and is not affected by the application of a resistive torque.

7. Conclusion
In this paper, we have studied the robustness of \( H_\infty \) control applied to an induction motor and by using the Luenberger observer for the observation of rotor flux. The obtained results showed the robustness of the variables flux and speed against external disturbances and uncertainties of modelling. This method enabled us to ensure a good robustness/stability compromise as well as satisfactory performances. The use of the Luenberger observer enables us to avoid the use of the direct methods of measurements weakening the mechanical engineering of the system.
Fig. 8. Luenberger observer (a) with no variation of $R_r$ and (b) with increase of $R_r$ 100%

Fig. 9. Robust control with Luenberger observer

8. References


McFarlane, D & Glover, K. (1989). Robust controller design using normalized coprime factor plant descriptions, *lecture Notes in Control and information Sciences*, Springer Verlag, (138), Berlin, Germany


This book collects fifteen relevant papers in the field of mechatronic systems. Mechatronics, the synergistic blend of mechanics, electronics, and computer science, integrates the best design practices with the most advanced technologies to realize high-quality products, guaranteeing at the same time a substantial reduction in development time and cost. Topics covered in this book include simulation, modelling and control of electromechanical machines, machine components, and mechatronic vehicles. New software tools, integrated development environments, and systematic design methods are also introduced. The editors are extremely grateful to all the authors for their valuable contributions. The book begins with eight chapters related to modelling and control of electromechanical machines and machine components. Chapter 9 presents a nonlinear model for the control of a three-DOF helicopter. A helicopter model and a control method of the model are also presented and validated experimentally in Chapter 10. Chapter 11 introduces a planar laboratory testbed for the simulation of autonomous proximity manoeuvres of a uniquely control actuator configured spacecraft. Integrated methods of simulation and Real-Time control aiming at improving the efficiency of an iterative design process of control systems are presented in Chapter 12. Reliability analysis methods for an embedded Open Source Software (OSS) are discussed in Chapter 13. A new specification technique for the conceptual design of self-optimizing mechatronic systems is presented in Chapter 14. Chapter 15 provides a general overview of design specificities including mechanical and control considerations for micro-mechatronic structures. It also presents an example of a new optimal synthesis method to design topology and associated robust control methodologies for monolithic compliant microstructures.

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