The Role of DSD and Radio Wave Scattering in Rain Attenuation

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1. Introduction

The (rain) drop size distribution (DSD) plays a very important role in meteorology (determination of radar reflectivity and consequently rain rate, classification of precipitation, flood prediction), in microwave radio-communication (determination of rain attenuation) and also in many other applications like agriculture and insurance business.

1.1 Quick overview of literature

There is a large number of papers devoted to the DSD problems. The classical and very important study by Marshall-Palmer (1948) has to be mentioned. Almost every scientist is using this DSD model for average rain. Rain intensity is a parameter of this DSD.

The most important recent papers concerning the DSD problem are focused on following topics:

Carolin Richter (1995) declared the Gamma function to be an appropriate analytical approximation of DSD. She has studied the dependence of rain rate, median diameter and shape factor ($\mu$-parameter in Gamma approximation) on synoptic events. Clear systematic trends in the drop spectra were found. It was possible to distinguish warm advection from the cold advection according to the numerical value of the shape factor $\mu$. Carolin has also found that the rain intensity does not influence the shape of DSD as no relation between rain rate and shape factor could be found.

Albert Waldvogel (1974) discussed the intercept parameter $N_o$ in the exponential approximation of DSD. Prof. Waldvogel has found that sudden variations of spectra can be recognised easily as $N_o$ jump versus the time axis. An empirical model is proposed for the relation between the type of raindrop spectra and the convective activity of the precipitating mass.

Tokay A. and Short D. (1996) have observed dramatic change in the intercept parameter $N_o$ in the Gamma approximation of DSD occurring during rainfall events with little change in rainfall rate. It can correspond to the transition from rain of convective origin to rain originating from the stratiform portion of tropical systems. The authors have presented an empirical stratiform-convective classification method based on $N_o$ and rain rate scatterplot. It is worth noting that this study is related only to tropical rains.
J. Joss and E. Gori (1978) have defined an “integral” shape factor of DSD different from the “µ” shape factor, which is a parameter in the Gamma approximation. They discussed the role of sampling time on the shape factor founding that adding many instant distributions from different conditions leads to an exponential distribution such as proposed by Marshall and Palmer (1948).

J. Joss and A. Waldvogel (1968) have modified the Marshal-Palmer DSD for drizzle, continuous rain, shower and thunderstorm for conditions of Switzerland. This modification cannot be accurate because it is based on short term DSD measurement.

O.Fišer and M.Hagen (1998) have discussed the influence of integration time on resulting DSD. For larger integration time the increasing agreement between experimental DSD and its exponential distribution was shown. Also variations of parameters defining DSD (N₀, λ, and µ) with rain rate was illustrated and discussed. Two methods to determine parameters of analytical DSDs were shown and discussed (linear regression, method of moments).

O.Fišer, D.Řezáčová, P. Pešice, Z. Sokol and O. Školoud (1998) realised an attempt to estimate the basic rain type from existing rain rate records using the rain event duration, rain amount, average and standard deviation of rain rate as predictors.

O. Fišer (2002b) discussed the role of particular rain drop size on resulting specific rain attenuation im micro and mm frequency bands.

O. Fišer (2003 a,b) compared existing methods for meteorological rain type identification using Czech DSD data. A lot of work has to be done because existing criterions were devoted for tropical regions only and the mentioned study has an introductory meaning.

O. Fišer (2004) derived the radar reflectivity-rain intensity (Z-R) analytical approximation using the Czech DSD data.

O. Fišer (2006) has published a preliminary study showing DSD variability and its frequency dependence.

O. Fišer (2007) analysed the DSD moments for the estimation of the bilateral relationships between DSD products (outputs).

Jameson, A. R., A. B. Kostinski (2002) have found power law relation between rain rate and radar reflectivity factor. It is found that apparently realistic but spurious nonlinear power-law relations still appear among rainfall parameters even though the rain is not only statistically homogeneous but purely random as well.

Řezáčová D., Kašpar M., Novák P., Setvák M., (2007) have published an overview of published and measured DSDs of of rain (Best, Marshall-Palmer and other distributions).

2. Drop size distribution

The rain drop size distribution (DSD, quantity symbol N) represents the probability density of equivolumetric drop diameter D being in the unity volume. The product N(D) dD gives the number of rain of the diameter between D and D+dD in the unity volume. Only rain drops of diameters below 7 mm can exist for physical reasons. Example of the DSD measured in the Czech Republic is shown in Figure 1.
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3. DSD measurement

Generally speaking, the measurement of the DSD is relatively rare. The greatest problem is
the cost of distrometer. The most developed and user friendly device for the DSD
measurement is a videodistrometer (videodistrometers developed at the Graz Technical
University, Austria under ESA contract, are well known). Other electromechanical distrometer
of the Joss-Waldvogel type, is used at the ETH Zurich and in Bavaria (in the DLR-
Oberpfaffenhofen and in the DWD-Hohenpeissenberg). Optical distrometers are or were used
in Oberpfaffenoefen and in Dubna (Russia). The Rutherford Appleton Laboratory (UK), has
performed a DSD measurement in Chilbolton (UK) and in Papua New Guinea. On the other
hand, there is a lot of DSD data which is not processed or which is processed only partially.
The DSD measurement is not very simple and it is not performed so often in comparison
with the rain rate measurement. Several DSD measurements were performed in the history
(e.g. Marshall and Palmer, 1948, Joss and Waldvogel, 1968, Lakomá, 1971, Federer and
Waldvogel, 1975, Li and Zhang, 1980) or more recently (e.g. Doelling et al., 1996, Hubert et
using different types of measurement technique (filter or glass catchment, electromechanical
distrometer, optical distrometer, videodistrometer and others). Many DSD measurements
have been made by the electromechanical distrometer the concept of which was developed
by Joss and Waldvogel (e.g. Joss and Waldvogel, 1967).
The results of the Czech DSD measurement (performed in 1998-1999) was published by

4. Analytical approximations of DSD

The exponential and Gamma distribution are the most frequently used analytical
approximations of the DSD because of their satisfactory correspondence with the typical
drop size distribution shape in the majority of experimental samples. There are also many other DSD models in the literature - for instance the log-normal model (Ajayi, Kozu, 1999). It should be reminded that also many various factors like the rain type, time of integration and others influence the analytical DSD modelling.

The equation (1) expresses the Gamma model of distribution function (DSD)

\[ N(D) = N_0 D^\mu \exp(-\lambda D) \]  

where

- \( D \) [mm] is the rain drop diameter
- \( N(D) \) [m\(^{-3}\)mm\(^{-1}\)] is the number of drops per unit volume per drop diameter interval (dD)
- \( N_0 \) [m\(^{-3}\)mm\(^{-1}\)] is the intercept parameter of DSD
- \( \lambda \) [mm\(^{-1}\)] is the slope parameter.
- \( \mu \) [-] is shape of the DSD, to avoid a mismatch it is preferred to call it as “\( \mu \) parameter”

Examples of numerical values of parameters are shown in Table 1 (Iguchi T., 1999) and plotted in Figure 2.

<table>
<thead>
<tr>
<th>Gamma</th>
<th>( N_0 )</th>
<th>( \lambda )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain type</td>
<td>mm(^{-3})mm(^{-1})</td>
<td>mm(^{-1})</td>
<td>-</td>
</tr>
<tr>
<td>Convective</td>
<td>6.29E5*R(^{-0.416})</td>
<td>8.35R(^{-0.185})</td>
<td>3</td>
</tr>
<tr>
<td>Stratiform</td>
<td>2.57E4*R(^{0.012})</td>
<td>5.5R(^{-0.129})</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 1. Examples of numerical values of the Gamma DSD model parameters (tropical region) where R is the rain rate in [mm/h]

The simpler Exponential distribution is used in the form given by the following expression

\[ N(D) = N_0 \exp(-\lambda D) \]  

As it is obvious, the exponential DSD model can be considered as a special case of the gamma DSD where the parameter \( \mu \) equals to zero.

The parameter \( \lambda \) in this model can be also expressed in the dependence on rain rate \( R \):

\[ \lambda = a + bR \]  

where \( b = -0.21 \).

Typical exponential DSD for various rain types (drizzle, thunderstorm and average rain in mild climate) is shown in Figure 3, which is plotted after Table 2. (source: Joss J. and Waldvogel A., 1968).

<table>
<thead>
<tr>
<th>Rain type</th>
<th>( N_0 )</th>
<th>( \lambda )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thunderstorm or shower</td>
<td>1 400</td>
<td>3*R(^{-0.21})</td>
<td></td>
</tr>
<tr>
<td>Continuous rain</td>
<td>7 000</td>
<td>4.1*R(^{-0.21})</td>
<td></td>
</tr>
<tr>
<td>Drizzle</td>
<td>30 000</td>
<td>5.7*R(^{-0.21})</td>
<td></td>
</tr>
<tr>
<td>Average rain</td>
<td>8 000</td>
<td>4.1*R(^{-0.21})</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Typical parameters of Exponential DSD model (Europe) where R is the rain rate in [mm/h]

Fig. 2. Gamma DSD model using parameters from Table 1 for certain rain rate value
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The simpler Exponential distribution is used in the form given by the following expression

\[ N(D) = N_0 \exp(-\lambda D) \]  (2)

As it is obvious, the exponential DSD model can be considered as a special case of the gamma DSD where the parameter \( \mu \) equals to zero.

The parameter \( \lambda \) in this model can be also expressed in the dependence on rain rate \( R \):

\[ \lambda \approx aR^b \]  (3)

where \( b = -0.21 \).

Typical exponential DSD for various rain types (drizzle, thunderstorm and average rain in mild climate) is shown in Figure 3, which is plotted after Table 2. (source: Joss J. and Waldvogel A., 1968).

<table>
<thead>
<tr>
<th>Rain type</th>
<th>( N_0 ) [mm(^{-1})m(^{-3})]</th>
<th>( \lambda ) [mm(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thunderstorm or shower</td>
<td>1400</td>
<td>3 ( \times ) ( R^{0.21} )</td>
</tr>
<tr>
<td>Continuos rain</td>
<td>7000</td>
<td>4.1( \times )( R^{0.21} )</td>
</tr>
<tr>
<td>Drizzle</td>
<td>30000</td>
<td>5.7( \times )( R^{0.21} )</td>
</tr>
<tr>
<td>Average rain</td>
<td>8000</td>
<td>4.1( \times )( R^{0.21} )</td>
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Table 2. Typical parameters of Exponential DSD model (Europe) where \( R \) is the rain rate in [mm/h]

![Fig. 3. Exponential DSD for various rain types (drizzle, thunderstorm and average rain in mild climate)](www.intechopen.com)
5. Determination of DSD parameters from measured values

In this part we describe two techniques determining the values of the model parameters $N_0$, $\lambda$, and $\mu$. After logarithmic linearization of the $N(D)$ approximation (1) the linear regression can be applied (least square method, Bartsch, 1996). The moment method uses the definition of the DSD moments, which, for the $n$-th moment, $M_n$, gives:

$$M_n = \int_0^\infty D^n N(D) dD$$

(4)

Using the method of moments following formulas for the parameters $N_0$ and $\lambda$ in the exponential DSD model (2) were derived:

$$N_0 = 98.65 * M_3 * \left(\frac{M_3}{M_6}\right)^{\frac{4}{3}}$$

(5)

$$\lambda = 4.93 * \left(\frac{M_3}{M_6}\right)^{\frac{1}{3}}$$

(6)

where $M_3$ and $M_6$ are the 3rd, and 6th DSD moments, respectively. Note that the same formulas were derived by Prof. Waldvogel (Waldvogel, 1974) where the liquid water content $W$ was preferred to the 3rd moment. The third and the sixth moments were chosen to approximate the DSD in order to determine the rain intensity (being approximately proportional to $M_3$) and the radar reflectivity factor (proportional to $M_6$) from the DSD model as accurate as possible.

To determine the parameters of the Gamma approximation (1) the formulas of Tokay and Short (Tokay, Short, 1996) can be used. The expressions using $M_3$, $M_4$, and $M_6$ are as follows:

$$\mu = \frac{11G - 8 + [\{G (G + 8)]^{1/2}}{2(1 - G)}$$

(7)

$$\lambda = \frac{(\mu + 4) M_3}{M_4}$$

(8)

$$N_0 = \frac{\lambda^{\mu+4} M_3}{\Gamma(\mu+4)}$$

(9)

where the auxiliary $G$ factor is given by

$$G = \frac{M_4^3}{M_2^2 * M_6}$$

(10)

The rain drop size distribution (some times it is called “rain drop spectrum”) determines uniquely its outputs (outputs can be called also “products”) as rain rate, radar reflectivity factor, rain attenuation and others (see next parts).
6. Outputs (products) of DSD

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6.1 DSD products of no frequency dependence

a. Rain intensity (rain rate)

\[ R_g = \frac{3.6}{10^3} \pi \int_0^\infty D^3 v(D) N(D)dD \]  

(11)

where \( R_g \) [mm/h] is the rain rate (corresponding to the rain rate derived from rain gauge measurement)

\( v \) is the terminal falling velocity

\( N(D) \) is the drop size distribution

\( D \) is the equivolumetric drop radius.

b. Radar reflectivity factor \( z \) [mm$^6$m$^{-3}$]

The radar reflectivity factor \( z \) (small letter “z”) is the 6th DSD moment, it can be computed from following expression:

\[ z = \int_0^\infty D^6 N(D)dD \]  

(12)

while for its logarithmic unit \( Z \) [dBZ] (capital letter “Z”) it is used:

\[ Z = 10 \log_{10} \{ \int_0^\infty D^6 N(D)dD \} \]  

(13)

If we study electromagnetic energy coming back from rain volume to the radar, we must be aware that the reflected energy is not dependent on frequency only in the “Rayleigh region” case (drop diameter \( D \) is much smaller in comparison with the wave length \( \lambda \), i.e. \( D \ll \lambda \)), more precisely the Rayleigh region is defined

\[ \frac{\pi D}{\lambda} \ll 1 \]  for \( n = 1 \) (n is the refractive index.) or \[ \frac{\pi nD}{\lambda} \ll 1 \]  for \( n > 1 \)

If we suppose rain with rain drops having diameter up to 4 mm (typical for mild climate), the Rayleigh region holds for frequencies below 2.5 GHz. But if we consider the existence of maximum rain drop diameter (\( D = 7 \) mm), the Rayleigh region is met at frequencies lower than 1.36 GHz. In practice, it is not so strict and the Rayleigh region is applicable for frequencies below 5 GHz.
6.2 DSD products dependent on frequency
The specific rain attenuation $A$ is strongly dependent on frequency, but rain attenuation is negligible for frequencies below 4 GHz.

For the specific rain attenuation $A$ in [dB/km] next equation is used:

$$A = 4.343 \cdot 10^{-3} \cdot \lambda \cdot \text{Im} \int f(D) \cdot N(D) \, dD$$

(14)

where $f$ is the complex forward scattering function (for exact definition see in part 7)

$\lambda$ is the wave length of the used transmission

$\text{Im}$ represents the imaginary part of complex number

The formula (14) is derived in part 9. As one can see, this expression strongly depends on the wave length $\lambda$ (i.e. frequency) in contrast to the no frequency dependence of the radar reflectivity factor.

Through the numerical simulations it is possible to search for relationships between mentioned quantities ($Z$, $R$ and $A$) called “DSD products.” The term frequency means the radio frequency of pertinent technical application in this paragraph.

7. Scattering functions

In literature different definitions of scattering functions are used, see, for instance [Uzunoglu et al., 1977]. One of the most used definition of the scattering function is the following one:

$$E^s = E^i \cdot f(K_1, K_1) \cdot r^{-1} \cdot e^{(jkr)}$$

(15)

where:

$E^s$ ... electric field of a scattered wave  [V/m]

$E^i$ ... electric field of a wave impressing on the raindrop  [V/m]

$k_0$ ... free space propagation constant [m$^{-1}$]

$r$ .... observation distance from the scattering drop [m]

$f(K_1, K_1)$ ... scattering function [m], $K_1$ is the direction of the incident field, $K_2$ is the direction of the scattered field especially:

if $K_1=K_2$, the forward scattering function is considered, or

if $K_1=-K_2$, the backward scattering function is considered

If the propagating radio wave direction is not parallel with horizontal drop axis, formula (16) can be used (see geometry in Figure 4).
6.2 DSD products dependent on frequency

The specific rain attenuation $A$ is strongly dependent on frequency, but rain attenuation is negligible for frequencies below 4 GHz.

For the specific rain attenuation $A$ in $[\text{dB/km}]$ next equation is used:

$$
A = \int \frac{dD}{NDA} \left( \frac{f}{\lambda} \right)^{-1} \left( \frac{\lambda}{\lambda_{ref}} \right)^{-3} \left( \frac{\lambda}{\lambda_{ref}} \right)^{3}
$$

(14)

where $f$ is the complex forward scattering function (for exact definition see in part 7)

$\lambda$ is the wave length of the used transmission

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7. Scattering functions

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$$
E_i \leftrightarrow E_s \rightarrow \frac{1}{\lambda_0} \left( \frac{\lambda_0}{\lambda_0} \right)^{-1} \left( \frac{\lambda_0}{\lambda_0} \right)^{3}
$$

(15)

where:

$E_s$ ... electric field of a scattered wave $[\text{V/m}]$

$E_i$ ... electric field of a wave impressing on the raindrop $[\text{V/m}]$

$\lambda_0$ ... free space propagation constant $[\text{m}]$

$r$ .... observation distance from the scattering drop $[\text{m}]$

$K_1, K_2$ ... scattering function $[\text{m}]$

$K_1$ is the direction of the incident field,

$K_2$ is the direction of the scattered field especially:

if $K_1 = K_2$, the forward scattering function is considered, or

if $K_1 = -K_2$, the backward scattering function is considered

If the propagating radio wave direction is not parallel with horizontal drop axis, formula (16) can be used (see geometry in Figure 4).

Fig. 4. Geometry of forward propagation direction in the general incidence angle $\alpha$.

$$
f_{h,v}(\alpha) \approx f_0(0^\circ) \cos^2(\alpha) + f_{h,v}(90^\circ) \sin^2(\alpha)
$$

(16)

where

$\alpha$ is angle between direction of propagation and zenith axis $[^\circ]$

$f_{h,v}(90^\circ)$ is the scattering function of horizontally respective vertically polarised wave for direction of propagation parallel with horizontal drop axis ($\alpha = 90^\circ$)

$f_0(0^\circ)$ is scattering function for the “zenith” propagation ($\alpha = 0^\circ$), it is not dependent on polarisation because of rain drop symmetry

8. Some methods computing scattering functions

8.1 Rayleigh scattering

The Rayleigh scattering theory was developed and published by Strutt (1871 a,b) – later Lord Rayleigh. The theory gives an approximation for the scattering of electro-magnetic radiation from spheres. It is valid for sphere diameters significantly smaller than the vacuum wavelength of the radiation., i.e. $\pi D/\lambda <<1$, see part 6.1. Lord Rayleigh derived a scattering function approximation for such case using elementary dipole theory.

The computation of the Rayleigh scattering is very simple (a handy calculator is sufficient) but we must be aware of strict limitations. It was proved that the frequency above about 5 GHz is owing to the usual rain drop diameter out of Rayleigh region.

The Rayleigh scattering helps us to study the frequency and temperature properties of attenuation on lower frequencies. It does not enable to compute depolarisation and angular dependencies.

8.2 Mie scattering

The Mie scattering theory was developed and published by Mie (1908). The scattering function $f$ (subscript “f” for forward, “b” for backward scattering) for spherical dielectric particles is given by the next formula:
\[
 f_f = -\frac{j \lambda^3}{\pi^3 D^2} \left[ \sum_{n=1}^\infty (2n+1) \left( a_n + b_n \right) \right]^*
\]

\[
 f_b = -\frac{j \lambda^3}{\pi^3 D^2} \left[ \sum_{n=1}^\infty (-1)^{n+1} (2n+1) \left( a_n - b_n \right) \right]^*
\]

where \( \lambda \) denotes the vacuum wavelength of the electro-magnetic radiation, \( j \) is the imaginary unit and \( D \) the diameter of the spherical drops, * is symbol for conjugate imaginary numbers. The coefficients \( a_n \) and \( b_n \) according to Mie depend on the complex relative refractivity \( \varepsilon_r = \varepsilon / \varepsilon_0 \) of the material (rain water in our case) and on the diameter \( D \) of the scattering sphere.

\( a_n, b_n \) are the Mie’s coefficient , its evaluation need not to be very complicated.

The Mie scattering calculation is also possible at this web page:
http://omlc.ogi.edu/software/mie/

For the Mie scattering computation a simple programmable computer is needed under the condition that it can work with complex variables. The Mie algorithm can be quite simple if the Bessel and Legendre polynomials were replaced by simple complex goniometric functions. The infinite series (the above printed formula) can be limited to the \( n \) being about 10 (or even less) of a perfect accuracy, see (Fiser, O., 1993).

Mie scattering helps us to study the frequency and temperature properties of rain attenuation if we accept that rain drop shape is spherical (for larger rain drops it is not true). Mie scattering does not enable to compute depolarisation and angular dependencies. On the other hand, there is no frequency limitation like in the Rayleigh scattering computation case.

### 8.3 Other methods computing scattering functions

For full utilisation (angular dependence, polarisation properties), numerical methods computing the scattering functions, are required.

The point matching method is much more complicated and general and it enables to study not only the properties of scattering functions but also the incident angle dependence, the bi-static scattering and depolarization phenomena as well. See, for instance Oguchi, T., 1973. Some studies to derive the scattering function using numerical methods are used. For instance the MultipleMultiPole (MMP) method was used by Hajny, Mazanek and Fiser at the Czech Technical University Prague , cf. Hajny, M. et al, 1998.

### 9. Derivation of formula for specific rain attenuation in rain volume

To derive a formula for specific rain attenuation we prepared next collection of formulas related to the Figure 5 (\( E_s \) is scattered field, \( E_i \) is incident field, \( E^0 \) is “free space” field, \( z_1 \rightarrow 0 \) (infinitesimal length of the rain volume between two parallel slabs), resulting electrical field is summed at the point \( P(z_o) \) - contributions of the original free space field \( E^0 \) and scattered field from rain drops in the thin rain volume. Incident wave is of the planar type in our
model (a very good approximation of spherical wave being very far from the transmitter).
Here are the formulas:

Resulting electrical field $E(z_o) \ [\text{V/m}]$ is a sum of free space field and scattered field:

$$E(z_o) = E^s(z_o) + E^o(z_o)$$  \hfill (18)

From the definition of scattering functions we have:

$$E^s(z_o) = E^i(z) \ f(D) \ \frac{e^{jkr}}{r}$$  \hfill (19)

For planar wave it is valid:

$$E^i(z) = E^i(0) \ e^{j kz}$$  \hfill (20)

Fig. 5. On specific rain attenuation

After some numerical simplifications

$$E^i(z_o) \sim E^i(0) \ f(D) \ \frac{e^{jkr}}{r}$$  \hfill (21)

$$E^s(z_o) \sim E^o(z_o) \ f(D) \ \frac{e^{jkr(z-z_o)}}{z_o}$$  \hfill (22)

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we obtain:

\[ E^{\nu}(z_o) = E^I(0) \ e^{jk_0z_o} \]  \hfill (23)

\[ r = \left( \rho^2 + z_o^2 \right)^{0.5} \sim z_o \left( 1 + \frac{1}{2} \rho^2 \right) \]  \hfill (24)

\[ r - z_o \sim \frac{\rho^2}{2z_o} \]  \hfill (25)

we obtain:

\[ E(z_o) = E^{\nu}(z_o) + E^\nu(z_o) \left\{ \int_{0}^{\infty} \int_{0}^{1} f(D)N(D)1dz'dD \right\} \]  \hfill (26)

where

\[ I = \int_{0}^{\pi} \int_{0}^{1} \rho \ e^{\frac{\pi}{2} \rho^2} \ d\rho \ d\varphi \]  \hfill (27)

and where F is the radius of the first Fresnel zone. This integral expresses the contributions of all rain drops in the infinite plane perpendicular to the direction of radiowave propagation. In fact, the contribution is considered from ashlar of the infinitesimal length in the direction of propagation (z axis in our picture):

\[ E(z_o) = E^{\nu}(z_o) \left[ 1 + j \frac{2\pi}{k_o}z_o C \right] \]  \hfill (28)

where

\[ C = \int_{0}^{\pi} f(D)N(D) \ dD \]  \hfill (29)

Finally

\[ \tau = \left\{ \lim_{\Delta z \to 0} \left( 1 + j \frac{2\pi}{k_o} \Delta z C \right)^{\frac{1}{\Delta z}} \right\}^L = e^{\frac{j\pi}{2}cL} \]  \hfill (30)

where
\[ \tau \] is the ratio of the attenuated field strength to the non attenuated one (transmission)
\[ f \] is the complex scattering function
Transmission $\tau$ was computed as a product of a number of $\left(\frac{L}{\Delta z}\right)$ ashlars of infinitesimal lengths arranging in the series of the certain length $L$. The formula for specific rain attenuation $\alpha$ in the [dB/km] unit is then given as $10 \log_{10}(\tau)$:

$$\alpha = 4.3434 \lambda \cdot 10^{\frac{\alpha}{36}} \left\{ \int_0^\infty \text{Im} f(D) N(D) dD \right\} \text{[dB / km]}$$

(31)

The next parametrical approximation for the drop size distribution – DSD (symbol N) after Marshall-Palmer is used very frequently for so called “average rain:”

$$N(D,R) = 80000 \cdot e^{-\frac{4.1\cdot D}{R}} \text{[m m^{-3} m^{-1}]}$$

(32)

where $R$ is rain rate (or rain intensity) in [mm/h] units and $D$ is equivolumetric rain drop diameter, see Marshall and Palmer, 1948. The same formula (31) was derived through similar way by Van de Hulst (1957).

The mostly used and very simple approximation for specific rain attenuation is this one:

$$\alpha \sim a \cdot R^b \text{[dB / km]}$$

(33)

where $a$ [k alternatively] and $b$ [$\alpha$ alternatively] are constants depending on frequency, polarisation and temperature. An example is shown in next table (ITU-R report):

<table>
<thead>
<tr>
<th>f [GHz]</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>To interpolate:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.0094</td>
<td>0.0177</td>
<td>0.0350</td>
<td>0.0722</td>
<td>0.1191</td>
<td>0.1789</td>
<td>Logarithmically</td>
</tr>
<tr>
<td>$b$</td>
<td>1.273</td>
<td>1.211</td>
<td>1.143</td>
<td>1.083</td>
<td>1.044</td>
<td>1.007</td>
<td>linearly</td>
</tr>
</tbody>
</table>

10. Variability of DSD

The majority of existing models estimating the radar reflectivity or microwave attenuation from rain intensity are rough ones (equations 33 or 34) because they are neglecting the DSD variability. By other words: two various rain events (shower and continuous rain, for instance) of the same rain intensity, say we 5 mm/h, can cause namely different numerical value of rain attenuation through the DSD variability (3 dB/km in shower and 2 dB/km in continuous rain in our example).
The radar reflectivity factor as well as the specific rain attenuation (of the radar signal, or of the microwave and mm wave link) depend on the rain rate only roughly. They both depend on the drop size distribution (DSD) primarily; this fact is frequently neglected (see equations 12, 13 and 14).

The DSD variability is obvious from Figure 6. This figure shows the probability density of the radar reflectivity factors computed through the equations 12 and 13 from measured DSDs corresponding to rain rates between 4.5 and 5.5 mm/h. After the usually used Marshall Palmer relation

\[ Z = 10 \log (300 R^{1.5}) \]  

the radar reflectivity factor would be 35.3 dBZ, but one can see, that Z varies from 27 to 42 dBZ (radar would announce rain rate between 1 and 6 mm/h in this case!)

\[
\begin{align*}
R &= 5 \text{ mm/h} \quad +/- 0.5 \quad \ldots \quad 35.2 \text{ dBZ after Marshall-Palmer} \\
26 \text{ dBZ} &\ldots \quad 1 \text{ mm/h} \\
41 \text{ dBZ} &\ldots \quad 6 \text{ mm/h}
\end{align*}
\]

Fig. 6. Radar reflectivity factor histogram corresponding to the rain rate between 4.5 and 5.5 mm/h.

A big dispersion of rain rate values R corresponding to the observed values of the radar reflectivity factor Z is also obvious from scatterplots (Figure 7). Again, it is due to the DSD variability.
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$$Z = 10 \log(300 R^{1.5}) \quad [\text{dBZ}]$$

the radar reflectivity factor would be 35.3 dBZ, but one can see, that $Z$ varies from 27 to 42 dBZ (radar would announce rain rate between 1 and 6 mm/h in this case!)

![Fig. 6. Radar reflectivity factor histogram corresponding to the rain rate between 4.5 and 5.5 mm/h.](image)

A big dispersion of rain rate values $R$ corresponding to the observed values of the radar reflectivity factor $Z$ is also obvious from scatterplots (Figure 7). Again, it is due to the DSD variability.

![Fig. 7. Scatterplot of the R-Z relation (DSD data and equations 11 and 13 were used while integration time being 1 minute).](image)

Similar scatterplots “attenuation versus rain rate” were done considering frequencies in the 10 - 100 GHz region. A big dispersion is observed, too, but the dispersion depends on the frequency of radio communication link. It was found that the rain attenuation at frequencies close to 40 GHz depends on the rain rate quite uniquely.

In Figure 8 there are shown imaginary parts of forward scattering functions being responsible for the specific rain attenuation (cf. equation 3). It is obvious, that the slope is varying with the frequency.

![Fig. 8. Imaginary parts of forward scattering functions being proportional to the rain attenuation](image)
Through the mathematical regression we have found the exponent “n” in the approximate relationship

$$\text{Im } f \sim D^n$$  \hspace{1cm} (34)

By other words it means that the specific rain attenuation $A$ is approximately proportional to the $n$-th DSD moment $M_n$ (see equation 4) while “n” varies with the frequency $f$ (unfortunately, there is used the same symbol “f” for both, frequency as well as for scattering function in the technical literature). The found exponent “n” for frequencies between 10 and 100 GHz is shown in Figure 9. Its value is decreasing with the frequency of the radio transmission.

![Figure 9. Exponent “n” in the $\text{Im } f \sim D^n$ dependence, where D is rain drop diameter](image)

After some speculations it could be concluded that the DSD variability courses the measure of the correspondence (uncertainty) between one DSD product ($Z, R$ and $A$) to another one, for instance the $Z - R$ relation.

It is known that the rain rate is given by the 3.67$^{th}$ DSD moment. As the exponent “n” in equation 34 approximates the 3.67 value in the case of frequencies within the 18 – 42 GHz interval, the $A - R$ relationship is quite unique for this frequency range. On the other hand, the 10 GHz specific rain attenuation approximates the 4.5$^{th}$ DSD moment, which is not very far from the 6$^{th}$ moment, i.e. from the radar reflectivity factor $Z$ (equation 13). That’s why the DSD variation does not very influence the $A - Z$ relation in the 10 GHz case and this relationship is not so ambiguous.

The particular contribution of rain drops of certain diameters to the rain attenuation (after equation 14) is varying considering varying frequency. More concretely: the role of small rain drops is increasing with the frequency. The prevailing contribution is caused by drops of the equivolumetric diameter close to 0.7–1.5 mm (see Figure 10).
Through the mathematical regression we have found the exponent “n” in the approximate relationship
\[ \text{Im} f \sim D^n \]  
(34)

By other words it means that the specific rain attenuation \( A \) is approximately proportional to the \( n \)-th DSD moment \( M_n \) (see equation 4) while “n” varies with the frequency \( f \) (unfortunately, there is used the same symbol “f” for both, frequency as well as for scattering function in the technical literature). The found exponent “n” for frequencies between 10 and 100 GHz is shown in Figure 9. Its value is decreasing with the frequency of the radio transmission.

![Figure 9](image1.png)

Fig. 9. Exponent “n” in the \( \text{Im} f \sim D^n \) dependence, where \( D \) is rain drop diameter

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![Figure 10](image2.png)

Fig. 10. Contribution of rain drops of diameter \( D \) to the specific rain attenuation; transmission frequency is a parameter

All results in this contribution were derived from the actual drop size distributions measurement performed by the videodistrometer of ESA, which was lent to the Institute of Atmospheric Physics Prague through its manufacturer Joaneum Research Graz (Austria) in the period July 1998–July 1999 [Fiser et al., 2002]. The time of integration was chosen to be 1 minute.

11. Conclusion

The applicability, importance and properties of the rain drop size distribution was demonstrated in this chapter. It was also shown that DSD determines rain rate, radar reflectivity and rain attenuation of microwave signal. A part of this chapter describes scattering functions describing radiowave reflection from rain drop.

One of the important results of this study is the following one: the radar reflectivity factor derived from the rain rate through the Z-R relation could be incorrect through the DSD variability. It is due to the fact, that the radar reflectivity is given by the 6\(^{th}\) DSD moment while the rain rate depends on 3.67\(^{th}\) DSD moment. A bigger rain drop contributes thus much more to the radar reflectivity than to the rain rate.

The rain attenuation at 10 GHz depends on the radar reflectivity factor quite uniquely and similarly the dependence of 42 GHz rain attenuation on the rain rate is unambiguous. For other relationships between DSD products we recommend the utilisation of the dependence of one DSD product on two other DSD products (or DSD moments), for instance to estimate the specific rain attenuation from both rain rate and radar reflectivity factor.

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Our planet is nowadays continuously monitored by powerful remote sensors operating in wide portions of the electromagnetic spectrum. Our capability of acquiring detailed information on the environment has been revolutionized by revealing its inner structure, morphology and dynamical changes. The way we now observe and study the evolution of the Earth’s status has even radically influenced our perception and conception of the world we live in. The aim of this book is to bring together contributions from experts to present new research results and prospects of the future developments in the area of geosciences and remote sensing; emerging research directions are discussed. The volume consists of twenty-six chapters, encompassing both theoretical aspects and application-oriented studies. An unfolding perspective on various current trends in this extremely rich area is offered. The book chapters can be categorized along different perspectives, among others, use of active or passive sensors, employed technologies and configurations, considered scenario on the Earth, scientific research area involved in the studies.

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