Home energy management problem: towards an optimal and robust solution

Duy Long Ha, Stéphane Ploix, Mireille Jacomino and Minh Hoang Le
G-SCOP lab (Grenoble Institute of Technology)
France

1. Introduction

A home automation system basically consists of household appliances linked via a communication network allowing interactions for control purposes (Palensky & Posta, 1997). Thanks to this network, a load management mechanism can be carried out: it is called distributed control in (Wacks, 1993). Load management allows inhabitants to adjust power consumption according to expected comfort, energy price variation and CO$_2$ equivalent rejection. For instance, during the consumption peak periods when power plants rejecting higher quantities of CO$_2$ are used and when energy price is high, it could be possible to decide to delay some services, to reduce some heater set points or to run requested services even so according to weather forecasts and inhabitant requests. Load management is all the more interesting that local storage and production means exist. Indeed, battery, photovoltaic panels or wind mills provide additional flexibilities. Combining all these elements lead to systems with many degrees of freedom that are very complex to manage by users.

The objective of this study is to setup a general mathematical formulation that makes it possible to design optimized building electric energy management systems able to determine the best energy assignment plan, according to given criteria. A building energy management system consists in two aspects: the load management and the local energy production management. (House & Smith, 1995) and (Zhou & Krarti, 2005) have proposed optimal control strategies for HVAC (Home Ventilation and Air Conditioning) system taking into account the natural thermal storage capacity of buildings that shift the HVAC consumption from peak-period to off-peak period. Zhou & Krarti (2005) has shown that this control strategy can save up to 10% of the electricity cost of a building. However, these approaches do not take into account the energy resource constraints, which generally depend on the autonomy needs of off-grid systems (Muselli et al., 2000) or on the total power production limits of the suppliers in grid connected systems.

The household load management problem can be formulated as a assignment problem where energy is considered as a resource shared by appliances, and tasks are energy consumptions of appliances. Ha et al. (2006a) presents a three-layers household energy control system that is both able to satisfy the maximum available electrical power constraint and to maximize user satisfaction criteria. This approach carries out more reactivity to adapt consumption to the energy provider requirements. Ha et al. (2006b) proposes a global solution for the household load management problem. In order to adapt the consumption to the available energy, the home automation system controls the appliances in housing by determining the
starting time of services and also by computing the temperature set points of HVAC systems. This problem has been formulated as a multi-objective constraint satisfaction problem and has been solved by a dynamic Tabu Search. This approach can carry out the coordination of appliance consumptions of HVAC system and of services in making it possible to set up a compromise between the cost and the user comfort criteria.

With an energy production management production point of view, Henze & Dodier (2003) has proposed an adaptive optimal control for an off-grid PV-hybrid system using a quadratic cost function and a Q-learning approach. It is more efficient than conventional control but it requires to be trained beforehand with actual data covering a long time period. Generally speaking, studies in literature focus only on one aspect of the home energy management problem: the load management or the local energy production but not on the joined load and production management problem.

This chapter formulates the global approach for the building energy management problem as a scheduling problem that takes into account the load consumption and local energy production points of view. The optimization problem of the building energy management is modeled using both continuous and discrete variables: it is modeled as a mixed integer linear problem.

2. Problem description

In this chapter, energy is restricted to electricity consumption and production. Each service is depicted by an amount of consumed/produced electrical power; it is supported by one or several appliances.

2.1 The concept of service

Housing with appliances aims at providing comfort to inhabitants thanks to services which can be decomposed into three kinds: the end-user services that produce directly comfort to inhabitants, the intermediate services that manage energy storage and the support services that produce electrical power to intermediate and end-user services. Support services deal with electric power supplying thanks to conversion from a primary energy to electricity. Fuel cells based generators, photovoltaic power suppliers, grid power suppliers such as EDF in France, belong to this class. Intermediate services are generally achieved by electrochemical batteries. Among the end-user services, well-known services such as clothe washing, water heating, specific room heating, cooking in oven and lighting can be found.

A service with index \( i \) is denoted \( SRV(i) \). Appliances are just involved in services: they are not central from an inhabitant point of view. Consequently, they are not explicitly modelled.

2.2 Caracterisation of services

Let us assume a given time range for anticipating the energy needs (typically 24 hours). A service is qualified as permanent if its energetic consumption/production/storage covers the whole time range of energy assignment plan, otherwise, the service is named temporary service. The following table gives some examples of services according to this classification.

<table>
<thead>
<tr>
<th></th>
<th>temporary services</th>
<th>permanent services</th>
</tr>
</thead>
<tbody>
<tr>
<td>support services</td>
<td>photovoltaic panels</td>
<td>power provider</td>
</tr>
<tr>
<td>intermediate services</td>
<td>-</td>
<td>storage</td>
</tr>
<tr>
<td>end-user services</td>
<td>washing</td>
<td>room heating</td>
</tr>
</tbody>
</table>

The services can also be classified according to the way their behavior can be modified.
Whatever the service is, an end-user, an intermediate or a support service can be modifiable or not. A service is qualified as *modifiable by a home automation system* if the home automation system is capable to modify its behavior (the starting time for example).

There are different ways of modifying services. Sometimes, modifiable services can be considered as continuously modifiable such as the temperature set points in room heating services or the shift of a washing. Some other services may be modified discretely such as the interruption of a washing service. The different ways of modifying services can be combined: for instance, a washing service can be considered both as interruptible and as continuously shiftable. A service modeled as discretely modifiable contains discrete decision variables in its model whereas a continuously modifiable service contains continuous decision variables.

Of course, a service may contain both discrete and continuous decision variables. A service can also be characterized by the way it is known by a home automation system. The consumed or produced power may be observable or not. Moreover, for end-user services, the impact of a service on the inhabitant comfort may be known or not. Obviously, a service can be taken into account by a home automation system if it is at least observable. Some services are indirectly observable. Indeed, all the not observable services can be gathered into a virtual non modifiable service whose consumption/production is deduced from a global power meter measurement and from the observable service consumptions and productions. In addition, a service can be taken into account for long term schedulings if it is predictable. In the same way as for observable services, all the unpredictable services can be gathered into a global no-modifiable predictable service. A service can be managed by a home automation system if it is observable and modifiable. Moreover, it can be long-term managed if it is predictable and modifiable.

![Diagram of services in housing](www.intechopen.com)
2.3 Principle of control mechanism

An important issue in home automation problems is the uncertainties in the model data. For instance, solar radiation, outdoor temperature or services requested by inhabitants may not be predicted with accuracy. In order to solve this issue, a three-layer architecture is presented in this chapter: a local layer, a reactive layer and an anticipative layer (see figure 2).

The **anticipative layer** is responsible for scheduling end-user, intermediate and support services taking into account predicted events and costs in order to avoid as much as possible the use of the **reactive layer**. The prediction procedure forecasts various informations about future user requests but also about available power resources and costs. Therefore, it uses information from predictable services and manage continuously modifiable and shiftable services. This layer has slow dynamics and includes predictive models with learning mechanisms, including models dealing with inhabitant behaviors. This layer also contains a predictive control mechanism that schedules energy consumption and production of end-user services several hours in advance. This layer computes plans according to available predictions. The sampling period of the anticipative layer is denoted $\Delta$. This layer relies on the most abstract models.

The **reactive layer** has been detailed in (Abras et al., 2006). Its objective is to manage adjustments of energy assignment in order to follow up a plan computed by the upper **anticipative layer** in spite of unpredicted events and perturbations. Therefore, this layer manages modifiable services and uses information from observable services (comfort for end-user services and power for others). This layer is responsible for decision-making in case of violation of predefined constraints dealing with energy and inhabitant comfort expectations: it performs arbitrations between services. The set-points determined by the plan computed by the upper **anticipative layer** are dynamically adjusted in order to avoid user dissatisfaction. The control actions may be dichotomic in enabling/disabling services or more gradual in adjusting...
set-points such as reducing temperature set point in room heating services or delaying a temporary service. Actions of the reactive layer have to remain transparent for the plan computed by the anticipative layer: it can be considered as a fast dynamic unbalancing system taking into account actual housing state, including unpredicted disturbances, to satisfy energy, comfort and cost constraints. If the current state is too far from the computed plan, the anticipative layer has to re-compute it.

The local layer is composed of devices together with their existing local control systems generally embedded into appliances by manufacturers. It is responsible for adjusting device controls in order to reach given set points in spite of perturbations. This layer abstracts devices and services for upper layers: fast dynamics are hidden by the controllers of this level. This layer is considered as embedded into devices: it is not detailed into this chapter.

This chapter mainly deals with the scheduling mechanism of the anticipative layer, which computes anticipative plans as shown in figure 3.

3. Modeling services

Modeling services can be decomposed into two aspects: the modeling of the behaviors, which depends on the types of involved models, and the modeling of the quality of the execution of services, which depends on the types of service. Whatever the type of model it is, it has to be
3.1 Modeling behavior of services

In order to model the behavior of the different kinds of services in housing, three different types of models have been used: discrete events are modeled by finite state machines, continuous behaviors are modeled by differential equations and mixed discrete and continuous evolutions are modeled by hybrid models that combine the two previous ones.

Using finite state machines (FSM)

A finite state machine dedicated to a service $SRV$ is composed of a finite number of states \( \{L_m; m \in \{1, \ldots, M\}\} \) and a set of transitions between those states \( \{T_{p,q} \in \{0,1\}; (p,q) \in S \subset \{1, \ldots, M\}^2\} \). Each state $L_m$ of a service $SRV$ is linked to a phase characterized by a maximal power production $P_m > 0$ or consumption $P_m < 0$.

A transition triggers a state change. It is described by a condition that has to be satisfied to be enabled. The condition can be a change of a state variable measured by a sensor, a decision of the anticipative mechanism or an elapsed time for phase transition. If it exists a transition between the state $L_m$ and $L_{m'}$ then $T_{m,m'} = 1$, otherwise $T_{m,m'} = 0$. An action can be associated to each state: it may be a modification of a set-point or an on/off switching. As an example, let’s consider a washing service.

The service provided by a washing machine may be modeled by a FSM with 4 states: the first state is the stand-by state $L_1$ with a maximal power of $P_1 = -5W$ (it is negative because it deals with consumed power). The transition towards the next state is triggered by the anticipative mechanism. The second state is the water heating state $L_2$ with $P_2 = -2400W$. The transition to the next state is triggered after $\tau_2$ time units. The next state corresponds to the washing characterized by $P_3 = -500W$. And finally, after a given duration $\tau_3$ depending on the type of washing (i.e. the type of requested service), the spin-drying state is reached with $P_3 = -1000W$. After a given duration $\tau_4$, the stand-by state is finally recovered. Considering that the initial state is $L_1$, this behavior can be formalized by:

\[
\begin{align*}
\begin{cases}
(state = L_1) \land (t = t_{start}) & \rightarrow state = L_2 \\
(state = L_2) \land (t = t_{start} + \tau_2) & \rightarrow state = L_3 \\
(state = L_3) \land (t = t_{start} + \tau_2 + \tau_3) & \rightarrow state = L_4 \\
(state = L_4) \land (t = t_{start} + \tau_2 + \tau_3 + \tau_4) & \rightarrow state = L_1
\end{cases}
\end{align*}
\]  

Using differential equations

In buildings, thermal phenomena are continuous phenomena. In particular, the thermal behavior of a HVAC system can be modeled by state space models:

\[
\begin{cases}
\frac{dx_c(t)}{dt} = A_c x_c(t) + B_c u_c(t) + F_c p_c(t) \\
y_c(t) = C x_c(t)
\end{cases}
\]

$x_c(t)$ contains state variables, usually temperature. $u_c(t)$ contains controlled input variables such as energy flows. $p_c(t)$ contains known but uncontrolled input variables such as outside temperature or solar radiance. A first order state space thermal model relevant for control purpose has been proposed in Nathan (2001) but the second order model based on an electric
analogy proposed in Madsen (1995) has been preferred for our control purpose because it models the dynamic of indoor temperature. For a room heating service \( SRV(i) \), it yields:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix} &= A_c \begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix} + B_c \begin{bmatrix} P(i,t) \\ \phi(i,t) \end{bmatrix} \\
T_{in}(i,t) &= C_c \begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix}
\end{align*}
\]

(3)

with \( A_c = \begin{bmatrix} -\frac{1}{\tau_{in}c_{in}} & \frac{1}{\tau_{in}c_{in} + \tau_{env}} \\ 0 & \frac{1}{\tau_{env}c_{in}} \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, F_c = \begin{bmatrix} 0 \\ \frac{w}{c_{in}} \end{bmatrix} \) and \( C_c = [1 \ 0] \).

This model allows a rather precise description of the dynamic variations of indoor temperature with:

- \( T_{in}, T_{out}, T_{env} \) the respective indoor, outdoor and housing envelope temperatures
- \( c_{in}, c_{env} \) the thermal capacities of first indoor environment and second the envelope of the housing
- \( r_{in}, r_{env} \) thermal resistances
- \( w \) the equivalent surface of the windows
- \( P \) the power consumed by the thermal generator, \( P \leq 0 \). In this chapter, this flow is assumed to correspond to an electric energy flow.
- \( \phi \) the energy flow generated by the solar radiance

In order to solve the anticipative problem, continuous time models have to be discretized according to the anticipation period \( \Delta \). Equation (2) modelling service \( SRV(i) \) becomes:

\[
\forall k \in \{1, \ldots, K\}, \\
\begin{bmatrix} T_{in}(i,k+1) \\ T_{env}(i,k+1) \end{bmatrix} = A_i \begin{bmatrix} T_{in}(i,k) \\ T_{env}(i,k) \end{bmatrix} + B_i \begin{bmatrix} E(i,k) \end{bmatrix} + F_i \begin{bmatrix} T_{out}(i,k) \\ \phi(i,k) \end{bmatrix}
\]

(4)

with \( A_i = e^{A_i \Delta}, B_i = (e^{A_i \Delta} - I_n)A_c^{-1}B_c, F_i = (e^{A_i \Delta} - I_n)A_c^{-1}F_c, E(i,k) = P(i,k)\Delta \) and \( E(i,k) \leq 0 \).

**Using hybrid models**

Some services cannot be modeled by a finite state machine nor by differential equations. Both approaches have to be combined: the resulting model is then based on a finite state machine where each state \( L_m \) actually becomes a set of states which evolution is depicted by a differential equation.

An electro-chemical storage service supported by a battery may be modeled by a hybrid model (partially depicted in figure 4). \( x(i) \) stands for the quantity of energy inside the battery and \( u(t) \) the controlled electrical power exchanged with the grid network.

**Using static models**

Power sources are usually modelled by static constraints. Local intermittent power resources, such as photovoltaic power system or local electric windmill, and power suppliers are considered here. Using weather forecasts, it is possible to predict the power production \( w(i,k) \)
Energy Management during each sampling period \([k\Delta, (k + 1)\Delta]\) of a support service \(SRV(i)\). The available energy for each sampling period \(k\) is then given by:

\[
E(i, k) = w(i, k)\Delta \forall k \in \{1, ..., K\}
\]

with \(w(i, k) \geq 0\)

According to the subscription between inhabitants and a power supplier, the maximum available power is given. It may depends on time. For a service of power supply \(SRV(i)\), it can be modelled by the following constraint:

\[
E(i, k) \leq p_{\text{max}}(i, k)\Delta \forall k \in \{1, ..., K\}
\]

where \(p_{\text{max}}(i, k)\) stands for the maximum available power.

3.2 Modeling quality of the execution of services

Depending on the type of service, the quality of the service achievement may be assessed in different ways. End-user services provide comfort to inhabitants, intermediate services provide autonomy and support services provide power that can be assessed by its cost and its impact on the environment. In order to evaluate these qualities different types of criteria have been introduced.

End-user services

Generally speaking, modifiable permanent services use to control a physical variable: the user satisfaction depends on the difference between an expected value and an actual one. Let’s consider for example the HVAC controlling a temperature. A flat can usually be split into several HVAC services related to rooms (or thermal zones) assumed to be independent.

According to the comfort standard 7730 (AFNOR, 2006), three qualitative categories of thermal comfort can be distinguished: A, B and C. In each category, (AFNOR, 2006) proposes typical value ranges for temperature, air speed and humidity of a thermal zone that depends on the type of environment: office, room,… These categories are based on an aggregated criterion named Predictive Mean Vote (PMV) modelling the deviation from a neutral ambience. The absolute value of this PMV is an interesting index to evaluate the quality of a HVAC service. In order to simplify the evaluation of the PMV, typical values for humidity and air speed are used. Therefore, only the ambient temperature corresponding to the neutral value of PMV (PMV=0) is dynamically concerned. Under this assumption, an ideal temperature \(T_{\text{opt}}\) is obtained. Depending on the environment, an acceptable temperature range coming from

\[
\begin{align*}
  & \frac{dx(t)}{dt} = \rho u(t) \\
  & \quad u(t) > 0 \\
  & u(t) = 0 \quad \text{discharging} \\
  & u(t) < 0 \quad \text{charging} \\
  & \frac{dx(t)}{dt} = \rho u(t) \\
  & u(t) = 0 \quad \text{stand-by} \\
  & u(t) < 0
\end{align*}
\]

Fig. 4. Hybrid model of a battery
the standard leads to an interval \([T_{\text{min}}, T_{\text{max}}]\). For instance, in an individual office in category A, with typical air speed and humidity conditions, the neutral temperature is \(T_{\text{opt}} = 22^\circ C\) and the acceptable range is \([21^\circ C, 23^\circ C]\).

Therefore, considering the HVAC service \(\text{SRV}(i)\), the discomfort criterion \(D(i,k)\), which is more usable than comfort criterion here, is modelled by the following formula where assumptions are depicted by two parameters \(a_1\) and \(a_2\):

\[
D(i,k) = |PMV(T_{\text{in}}(i,k))| = \begin{cases} 
  a_1 \times \frac{(T_{\text{opt}} - T_{\text{in}}(i,k))}{T_{\text{opt}} - T_{\text{Min}}} & \text{if } T_{\text{in}}(i,k) \leq T_{\text{opt}} \\
  a_2 \times \frac{(T_{\text{opt}} - T_{\text{in}}(i,k))}{T_{\text{Max}} - T_{\text{opt}}} & \text{if } T_{\text{in}}(i,k) > T_{\text{opt}}
\end{cases}
\] (7)

The global comfort criterion is defined as following:

\[
D(i) = \sum_{k=1}^{K} D(i,k)
\] (8)

Generally speaking, modifiable temporary end-user services do not aim at controlling a physical variable. Temporary services such as washing are expected by inhabitants to finished at a given time. Therefore, the quality of achievement of a temporary service depends on the amount of time it is shifted. Therefore, in the same way as for permanent services, a user dissatisfaction criterion for a service \(\text{SRV}(i)\) is defined by:

\[
D(i) = \begin{cases} 
  \frac{f(i) - f_{\text{opt}}(i)}{f_{\text{max}}(i) - f_{\text{opt}}(i)} & \text{if } f(i) > f_{\text{opt}}(i) \\
  \frac{f_{\text{opt}}(i) - f(i)}{f_{\text{opt}}(i) - f_{\text{min}}(i)} & \text{if } f(i) \leq f_{\text{opt}}(i)
\end{cases}
\] (9)

where \(f_{\text{opt}}\) stands for the requested end time and \(f_{\text{min}}\) and \(f_{\text{max}}\) stand respectively for the minimum and maximum acceptable end time.

**Intermediate services**

Intermediate services are composed of two kinds of services: the power storage services, which store energy to be able to face difficult situations such as off-grid periods, and then lead to the availability of the stored power supplier services (see figure 1). A power storage service \(\text{SRV}(i)\) and a stored power service \(\text{SRV}(j); j \neq i\) are associated to each storage system. The quality of a power storage service has to be evaluated: it is related to the amount of stored energy. This quality is called autonomy.

Let us consider a electric storage system modelled by a power storage service \(\text{SRV}(i)\) and by a stored power supplier service \(\text{SRV}(j)\). The stock \(E_{\text{stock}}(k)\) of the storage system is modelled by:

\[
E_{\text{stock}}(k) = E_{\text{stock initial}} - \sum_{i=1}^{k} (E(i, \xi) + E(j, \xi))
\] (10)

with \(E(i, \xi) \leq 0\) and \(E(j, \xi) \geq 0\).

Let \(P_{\text{ref}}\) be the reference power taken into account for the computation of the autonomy duration \(\tau_{\text{autonomy}}\). The autonomy objective \(A(k)\) can be defined by:

\[
A(k) = \sum_{k \in \{1,...,K\}} E_{\text{stock}}(k)
\] (11)
Depending on the inhabitant expectations, autonomy can also be formulated by constraints to be satisfied at any sample time: \( P_{\text{ref}}^{\text{autonomy}} - E^\text{stock}(k) = 0, \forall k \in \{1, \ldots, K\} \).

Let’s now focus on stored power supplier service. What is the quality for this service i.e. the service that provides stored energy to the housing. It is not a matter of economy nor of ecology because costs is already taken into account when power production services provide power to the storage system. It is not also a matter of stored energy: there is no quality of service defined for stored power supplier service.

### Support services

Support services dealing with power resources do not interact directly with inhabitants. However, inhabitants do care about their cost and their environmental impact. These two aspects have to be assessed.

In most cases, the economical criterion corresponds to the cost of the provided, stored or sold energy. This cost may contain depreciation of the device used to produce power.

Let \( \text{SRV}(0) \) be a photovoltaic support service and \( \text{SRV}(1) \) be a power supplier service. Let’s examine the case of power provider such as EDF in France. Energy is sold at a given price \( C(1,k) \) to the customer for each consumed kWh at time \( k \). In order to promote photovoltaic production, power coming from photovoltaic plants is bought by the supplier at higher price \( C(0,k) > C(1,k) \).

Different power metering principles can be subscribed with a French power supplier. Only the most widespread principle is addressed. The energy cost is thus given by the following equation:

\[
C(k) = C(1,k)E(1,k) - C(0,k)E(0,k), \forall k \in \{1, \ldots, K\} \tag{12}
\]

The equivalent mass of carbon dioxide rejected in the atmosphere has been used as ecological criterion for a support service. This criterion is easy to establish for most power devices: photovoltaic cells, generator and even for energy coming from power suppliers. Powernext energy exchange institution publishes the equivalent mass of carbon dioxide rejected in the atmosphere per power unit in function of time (see [http://www.powernext.fr](http://www.powernext.fr)). For instance, in France, electricity coming from the grid network produces 66g/kWh of CO\(_2\) during off-peak periods and 383g/kWh during peak period (Angioletti & Despretz, 2003). Energy coming from photovoltaic panels is considered as free of CO\(_2\) rejection (grey energy is not taken into account). For each support service \( \text{SRV}(i) \), a CO\(_2\) rejection rate \( \tau_{\text{CO}_2}(i,k) \) can be defined as the equivalent volume of CO\(_2\) rejected per kWh. Therefore, the total rejection for a support service \( \text{SRV}(i) \) during the sampling period \( k \) is given by \( \tau_{\text{CO}_2}(i,k)E(i,k) \) where \( E(i,k) \) corresponds to the energy provided by the support service \( \text{SRV}(i) \) during the sampling period \( k \).

### 4. Formulation of the anticipative problem as a linear problem

The formulation of the energy management problem contains both behavioral models with discrete and continuous variables, differential equation and finite state models, and quality models with nonlinearities such as in the PMV model. In order to get mixed linear problems which can be solved by well known efficient algorithms, transformations have to be done. The ones that have been used are summarized in the next section.

#### 4.1 Transformation tools

Basically, a proposition denoted \( \mathcal{X} \) is either true or false. It can result from the combination of propositions thanks to connecting operators such as "\&" (and), "\lor" (or), "\oplus" (exclusive or), "\Rightarrow"

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(not), "→" (implies), "↔" (if and only if)... Whatever the proposition \( \mathcal{X} \) is, it can be associated to a binary variable \( \delta \in \{0, 1\} \) such as: \( \mathcal{X} = (\delta = 1) \).

Therefore, (Williams, 1993) has shown that, in integer programming, connecting operators may be modelled by:

\[
\begin{align*}
\forall \mathcal{X} & \quad \leftrightarrow \quad \delta = 0 \\
\mathcal{X}_1 \land \mathcal{X}_2 & \quad \leftrightarrow \quad \delta_1 + \delta_2 = 2 \\
\mathcal{X}_1 \lor \mathcal{X}_2 & \quad \leftrightarrow \quad \delta_1 + \delta_2 \geq 1 \\
\mathcal{X}_1 \oplus \mathcal{X}_2 & \quad \leftrightarrow \quad \delta_1 + \delta_2 = 1 \\
\mathcal{X}_1 \rightarrow \mathcal{X}_2 & \quad \leftrightarrow \quad \delta_1 - \delta_2 \leq 0 \\
\mathcal{X}_1 \leftrightarrow \mathcal{X}_2 & \quad \leftrightarrow \quad \delta_1 - \delta_2 = 0
\end{align*}
\]

(13)

According to (Bemporad & Morari, 1998), the transformation into a standard linear problem can be achieved using lower and upper bounds of \( \text{dom}(f(x); x \in \text{dom}(x)) = \text{dom}(ax - b; x \in \text{dom}(x)) \subset [m, M] \). Then, Binary variables can be connected to linear conditions as follows:

\[
\delta = (ax - b \leq 0) \leftrightarrow \begin{cases} 
ax - b \leq M(1 - \delta) \\
ax - b > m\delta
\end{cases}
\]

(14)

Consider for instance the statement \( a_1x \leq b_1 \leftrightarrow a_2x' \leq b_2 \). Using the previous transformation, it can be formulated as:

\[
\begin{align*}
a_1x - b_1 & \leq M(1 - \delta) \\
a_1x - b_1 & \leq m\delta \\
a_2x' - b_2 & \leq M(1 - \delta) \\
a_2x' - b_2 & \leq m\delta
\end{align*}
\]

with \( \text{dom}(a_1 x - b_1; x \in \text{dom}(x)) \cup \text{dom}(a_2 x' - b_2; x' \in \text{dom}(x')) \subset [m, M] \).

In many cases, such as in presence of absolute values like in PMV evaluation, products of discrete and continuous variables appear. They have to be reformulated in order to get mixed linear problems. Auxiliary variables may be used for this purpose. First consider the product of 2 binary variables \( \delta_1 \) and \( \delta_2 \): \( \delta_3 = \delta_1 \times \delta_2 \). It can be transformed into a discrete linear problem:

\[
\delta_3 = \delta_1 \times \delta_2 \leftrightarrow \begin{cases} 
-\delta_1 + \delta_3 \leq 0 \\
-\delta_2 + \delta_3 \leq 0 \\
\delta_1 + \delta_2 - \delta_3 \leq 1 \\
\delta_1, \delta_2, \delta_3 \in \{0, 1\}
\end{cases}
\]

(15)

Consider now the product of a binary variable with a continuous variable: \( z = \delta \times x \) where \( \delta \in \{0, 1\} \) and \( x \in [m, M] \). It means that \( \delta = 0 \rightarrow z = 0 \) and \( \delta = 1 \rightarrow z = x \). Therefore, the semi-continuous variable \( z \) can be transformed into a mixed linear problem:

\[
z = \delta \times x \leftrightarrow \begin{cases} 
z \leq M \times \delta \\
z \geq m\delta \\
z \leq x - m(1 - \delta) \\
z \geq x - M(1 - \delta)
\end{cases}
\]

(16)

These transformations can now be used to remove nonlinearities from the PMV computations, time shifting of services and power storage.
4.2 Linearization of PMV

Generally speaking, behavioral models of HVAC systems is given by Eq. (2) and an example is given by (3). Model (4) is already linear but nonlinearities come up with the absolute value of the PMV evaluation. Let’s introduce a binary variable $\delta_a(k)$ satisfying $\delta_a(k) = 1 \Leftrightarrow T_{in}(k) \leq T_{opt} \forall k$. Then, the PMV function (7) can be reformulated into a mixed linear form for every service $SRV(i)$:

$$|PMV(T_{ia}(k))| = \delta_a(k) \times a_1 \times \frac{(T_a(i,k) - T_{opt})}{T_{max} - T_{opt}} + (1 - \delta_a(k)) \times a_2 \times \frac{(T_{opt} - T_a(k))}{T_{max} - T_{opt}}$$

Using eq. (14) to transform the absolute value, the equivalent form of the condition that contains $T_a(k) \leq T_{opt}$ is given by:

$$\left\{ \begin{array}{l}
T_a(k) - T_{opt} \leq (T_{max} - T_{opt})(1 - \delta_a(k)) \\
T_a(k) - T_{opt} \geq \epsilon + (T_{min} - T_{opt} - \epsilon)\delta_a(k)
\end{array} \right.$$  

A semi-continuous variable $z_a(k)$ is added to take place of the product $\delta_a(k) \times T_{in}(k)$ in eq. (17). According to eq. (16), the transformation of $z_a(k) \equiv \delta_a(k) \times T_{in}(k)$ leads to:

$$\left\{ \begin{array}{l}
z_a(k) \leq (T_{max} - T_{opt})\delta_a(k) \\
z_a(k) \geq (T_{min} - T_{opt})\delta_a(k) \\
z_a(k) \leq T_{in}(k) - (T_{min} - T_{opt})(1 - \delta_a(k)) \\
z_a(k) \geq T_{in}(k) - (T_{max} - T_{opt})(1 - \delta_a(k))
\end{array} \right.$$  

After the linearization of PMV, let’s now consider the linearization of the time shifting of services.

4.3 Formalizing time shifting

Temporary services are modelled by finite state machines. The consumption of a state can be shifted such as task in scheduling problems. The starting and ending times of services can be synchronized to an anticipative period such as in (Duy Ha, 2007). It leads to a discrete-time formulation of the problem. However, this approach is both a restriction of the solution space and an approximation because the length of a time service has to be a multiple of $\Delta$. The general case has been considered here.
In the scientific literature, continuous time formulations of scheduling problems exist (Castro & Grossmann, 2006; Pinto & Grossmann, 1995; 1998). However, these results concern scheduling problems with disjunctive resource constraints. Instead of computing the starting time of tasks, the aim is to determine the execution sequence of tasks on shared resources. In energy management problems, the matter is not restricted to determine such sequence because several services can be achieved at the same time.

An alternative formulation based on transformations (14) and (16), suitable for the energy management in housings, is introduced.

Temporary services can be continuously shifted. Let $DUR(i,j)$, $f(i,j)$ and $p(i,j)$ be respectively the duration of the state $j$ of service $SRV(i)$, the ending time and the power related to the service $SRV(i)$ during the state $j$. $f(i,j)$ is defined according to inhabitant comfort models: they correspond to extrema in the comfort models presented in section 3.2.

According to (Esquirol & Lopez, 1999), the potential consumption/production duration (effective duration if positive) $d(i,j,k)$ of a service $SRV(i)$ in state $j$ during a sampling period $[k\Delta, (k+1)\Delta]$ is given by (see figure 5):

$$d(i,j,k) = \min(f(i,j), (k+1)\Delta) - \max(f(i,j) - DUR(i,j), k\Delta) \tag{20}$$

Therefore, the consumption/production energy $E(i,j,k)$ of the service $SRV(i)$ in state $j$ during a sampling period $[k\Delta, (k+1)\Delta]$ is given by:

$$E(i,j,k) = \begin{cases} d(i,j,k)p(i,j) & \text{if } d(i,j,k) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

$E(i,j,k)$ can be modelled using a binary variable: $\delta_{10}(i,j,k) = (d(i,j,k) \geq 0)$ and a semi-continuous variable $z_{10}(i,j,k) = \delta_{10}(i,j,k)d(i,j,k)$ such as in (14) and in (16). It leads to the following inequalities:

$$d(i,j,k) \leq \delta_{10}(i,j,k)\Delta \tag{22}$$

$$d(i,j,k) > (\delta_{10}(i,j,k) - 1)\Delta \tag{23}$$

$$E(i,j,k) = z_{10}(i,j,k)p(i,j) \tag{24}$$

$$z_{10}(i,j,k) \leq \delta_{10}(i,j,k)\Delta \tag{25}$$

$$z_{10}(i,j,k) \geq -\delta_{10}(i,j,k)\Delta \tag{26}$$

$$z_{10}(i,j,k) \leq d(i,j,k) + (1 - \delta_{10}(i,j,k))\Delta \tag{27}$$

$$z_{10}(i,j,k) \geq d(i,j,k) - (1 - \delta_{10}(i,j,k))\Delta \tag{28}$$

But the model still contains nonlinear functions min and max in the expression of $d(i,j,k)$. Therefore, equation (20) has to be transformed into a mixed-linear form. Let’s introduce 2 binary variables $\delta_{11}(i,j,k)$ and $\delta_{12}(i,j,k)$ defined by:

$$\delta_{11}(i,j,k) = (f(i,j) - k\Delta \geq 0)$$

$$\delta_{12}(i,j,k) = (f(i,j) - DUR(i,j) - k\Delta \geq 0)$$

Using (14), it yields:

$$f(i,j) - k\Delta \leq \delta_{11}(i,j,k)\Delta \tag{29}$$

$$f(i,j) - k\Delta \geq (\delta_{11}(i,j,k) - 1)\Delta \tag{30}$$

$$f(i,j) - DUR(i,j) - k\Delta \leq \delta_{12}(i,j,k)\Delta \tag{31}$$

$$f(i,j) - DUR(i,j) - k\Delta \geq (\delta_{12}(i,j,k) - 1)\Delta \tag{32}$$
Therefore, min and max of equation (20) become:

\begin{align*}
 f_{\min}(i, j, k) &= \delta_{t_1}(i, j, k, k+1)(k+1)\Delta + (1 - \delta_{t_1}(i, j, k+1)) f(i, j) \\
 s_{\max}(i, j, k) &= \delta_{t_2}(i, j, k)(f(i, j) - DUR(i, j)) + (1 - \delta_{t_2}(i, j, k)) k\Delta
\end{align*}

with \( f(i, j), (k + 1)\Delta = f_{\min}(i, j, k) \) and \( f(i, j) - DUR(i, j), k\Delta = s_{\max}(i, j, k) \).

The duration \( d(i, j, k) \) can then be evaluated:

\[ d(i, j, k) = f_{\min}(i, j, k) - s_{\max}(i, j, k) \] (35)

Equations (22) to (35) model the time shifting of a temporary service.

Let’s now consider nonlinearities inherent to power storage services modelled by hybrid models.

### 4.4 Linearization of power storage

A storage service \( SRV(i) \) with a maximum capacity of \( E_{\text{stock}}^{\text{max}} \) can be modelled at time \( k \) by:

\[ E_{\text{stock}}(i, k) = \max(\min(E_{\text{stock}}^{\text{max}}, E_{\text{stock}}(i, k-1) + E(i, k-1)), 0) \]

Let’s define the following binary variables: \( \delta_1(i, k) = (E_{\text{stock}}(i, k) \leq E_{\text{stock}}^{\text{max}}) \) and \( \delta_2(i, k) = (E_{\text{stock}}(i, k) \geq 0) \). Using (14), it yields:

\begin{align*}
 E_{\text{stock}}(i, k) - E_{\text{stock}}^{\text{max}} &\leq (1 - \delta_1(i, k)) E_{\text{stock}}^{\text{max}} \\
 E_{\text{stock}}(i, k) - E_{\text{stock}}^{\text{max}} &\geq -\delta_1(i, k) E_{\text{stock}}^{\text{max}} \\
 E_{\text{stock}}(i, k) &\leq \delta_2(i, k) E_{\text{stock}}^{\text{max}} \\
 E_{\text{stock}}(i, k) &\geq (\delta_2(i, k) - 1) E_{\text{stock}}^{\text{max}}
\end{align*}

The stored energy can then be written:

\[ E_{\text{stock}}(i, k) = \delta_1(i, k-1)\delta_2(i, k-1) (E_{\text{stock}}(i, k-1) + E(i, k-1)) \ldots + (1 - \delta_1(i, k)) E_{\text{stock}}^{\text{max}} \]

With variables \( \delta_3(i, k) = \delta_1(i, k)\delta_2(i, k), z_1(i, k) = \delta_3(i, k)E_{\text{stock}}(i, k) \) and \( z_2(i, k) = \delta_3(i, k)E(i, k) \) and using transformations (15) and (16), the energy \( E_{\text{stock}}(i, k) \) can be rewritten into a linear form:

\[ E_{\text{stock}}(i, k) = z_1(i, k-1) + z_2(i, k-1) + (1 - \delta_1(i, k)) E_{\text{stock}}^{\text{max}} \]

The following constraints must be satisfied:

\begin{align*}
 -\delta_1(i, k) + \delta_3(i, k) &\leq 0 \\
 -\delta_2(i, k) + \delta_3(i, k) &\leq 0 \\
 \delta_1(i, k) + \delta_2(i, k) - \delta_3(i, k) &\leq 1 \\
 z_1(i, k) &\leq \delta_3(i, k) E_{\text{stock}}^{\text{max}} \\
 z_1(i, k) &\geq -\delta_3(i, k) E_{\text{stock}}^{\text{max}} \\
 z_1(i, k) &\leq E_{\text{stock}}(i, k) + (1 - \delta_3(i, k)) E_{\text{stock}}^{\text{max}} \\
 z_1(i, k) &\geq E_{\text{stock}}(i, k) - (1 - \delta_3(i, k)) E_{\text{stock}}^{\text{max}} \\
 z_2(i, k) &\leq \delta_3(i, k) E_{\text{stock}}^{\text{max}} \\
 z_2(i, k) &\geq -\delta_3(i, k) E_{\text{stock}}^{\text{max}} \\
 z_2(i, k) &\leq E(i, k) + (1 - \delta_3(i, k)) E_{\text{stock}}^{\text{max}} \\
 z_2(i, k) &\geq E(i, k) - (1 - \delta_3(i, k)) E_{\text{stock}}^{\text{max}}
\end{align*}
Equations (40) to (51) are a linear model of a power storage service. Main services have been modelled by mixed integer linear form. Other services can be modelled in the same way. Let’s now focus on how to solve the resulting mixed integer linear problem.

5. Solving approach

Anticipative control in home energy management can be formulated as an multicriteria mixed-linear programming problem represented by a set of constraints and optimization criteria.

5.1 Problem summary

In an actual problem, the number of constraints is so large they cannot be detailed in this chapter. Nevertheless, the fundamental modelling and transformation principles have been presented in sections 3 and 4. HVAC services are representative examples of permanent services. They have been modelled by equations like (4) and (19). The decision variables are heating powers $\Phi_s(i, k)$. Temporary services, such as clothes washing, are modelled by equations like (22) to (35). The decision variables are the ending times: $f(i, j)$. Storage services are modelled by equations like (40) to (51). The decision variables are energy exchange with the storage systems: $E(i, j)$. Power supplier services are modelled by equations like (5). There is no decision variable for these services.

These results can be adapted to fit most situations. If necessary, more details about modelling can be found in (Duy Ha, 2007). As a summary, the following constraints may be encountered:

- linearized behavioral models of services
- linearized comfort models related to end-user services

In addition, a constraint modelling the production/consumption balance has to be added. Generally speaking, this constraint can be written:

$$\forall k \in \{1, \ldots, K\}, \sum_{i \in I} E(i, k) = 0$$

where $I$ contains the indexes of available predictable services.

If there is a grid power supplier modelled by a support service $SRV(0)$, the imported energy can be adjusted to effective needs (it is also true for fuel cells based support services). Therefore, $E(0, k)$ has to be set to the maximum available energy for a sampling period: $E(0, k) = P^{\text{max}}(0, k) \Delta$ where $P^{\text{max}}(0, k)$ stands for the maximum available power during sampling period $k$. Consequently, (52) becomes:

$$\forall k \in \{1, \ldots, K\}, \sum_{i \in \mathcal{I}} E(i, k) \geq 0$$

All the predictable but not modifiable services provide data to the optimization problem. Their indexes are contained in $\mathcal{T}^{\text{modifiable}} \subset \mathcal{I}$. Decision variables are all related to predictable and modifiable services: they may be binary or continuous decision variables. The problem to be solved is thus a mixed-linear programming problem. Moreover, the optimization problem is a multi-criteria problem using the following criteria: economy, dissatisfaction, CO2eq and autonomy criteria.
Economy criterion is given by (12) when there is only a grid power supplier and a photovoltaic power supplier. Depending of the predictable support services $T_{support}$ excluding photovoltaic power supplier and on the existence of photovoltaic power supplier $SRV(0)$, 

$$f_{\text{autonomy}} = \sum_{k=1}^{K} \left( \sum_{i \in T_{support}} C(i,k)E(i,k) - C(0,k)E(0,k) \right)$$

(54)

where $C(i,k)$ stands for the kWh cost of the support service $i$. Dissatisfaction criterion comes from expressions like (7) and (9). Let $T_{\text{end-user}} \subset I$ be the indexes of predictable end-user services. The comfort criteria may be given by:

$$f_{\text{discomfort}} = \sum_{i \in T_{\text{end-user}}} \sum_{k \in \{1,...,K\}} D(i,k)$$

(55)

The autonomy criterion comes from (11). It is given by:

$$f_{\text{autonomy}} = \sum_{k \in \{1,...,K\}} A(k)$$

(56)

If there are several storage systems, the respective $A(k)$ have to be summed up in the criterion $f_{\text{autonomy}}$.

Finally, the CO2 equivalent rejection can be computed like the autonomy criteria:

$$f_{\text{CO2eq}} = \sum_{k=1}^{K} \sum_{i \in T_{support}} \tau_{CO2}(i,k)E(i,k)$$

(57)

where $\tau_{CO2}(i,k)$ stands for the CO2 equivalent volume rejection for 1 kWh consummed by the support service $i$ and $T_{support}$ gathers the indexes of predictable support services. All these criteria can be aggregated into a global criterion. $\alpha$-criterion approaches can also be used.

5.2 Decomposition into subproblems

In section 2.2, services have been split into permanent and temporary services. Let $T_{\text{temporary}}$ be the indexes of modifiable and predictable temporary services. It is quite usual in housing that some modifiable and predictable temporary services cannot occur at the same time, whatever the solution is. Using this property, the search space can be reduced.

Let’s defined the horizon of a service.

**Definition 1.** The horizon of a service $SRV(i)$, denoted $H(SRV(i))$, is a time interval in which $SRV(i)$ may consume or produce energy.

The horizon of a service $SRV(i)$ is denoted: $[H(SRV(i)), \bar{H}(SRV(i))] \subseteq [0, K\Delta]$. A permanent service has an horizon equal to $[0, K\Delta]$. A temporary service $SRV(i)$ has an horizon given by $H(SRV(i)) = s_{\text{min}}(i)$ (the earliest starting of the service) and $\bar{H}(SRV(i)) = f_{\text{max}}(i)$ (the latest ending of the service).

Only predictable and modifiable services are considered in the following because they contain decision variables. Two predictable and modifiable services may interact if and only if there is a non empty intersection between their horizons.

**Definition 2.** Two predictable and modifiable services $SRV(i)$ and $SRV(j)$ are in direct temporal relation if $H(SRV(i)) \cap H(SRV(j)) \neq \emptyset$. The direct temporal relation between $SRV(i)$ and $SRV(j)$ is denoted $SRV(i), SRV(j) = 1$ if it exists, and $SRV(i), SRV(j) = 0$ otherwise.
If \( H(SRV(i)) \cap H(SRV(j)) = \emptyset \), \( SRV(i) \) and \( SRV(j) \) are said temporally independent. Even if two services \( SRV(i) \) and \( SRV(j) \) are not in direct temporal relation, it may exists an indirect relation that can be found by transitivity. For instance, consider an additional service \( SRV(l) \). If \( SRV(i), SRV(l) = 1, SRV(i), SRV(l) = 1 \) and \( SRV(i), SRV(j) = 0 \), \( SRV(i) \) and \( SRV(j) \) are said to be indirect temporal relation.

Direct temporal relations can be represented by a graph where nodes stands for predictable and modifiable services and edges for direct temporal relations. If the direct temporal relation graph of modifiable and predictable services is not connected, the optimization problem can be split into independent sub-problems. The global solution corresponds to the union of sub-problem solutions (Diestel, 2005). This property is interesting because it may lead to important reduction of the problem complexity.

### 6. Application example of the mixed-linear programming

After the decomposition into independent sub-problems, each sub-problem related to a specific time horizon can be solved using Mixed-Linear programming. The open source solver GLKP (Makhorin, 2006) has been used to solve the problem but commercial solver such as CPLEX (ILOG, 2006) can also be used. Mixed-Linear programming solvers combined a branch and bound (Lawler & Wood, 1966) algorithm for binary variables with linear programming for continuous variables.

Let’s consider a simple example of allocation plan computation for a housing for the next 24h with an anticipative period \( \Delta = 1h \). The plan starts at 0am. Energy coming from a grid power supplier has to be shared between 3 different end-user services:

- \( SRV(1) \) is a room HVAC service whose model is given by (3). According to the inhabitant programming, the room is occupied from 6pm to 6am. Out of the occupation periods, the inhabitant dissatisfaction \( D(1,k) \) is not taken into account. Room HVAC service is thus considered here as a permanence service. The thermal behavior is given by:

\[
\begin{bmatrix}
T_{in}(1,k+1) \\
T_{env}(1,k+1)
\end{bmatrix} =
\begin{bmatrix}
0.299 & 0.686 \\
0.203 & 0.794
\end{bmatrix}
\begin{bmatrix}
T_{in}(1,k) \\
T_{env}(1,k)
\end{bmatrix}
+ 
\begin{bmatrix}
1.264 & 0.336 \\
0.015 & 0.004
\end{bmatrix}
E(1,k) + 
\begin{bmatrix}
0.44 \\
0.116
\end{bmatrix}
\phi_s(1,k)
\]

(58)

The comfort model of service \( SRV(1) \) in period \( k \) is

\[
D(1,k) = \begin{cases} 
\frac{22 - T_{in}(i,k)}{5} & \text{if } T_{in}(i,k) \leq 22 \\
\frac{T_{in}(i,k) - 22}{5} & \text{if } T_{in}(i,k) > 22 
\end{cases}
\]

(59)

The global comfort of service \( SRV(1) \) is the sum of comfort model of the whole period:

\[
D(1) = \sum_{k=1}^{K} D(1,k)
\]

(60)

- Service \( SRV(2) \) corresponds to an electric water heater. It is considered as a temporary preemptive service. Its horizon is given by \( H(SRV(2)) = [3,22] \). The maximal power consumption is 2kW and 3.5kWh can be stored within the heater.
• SRV(3) corresponds to a cooking in an oven that lasts 1h. It is considered as a temporary and modifiable but not preemptive service. It just can be shifted providing that the following comfort constraints are satisfied: \( f_{\text{min}}(3) = 9 : 30 \text{am}, f_{\text{max}}(3) = 5 \text{pm}, f_{\text{opt}} = 2 \text{pm} \) where \( f_{\text{min}}, f_{\text{max}} \) and \( f_{\text{opt}} \) stand respectively for the earliest acceptable ending time, the latest acceptable ending time and the preferred ending time. The cooking requires \( 2kW \). The global comfort of service SRV(2) is:

\[
D(3) = \begin{cases} 
\frac{f(3) - 14}{2(14 - f(3))} & \text{if } f(3) > 14 \\
\frac{3}{9} & \text{if } f(3) \leq 14
\end{cases}
\]  

(61)

• SRV(4) is a grid power supplier. There is 2 prices for the kWh depending on the time of day. The cost is defined by a function \( C(4, k) \). The energy used is modelled by \( E(4, k) \). The maximum subscribed power is \( E_{\text{max}}(4) = 4kW \).

The consumption/production balance leads to:

\[
\sum_{i=1}^{3} E(i, k) \leq E_{\text{max}}(4)
\]  

(62)

The objective here is to minimize the economy criterion while keeping a good level of comfort for end-user services. The decision variables correspond to:

• the power consumed by SRV(1) that correspond to a room temperature
• the interruption SRV(2)
• the shifting of service SRV(3)

The chosen global criterion to be minimized is:

\[
J = \sum_{k=1}^{K} (E(4, k)C(4, k)) + D(1) + D(3)
\]  

(63)

The analysis of temporal relations points out a strongly connected direct temporal relation graph: the problem cannot be decomposed. The problem covering 24h yields a mixed-linear program with 470 constraints with 40 binary variables and 450 continuous variables. The solving with GLPK led to the result drawn in figure 6 after 1.2s of computation with a 3.2Ghz Pentium IV computer. Figure 6 points out that the power consumption is higher when energy is cheaper and that the temperature in the room is increased before the period where energy is costly in order to avoid excessive inhabitant dissatisfaction where the room is occupied.

In this case of study, a basic energy management is also simulated. In assuming that: the service SVR(1) is managed by the user; the heater is turned on when the room is occupied and turned off in otherwise. The set point temperature is set to 22°C. The the water heating service SVR(2) is turned on by the signal of off-peak period (when energy is cheaper). The cooking service SVR(3) is programmed by user and the ending of service is 2pm. The result of this simulation is presented in figure 8.

The advanced management reaches the objective of reducing the total cost of power consumption (-22%). The dissatisfaction of the services SVR(1) and SVR(3) reach a good level in comparison with the basic management strategy. Indeed, a 1°C shift from the desired temperature during one period leads to a dissatisfaction of 0.2 and a dissatisfaction of 0.22 corresponds to
Fig. 6. Considered weather and energy cost forecasts

Fig. 7. Results of the advanced energy management strategy computed by GLPK
a 1 hour delay for the cooking service. The basic management lead to an important dissatisfaction regarding the service \( SVR(1) \), the heater is turned on only when the room is occupied. It lead to a dissatisfaction in period [6pm, 7pm]. The cooking service \( SVR(3) \) is shifted one hour sooner by the advanced management strategy for getting the off-peak tariff. The total energy consumption of advanced management is slightly higher than the one of basic management strategy (+3%) but in terms of carbon dioxide emission, an important reduction (-65%) is observed. Thanks to an intelligent energy management strategy, economical cost and environmental impact of the power consumption have been reduced.

In addition, different random situations have been generated to get a better idea of the performance (see table 1). The computation time highly increases with the number of binary variables. Examples 3 and 4 show that the computation time does not only depend on the

<table>
<thead>
<tr>
<th>Strategy of energy management</th>
<th>Total cost</th>
<th>Energy consumption</th>
<th>CO2 emission</th>
<th>( D(1) )</th>
<th>( D(3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic management</td>
<td>1.22euros</td>
<td>13.51kWh</td>
<td>3452.2g</td>
<td>0.16</td>
<td>0.00</td>
</tr>
<tr>
<td>Advanced management</td>
<td>0.95euros</td>
<td>13.92kWh</td>
<td>1216.2g</td>
<td>0.20</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 1. Comparison between the two strategies of energy management
number of constraints and of variables. Example 5 fails after one full computation day with an out of memory message (there are 12 services in this example).

Mixed-linear programming manages small size problems but is not very efficient otherwise. The hybrid meta-heuristic has to be preferred in such situations.

<table>
<thead>
<tr>
<th>Example number</th>
<th>Number of variables</th>
<th>Number of constraints</th>
<th>Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>201 continuous, 12 binary</td>
<td>204</td>
<td>1.2s</td>
</tr>
<tr>
<td>2</td>
<td>316 continuous, 20 binary</td>
<td>318</td>
<td>22s</td>
</tr>
<tr>
<td>3</td>
<td>474 continuous, 24 binary</td>
<td>479</td>
<td>144s</td>
</tr>
<tr>
<td>4</td>
<td>474 continuous, 24 binary</td>
<td>479</td>
<td>32m</td>
</tr>
<tr>
<td>5</td>
<td>1792 continuous, 91 binary</td>
<td>1711</td>
<td>&gt;24h</td>
</tr>
</tbody>
</table>

Table 2. Results of random problems computed using GLPK

7. Taking into account uncertainties

Many model parameters used for prediction, such as predicting the weather information, are uncertain. The uncertainties are also present in the optimization criterion. For example, the criterion corresponding to thermal sensation depends on air speed, the metabolism of the human body that are not known precisely.

7.1 Sources of uncertainties in the home energy management problem

There are two main kinds of uncertainties. The first one comes from the outside like the one related to weather prediction or to the availability of energy resources. The second one corresponds to the uncertainty which come from inside the building. Reactive layer of the control mechanism manages uncertainties but some of them can be taken into account during the computation of robust anticipative plans.

The weather prediction naturally contains uncertainties. It is difficult to predict precisely the weather but the outside temperature or the level of sunshine can be predicted with confident intervals. The weather prediction has a significant impact on the local production of energy in buildings. In literature, effective methods to predict solar radiation during the day are proposed. Nevertheless, the resulting predictions may be very different from the measured values. It is indeed difficult to predict in advance the cloud in the sky. Uncertainties about the prediction of solar radiation have a direct influence on the consumption of services such as heating or air conditioning systems. Moreover, it can also influence the total available energy resource if the building is equipped with photovoltaic panels.

The disturbances exist not only outside the building but also in the building itself. A home energy management system requires sensors to get information on the status of the system. But some variables must be estimated without sensor: for example metabolism of the body of the inhabitants or the air speed in a thermal zone. More radically, there are energy activities that occur without being planned and change the structure of the problem. In the building, the user is free to act without necessarily preventing the energy management system. The consumption of certain services such as cooking, lighting, specifying the duration and date of execution remain difficult to predict. The occupation period of the building, which a strong energy impact, also varies a lot.

Through a brief analysis, sources of uncertainties are numerous, but the integration of all
sources of uncertainties in the resolution may lead to very complex problem. All the uncertainties cannot be taken into account at the same time in the anticipative mechanism: it is better to deal firstly with disturbances that has a strong energy impact. The sources of uncertainty have been classified according to two types of disturbances:

- The first type of uncertainty corresponds to those who change the information on the variables of the problem of energy allocation. The consequence of such disturbances is generally a deterioration of the actual result compared to the computed optimal solution.

- The second type corresponds to the uncertainties that cause the most important disturbances. They change the structure of the problem by adding and removing strong constraints. The consequence in the worst case is that the current solution is no longer relevant.

In both cases, the reactive mechanism will manage the situations in decreasing user satisfaction. If the anticipative plan is robust, it will be easier for the reactive mechanism to keep user satisfaction high.

7.2 Modelling uncertainty

A trail of research for the management of uncertainties is stochastic optimization, which amounts to represent the uncertainties by random variables. These studies are summarized in Greenberg & Woodruf (1998). Billaut et al. (2005b) showed three weak points of these stochastic methods in the general case:

- The adequate knowledge of most problems is not sufficient to infer the law of probability, especially during initialization.

- The source of disturbances generally leads to uncertainty on several types of data at once. The assumption that the disturbances are independent of each other is difficult to satisfy.

- Even if you come to deduce a stochastic model, it is often too complex to be used or integrated in a optimization process.

An alternative approach to modelling uncertainty is the method of intervals for continuous variables: it is possible to determine an interval pillar of their real value. You can find this approach to the problem of scheduling presented in Dubois et al. (2003; 2001). Aubry et al. (2006); Rossi (2003) have used the all scenarios-method to model uncertainty in a problem of load-balancing of parallel machines. The combination of three types of models (stochastic model, scenario model, interval model) is also possible according to Billaut et al. (2005b).

In the context of the home energy management problem, stochastic methods have not been used because ensuring an average performance of the solution is not the target. For example, an average performance of user’s comfort can lead to a solution which is very unpleasant at a time and very comfortable at another time. The methods based on intervals appear to be an appropriate method to this problem because it is a min-max approach. For example, uncertainty about weather prediction as the outside temperature $T_{ext}$ can be modelled by an interval $T_{ext} \in [T_{ext}, T_{ext}]$. The modelling of an unpredictable cooking whose duration is $p \in [0.5h, 3h]$ and the execution date is in the interval $s(i) \in [18h, 22h]$. Similarly, the uncertainty of the period of occupation of the building or other types of disturbances can be modelled.
7.3 Introduction to multi-parametric programming

The approach taking into account uncertainties is to adopt a three-step procedure like scheduling problems presented in Billaut et al. (2005a):

- Step 0: Solving the problem in which the parameters are set to predict their most likely value.
- Step 1: Solving the problem, where uncertainties are modelled by intervals, to get a family of solutions.
- Step 2: Choosing a robust solution from among those which have been computed at step 1.

The main objective is to seek a solving method for step 1. A parametric approach may be chosen for calculating a family of solutions that will be used by step 2.

The parametric programming is a method for solving optimization problem that characterizes the solution according to a parameter. In this case, the problem depends on a vector of parameters and is referred to as a Multi-Parametric programming (MP). The first method for solving parametric programming was proposed in Gass & Saaty (1955), then a method for solving multi-parametric has been presented in Gal & J.Nedoma (1972). Borrelli (2002); Borrelli et al. (2000) have introduced an extension of the multi-parametric programming for the multi-parametric mixed-integer programming: a geometric method programming. The multi-parametric programming is used to define the variables to be optimized according to uncertainty variables.

Formally, a MP-MILP is defined as follows: let \( x_c \) be the set of continuous variables, and \( x_d \) be the set of discrete variables to be optimized. The criterion to be minimized can be written as:

\[
J(x_c, x_d) = Ax_c + Bx_d
\]

subject to

\[
\begin{bmatrix}
F & G & H
\end{bmatrix}
\begin{bmatrix}
x_c \\
\theta \\
x_d
\end{bmatrix} \leq W
\]

where \( \theta \) is a vector of uncertain parameters.

**Definition 1** A polytope is defined by the intersection of a finite number of bounded half-spaces. An admissible region \( P \) is a polytope of \( \begin{bmatrix} x_c \\ \theta \\ x_d \end{bmatrix} \) on which each point can generate an admissible solution to the problem 64. \( \begin{bmatrix} x_c \\ \theta \\ x_d \end{bmatrix} \) belongs to a family of polytopes defined by the values of \( x_d \in \text{dom}(x_d) \):

\[
P(x_d) = \left\{ (x_c, \theta) \mid \begin{bmatrix} F & G & H \end{bmatrix} \begin{bmatrix} x_c \\ \theta \\ x_d \end{bmatrix} \leq W \right\}
\]

In this family of polytopes, the optimal regions are defined as follows:

**Definition 2** The optimal region \( P^*(x_d) \subseteq P \) is the subset of \( P(x_d) \), in which the problem 64 admits at least one optimal solution. \( P^*(x_d) \) is necessarily a polytope because:

- a polytope is bounded by hyperplans which can lead to edges that are polytopes
- a polytope is a convex hypervolume
The family of the optimal region $P^*(x_d)$:

$$P^*(x_d) = \left\{ (x_c, \theta) \mid \left[ \begin{array}{ccc} F & G & H \\ \theta & x_d \\ x_c \end{array} \right] \leq W \right\}$$

This family of spaces $P^*(x_d)$ with $x_d \in \text{dom}(x_d)$ can be described by an optimal function $Z(x_c, x_d)$.

To determine this function $Z$, different spaces are defined, some of which correspond to the space of definition of this function $Z$.

**Definition 3** The family of the admissible regions for $\theta$ is defined by:

$$\Theta_a(x_d) = \left\{ \theta \mid \exists x_c \text{ sbj. to } \left[ \begin{array}{ccc} F & G & H \\ \theta & x_d \\ x_c \end{array} \right] \leq W \right\}$$

**Definition 4** The family of the optimal regions for $\theta$ is a subset of the family $\Theta_a(x_d)$:

$$\Theta^*_a(x_d) = \left\{ \theta \mid \exists x_c^* \text{ sbj. to } \left[ \begin{array}{ccc} F & G & H \\ \theta & x_d \\ x_c \end{array} \right] \leq W \right\}$$

**Definition 5** The family of the admissible regions for $x_c$ is defined by:

$$X_a(x_d) = \left\{ x_c \mid \exists \theta \text{ sbj. to } \left[ \begin{array}{ccc} F & G & H \\ \theta & x_d \\ x_c \end{array} \right] \leq W \right\}$$

**Definition 6** The family of the optimal regions for $x_c$ is a subset of the family $X_a(x_d)$:

$$X^*_a(x_d) = \left\{ x_c^* \mid \exists \theta \text{ sbj. to } \left[ \begin{array}{ccc} F & G & H \\ \theta & x_d \\ x_c \end{array} \right] \leq W \right\}$$

**Definition 7** The objective function represents the family of optimal regions $P^*(x_d)$ which was defined in 65. It is defined by $X^*_a(x_d)$ to $\Theta^*_a(x_d)$, which were defined in 70 and 68 respectively:

$$Z(x_c, x_d) : X^*_a(x_d) \rightarrow \Theta^*_a(x_d)$$

**Definition 8** The critical region $RC_m(x_d)$ is a subset of the space $P^*(x_d)$ where the local conditions of optimality for the optimization criterion remain immutable, i.e. that the function optimizer $Z_m(x_c, x_d) : X^*_a(x_d) \rightarrow \Theta^*_a(x_d)$ is unique. $RC_m(x_d)$ is determined by doing the union of different optimal regions $P^*(x_d)$ which has the same optimizer function.

The purpose of the linear multi-parametric mixed-integer programming is to characterize the variables to optimize $x_c, x_d$ and the objective function according to $\theta$. The principle for solving the MP-MILP is summarized by two next steps:
• First step: search in the region of parameters $\theta$ the smallest sub-space of $\mathbb{P}$ which contains the optimal region $P^*(x_d)$. Then, determine the system of linear inequalities according to $\theta$ which defines $\mathbb{P}$.

• Second step: determine the set of all critical regions: the region $\mathbb{P}$ is divided into sub-spaces $RC_m(x_d) \in P^*(x_d)$. In the critical region $RC_m(x_d)$, the objective function $Z_m^*(x_c, x_d)$ remains a unique function. After determining the family of critical regions $RC_m(x_d)$, the piecewise affine functions of $Z_m^*(x_c, x_d)$ that characterize $x_c, x_d$ according to $\theta$ is found. After refining the critical regions by grouping sub-spaces $RC_m$, we can get minimal facades which characterize the critical region.

7.4 Application to the home energy management problem

After having introduced multi-parametric programming, the purpose of this section is to adapt this method to the problem of energy management. As shown before, the problem of energy management in the building can be written as:

$$J = (A_1.z + B_1.\delta + D_1)$$
$$A_2.z + B_2.\delta + C_2.x \leq C$$

(72)

where $z \in \mathbb{Z}$ is the set of continuous variables and $\delta \in \Delta$ is the set of binary variables resulting from the logic transformation see section 4. Uncertainties can be modelled by intervals $\theta \in \Theta$. Assuming that the uncertainties are bounded, so

$$\underline{\theta} \leq \theta \leq \overline{\theta}$$

(73)

The family of solutions of the problem taking into account the uncertainties is generated by parametric programming. To illustrate this method, two examples are proposed.

Example 1. Consider a thermal service supported by an electric heater with a maximum power of 1.5 kW. $T_a$ is the indoor temperature and $T_m$ is the temperature of the building envelope with an initial temperatures $T_a(0) = 22^\circ$C and $T_m(0) = 22^\circ$C. A simplified thermal model of a room equipped with a window and a heater has been introduced in Eq. (3). The initial temperatures are set to $T_a(0) = 21^\circ$C, $T_m(0) = 22^\circ$C. The thermal model of the room after discretion with a sampling time equal to 1 hour is:

$$\begin{bmatrix}
T_a(k+1) \\
T_m(k+1)
\end{bmatrix}
= \begin{bmatrix}
0.364 & 0.6055 \\
0.359 & 0.625
\end{bmatrix}
\begin{bmatrix}
T_a(k) \\
T_m(k)
\end{bmatrix}
+ \begin{bmatrix}
0.0275 & 1.1966 & 0.4193 \\
0.016 & 0.7 & 0.2434
\end{bmatrix}
\begin{bmatrix}
T_{ext} \\
\phi_r \\
\phi_s
\end{bmatrix}$$

(74)

Supposing that the function of thermal satisfaction is written in the form:

$$U(k) = \delta_a(k).a_1.\frac{T_{opt} - T_a(k)}{T_{opt} - T_{min}} + (1 - \delta_a(k)).a_2.\frac{T_{opt} - T_a(k)}{T_{opt} - T_{Max}}$$

(75)

where:

• $\delta_a(k)$: binary variable verifying $[\delta_a(k) = 1] \iff [T_a(k) \leq T_{opt}], \forall k$
• $T_{opt}$: ‘ideal’ room’s temperature for the user.
• $[T_{min}, T_{Max}]$: the area of the value of room’s temperature.
• $a_1, a_2$: are two constant that reflect the different between the sensations of cold or hot. 

with $T_{opt} = 22^\circ C$, $T_{min} = 20^\circ C$, $T_{Max} = 24^\circ C$ and $a_1 = a_2 = 1$.

It is assumed that there was not a precise estimate of the outdoor temperature $T$ but it is possible to set that the outdoor temperature varies within a range: $[-5^\circ C, +5^\circ C]$. The average energy assigned to the heater over a period of 4 hours to minimize the objective function is:

$$J = \left( \sum_{k=1}^{4} U(k) \right)$$  \hspace{1cm} (76)

The parametric programming takes into account uncertainties on the outdoor temperature. An implementation of multi-parametric solving may be done using a toolbox called Multi Parametric Toolbox MPT with the programming interface named YALMIP solver developed by Lofberg (2004). The resolution of the example 1 takes 3.31 seconds on using a computer Pentium IV 3.4 GHz. The average energy assigned to the heater according to the temperature outside is:

$$\phi_r(i) = \begin{cases} 
1.5 & \text{if} \ - 5 \leq T_{ext} \leq -0.875 \\
-0.097 \times T_{ext} + 1.415 & \text{if} \ - 0.875 < T_{ext} \leq 5 
\end{cases}$$  \hspace{1cm} (77)

The parametric programming divided the uncertain region into two critical regions. The first region corresponds to the zone: $-5 \leq T_{ext} \leq -0.875$. The optimal solution is to put the heater to the maximum level in order to approach the desired temperature. In the second critical region, $-0.875 \leq T_{ext} \leq 5$, the energy assigned to the heater is proportional to the outdoor temperature. The higher the outside temperature is, the less energy is assigned to the radiator. In fact, $T_{ext} = -0.875$ is the point of the system where the maximum power generated by the radiator can compensate the thermal flow lost through the building envelope.

Example 2. This example is based on example 1 but with additional uncertainties on sources. In this example, the disturbance caused by the user have been simulated. It is assumed that in the 3rd and 4th periods of the resource assignment plan, it is likely that a consumption may occur. Accordingly, the available energy during the periods 3 and 4 is between 0 and 2kWh. A parametric variable $E_{max} \in [0, 2]$ and a constraint are added as follows:

$$\phi_r(3) + \phi_r(4) \leq E_{max}$$  \hspace{1cm} (78)

The optimal solution of the problem must be computed based on two variables $[T_{ext}, E_{max}]$. This example has still been solved using the MPT tool. This time, the solver takes 5.2 seconds. The average energy assigned for the period 1, $\phi_r(1)$, is independent of the variable $E_{max}$. It means that whatever happens on the energy available during periods 3 and 4, the decision to the period 1 can not improve the situation:

$$\phi_r(1) = \begin{cases} 
1.5 & \text{if} \ - 5 \leq T_{ext} \leq -0.875 \\
0 & \text{if} \ - 0.875 < T_{ext} \leq 5
\end{cases}$$  \hspace{1cm} (79)

The energy assigned to the heater in the second period $\phi_r(2)$ is a piecewise function which consists of five different critical areas. Among these five regions (fig.9), we see that the optimal solution assigns the maximum energy to the heater in three regions. By anticipating the availability of resources in periods 3 and 4, the comfort is improved in the heating zone. This result corresponds to the conclusion found in Ha et al. (2006a). During periods 3 and 4, the
consumption of radiator is less important than for the periods 1 and 2. A robust solution is obtained despite the disturbance of the resource and the outside temperature. However, in the critical region 5 (Fig. 9), there is an extreme case in which it is very cold outside and there is simultaneously a large disturbance on the availability of the resource. The only solution is to put $\phi_r(k)$ to the maximum value although there is a deterioration in the comfort of user.

After generating the family of solutions at step 1, an effective solution must be chosen during step 2. Knowing that the optimal solutions of step 1 are piecewise functions limited by critical regions, therefore the procedure of selecting a solution now is to select a piecewise function. The area of research is therefore reduced and the algorithm of step 2 requires few computations. A min-max approach is used to find a robust solution among the family of solutions. A polynomial algorithm that comes in the different critical regions to find a solution that optimizes the criterion is used:

$$J^* = (\max(J(\theta)) | \theta \in \mathcal{P}^*)$$

8. Conclusion

This chapter presents a formulation of the global home electricity management problem, which consists in adjusting the electric energy consumption/production for habitations. A service oriented point of view has been justified: housing can be seen as a set of services. A 3-layer control mechanism has been presented. The chapter focuses on the anticipative layer, which computes optimal plannings to control appliances according to inhabitant request and weather forecasts. These plannings are computed using service models that include behavioral, comfort and cost models.

The computation of the optimal plannings has been formulated as a mixed integer linear programming problem thanks to a linearization of nonlinear models. A method to decompose the whole problem into sub-problems has been presented. Then, an illustrative application example has been presented. Computation times are acceptable for small problems but it increases up to more than 24h for an example with 91 binary variables and 1792 continuous ones. Heuristics has to be developed to reduce the computation time required to get a good solution.
Even if uncertainties can be managed by the reactive layer, an approach that takes into account uncertainties model by intervals from the anticipative step has been presented. It is an adaptation of the multi-parametric programming. It leads to robust anticipative plans. But this approach is useful for biggest uncertainties because it is difficult to apprehend a large number of uncertainties because of the induced complexities.

9. References


Forecasts point to a huge increase in energy demand over the next 25 years, with a direct and immediate impact on the exhaustion of fossil fuels, the increase in pollution levels and the global warming that will have significant consequences for all sectors of society. Irrespective of the likelihood of these predictions or what researchers in different scientific disciplines may believe or publicly say about how critical the energy situation may be on a world level, it is without doubt one of the great debates that has stirred up public interest in modern times. We should probably already be thinking about the design of a worldwide strategic plan for energy management across the planet. It would include measures to raise awareness, educate the different actors involved, develop policies, provide resources, prioritise actions and establish contingency plans. This process is complex and depends on political, social, economic and technological factors that are hard to take into account simultaneously. Then, before such a plan is formulated, studies such as those described in this book can serve to illustrate what Information and Communication Technologies have to offer in this sphere and, with luck, to create a reference to encourage investigators in the pursuit of new and better solutions.

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