

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



---

# **Initial Shapes of Cable-Stayed Bridges during Construction by Cantilever Methods – Numerical Simulation and Validation of the Kao Ping Hsi Bridge**

---

Ming-Yi Liu and Pao-Hsii Wang

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/59816>

---

## **1. Introduction**

Cable-stayed bridges have become popular because of their aesthetic appearance, structural efficiency, ease of construction and economic advantage over the past several decades. However, this kind of bridge is light, flexible and with low inherent damping. Accordingly, they are sensitive to ambient excitations from seismic, wind and traffic loads. Because the geometric and dynamic properties of the bridges as well as the characteristics of the excitations are complicated, it is important to fully understand the system behaviors with reasonable bridge shapes at each erection stage during construction by the cantilever method, which is used to provide the necessary information to accurately calculate the dynamic responses of the bridges under the complex excitations.

A lot of studies on this kind of bridges have been done in the last half century [1-3]. However, few analytical techniques have been presented for the cable-stayed bridges during erection stages in construction. To the authors' best knowledge, a number of papers have investigated the erection procedure of cable-stayed bridges, focusing primarily on the improvement of construction technology [4-6], but not for the analytical purpose. Since the initial shape of a cable-stayed bridge not only reasonably provides the geometric configuration as well as the prestress distribution of the whole bridge under the weight of the deck-tower system and the pretension forces in the stay cables, but also definitely ensures the satisfaction of the relationships for the equilibrium conditions, boundary conditions and architectural design requirements [7-11], it is essential to research the system behaviors with the appropriate initial shapes of cable-stayed bridges.

The objective of this chapter is to fully understand the system behaviors with the appropriate initial shapes of cable-stayed bridges at each erection stage during construction by the

cantilever method. Two computational procedures during erection stages: a forward process analysis and a backward process analysis, are presented for this reason [10]. On the basis of the two procedures, a series of initial shape analyses are conducted to investigate the bridge shapes at each erection stage. Numerical examples based on finite element models of the Kao Ping Hsi Bridge in Taiwan [12] are developed to validate the two proposed approaches. The initial shape at each erection stage provides the necessary data for checking and controlling the real-time erection procedure of a cable-stayed bridge during construction. The designed shape, i.e., the geometric configuration and the prestress distribution, of the whole bridge can then be achieved.

## 2. Finite element formulation

A cable-stayed bridge can be considered as an assembly of a finite number of cable elements for the stay cables and beam-column elements for both the decks and towers based on the finite element concepts. A number of assumptions are used in this study: the material is homogeneous and isotropic; the stress-strain relationship of the material remains within the linear elastic range during the whole nonlinear response; the external forces are displacement independent; large displacements and large rotations are allowed, but strains are small; each stay cable is fixed to both the deck and tower at their joints of attachment. Under the above assumptions, the initial shape analysis of cable-stayed bridges is conducted according to the system equations with the consideration of geometric nonlinearities.

### 2.1. Geometric nonlinearities

Three types of geometric nonlinearities: the cable sag, beam-column and large displacement effects, are considered to reasonably simulate cable-stayed bridges in this study.

A stay cable will sag into a catenary shape because of its weight and tensile force. Such cable sag effect has to be considered when the stay cable is represented by a single straight cable element. A stay cable with tensile stiffness is assumed to be perfectly elastic. The compressive, shear and bending stiffnesses of the stay cable are neglected. The cable sag nonlinearity can be simulated according to the equivalent modulus of elasticity of the stay cable [13]

$$E_{eq} = \frac{E_c}{1 + \frac{(wl_c)^2 A_c E_c}{12T^3}}, \quad (1)$$

where  $E_c$ ,  $A_c$  and  $l_c$  are the effective modulus of elasticity, the cross-sectional area and the horizontal projected length of the stay cable, respectively;  $w$  is the weight of the stay cable per unit length;  $T$  is the tension in the stay cable. The stiffness matrix of a cable element in Figure 1 can be expressed as

$$KE_{jk} = \begin{cases} \begin{bmatrix} \frac{E_{eq} A_c}{L_c} \\ 0 \end{bmatrix}, & u_1 > 0 \\ [0], & u_1 \leq 0 \end{cases}, \quad (2)$$

where  $u_1$  is the element coordinate for the relative axial deformation;  $L_c$  is the chord length of the stay cable.

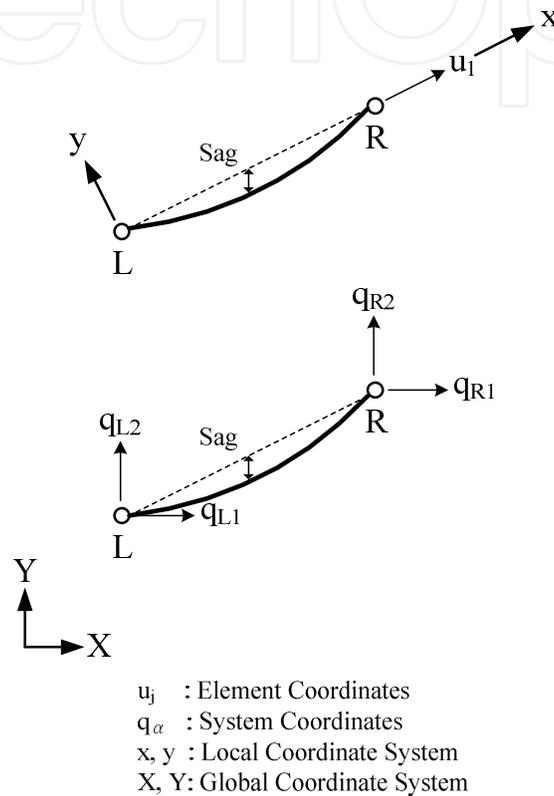


Figure 1. Cable element.

Large compressive forces in the deck-tower system of a cable-stayed bridge occur due to high pretension forces in the stay cables. For this reason, the beam-column effect between such compressive forces and bending moments has to be taken into consideration when beam-column elements are used to simulate both the decks and towers. Shear strains of a beam-column element in Figure 2 are negligible according to the Euler-Bernoulli beam theory.  $u_1$ ,  $u_2$  and  $u_3$  are the element coordinates for the left end rotation, the right end rotation and the relative axial deformation, respectively. The stiffness matrix of the beam-column element can be written as

$$KE_{jk} = \frac{E_b I_b}{L_b} \begin{bmatrix} C_s & C_t & 0 \\ C_t & C_s & 0 \\ 0 & 0 & R_t A_b / I_b \end{bmatrix}, \quad (3)$$

where  $E_b$ ,  $A_b$ ,  $I_b$  and  $L_b$  are the modulus of elasticity, the cross-sectional area, the moment of inertia and the length of the beam-column element, respectively;  $C_s$ ,  $C_t$  and  $R_t$  are the stability functions representing the interaction between the axial and bending stiffnesses of the beam-column element [14].

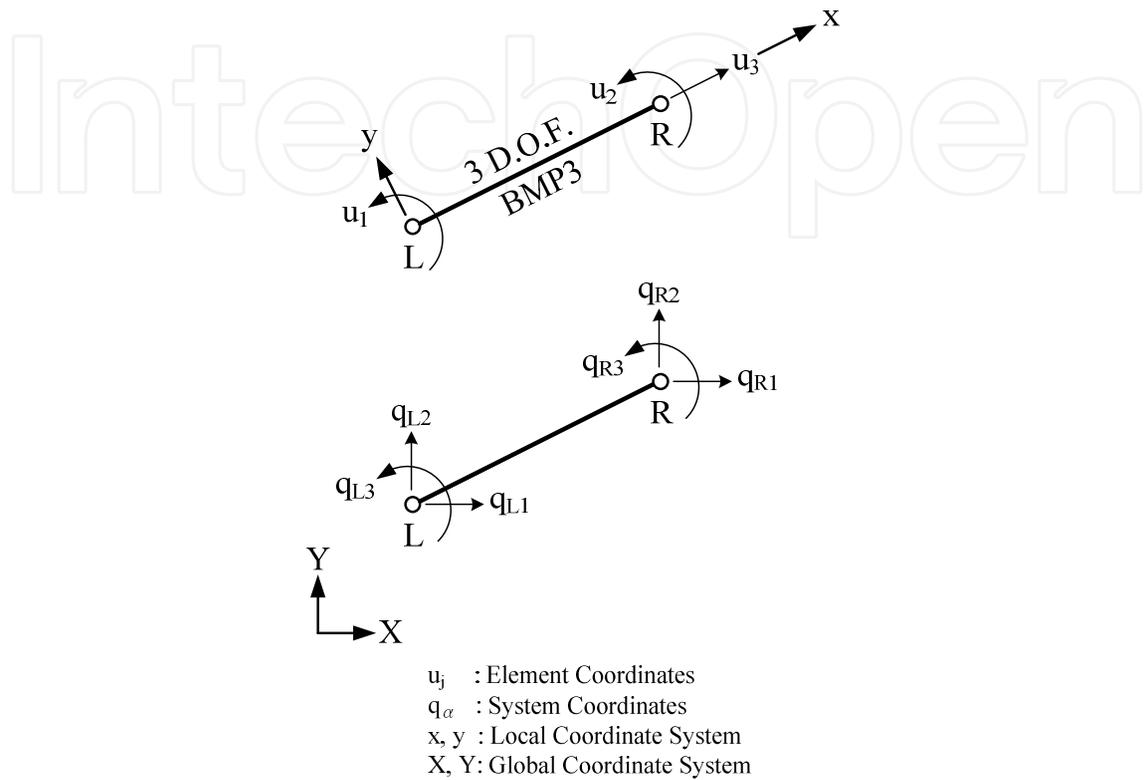


Figure 2. Beam-column element.

Large displacements occur in the deck-tower system due to the large span as well as less weight of a cable-stayed bridge. Such effect has to be considered when the equilibrium equations are derived from the deformed position. Under these conditions, the element coordinate  $u_j$  can be expressed as a nonlinear function of the system coordinate  $q_\alpha$  in both Figure 1 and Figure 2, i.e.,  $u_j = u_j(q_\alpha)$ . By differentiating  $u_j$  with respect to  $q_\alpha$ , the first-order and second-order coordinate transformation coefficients can be individually written as

$$a_{j\alpha} = \frac{\partial u_j}{\partial q_\alpha}, \tag{4}$$

$$a_{j\alpha,\beta} = \frac{\partial a_{j\alpha}}{\partial q_\beta} = \frac{\partial^2 u_j}{\partial q_\alpha \partial q_\beta}. \tag{5}$$

$a_{j\alpha}$  and  $a_{j\alpha,\beta}$  for the stiffness matrices of the cable and beam-column elements can be found in [7], which are used to develop the tangent system stiffness matrix in Section 2.2.

## 2.2. System equations

The system equations in generalized coordinates of a nonlinear finite element model of a cable-stayed bridge can be derived from the Lagrange's virtual work principle

$$\sum_{EL} S_j a_{j\alpha} = P_\alpha, \alpha = 1, 2, 3, \dots, N, \quad (6)$$

$$S_j = KE_{jk} u_k + S_j^0, \quad (7)$$

$$P_\alpha = \bar{K}^j \cdot \bar{b}_\alpha^j, \quad (8)$$

$$\bar{b}_\alpha^j = \frac{\partial \bar{W}^j}{\partial q_\alpha}, \quad (9)$$

where  $S_j$  is the element force vector;  $P_\alpha$  is the external force vector;  $S_j^0$  is the initial element force vector;  $\bar{K}^j$  is the external nodal force vector;  $\bar{b}_\alpha^j$  is the basis vector;  $\bar{W}^j$  is the displacement vector corresponding to  $\bar{K}^j$ ;  $N$  is the number of degrees of freedom; the subscript  $\alpha$  denotes the number of the system coordinate; the subscripts  $j$  and  $k$  represent the numbers of the element coordinates; the superscript  $j$  denotes the nodal number;  $\sum_{EL}$  represents the summation over all elements.

Under consideration of three types of geometric nonlinearities mentioned in Section 2.1,  $KE_{jk}$  of a cable element and that of a beam-column element can be determined from Eq. (2) and Eq. (3), respectively. The former and the latter are individually due to the cable sag effect and the beam-column effect.  $u_j$ ,  $a_{j\alpha}$  and  $\bar{b}_\alpha^j$  are nonlinear functions of  $q_\alpha$  when the large displacement effect occurs.  $\bar{K}^j$  can be written as a function of  $q_\alpha$  if they are displacement dependent forces.

Eq. (6) is a set of simultaneous nonlinear equations. In order to incrementally solve these equations, the linearized system equations in a small force interval are derived based on the first-order Taylor series expansion of Eq. (6)

$${}^2 K_{\alpha\beta}^n \Delta q_\beta^n = {}_u P_\alpha^n + \Delta P_\alpha^n, P_\alpha^n \leq P_\alpha \leq P_\alpha^{n+1}, \quad (10)$$

$${}^2 K_{\alpha\beta}^n = \sum_{EL} KE_{jk}^n a_{j\alpha}^n a_{k\beta}^n + \sum_{EL} S_j^n a_{j\alpha,\beta}^n - {}^n \bar{K}^j \cdot {}^n \bar{b}_{\alpha,\beta}^j - {}^n \bar{K}_\beta^j \cdot {}^n \bar{b}_\alpha^j, \quad (11)$$

$$\bar{b}_{\alpha,\beta}^j = \frac{\partial \bar{b}_\alpha^j}{\partial q_\beta}, \quad (12)$$

$$\bar{K}_\alpha^j = \frac{\partial \bar{K}^j}{\partial q_\alpha}, \quad (13)$$

$${}_u P_\alpha^n = P_\alpha^n - \sum_{EL} S_j^n a_{j\alpha}^n, \quad (14)$$

$$\Delta P_\alpha^n = P_\alpha^{n+1} - P_\alpha^n, \quad (15)$$

$$\Delta q_\alpha^n = q_\alpha^{n+1} - q_\alpha^n, \quad (16)$$

where  ${}^2K_{\alpha\beta}^n$  is the tangent system stiffness matrix;  ${}_u P_\alpha^n$  is the unbalanced force vector;  $\Delta P_\alpha^n$  is the increment of the external force vector;  $\Delta q_\alpha^n$  is the increment of the system coordinate vector; the superscripts  $n$  and  $n + 1$  denote the numbers of the force steps; the superscript 2 represents the second-order iteration matrix.

${}^2K_{\alpha\beta}^n$  in Eq. (11) includes four terms. The first term is the elastic stiffness matrix, while the second and third terms are the geometric stiffness matrices induced by large displacements. In addition, the fourth term is the geometric stiffness matrix induced by displacement dependent forces, which is negligible in this study.

Eq. (10) is a set of simultaneous linear equations in a small force interval, which can be solved by the Newton-Raphson method [7-11].

### 2.3. Initial shape analysis

The initial shape of a cable-stayed bridge provides the geometric configuration as well as the prestress distribution of such bridge under the weight of the deck-tower system and the pretension forces in the stay cables. The relationships for the equilibrium conditions, boundary conditions and architectural design requirements have to be achieved for the bridge shape. Under these conditions, the initial shape analysis of cable-stayed bridges is presented by considering three types of geometric nonlinearities including the cable sag, beam-column and large displacement effects.

For the initial shape analysis of a cable-stayed bridge, the weight of the deck-tower system is considered, whereas the weight of the stay cables is assumed to be negligible. The shape finding computation is conducted using a two-loop iteration method: an equilibrium iteration and a shape iteration [7-11]. It can be started with an estimated initial element force, i.e., pretension force, in the stay cables. By incrementally solving Eq. (10), i.e., the equilibrium iteration, the equilibrium configuration of the whole bridge under the weight of the deck-tower system can be obtained based on the reference configuration, i.e., architectural design form, with no deflection and zero prestress in the deck-tower system.

The bridge configuration satisfies the equilibrium and boundary conditions after the above equilibrium iteration. However, the architectural design requirements are generally not achieved due to the fact that large displacements and variable bending moments occur in the deck-tower system from the large bridge span. Under these conditions, the appropriate initial shape can be determined by conducting the shape iteration to reduce the displacements as well as to smooth the bending moments.

Several control points are selected for insuring that both the deck and tower displacements achieve the architectural design requirements in the shape iteration

$$\left| \frac{q_{\alpha}}{L_r} \right| \leq \varepsilon_r, \quad (17)$$

where  $q_{\alpha}$  is the displacement in a certain direction of the control point;  $L_r$  is the reference length;  $\varepsilon_r$  is the convergence tolerance. Each control point is the node intersected by the deck and the stay cable for checking the deck displacement.  $q_{\alpha}$  and  $L_r$  represent the vertical displacement of the control point and the main span length, respectively. Similarly, each node intersected by the tower and the stay cable, or located on the top of the tower is selected as the control point for checking the tower displacement.  $q_{\alpha}$  and  $L_r$  individually denote the horizontal displacement of the control point and the tower height.

If Eq. (17) is not satisfied, the element axial forces calculated in the previous equilibrium iteration will be used as the initial element forces in the new equilibrium iteration, and the corresponding equilibrium configuration of the whole bridge under the weight of the deck-tower system will be obtained again. The shape iteration will then be repeated until Eq. (17) is satisfied. Under these conditions, the convergent configuration can be regarded as the initial shape of the cable-stayed bridge.

### 3. Initial shape analysis during erection stages

The initial shapes, i.e., the geometric configuration and the prestress distribution, of a cable-stayed bridge during erection stages provide the essential information for the bridge construction. According to the cantilever method, two finite element computational procedures: the forward process analysis and the backward process analysis, are presented for the shape finding of the bridge at each erection stage during construction.

#### 3.1. Cantilever methods

The cantilever method has been widely used for the girder erection of cable-stayed bridges with self-anchored cable systems. Such method provides the natural and logical solution for the construction of large-span cable-stayed bridges during erection stages. New girder segments are installed and then supported by new stay cables at each erection stage, and the process keeps going stage-by-stage until the bridge construction is completed. The cantilever method can be further categorized into the single cantilever method and the double cantilever method. In the former, the side span girders are erected on temporary supports, while the main span girders are erected by one-sided free cantilevering until the span center or the anchor pier on the far end is reached. In the latter, the bridge girders are erected by double-sided free cantilevering from both sides of the tower toward the main span center and the anchor pier. Figure 3 illustrates the erection stages of cable-stayed bridges by the single cantilever method, which is considered in this study. Its concept can also be applied to the double cantilever method.

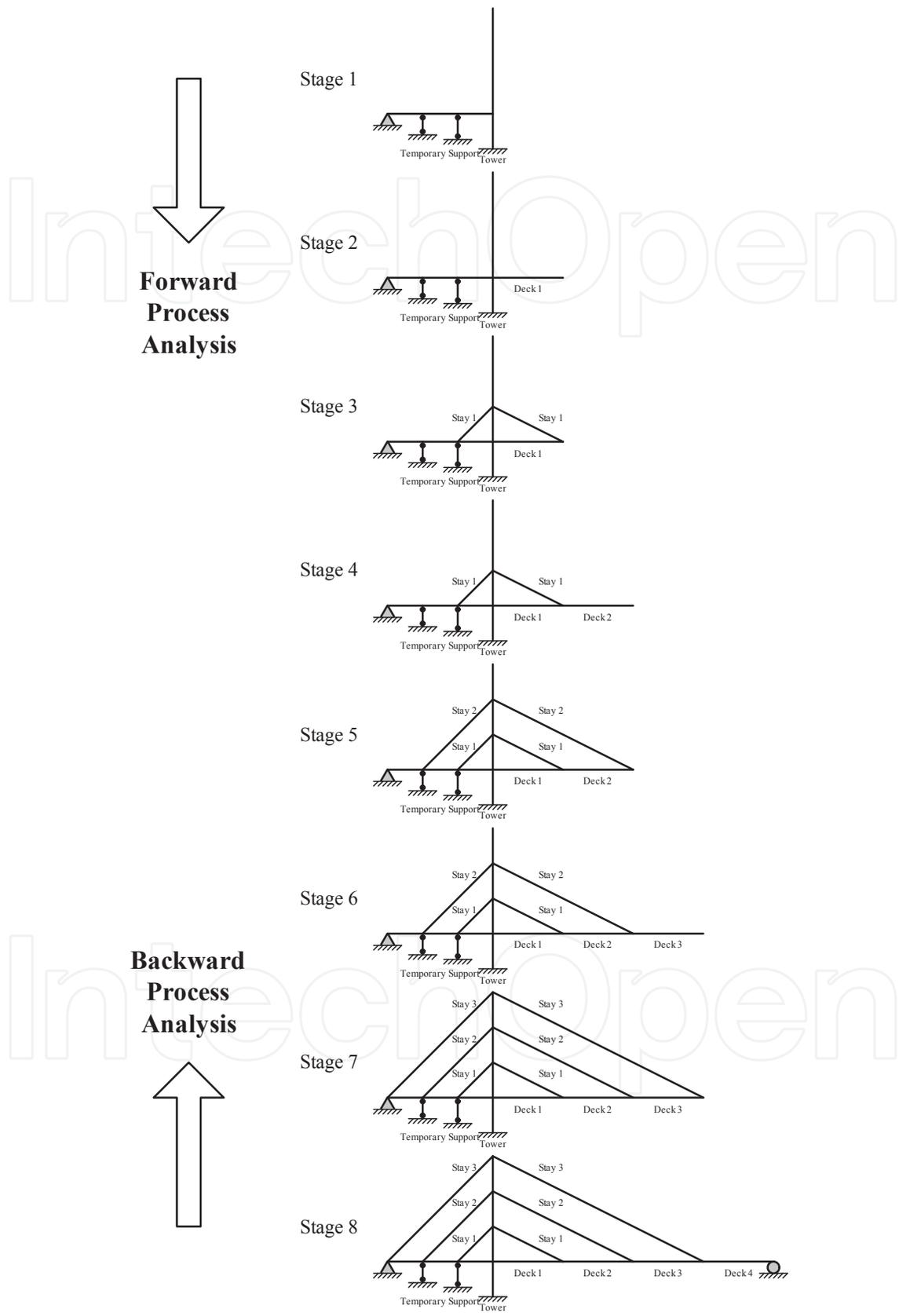


Figure 3. Erection stages of cable-stayed bridges.

### 3.2. Forward process analysis

The forward process analysis of cable-stayed bridges during construction by the single cantilever method is conducted based on the sequence of erection stages in Figure 3. The geometric configuration and the element forces of a cable-stayed bridge at each erection stage can therefore be estimated. At erection stages with even number (2, 4, 6), the new girder segments are installed and the corresponding relatively large vertical displacements and bending moments of the girders occur due to the lack of the exterior stay cables. While at erection stages with odd number (3, 5, 7), the new exterior stay cables are installed at the tip of the new girder segments. Under these conditions, the stay cables are stressed to lift the girders to a certain elevation, which can keep the desired correct position as well as reduce the bending moments of the girders. The pretension forces in stay cables and the girder elevation at each erection stage can be obtained by the initial shape analysis presented in Section 2.3. For the forward process analysis, the shape iteration has to be performed to keep the girders in a horizontal position, implying that an upward precamber is allowed during construction. As illustrated in Figure 4, the shape iteration can be started with an estimated initial cable force  $\overset{\circ}{T}$  by setting the tip displacement of the girder segment under its dead load  $w$  and the weight of the machine equipments  $W_{eq}$  to be equal to that resulting from  $\overset{\circ}{T}$ , which can be written as

$$\overset{\circ}{T} = \frac{3wl + 8W_{eq}}{8 \sin \alpha}, \quad (18)$$

where  $l$  is the length of the girder segment;  $\alpha$  is the inclined angle of the stay cable.

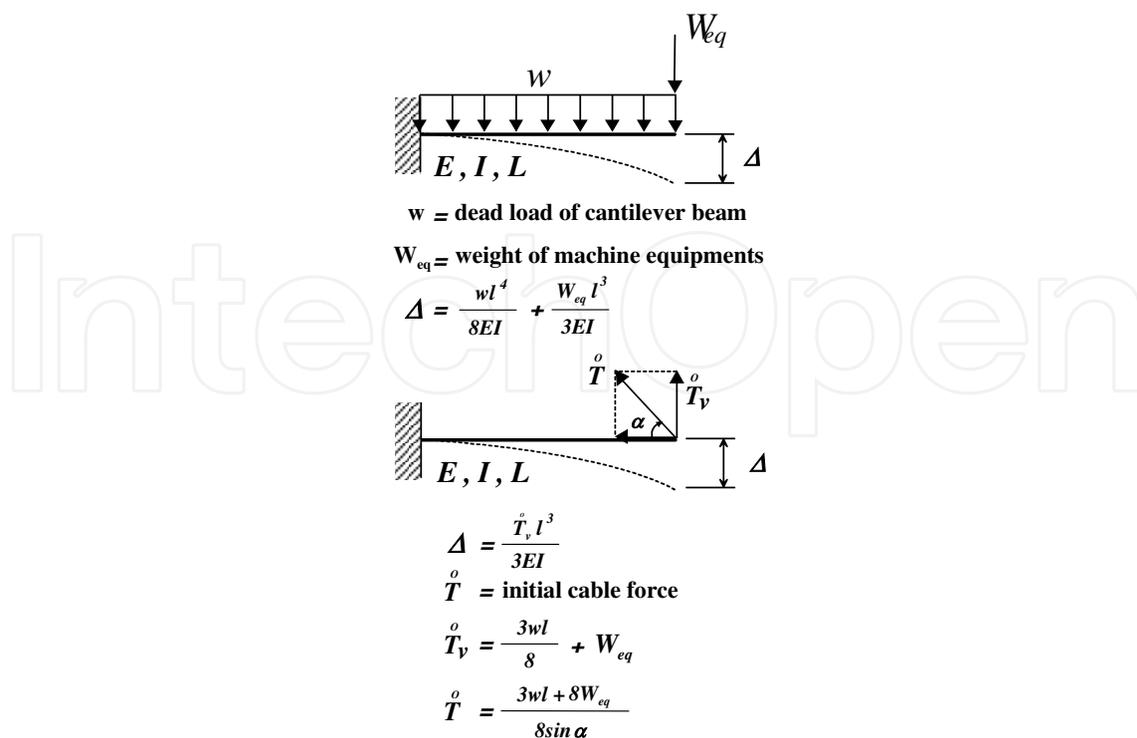


Figure 4. Estimation of initial cable force.

Figure 5 illustrates the flowchart of the forward process analysis following the actual sequence of erection stages of cable-stayed bridges during construction. The advantage of the forward process analysis is that the real-time factors of the bridges, such as creep and shrinkage of concrete, any alteration in design, etc., can be taken into consideration during construction.

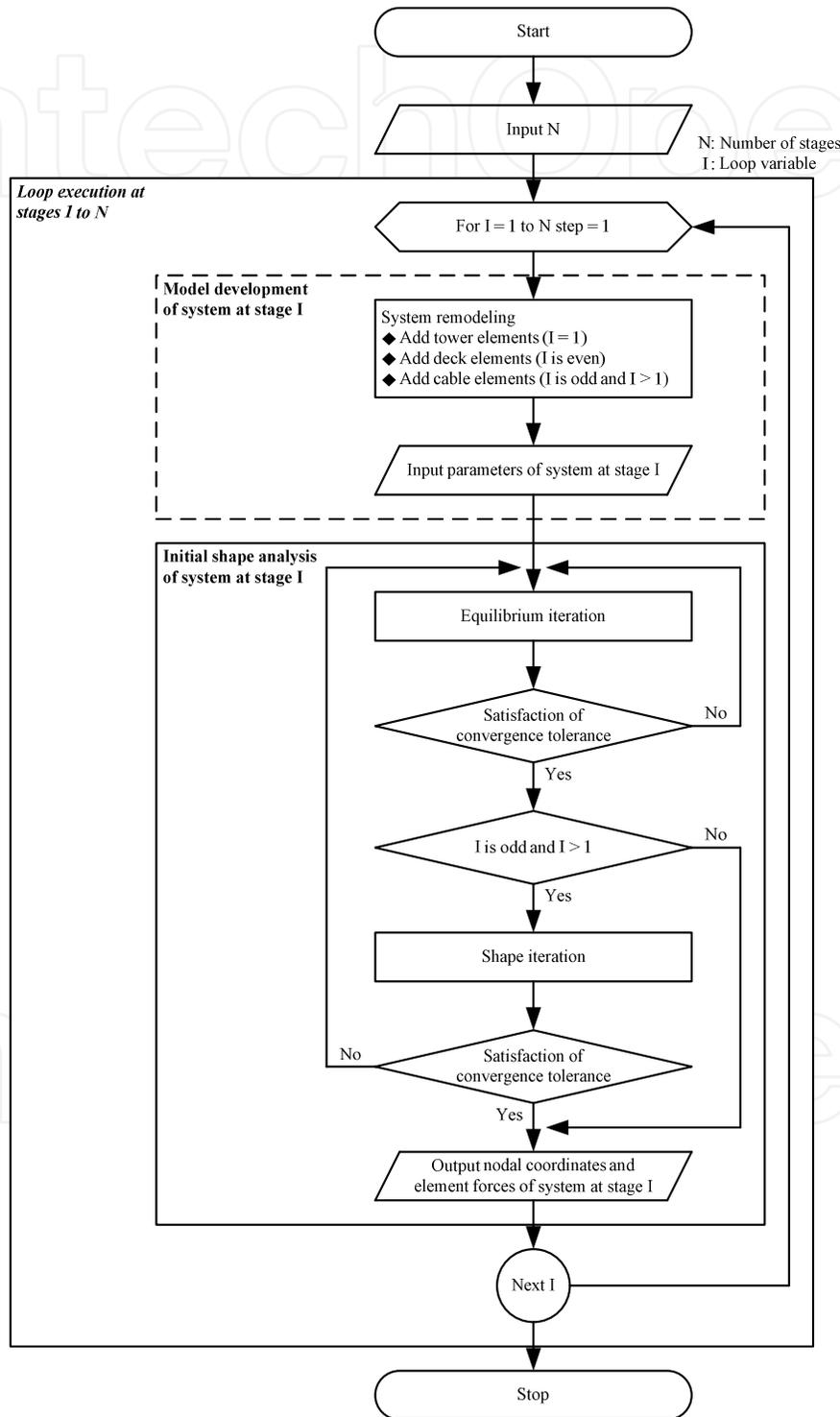


Figure 5. Flowchart of forward process analysis.

### 3.3. Backward process analysis

In contrast to the forward process analysis, the backward process analysis of cable-stayed bridges during construction by the single cantilever method is conducted based on the reverse sequence of erection stages in Figure 3. The computation is started with the whole bridge at the first erection stage, i.e., stage 8 in Figure 3. After removing the existing girder segments and the adjoining stay cables, the system can be remodeled and reanalyzed to estimate the geometric configuration and the member forces of the bridge at the current erection stages, i.e., stages 7 to 2 in Figure 3. The computation is continued repeatedly until the final erection stage, i.e., stage 1 in Figure 3. For the backward process analysis, the initial shape of the whole bridge has to be obtained first at the first erection stage by the initial shape analysis presented in Section 2.3. After removing the existing girder segments and the adjoining stay cables, the geometric configuration and the member forces of the bridge at each erection stage can be determined anew by solving the static system equations. On the basis of the linearized system equations from the nonlinear theory, the equilibrium iteration is performed using the Newton-Raphson method.

Figure 6 illustrates the flowchart of the backward process analysis following the reverse sequence of erection stages of cable-stayed bridges during construction. The advantage of the backward process analysis is that the initial shape of a cable-stayed bridge at each erection stage can be obtained by the equilibrium iteration without the shape iteration. Such initial shape determined by the equilibrium conditions can be considered as the desired correct position of the bridge for the next erection stage, in which the girder is precambered upwards. In other words, the computational efficiency of the backward process analysis without the shape iteration is better than that of the forward process analysis with the shape iteration. In contrast, the disadvantage of the backward process analysis is that the real-time factors of the bridge, such as creep and shrinkage of concrete, any alteration in design, etc., can not be taken into consideration during construction due to the fact that the computation is performed backwards from the whole bridge.

## 4. Numerical examples

As an example, the Kao Ping Hsi Bridge in Figure 7 is taken for the shape finding analysis of the bridge during the erection procedure using the single cantilever method. This bridge is an unsymmetrical single-deck cable-stayed bridge with a main span of 330 m and a side span of 184 m. The deck consisting of steel box girders in the main span and concrete box girders in the side span is supported by 28 stay cables arranged in a central plane originated at the 184 m tall, inverted Y-shaped, concrete tower. More detailed information of the Kao Ping Hsi Bridge is available in [12].

Figure 7 illustrates the two-dimensional finite element model of the bridge. This model contains 48 beam-column elements for the deck and tower, 28 cable elements for the stay cables and 49 nodes. A hinge, roller and fixed supports are used to model the boundary conditions of the left and right ends of the deck and the tower, respectively, and a rigid joint is employed to simulate the deck-tower connection. The Kao Ping Hsi Bridge was erected by single

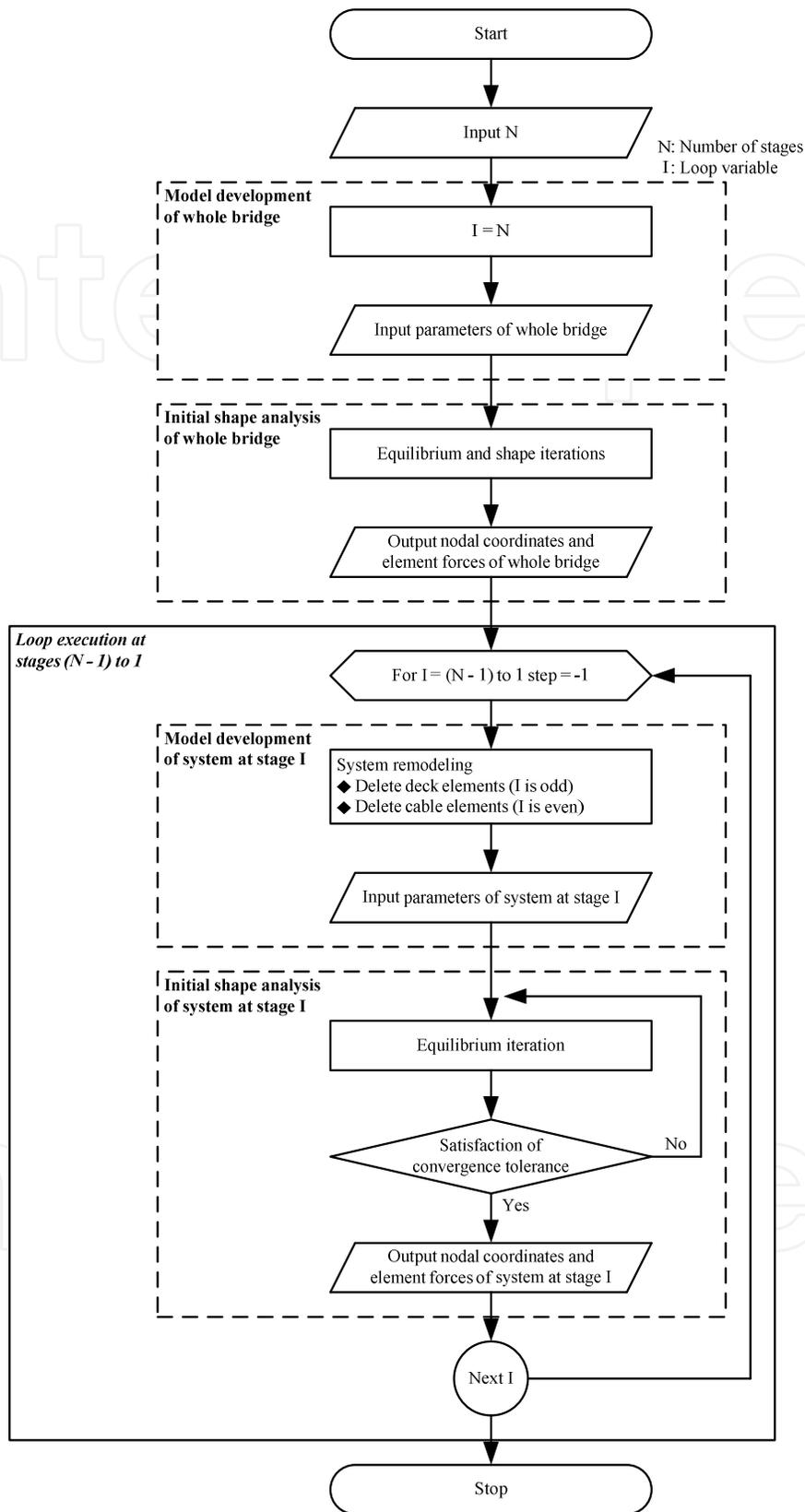


Figure 6. Flowchart of backward process analysis.

cantilever method and there are 30 erection stages for constructing the girder of the bridge. On the basis of the computational procedures during erection stages presented in Section 3, the whole bridge analysis, forward process analysis and backward process analysis are conducted in this study.

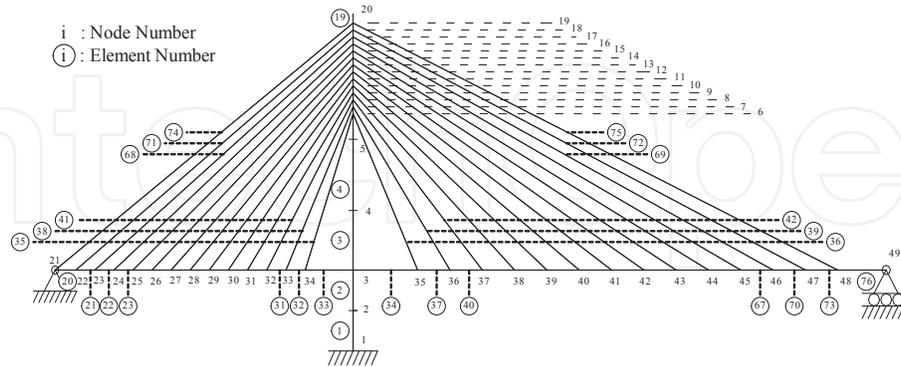


Figure 7. Finite element model of the Kao Ping Hsi Bridge.

#### 4.1. Whole bridge analysis

Based on the finite element procedures presented in Section 2.3, the initial shape analysis is conducted to reasonably provide the geometric configuration of the whole Kao Ping Hsi Bridge. In Figure 7, nodes 27, 35, 42 and 45 are selected as the control points for checking the deck vertical displacement, while node 19 is selected as the control point for checking the tower horizontal displacement. The convergence tolerance  $\epsilon_r$  is set to  $10^{-4}$  in this study.

Figure 8 shows the initial shape of the whole Kao Ping Hsi Bridge (solid line), which indicates that the maximum vertical and horizontal displacements measured from the reference configuration (dashed line) are 0.033 m at node 42 in the main span of the deck and -0.024 m at node 8 in the tower, respectively. In addition, Figure 8 illustrates that the overall displacement obtained by the two-loop iteration method, i.e., the equilibrium and shape iterations, is relatively smaller than that during the shape iteration (dot-dashed line). Consequently, the initial shape obtained by the two-loop iteration method can be used to reasonably describe the geometric configurations of cable-stayed bridges.

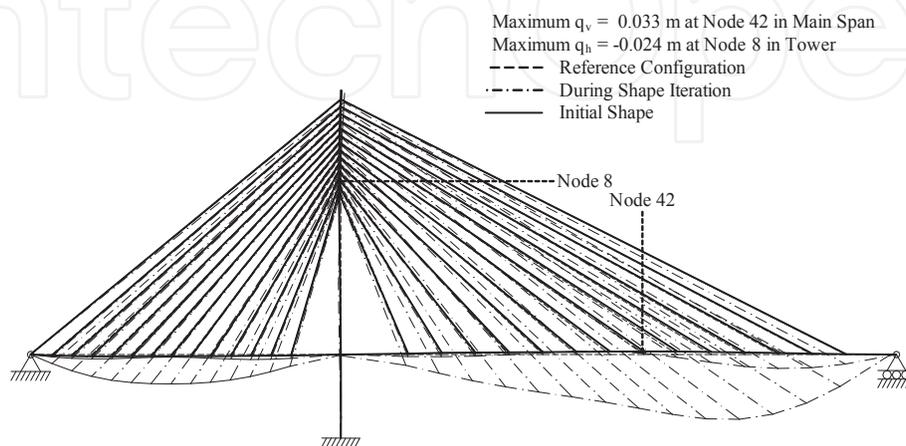
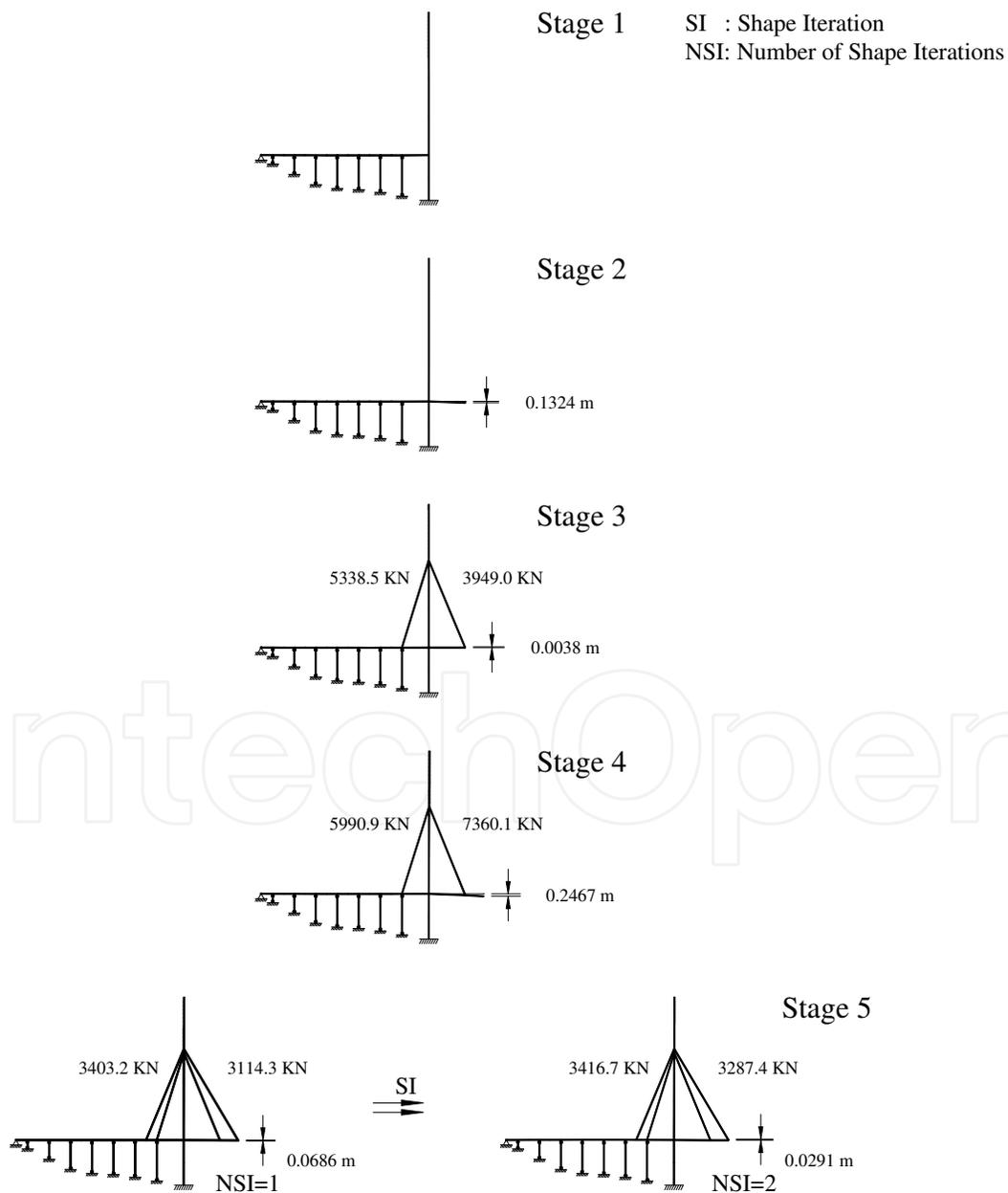
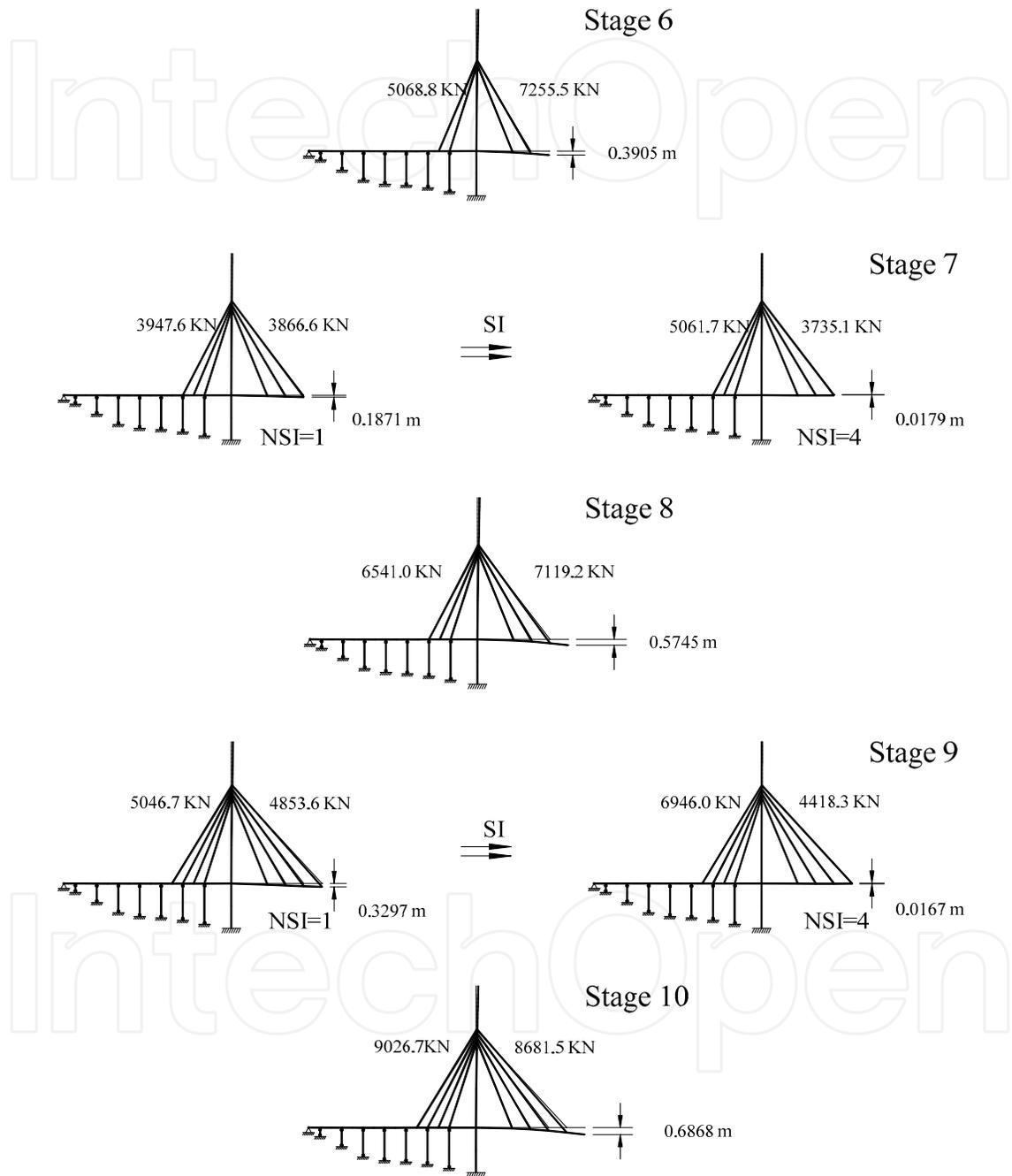


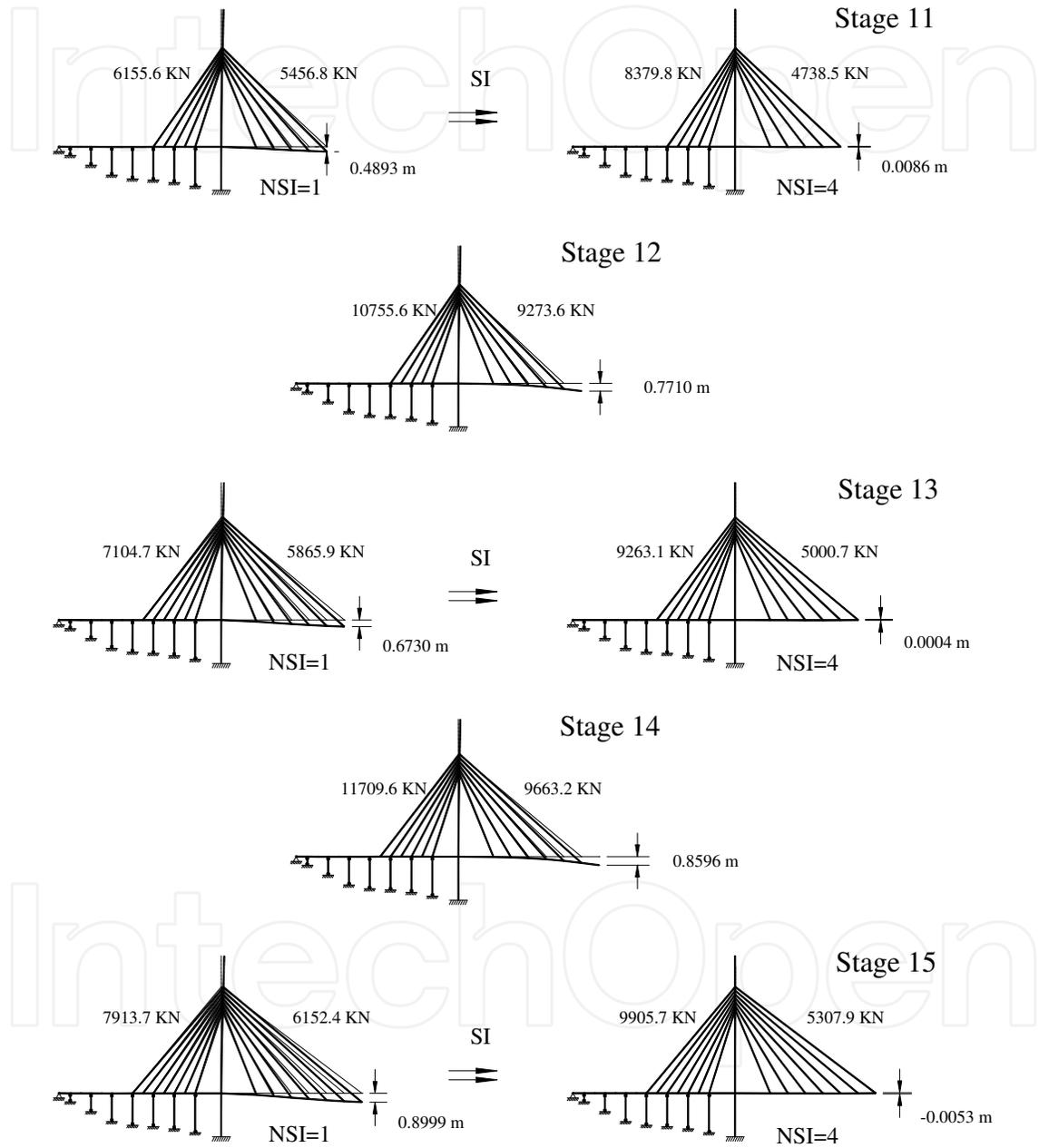
Figure 8. Initial shape of the whole Kao Ping Hsi Bridge.

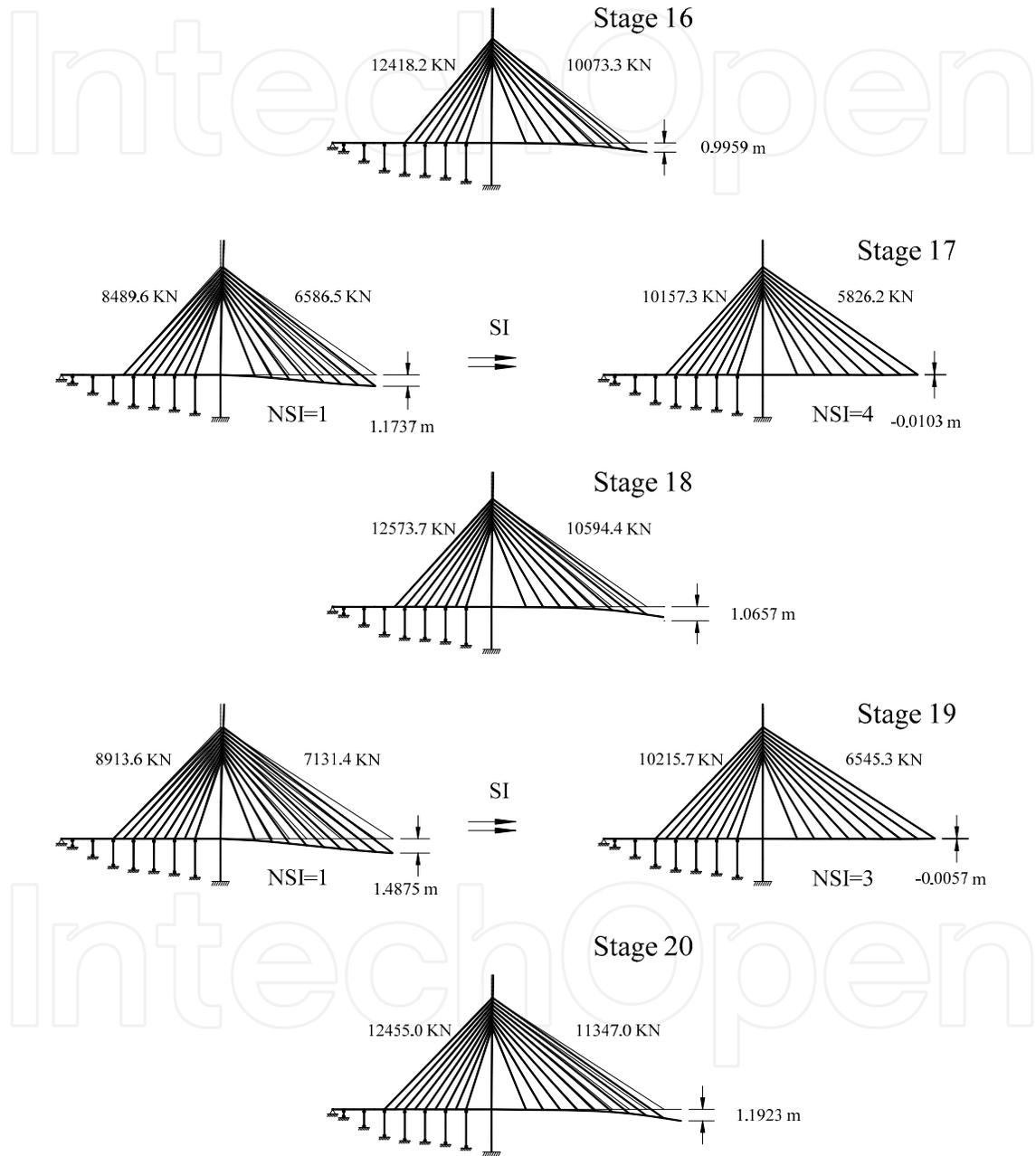
### 4.2. Forward process analysis

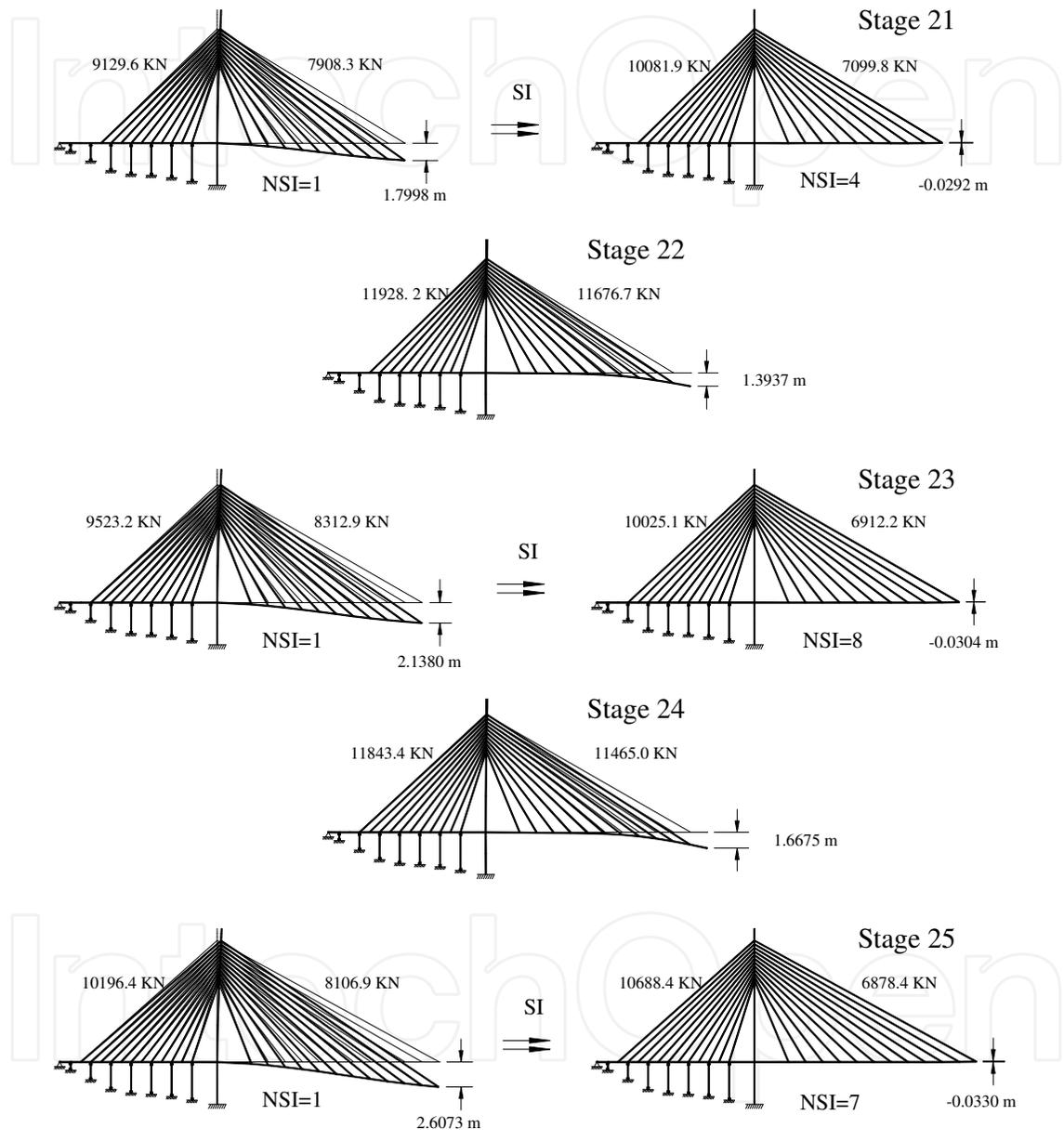
According to the forward process analysis of cable-stayed bridges during construction by the single cantilever method in Section 3.2, the initial shape of the Kao Ping Hsi Bridge at each erection stage for a total of 30 stages is obtained, as shown in Figure 9. The geometric configuration of the bridge and the corresponding vertical displacement of the girder tip in the main span as well as the element forces of the exterior stay cables are illustrated at each erection stage, where NSI represents the number of shape iterations (SI) for the convergent solution. At erection stages with even number (2, 4, 6,...), the new girder segments are installed. Under these conditions, the equilibrium position can be determined anew without the shape iteration. While at erection stages with odd number (3, 5, 7,...), the new

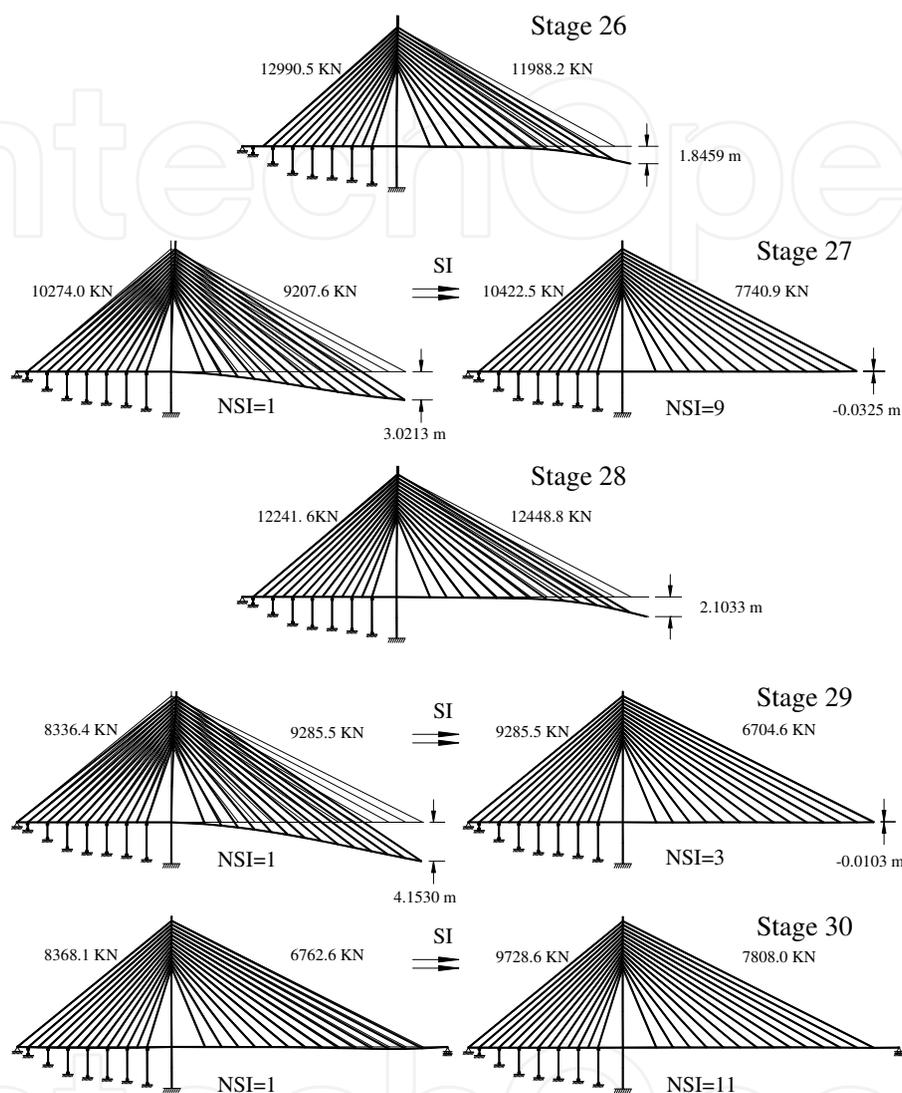










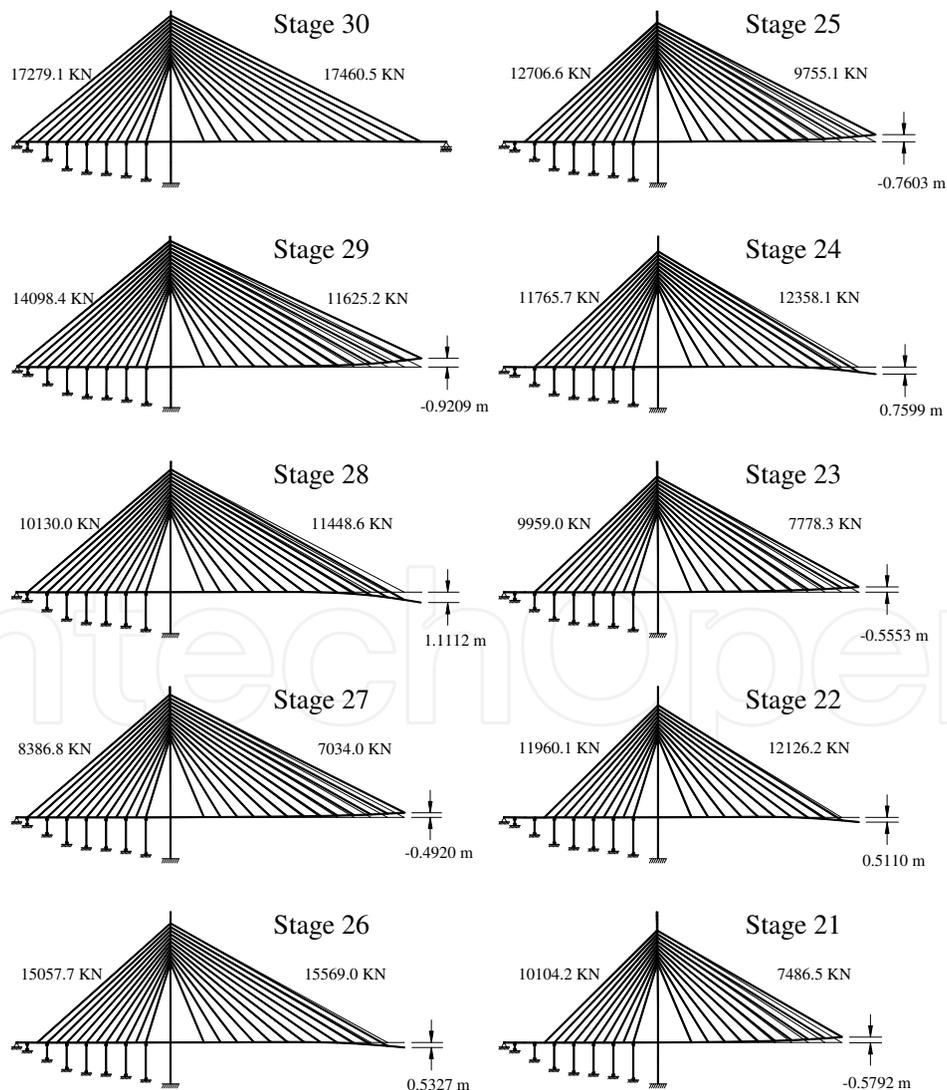


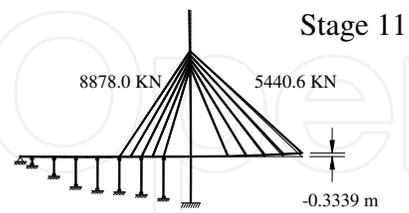
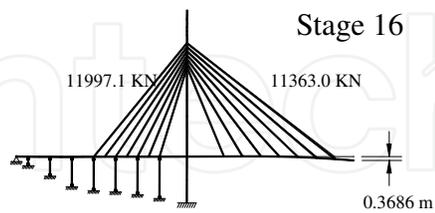
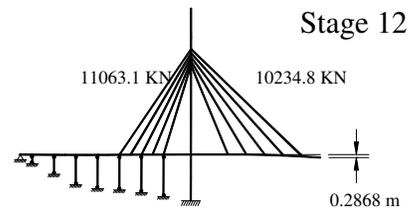
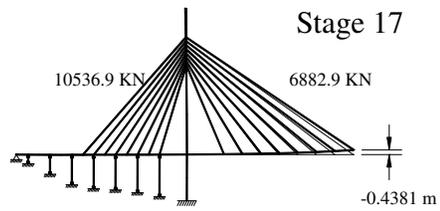
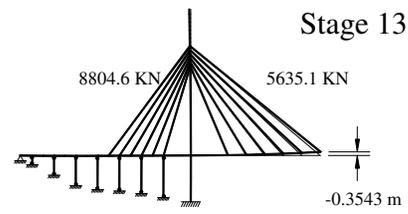
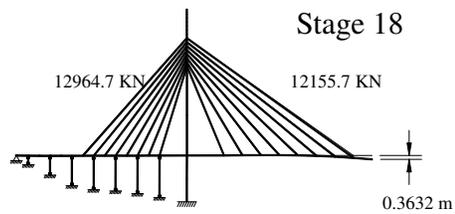
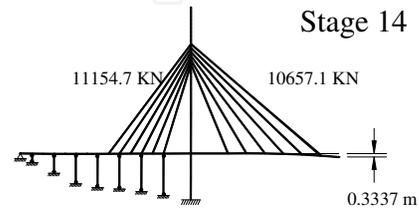
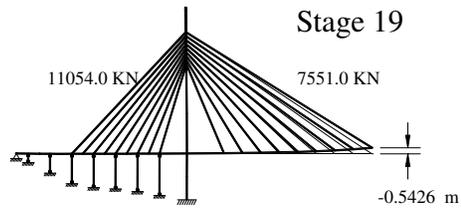
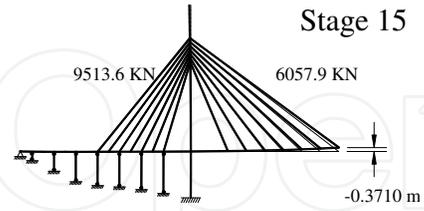
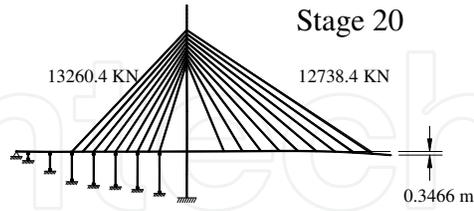
**Figure 9.** Initial shape of the Kao Ping Hsi Bridge at each erection stage based on forward process analysis.

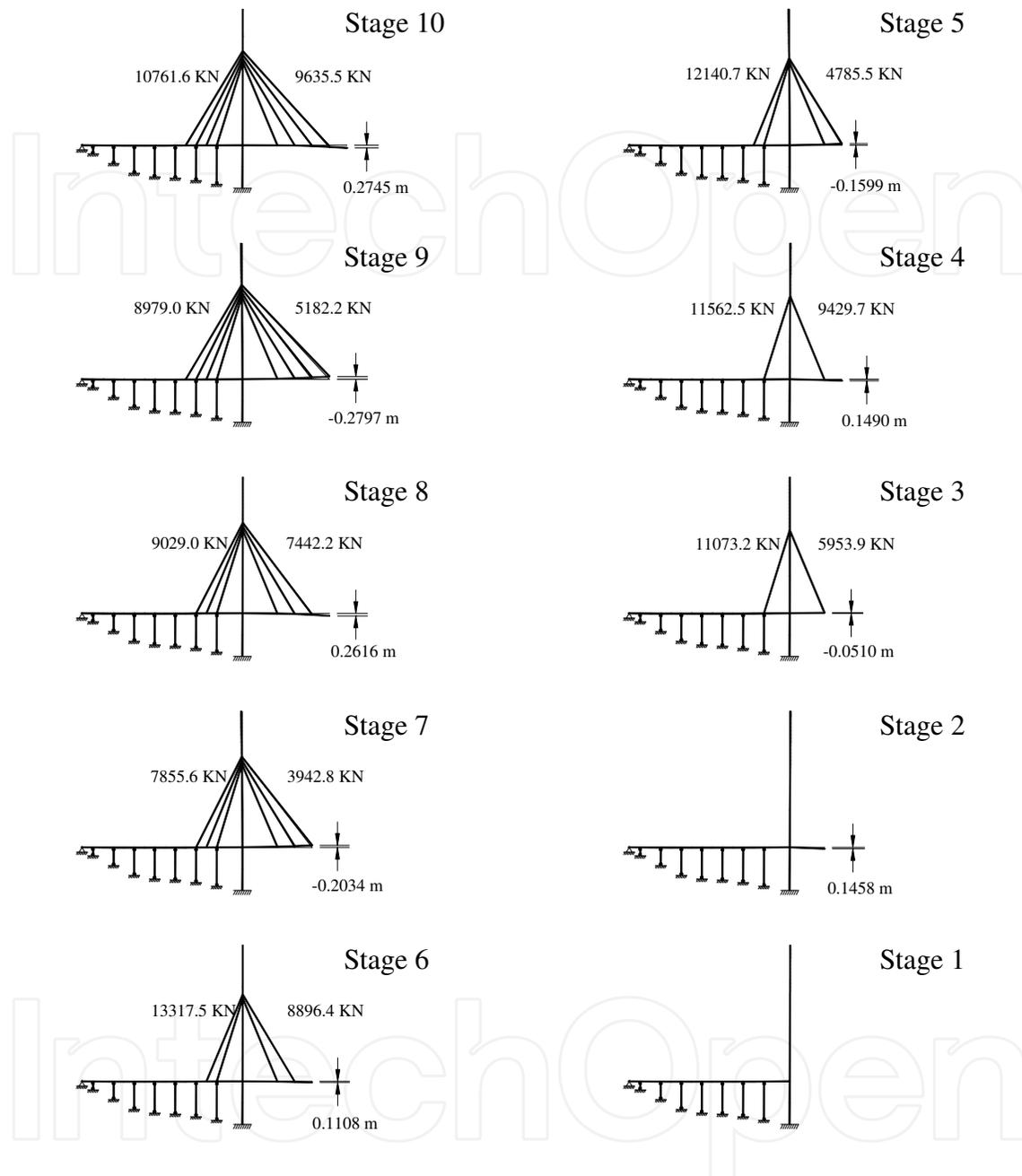
exterior stay cables are installed and stressed. Under these conditions, the new equilibrium position can be estimated by the shape iteration. The girders can therefore be kept in a horizontal position and the corresponding bending moments can also be reduced. At stage 30, the shape iteration has to be conducted to ensure that the designed shape of the whole bridge for the construction completion is almost identical to that from the whole bridge analysis. This is because the initial shapes of the bridge at erection stages by the shape iteration associated with the estimated initial cable forces are not unique. In summary, Figure 9 provides the necessary data for checking and controlling the real-time erection procedure of the bridge during construction.

### 4.3. Backward process analysis

According to the backward process analysis of cable-stayed bridges during construction by the single cantilever method in Section 3.3, the initial shape of the Kao Ping Hsi Bridge at each erection stage for a total of 30 stages is obtained, as shown in Figure 10. The geometric configuration of the bridge and the corresponding vertical displacement of the girder tip in the main span as well as the element forces of the exterior stay cables are illustrated at each erection stage. No shape iteration is needed for the backward process analysis, implying that the initial shapes of the bridge at erection stages are unique. The computation is started







**Figure 10.** Initial shape of the Kao Ping Hsi Bridge at each erection stage based on backward process analysis.

with the whole bridge at stage 30. After removing the existing girder segments at erection stages with odd number (29, 27, 25,...), the upward displacement of the girder tip in the main span occur, which can be considered as the precamber of the girder for the next erection stage. Such precamber can keep the girders in the original designed configuration as well as reduce the bending moments of the girders. After removing the existing exterior stay cables at erection stages with even number (28, 26, 24,...), the large downward displacement of the girder tip in the main span occur and the corresponding bending moments become

quite large. In summary, Figure 10 provides the necessary data for checking and controlling the real-time erection procedure of the bridge during construction.

## 5. Conclusions

The objective of this chapter is to fully understand the system behaviors with the appropriate initial shapes of cable-stayed bridges at each erection stage during construction by the cantilever method. Two computational procedures during erection stages: a forward process analysis and a backward process analysis, are presented for this reason. On the basis of the two procedures, a series of initial shape analyses are conducted to investigate the bridge shapes at each erection stage. Numerical examples based on finite element models of the Kao Ping Hsi Bridge in Taiwan are developed to validate the two proposed approaches. Based on the numerical analysis in this study, some conclusions are made as follows:

1. Both the forward process analysis and the backward process analysis provide the necessary data for checking and controlling the real-time erection procedure of a cable-stayed bridge during construction. The designed shape, i.e., the geometric configuration and the prestress distribution, of the whole bridge can then be achieved.
2. The advantage of the forward process analysis is that the real-time factors of a cable-stayed bridge, such as creep and shrinkage of concrete, any alteration in design, etc., can be taken into consideration during construction. However, the shape iteration at the final erection stage has to be conducted to ensure that the designed shape of the whole bridge for the construction completion is almost identical to that from the whole bridge analysis. This is because the initial shapes of the bridge at erection stages by the shape iteration associated with the estimated initial cable forces are not unique.
3. The advantage of the backward process analysis is that the initial shape of a cable-stayed bridge at each erection stage can be obtained by the equilibrium iteration without the shape iteration. Such initial shape determined by the equilibrium conditions can be considered as the desired correct position of the bridge for the next erection stage, in which the girder is precambered upwards. Under these conditions, the initial shapes of the bridge at erection stages are unique. In other words, the computational efficiency of the backward process analysis without the shape iteration is better than that of the forward process analysis with the shape iteration. In contrast, the disadvantage of the backward process analysis is that the real-time factors of the bridge, such as creep and shrinkage of concrete, any alteration in design, etc., can not be taken into consideration during construction due to the fact that the computation is performed backwards from the whole bridge.
4. Both the forward process analysis and the backward process analysis of cable-stayed bridges during construction by the single cantilever method based on the sequence of erection stages are presented in this study. These concepts can also be applied to the double cantilever method.

## Acknowledgements

The authors would like to acknowledge the support of the National Science Council, Taiwan, under Grant No. NSC 89-2211-E-033-009. Parts of the chapter are produced from the authors' previous publication [11].

## Author details

Ming-Yi Liu\* and Pao-Hsui Wang

\*Address all correspondence to: myliu@cycu.edu.tw

Department of Civil Engineering, Chung Yuan Christian University, Jhongli City, Taoyuan County, Taiwan

## References

- [1] Troitsky, M.S. (1988). "Cable-stayed bridges: Theory and design." Second Edition, Blackwell Scientific Publications, Oxford, UK.
- [2] Walther, R., Houriet, B., Isler, W., Moïa, P., and Klein, J.F. (1999). "Cable stayed bridges." Second Edition, Thomas Telford, Ltd, London, UK.
- [3] Gimsing, N.J., and Georgakis, C.T. (2012). "Cable supported bridges: Concept and design." Third Edition, John Wiley & Sons, Ltd, Chichester, UK.
- [4] Leonhardt, F., and Zellner, W. (1991). "Past, present and future of cable-stayed bridges." *Proceedings of the Seminar - Cable-Stayed Bridges: Recent Developments and their Future*, Yokohama, Japan, 1-33, December 10-11.
- [5] Virlogeux, M. (1991). "Erection of cable-stayed bridges." *Proceedings of the Seminar - Cable-Stayed Bridges: Recent Developments and their Future*, Yokohama, Japan, 77-106, December 10-11.
- [6] Nakagawa, T., Sugii, K., Nagai, M., and Mochizuki, H. (2000). "Design and erection of a cable trussed bridge." *Proceedings of the Thirteenth KKNN Symposium on Civil Engineering*, Taipei, Taiwan, 107-112, December 7-8.
- [7] Wang, P.H., Tseng, T.C., and Yang, C.G. (1993). "Initial shape of cable-stayed bridges." *Computers and Structures*, 46(6), 1095-1106.
- [8] Wang, P.H., and Yang, C.G. (1996). "Parametric studies on cable-stayed bridges." *Computers and Structures*, 60(2), 243-260.

- [9] Wang, P.H., Lin, H.T., and Tang, T.Y. (2002). "Study on nonlinear analysis of a highly redundant cable-stayed bridge." *Computers and Structures*, 80(2), 165-182.
- [10] Wang, P.H., Tang, T.Y., and Zheng, H.N. (2004). "Analysis of cable-stayed bridges during construction by cantilever methods." *Computers and Structures*, 82(4-5), 329-346.
- [11] Liu, M.Y., and Wang, P.H. (2012). "Finite element analysis of cable-stayed bridges with appropriate initial shapes under seismic excitations focusing on deck-stay interaction." In Sezen, H. (Ed.), Chapter 9 of *Earthquake Engineering* (pp. 231-256). InTech, Rijeka, Croatia. ISBN: 978-953-51-0694-4.
- [12] Cheng, W.L. (2001). "Kao Ping Hsi Bridge." Taiwan Area National Expressway Engineering Bureau, Ministry of Transportation and Communications, Taipei, Taiwan.
- [13] Ernst, H.J. (1965). "Der E-Modul von Seilen unter Berücksichtigung des Durchhanges." *Der Bauingenieur*, 40(2), 52-55. (in German).
- [14] Fleming, J.F. (1979). "Nonlinear static analysis of cable-stayed bridge structures." *Computers and Structures*, 10(4), 621-635.

