A Panoramic 3D Reconstruction System Based on the Projection of Patterns

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Received 30 Aug 2013; Accepted 13 Jan 2014

DOI: 10.5772/58227

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Abstract This work presents the implementation of a 3D reconstruction system capable of reconstructing a 360-degree scene with a single acquisition using a projection of patterns. The system is formed by two modules: the first module is a CCD camera with a parabolic mirror that allows the acquisition of catadioptric images. The second module consists of a light projector and a parabolic mirror that is used to generate the pattern projections over the object that will be reconstructed. The projection system has a 360-degree field of view and both modules were calibrated to obtain the extrinsic parameters. To validate the functionality of the system, we performed 3D reconstructions of three objects, and show the reconstruction error analysis.

Keywords Panoramic 3D Reconstruction, Calibration, Catadioptric Camera, Catadioptric Projector

1. Introduction

In recent times, 3D reconstruction systems have become fundamental for object modelling applications such as virtual environments development for archaeology [1, 2], video surveillance [3, 4], quality control processes [5, 6], disease diagnosis [7, 8], and measurement [9, 10].

The 3D reconstruction systems that incorporate lenses and mirrors into acquired images with an omnidirectional field of view are called catadioptric systems. A common approach for 3D reconstruction is the use of structured light projection with a 360-degree laser line combined with omnidirectional stereo cameras that take images with multiple views [11, 12]. The disadvantage of these approaches is that we can only recover 3D information in the area of the laser incidence, limiting the reconstruction area. Another approach is the use of an omnidirectional camera constructed by a traditional perspective camera and a pyramid reflector [13].

In [14], the authors propose a panoramic sensor using a convex mirror capable of producing panoramic depth maps by feature matching from two panoramic images. However, this technique fails with textureless surfaces, being unable to match the texture features. A stereo system with a texture projector can eliminate or reduce error matching. For this reason we propose a new 3D reconstruction system based on a projection system and a single catadioptric camera.

In order to obtain a good reconstruction, it is necessary to calibrate the catadioptric cameras, estimating the geometric model of the camera. In recent decades, several algorithms have been proposed for the calibration of omnidirectional cameras [14–18].

The 3D reconstruction systems based on conventional cameras have been well-studied [19–22]. However, for
applications where we need to process a 360-degree field of view (for example inside hollow objects), conventional systems implement a piecemeal reconstruction. In this paper, we describe the 360-degree 3D reconstruction of a scene from a single acquisition using a projection of patterns.

2. Geometrical model of the catadioptric vision system

2.1 Experimental setup

Figure 1 shows a schematic overview of the experimental setup for the proposed panoramic 3D reconstruction system, which consists of two parabolic mirrors, a Marlin F-080 CCD camera, a composite lens (L), and a Sony VPL-ES5 3LCD light projector.

![Figure 1. Schematic drawing of the experimental setup for the proposed panoramic 3D reconstruction system.](image1)

First, a catadioptric camera was assembled, aligning the mirror (PM1) with the CCD camera in order to capture the full environment that is projected onto the mirror. Using a chessboard calibration pattern, we calibrate the catadioptric camera obtaining the intrinsic and extrinsic parameters. Once the camera is calibrated, we design the optical setup of the pattern projection system, placing the mirrors (PM1, PM2) back-to-back. In front of the projector, a composite lens (L) was inserted in order to reduce the projected image towards the mirror (PM2), as seen in Figure 2.

![Figure 2. Experimental setup for the panoramic 3D reconstruction system. The system consists of a Marlin F-080 CCD camera, two parabolic mirrors (PM1 and PM2) on a back-to-back configuration, and a Sony VPL-ES5 3LCD light Projector with a composite lens (L).](image2)

We focused our work on two areas: calibration and 3D reconstruction. For calibration, we used a plane with a printed calibration pattern and a second calibration pattern projected by the PM2-projector system. The printed pattern was used to calibrate the camera-PM1 system, while the projected pattern was used to calibrate the PM2-projector, and to calculate the rotation and translation matrix between the PM1 and PM2 mirrors. For reconstruction, we performed matching between the projected image and the acquired image. For matching, we used the epipolar curves constraints. Finally, we use three objects to evaluate the reconstruction system: two perpendicular planes, a cylinder, and a semi-sphere.

2.2 Geometrical model description for the catadioptric camera calibration

In order to calibrate the catadioptric camera, we use the geometric model proposed in [18]. To solve this model, we use a chessboard pattern to obtain the intrinsic parameters: mirror curvature and image centre, as well as the extrinsic parameters: rotation and translation between the camera reference system and the pattern.

The general model for the catadioptric camera has two reference systems: the camera image plane, represented by \((u', v')\), and the sensor plane, represented by \((u'', v'')\). Let \(X\) be a point in the scene, then we assume that \(u'' = [u'', v'']^T\) is the projection of \(X\) onto the sensor plane and \(u' = [u', v']^T\) is the image in the camera plane (see Figure 3). As described in [23], both systems are related by an affine transformation \(u'' = Au' + t\), \(A \in \mathbb{R}^{2 \times 2}\) and \(t \in \mathbb{R}^2\).

An imaging function \(g\) relates a point \(u''\) in the sensor plane with the vector \(p = [u'', v'', w'']\) coming from the viewpoint \(O\) toward a scene point \(X\) (see Figure 3). The relation between a point in pixels \(u'\) and a point in the scene \(X\) is given by eq. (1).

\[
\lambda \cdot p = \lambda \cdot g(u'') = \lambda \cdot g(Au' + t) = PX
\]  

(1)

![Figure 3. General model for the catadioptric camera. A point \(X\) in the scene is projected in the sensor plane \((u'')\), in metric coordinates, and in the image plane \((u')\), in pixel coordinates. \(p\) is the point \(X\) seen from the viewpoint \(O\). \(R\) and \(T\) are the extrinsic parameters (rotation matrix and translation vector) that relates the camera’s coordinate system with the world coordinate system.](image3)
where $X \in \mathbb{R}^4$ is represented in homogeneous coordinates and $P \in \mathbb{R}^{3\times 4}$ is the perspective projection matrix, and $\lambda$ is a scaling factor. Through camera calibration, we estimate the matrix $A$, the vector $t$, and the non-linear function $g$, such that all the vectors $g(Au' + t)$ satisfy eq. (1). Eq. (2) is proposed for $g$.

$$g(u'', v'') = (u'', v'', f(u'', v''))^T$$

The function $f$ depends on $u''$ and $v''$ only and can be described in polynomial form as shown in Eq. (3).

$$f(u'', v'') = a_0 + a_1 p'' + a_2 p''^2 + \ldots + a_N p''^N$$

where $p'' = \sqrt{u''^2 + v''^2}$. The coefficients $a_i$, $i = 0, 1, 2, \ldots, N$ and the polynomial degree $N$ are determined by the calibration process. Thus, eq. (1) can be rewritten as eq. (4).

$$\lambda \cdot \begin{bmatrix} u'' \\ v'' \\ t'' \end{bmatrix} = \lambda \cdot g(Au' + t) = \lambda \begin{bmatrix} (Au' + t) \\ f(u'', v'') \end{bmatrix} = P \cdot X, \quad \lambda > 0$$

For cameras with parabolic mirrors we use eq. (5) as proposed in [25]. This simplification allows us to assume $a_1 = 0$, and rewriting eq. (3) we obtain eq. (6).

$$\frac{df}{dp} \bigg|_{p=0} = 0$$

$$f(u'', v'') = a_0 + a_2 p''^2 + \ldots + a_N p''^N$$

From the camera’s calibration process, the parameters $[A, t_0, t_2, \ldots, t_N]$ are estimated with an iterative procedure. Parameters $A$ and $t$ describe an affine transformation that relates the sensor plane with the image plane of the camera. $A$ is the rotation matrix and $t$ is a translation vector between $I$ and $O$, in the image plane of the camera (see Figure 3).

According to this, we assume that $A = I$, and $t = 0$, therefore $u'' = u'$. Replacing this relation in eq. (4) and using eq. (6), we get the projection shown in eq. (7).

$$\lambda \cdot \begin{bmatrix} u'' \\ v'' \\ t'' \end{bmatrix} = \lambda \cdot g(u') = \lambda \begin{bmatrix} u' \\ v' \\ f(p') \end{bmatrix} = \lambda \begin{bmatrix} u' \\ v' \\ a_0 + a_2 p'^2 + \ldots + a_N p'^N \end{bmatrix} = P \cdot X, \quad \lambda > 0$$

To eliminate the depth scale dependency $\lambda_{ij}$, both sides of eq. (8) are multiplied vectorially by $p_{ij}$ resulting in eq. (9).

$$\lambda_{ij} \cdot p_{ij} = \lambda_{ij} \begin{bmatrix} u_{ij} \\ v_{ij} \\ p_{ij}^N \end{bmatrix} = P^i \cdot X_W$$

$$= \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \cdot \begin{bmatrix} x_{ij} \\ y_{ij} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x_{ij} \\ y_{ij} \\ 1 \end{bmatrix}$$

For the calibration process, we obtain the extrinsic parameters for each image that indicate the position of the calibration pattern with respect to the camera reference system.

To perform the calibration, we used a printed chessboard pattern in different positions, which relates the camera’s coordinate system with the world $W$ through a rotation matrix $R_{W PM1}$ and a translation vector $T_{W PM1}$, called extrinsic parameters. Let $l^i$ be the calibration pattern image, $X_W = [x, y, z]$ representing the 3D coordinates of a point in the calibration plane reference system, and $m_{ij} = [u_{ij}, v_{ij}]$ are the pixel coordinates in the image plane. Considering a calibration pattern with a flatness shown in Figure 4, we assume that $z_{ij} = 0$. Consequently, we obtain eq. (8).
From eq. (9), for each point \( p_i \) of the calibration pattern, we obtain the three homogeneous equations:

\[
\begin{align*}
  v_j \cdot q_3 - f(\rho_j) \cdot q_2 &= 0 \\
  f(\rho_j) \cdot q_1 - u_j \cdot q_3 &= 0 \\
  u_j \cdot q_2 - v_j \cdot q_1 &= 0
\end{align*}
\]

(10.1) \hspace{1cm} (10.2) \hspace{1cm} (10.3)

where \( x_j, y_j \) and \( z_j \) and \( u_j, v_j \) are known, and correspond to the mirror points and image points, respectively. \( q_1, q_2 \) and \( q_3 \) are given by:

\[
\begin{align*}
  q_1 &= (r_{11}x_j + r_{12}y_j + t_1) \\
  q_2 &= (r_{21}x_j + r_{22}y_j + t_2) \\
  q_3 &= (r_{31}x_j + r_{32}y_j + t_3)
\end{align*}
\]

Eq. (10.3) is linear and has the unknown parameters \( r_{11}, r_{12}, r_{21}, r_{22}, t_1, t_2 \). Rewriting eq. (10.3) for \( L \) points of the calibration pattern as a system of linear equations, we get eq. (11).

\[
M \cdot H = 0
\]

where:

\[
H = \begin{bmatrix} r_{11}, r_{12}, r_{21}, r_{22}, t_1, t_2 \end{bmatrix}^T
\]

\[
M = \begin{bmatrix} -v_1x_1 & -v_1y_1 & u_1x_1 & u_1y_1 & -v_1 & u_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-v_Lx_L & -v_Ly_L & u_Lx_L & u_Ly_L & -v_L & u_L \end{bmatrix}
\]

Using singular value decomposition (SVD), we can represent \( M = U \Sigma V^T \), the solution to the system can be found in the singular vector \( v_k \) associated with the least singular vector \( c_0 \).

Once eq. (10.3) is solved to find the camera extrinsic parameters, we substitute the estimated values into the equations (10.1) and (10.2), and solve for the camera intrinsic parameters \( a_0, a_2, \ldots, a_N \) that describe the shape of the imaging function \( g \).

Equations (10.1) and (10.2) are rewritten again as a linear system of equations. But now, we incorporate all the \( k \) calibration pattern observations to obtain eq. (13).

\[
\begin{bmatrix} A_1 & A_1\rho_1^2 & \ldots & A_1\rho_N^2 & -v_1 & 0 & \ldots & 0 \\
C_1 & C_1\rho_1^2 & \ldots & C_1\rho_N^2 & -u_1 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
A_k & A_k\rho_1^2 & \ldots & A_k\rho_N^2 & 0 & 0 & \ldots & -v_k \\
C_k & C_k\rho_1^2 & \ldots & C_k\rho_N^2 & 0 & 0 & \ldots & -u_k \end{bmatrix} \cdot r = B
\]

(13)

Fig. 5. a) Projected calibration pattern, b) Image captured by the catadioptric camera showing two calibration patterns in a white plane: the printed pattern is used to calibrate the catadioptric camera (camera CCD-PM1 mirror), and the projected pattern is used to calibrate the projection system (PM2 mirror-projector).

The solution of the system is obtained using the pseudoinverse matrix. Thus, the intrinsic parameters \( a_0, a_2, \ldots, a_N \) which describe the model, are now known. In order to compute the polynomial degree \( N \), we actually start from \( N=2 \), then we increase \( N \) by unitary steps and we compute the average value of the reprojection error of all calibration points. The procedure stops when the minimum error is found.

2.3 Calibration of the catadioptric camera with the light projection system

In order to calibrate the catadioptric camera with the projection system (see Figure 6), we develop the following procedure:

A calibration pattern image was projected (see Figure 5a)) in different positions over the mirror PM2. The mirror PM2 reflects the image until the calibration plane \( W \) is found. The calibration plane position is known with respect to PM1, (as seen in Figure 5b)) because the calibration plane contains the printed pattern used to calibrate the camera-PM1 system. Once a catadioptric image is acquired by the camera-PM1, a subpixel algorithm provides the points of the projected calibration pattern. These points are represented as \( u_{PM1} \).

The plane where the projection pattern is located is defined by \( \Pi_w \), and is calculated with eq. (14) [24].

\[
\Pi_w = \begin{bmatrix} n_{x}^T \\ D_{f}^T \end{bmatrix} = \begin{bmatrix} R_{W \rightarrow PM1}^{-1} \\ -(R_{W \rightarrow PM1}^{-1} \cdot T_{W \rightarrow PM1})^T \end{bmatrix} \begin{bmatrix} 0^T \\ 1 \end{bmatrix} \begin{bmatrix} n_{x}^T \\ D_{f}^T \end{bmatrix}
\]

where \( R_{W \rightarrow PM1} \) and \( T_{W \rightarrow PM1} \) represent the rotation matrix and translation vector respectively between the world and the mirror PM1. 0 denotes the null vector, \( n_{x} \) and \( D_{f} \) are unit normal vectors to the plane \( \Pi_w \), and finally
$D_1$ and $D_2$ denote the distance from the mirror PM1 to the plane $\Pi_{iw}$. The points $u_{PM1}$ are mapped to the plane $\Pi_{iw}$, and are denoted as $X_{PM1}$. These points are defined on the mirror PM1. A rigid transformation is required to represent the points $X_{PM1}$ on the mirror PM2 coordinate system, as described in eq. (15).

$$x_{PM2} = R_{PM1\rightarrow PM2}x_{PM1} + T_{PM1\rightarrow PM2} \tag{15}$$

When the points $X_{PM2}$ are projected onto the mirror PM2, they are represented by $x_{PM2}$. Eq. (7) is used to define the points $x_{PM2}$ in the projector image as $v_{PM2}$. However, as the rotation matrix $R_{PM1\rightarrow PM2}$ and the translation vector $T_{PM1\rightarrow PM2}$ are unknown parameters, a function that optimizes the value for these parameters was developed.

This function represented by $g^2$ has known input parameters such as: the points $X_{PM1}$, the extracted points $u_{PM2}$ from the projected pattern (see Figure 5a)), and the unknown parameters: rotation $R_{PM1\rightarrow PM2}$ and translation $T_{PM1\rightarrow PM2}$.

The $v_{PM2}$ points are used to calculate the error with eq. (16). This error is minimized according to $\delta_2(\theta_1\theta_2\theta_3, tx, ty, tz)$ using the Levenberg-Marquardt optimization method in MATLAB®, resulting in the rotation and translation between the camera-mirror (PM1) and the mirror(PM2)-projector.

$$e = (u_{PM2} - v_{PM2})^2 \tag{16}$$

2.4 Epipolar geometry for panoramic cameras

Epipolar geometry describes the relationship between positions of corresponding points in a pair of images acquired by central catadioptric cameras [14, 25], as seen in Figure 7. In this section, the alignment parameters (rotation and translation) between the camera-mirror (PM1) and the mirror(PM2)-projector are described.

![Figure 6. Schematic diagram of the panoramic 3D reconstruction system. The projections of 3D points $X_W$ on the mirrors (PM1, PM2) are denoted by $x_{PM1}$ and $x_{PM2}$. $u_{PM1}$ are the image points acquired by the catadioptric camera, and $u_{PM2}$ are the points of the projected image.](image)

![Figure 7. Epipolar geometry between two catadioptric cameras with a parabolic mirror. $R_{PM1\rightarrow PM2}$ is the rotation matrix and $T_{PM1\rightarrow PM2}$ the translation vector between both mirrors (PM1-PM2). $\Pi$ is the epipolar plane.](image)
Thus, eq. (23) is denoted by eq. (24) as:

\[ n_2 = [p, q, s]^T \]  

(24)

we can write the equation of the plane Π in the projector’s coordinate system resulting in eq. (25):

\[ px + qy + sz = 0 \]  

(25)

The derivation of \( A_2(E, u_1) \) from eq. (21) based on a parabolic mirror, \( z \) is substituted into eq. (25) resulting in eq. (26) that allows the calculation of the epipolar conics for this type of mirror:

\[ sx^2 + 2b_2px + sy^2 + 2b_2qy - sb^2 = 0 \]  

(26)

where \( b_2 \) is the mirror parameter and \( p, q, s \) are defined by eq. (24).

2.5 3D reconstruction

Knowing the rotation \( R_{PM1\rightarrow PM2} \) and translation \( T_{PM1\rightarrow PM2} \) between both mirrors, we can find the relationship between the pixel coordinates in the image and the angles defined in the parabolic mirror reference system (see Figure 8). Having this information, 3D point coordinates are calculated from the correspondence of points in the projected image and the acquired image by the catadioptric camera [14].

Given two points \( x_{PM1}^i \) and \( x_{PM2}^i \) on the surfaces of both mirrors PM1 and PM2, and the coordinates \( [x, y, z] \) from a point \( x_{PM2}^i \) we have eq. (27):

\[ \phi_1 = \arctan \left( \frac{y}{x} \right) \]

\[ \theta_1 = \arctan \left( \frac{\sqrt{x^2 + y^2}}{z} \right) \]  

(27)

\[ \omega_1 = \arccos \left( \frac{(x_{PM1}^i)^T (-R_{PM1\rightarrow PM2}T_{PM1\rightarrow PM2})}{||x_{PM1}^i|| ||T_{PM1\rightarrow PM2}||} \right) \]

\[ \omega_2 = \arccos \left( \frac{(x_{PM2}^i)^T T_{PM1\rightarrow PM2}}{||x_{PM2}^i|| ||T_{PM1\rightarrow PM2}||} \right) \]

3. Results

3.1 Calibration

We used 26 images to calibrate the catadioptric camera, some of these are shown in Figure 9. As we can see, the white plane (\( \Pi_{w} \)) contains two calibration patterns: one printed and the other projected.

The printed pattern is used to calibrate the catadioptric camera formed by the CCD camera and the PM1 mirror, and to compute the plane \( \Pi_{w} \). Once the camera-PM1 setup is calibrated, the projected pattern is mapped to the mirror and then to plane \( \Pi_{w} \). This pattern is used to compute the rotation and translation between the mirrors PM1 and PM2, as explained in section 2.5, and to calibrate the PM2-projector setup.

\[ D = \frac{T_{PM1\rightarrow PM2} \sin \omega_2}{\sin(\omega_1 + \omega_2)} \]

\[ X = \begin{bmatrix} D \sin \theta_1 \cos \phi_1 \\ D \sin \theta_1 \sin \phi_1 \\ D \cos \theta_1 \end{bmatrix} \]  

(28)

Figure 9. Calibration images. Twenty-six images were used to calibrate the catadioptric camera. Each image contains a white plane with the two calibration patterns in different positions and orientations.

The intrinsic parameters from the catadioptric camera are presented in Table 1. The values of the mirror’s curvature allow us to confirm that it is a parabola.

<table>
<thead>
<tr>
<th>Polynomial coefficients ( f(\rho) )</th>
<th>Image centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>-147.8562</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 1. Intrinsic parameters of the camera-PM1

The intrinsic parameters obtained for the PM2-projector system are presented in Table 2.

<table>
<thead>
<tr>
<th>Polynomial coefficients ( f(\rho) )</th>
<th>Image centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>55.1000</td>
<td>-0.0056</td>
</tr>
</tbody>
</table>

Table 2. Intrinsic parameters of the PM2-projector
The rotation and translation matrix between the mirror PM1 and the mirror PM2 as described in section 2.4, correspond to the values shown in eq. (29).

\[
R = \begin{bmatrix}
-0.3924 & 0.9184 & 0.0505 \\
-0.8897 & -0.3929 & 0.2324 \\
0.2333 & 0.0463 & 0.9713 \\
\end{bmatrix}
\quad (29)
\]

\[
[T]_x = \begin{bmatrix}
0 & -108.2657 & -0.4395 \\
108.2657 & 0 & -44.9961 \\
0.4395 & 44.9961 & 0 \\
\end{bmatrix}
\]

When building the optical setup for the panoramic 3D reconstruction system, we carefully aligned the PM1 and PM2 mirrors. However, as can be seen in the rotation matrix R, a small misalignment exists.

To calculate the calibration error in the camera–mirror PM1 system, we considered the position of the 3D points on the printed calibration pattern in the catadioptric image. With the rotation and translation parameters, these points are projected towards the PM1 mirror, using the intrinsic parameters (shown in Table 1) that are mapped to the image. This way, we compared the points from the printed pattern projected to the catadioptric image with the points extracted from the acquired image by the camera.

The calibration error in the catadioptric projection system (PM2-projector) is obtained as follows: we extract the positions of the 3D points from the projected calibration pattern in the catadioptric image, and then these points are projected towards the mirror PM1 and are represented in the plane \( \Pi_{L_0} \). With the 3D points in the plane, we calculate the rotation and translation matrix between the PM1 and PM2 mirrors in order to represent the points in the PM2 mirror reference system, using the intrinsic parameters (shown in Table 2) that are mapped towards the projector. Finally, we compared these points from the projected pattern to obtain the error.

The calibration error from the catadioptric camera (camera-PM1) is about 0.4 pixels, and the calibration error from the catadioptric projection system (PM2-projector) is 1.56 pixels.

### 3.2 Angle of view

Figure 10 shows the direction of \( T_1 \) as function of the camera focus (\( F_1 \)) and the sensor size (\( L_4 \)). \( T_1 \) is defined by eq. (30):

\[
T_1 = \lambda \begin{bmatrix} F_1 \\ L_4 \end{bmatrix} = \begin{bmatrix} Z_1 \\ \rho^m \end{bmatrix}
\quad (30)
\]

where \( Z_1 = a_0 + a_2 (\rho^m)^2 \), \( \rho^m \) corresponds to the largest radius of the mirror that can be seen by the camera. \( \lambda \) is a scale factor, \( a_0, a_2 \) are the polynomial coefficients that describe the PM1 mirror curvature.

There are two solutions for \( \rho^m \) and \( \lambda \), the solution associated with our experimental setup is eq. (31):

\[
\lambda = \frac{F_1 - \sqrt{F_1^2 - 4a_2L_4^2}}{2a_2L_4}, \quad \rho^m = \frac{F_1 - \sqrt{F_1^2 - 4a_2L_4^2}}{2a_2L_4}
\quad (31)
\]

Considering that the angle formed by \( T_1, N_1 \) is the same as the one formed by \( N_1, S_1 \), we obtain the eq. (32):

\[
S_1 = 2(T_1^T n_1)n_1 - T_1; \quad (32)
\]

Approximating \( R_{PM1→PM2} = I \), we have the eq. (33):

\[
S_2 = 2(T_2^T n_2)n_2 - T_2; \quad (33)
\]

where:

\[
S_2 = \frac{N_2}{\sqrt{N_2^T N_2}}\begin{bmatrix} -2a_0 \rho^m \\
-2a_2 \rho^m \end{bmatrix} \quad \text{and} \quad T_2 = \begin{bmatrix} -a_0' \rho^m - \rho^m \end{bmatrix};
\]

The angle of view \( \theta \) of the reconstruction system is given by:

\[
\theta = \cos^{-1}\left(\frac{S_1^T S_2}{\sqrt{S_1^T S_1} \sqrt{S_2^T S_2}}\right)
\]

Substituting the values of the camera, projector and the mirrors PM1 and PM2, we obtain \( \theta = 125.4125 \) degrees.

---

Figure 10. Real dimensions of the experimental setup. \( L_1, F_1, L_2 \) and \( F_2 \) are shown at a larger size for visualization purposes. \( L_1, \), \( F_1, L_2 \) and \( F_2 \) are the sensor size, camera focus, display size and projector focus, respectively.
3.3 Reconstruction

In order to test the functionality of the system, we reconstruct the object shown in Figure 11 containing two perpendicular planes.

![Figure 11](image1). Test object 1: Perpendicular planes. A set of two perpendicular planes was used to test the functionality of the system.

The perpendicularity of the two planes was measured using the Mitutoyo BJ1015 Coordinate-measuring machine and Metrolog XG® software (see Figure 12) obtaining an angle of 89.192 degrees between them.

To perform the reconstruction, we projected a pattern of points onto the perpendicular planes (see Figure 13), and then we acquired an image from this projection using the catadioptric camera (see Figure 14). The projected pattern points have random colours. We perform the correspondence between the acquired image and the projected image using the colour of the projected points and the epipolar curves constraints (described in section 2.4).

Epipolar geometry from the 3D reconstruction system is shown in Figure 15. Here we can see that the conic lines obtained with eq. (26) intersect in the epipole and passed through the corresponding points. Once the correspondence is found, we build the 3D reconstruction of the object. The reconstructed planes can be seen in Figure 16.

![Figure 12](image2). Measuring perpendicularity of the two planes using the Mitutoyo BJ1015 Coordinate-measuring machine.

![Figure 13](image3). Projected pattern. The pattern projected by the PM2-projector system over the perpendicular planes consists of a set of points with random colours.

![Figure 14](image4). Catadioptric image with the points projected on the perpendicular planes, captured by the camera CCD-mirror PM1.

![Figure 15](image5). Epipolar geometry. Coloured projected points with their corresponding conics (epipolar lines).

Having previous knowledge about the object formed by the two planes, we segmented the reconstructed points in the objects. Each object corresponds to one of the two perpendicular planes $\Pi_1$ and $\Pi_2$. Then, we calculate the error, considering the perpendicular distance $d_i$ between the points and the corresponding plane using eq. (34).
Figure 16. 3D reconstruction of the perpendicular planes.

Figure 17. Reconstruction error in perpendicular planes. Perpendicular distance error in millimetres of the points in plane one (red) and plane two (blue).

\[ \tilde{e} = \frac{1}{n} \sum_{i=1}^{n} d_i(p_i, \Pi_j) \] (34)

In Figure 17, the perpendicular distance in millimetres from each point with the corresponding plane is shown. The graph shows a mean error of 2.4 mm on plane 1 and 3.6 mm on plane 2. The angle between the two planes is 89.8488 degrees.

We performed two additional reconstructions test using a cylindrical object (see Figure 18) and a semi-sphere (see Figure 19).

A pattern of points with random colours was projected inside the cylinder to obtain the results shown in Figure 20.

In order to know the real values of the cylinder, we perform a curve fitting with the 3D reconstructed points using Metrolog XG® software. The results indicate an error of 4.7657 mm (see Table 3) with respect to the real object radius.

<table>
<thead>
<tr>
<th>Real radius [mm]</th>
<th>Radius [mm]</th>
<th>Error [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>115.79</td>
<td>111.02</td>
<td>4.7657</td>
</tr>
</tbody>
</table>

Table 3. Curve fitting results of reconstructed 3D points using Metrolog XG® software
Figure 21. 3D reconstruction of the semi-sphere.

The reconstruction of the semi-sphere shown in Figure 21.

4. Conclusions and future work

The panoramic 3D reconstruction systems found in the literature are based on two catadioptric cameras. Thus, the 3D reconstruction is based on the matching of corresponding points between two views. However, these methods fail in applications with textureless objects. In this work, we proposed a new reconstruction system based on a single catadioptric camera and a catadioptric projection system. The reconstruction can be done with a single acquisition or with multiple acquisitions moving the position of the projected points. For this, we first calibrate the catadioptric vision system, and then we calibrate the catadioptric projector. The reconstruction error in an object composed by two perpendicular planes was 2.4 mm on the first plane, and 3.6 mm on the second plane, with an angle of 89.8488 degrees between planes. The reconstruction error in the cylindrical object was 4.7 mm. Our future work will focus on obtaining denser 3D reconstructions and characterization of reconstruction uncertainty. Moreover, increasing the reconstruction area by moving the system inside the object.

5. Acknowledgements

The authors wish to acknowledge the financial support for this work by the Consejo Nacional de Ciencia y Tecnología (CONACYT) through project SEP-2005-O1-51004/25293 and the financial support scholarship number 256319, and by the Instituto Politécnico Nacional through project SIP-20130165.

6. References


