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Theoretical Investigations of the Control
Movement of the CLAWAR at Statically Unstable
Regimes

Alexander Gorobtsov
Volgograd State Technical University
Russia

1. Introduction

Dynamic control of constrained spatial mechanical systems, such as manipulators, CLAWAR, multifingered robotic hands and etc have been classic problem in robotics research. The theory of control based on fundamental idea of dynamic inverse. One of the best control schemes is hybrid position/force (Mayorga R. V., Wong A. K, Паписон В.А.). Ones works has been exploited the hybrid control to manipulators with open constraints. The theory of the control of robotics with closed constraints (Vukobratovic M., Kircanski M.) assumed that the rectangular matrix of the coefficients of constraints is the matrix of full rank, and not include redundant constraints. The problem of the redundant constraint raised in walking machines and systems with redundantly drive parallel mechanisms too (Jongwon K., Frank C. P.).

The mentions methods are references at two classes of mechanical systems. First it is spatial system with unmovable body and seconds is a spatial system without unmovable body. The dynamics of the systems first class is described by equations, expressed at explicit form at relative or absolute coordinates. That form of equation is permitted simple using classical method of stability analysis. The method of this type well compliance to control and stability analysis of the manipulators with end effectors constrained, parallel manipulators and multifingered robotic hands with closed constrained. Walking machines (CLAWAR) have not unmovable body. So the dynamics equations can not be expressed in generalized coordinates. At any case to equations in this form it is difficult to apply the classical methods of stability analysis. To avoid it constrain researcher used linearization equations of motion (Jong H. P.). Linearization equations made possible to provide stable analysis only at small domain of the point of linearization. To create robust CLAWAR it is necessary to accomplish stability analysis of nonlinear equation of motion at large neighborhood of current position.

At present work describes the mathematical formulation of control dynamic of a class of spatial mechanical system. The proposed mathematical formulation is realized at computer software of simulation dynamic linked multibody systems. For CLAWAR applications are investigated strong instability locomotion regimes at four feet and two fingers simulation model.
2. Mathematical define problem

The solutions described problem conviently to find at numerical form based on Lagrange equation 1 type or in other words Euler - Lagrange equation:

\[
\begin{align*}
\mathbf{M}_x - \mathbf{D}_x^T \mathbf{p} &= f(x,x,t) + \mathbf{u}(t) \\
\mathbf{D}_x &= \mathbf{h}(x,x)
\end{align*}
\]  

(1)

In above equations \(x\) is the state vector dimension \(n\), \(\mathbf{M}\) is a inertia matrix, \(f(x,x,t)\) is the external forces vector, \(\mathbf{u}(t)\) is a control forces vector, \(\mathbf{D}\) is the matrix of the variable coefficients of the constraints equations with dimension \(k \times n\), \(k\) - number of constraints equations, \(\mathbf{h}(x,x)\) is right side constraints equations vector, \(\mathbf{p}\) is Lagrange multipliers vector. Eq. (1) is the generalized equation of motion of any control mechanical system and well known at multibody dynamics. Assume that the control motion is the movement some prescribe point of the system along program trajectories \(w(t)\), where \(w(t)\) is vector with dimension \(m\). Eq. (1) can be transforming by substitute control forces \(\mathbf{u}(t)\) on constraints equation due program trajectories \(w(t)\):

\[
\begin{align*}
\mathbf{M}_x - \mathbf{D}_x^T \mathbf{p} - \mathbf{D}_x^T \mathbf{p}_w &= f(x,x,t) \\
\mathbf{D}_x &= \mathbf{h}(x,x) \\
\mathbf{D}_x x &= w(t)
\end{align*}
\]  

(2)

where \(\mathbf{D}_w\) is the matrix of the variable coefficients constraints equations according program trajectories \(w(t)\) with dimension \(k \times m\), \(\mathbf{w}(t)\) is second derivation of a program trajectories, \(\mathbf{p}_w\) is the Lagrange multipliers vector due \(\mathbf{w}(t)\). Eq. (2) are equivalent Eq. (1), but (2) haven’t contain unknown control forces \(\mathbf{u}(t)\). Hence numeric integration (2), provide kinematics parameters of the program movement \(x^*(t)\) and velocities and accelerations too.

To find control forces \(\mathbf{u}(t)\) at arbitrary time lets apply to system of Eq. (1) the constraints according to degrees of freedom \(n-k\). At this condition Eq. (1) become static and have zero degree of freedoms. To summarize, we can transform Eq. (1) to:

\[
\begin{align*}
\mathbf{M}_x D^T \mathbf{p}_o &= f(x^*,x^*,t) - \mathbf{M}_x^* + \mathbf{D}_x^* \mathbf{p}^* \\
\mathbf{D}_x &= \mathbf{h}(x^*,x^*) \\
\mathbf{D}_x x &= \mathbf{h}_o(x^*,x^*)
\end{align*}
\]  

(3)

Where \(x^*,x^*,x^*\) are accelerations, velocities and displacements from Eq. (2), \(\mathbf{D}_o\) is the matrix of the variable coefficients constraints equations accord degrees of freedom system by Eq. (1), \(\mathbf{h}_o(x^*,x^*)\) is right side vector and \(\mathbf{p}_o\) is the Lagrange multipliers vector for ones constraints equations, \(\mathbf{p}^*\) is the Lagrange multipliers vector due Eq. (2).

From Eq. (3) for current time moment it can be obtain \(\mathbf{p}_o\) vector. To compare Eq. (1) and Eq. (3) with account \(\mathbf{M}x = 0\) at Eq. (3), we can write drives forces equations:
Eq. (3) have solution in case $m = n - k$, i.e. dimension matrix $D_o$ is $n - k \times n$. Additional conditions for eq. (3) are $\text{rank}(D) = k$, $\text{rank}(D_o) = m$, $\text{rank}\left(\begin{array}{c} D \\ D_o \end{array}\right) = n$. The $\text{rank}(D) < k$ is the case redundant constraints, $\text{rank}(D_o) < m$ is expensive determinate program motion, $\text{rank}\left(\begin{array}{c} D \\ D_o \end{array}\right) < n$ is redundant actuators. The case of expensive determinate program motion can be simple solve by changing of the function $w(t)$. More complex problems are the cases of redundant constraints and actuators. Some methods for solving ones described low.

If the $\text{rank}(D) = k_1$ and $k_1 < k$, then the mechanical system has $k - k_1$ redundant constraints. For define the constrains reactions dues all $k$ constraints proposed to cut redundant constraints and insert into cutting some equivalent kinematical subchain. Beside ones it can necessary change $k - k_1$ constraints and assign to system $k - k_1$ appended degree of freedom, which compliant every redundant constrain. Under this transformations eq. (1) may be writing

$$\begin{bmatrix} M \dot{x} - D^T p = f(x, x_{\omega}) + u(t) \\ D \dot{x} = h(x, x_{\omega}) \end{bmatrix}$$

Where $x = \begin{bmatrix} x \\ x_{\omega} \end{bmatrix}$, $x_{\omega}$ is the state vector of the subchains bodies, $M_{\omega} = \begin{bmatrix} M \\ M_{\omega} \end{bmatrix}$, $M_{\omega}$ is inertia matrix of the subchains bodies, $D_{\omega} = \begin{bmatrix} D \\ D_{\omega} \end{bmatrix}$, $D_{\omega}$ is the matrix $D$ without $(k - k_1) \times 2$ constraints equations, which dues to redundant links and deleted constraints, provided appended degree of freedom, $D_o$ is the matrix of the variable coefficients constraints equations corresponding to links of the subchains bodies, $p_o$ is the constraints reactions of the transformed scheme. The estimate of the accuracy for Lagrange multipliers $p_o$ may be written at form:

$$|p - p_o| \leq |M||M_{\omega}|$$

The matrix norm is the module of the maximal coefficient of the matrix. If $\text{rank}\left(\begin{array}{c} D \\ D_o \end{array}\right) = n_1$ and $n_1 < n$, then the mechanical system has $n - n_1$ redundant actuators and the eq. (3) haven’t solution. It can be possible to transform eq. (3) to form (5) in this case too. Then from ones equations we can provide actuators forces $p_o$ with accuracy estimate at form (6).

To transform equations (1) and (3) to form (5) is need find redundant constraints. Universal algorithm is the step by step including constraint equation to full system and estimate
spectral radius matrix \( \begin{pmatrix} \mathbf{M} & -\mathbf{D}^T \\ \mathbf{D} & 0 \end{pmatrix} \) at every step. This method have realized at universal software FRUND.

The essential of proposed approach is substitution redundant reactions forces to inertial forces. The main advantage of proposed method in comparison with existing methods is used the most common form of the equations of motion (1), that can be applied to system with arbitrary structure.

3. Computer Realization of the Method

The inverse dynamic analysis written at form of Eq. 1-3 was realized at universal multibody software FRUND (http://frund.vstu.ru). The algorithm of the software based on numeric integration of the system of differential-algebraic equations (1) - (3). The numeric integration including the sequential solution of three subtasks. At every step of integration the first is calculated Eq. 2. That equation produced the vectors of displacement, velocity and acceleration of the program movement of the system. The data of the program movement are sending to second module of calculated program reactions at constraints and drives via the Eq. 3. After that, all data passing to third module. Third module provides the parameters of the control movement of mechanical system by integrated Eq. 1. Input data for simulation the control movement are the parameters of mechanical system and the special subroutines, described program movement \( \mathbf{w}(t) \). Output data are presented at plot or animation forms.

4. The Theoretical Analysis and Simulation Results

The hypothetic walking machine used in this chapter has four legs – Fig.1. Each leg has five parts. The foot of the leg has two fingers. Total quantity drives at every leg are six. In this model, it is assumed that there exists an elastic pad at the sole of each foot of the robot. Each elastic pad is assumed to be composed of three dimensional nonlinear spring and damper units. Vertical and lateral forces at the pad are zero when no contact with the ground is made. Lateral forces are depended from friction ratio. Each elastic pad is connected to finger. The model of elastic pad is used for describe control motion of the machine Eq. 1. To obtain the program motion is used rigid model of surface contact. The program motion of the model is defined by describe kinematics parameters for three points of the cab along summary six translation directions.

Parameters walking machine: total mass 233 kg, leg mass 8 kg, longitudinal distance between legs 1.5 meters, lateral distance between legs 2.5 meters, height center of gravity of the cab 1.1 meters, distance between fingers of the same foot 0.4 meters. Maximal forces at the drives are 4 kN, drives feed back proportional coefficient 80, drives feed back differential coefficient 2.4, nominal friction ratio 0.8.

Model contain adaptive algorithm of the foot movement relative the cab of machine. The parameters of the algorithm are defined the step length and the direction of the displacement of the foot at horizontal plane. Above it the algorithm can change initial position of the foots at three direction. The length of the foot can change at diapason 0 – 2 meters, the foot lift 0.1 meters, minimal step time period 0.4 s. The type of the walking defined initial phase for every legs.
At first stage of investigation is considered the accelerate motion of the robot to predefined velocity and after that the movement with the constant velocity. The walking of the movement is synchronous step of the two diagonal legs of the other sides. It walking we can classify how static quasistability. The time period, when the total vector of the vertical reactions of the contact fingers is out of static polygon is less of the time of legs shift. At this case the results of computer simulation have make conclusion that some coordinates of the equation have asymptotic stability. Fig. 2 shows convergence at drive displacement for program and control movement.

Figure 1. Computer model of walking machine

Figure 2. Drive displacement at left front leg, hip – leg actuator
As we can see from Fig. 3 the displacement of the cab at control motion is not convergence to one at program motion. That kinematics parameter have constant shift. As show the comparison of the velocity of program and control motion the maximal difference is at velocity of the cab Fig.4. The control movement in sense of velocity of the cab is stability but is not asymptotic stability. The simulation has obtained that the actuators velocity are asymptotic stability.
The periodic type of the cab velocity has peak dependence from surface friction ratio. The main cause of one phenomenon is static quasistability.

The second investigated regime of motion is the motion with a shift legs middle position to front as show in Fig. 5. At Fig. 6 is presented program vertical reaction at one of the finger for some shift values. Note, from Fig. 6, that the sign of reaction at finger for zero shift is positive. One case due results on Fig. 2 – 4. At case of shift legs middle position the reactions may have negative sign – Fig. 6. The negative sign of the vertical reaction is denote physical unrealizability at control movement. So at this case the control movement may be unstable.

At Fig. 7 shows the animation of the control movement for zero legs shift and shift 10 sm. It can see that the variant of the shift legs is unstable. The essential of unstable is the trend of the movement direction of the cab and less value of average velocity. Graphical presentation of one is reported in Fig. 8. At the computer simulation have obtain full unstable regimes with crash for more values front shift.
Figure 7. The animation of control motion with zero shift legs (left) and front shift legs 10 sm (right)

To stabilise control movement is introduce the method of the modify program motion. For this purpose to cab of the machine is applied added acceleration, which defined from condition to change the signs of the vertical reaction at contact fingers. The spatial vector of the added acceleration is needed to cross the static polygon and to lie inside of the friction cone. To provide the acceleration vectors a program reaction at the contact finger is used. The program reactions are obtained form Eq. 2. The algorithm of the modify program movement was made for case four legs machine and walking with two foots at contact. The added accelerations were calculated only for time moments, when one or some of the vertical reactions of program movement are negative. If the vertical reactions were positive at some describe long time, then the added acceleration assign new values, calculated from the vector of the forces, backing trajectory to initial program motion. Let name this forces back forces.

Figure 8. Longitudinal displacement of the cab, control motion is for front shift legs 10 sm
The method of the modified program movement make possible the stabilization of the control movement. The results of the simulation are reported in Fig. 9. It is presented two variants of parameter back forces of the method of modify program motion. Variant 1 is the nominal value back force 20, variant 2 is double nominal variant. Note, from Fig. 9, that back forces have significant influence on stability of program motion. The added acceleration provides stability motion, but do not provide convergence the control motion to program motion - Fig. 10.

Figure 9. Longitudinal displacement of the cab for control motion for front legs shift 10 sm

Figure 10. Longitudinal velocity of the cab for program motion
The sensitivity of the system to solution accuracy is reported in Fig. 11. The trajectories of the control movement are different for big and small errors integration. The proposed method of the modify program movement is applied to strong instability regimes. For example, is investigated the movement of the machine at program motion by the low at Fig. 3-4, only the sign of the program velocity is negative. The walking of machine at this case is synchronous shift both the front legs and then the rear legs. This walking is analog gallop walking. One walking is strong instability, because the lift legs from the same side is provide machine falling. At Fig. 12 (right) is presented the unstable control motion of the machine with falling.

Figure 11. Longitudinal displacement of the cab

Figure 12. Control motion for modified program motion at statically unstable regime (left), control motion for nominal program motion (right)
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To stabilize the control motion is used the modified program motion. The modify of program motion at this regime have evident interpretation. That is appended longitudinal acceleration on locomotion direction, and so the result program motion is the movement with grew velocity - Fig. 13. The longitudinal acceleration has corrected negative vertical reactions at the fingers to positive – Fig. 14. Note that longitudinal acceleration provided unlimited speed grows, and indicate changes the parameters of robot walking. It method is produced the stability control locomotion – Fig 12. (left).

![Graph showing longitudinal velocity of the cab](image1)

**Figure 13.** Longitudinal velocity of the cab

The stable locomotion at that regime is depended from step length, for short step length doesn’t provide the stable locomotion. The achieved results may be used at formulation the type of stability of the spatial complex mechanical system. It is necessary note, that the problem of stability mechanical system, described Eq. 1-3 can not be solved by the first
Lyapunov method, because the system is strong nonlinearity. The asymptotic stability of the part of state variable of the Eq.1-3, defined the relative displacement and velocities at the drives is provide by proportional and differential feedback gains - Fig. 2. Global stability may be formulated at the terms of second Lyapunov method. This method very difficult to apply for case of multidimensional differential-algebraic equations. The complexity of the problem is grow by the property of numerical integrations methods, needed to solve the equations. The nonlinearity of problem of the synthesis of control motion illustrated in Fig. 11, where presented two trajectories of the cab for two values of integration accuracy. Note, that both trajectories are stable at local sense.

It may be conclusion that the analytical solving the problem of global stability of the system Eq. 3-1 is difficult. So it needed to develop numerical methods, based on precise nonlinear models.

5. Conclusion

A new algorithm for the control of CLAWAR at statically unstable regimes was introduced and its effectiveness investigated via numerical simulation. The essential advantages of the proposed approach may be summarized as follows. The proposal algorithm was shown to be capable of providing the control movement of the spatial machines with arbitrary structures and redundant constraints and actuators. The performance of method of modify the program movement was shown a steady state locomotion of the CLAWAR. Based on proposed method can be created the virtual prototypes of the CLAWAR with stable locomotion at any type of walking - jump, run and etc. To solve that task is need to develop more effective software, based on parallel processing, and create adaptive algorithms for procedures of program motion modification and adaptive parameters relative foots motion.

6. References


Nature has always been a source of inspiration and ideas for the robotics community. New solutions and technologies are required and hence this book is coming out to address and deal with the main challenges facing walking and climbing robots, and contributes with innovative solutions, designs, technologies and techniques. This book reports on the state of the art research and development findings and results. The content of the book has been structured into 5 technical research sections with total of 30 chapters written by well recognized researchers worldwide.

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