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A designing method of the passive dynamic walking robot via analogy with Phase locked loop circuits

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1. Introduction

Recently many biped walking robots are developed, and almost all the researches about those robots adopt mass-link models to control and referring zero moment point not to fall over while walking. Here we position these conventional control methods as “model-based control”. Features of the method are that the robot moves accurate and rigid at low efficiency. So it is also important to remember that these researches also adopt other ideas to decrease their energy consumption, to absorb the impact shock of the contact, or to control the joint stiffness …etc. For example, Honda robot “ASIMO” adopts soft material soles (Takenaka, 2001), Sony robot “SDR-4X” and Waseda university’s robot controls its joint stiffness (Iribe et al, 2007) (Yamaguchi & Takanishi, 1997), and so on. So it seems important and valuable for the model-based control method to import the ideas based on the dynamics of the robots for their performance gain.

On the other hand, we focus the passive dynamic walking robot which walks only by the dynamics caused by the gravity (McGeer, 1990) (Goswani et al, 1996) (Garcia et al, 1998) (Sugimoto & Osuka, 2005). Here we also position this control method which is operated by the dynamics as “dynamics-based control” method. Features of the method are that the robot moves soft and smooth at high efficiency. If the principle of the dynamics-based control is analyzed and elucidated, we will be able to apply the essence of this system to the conventional walking robot control, and also able to contribute the performance gain of walking robot control (Iribe & Osuka, 2006-2). Therefore we think that well-designed walking robots have both two above-mentioned ideas as shown in Fig.1. However, the dynamics-based control method doesn’t seems to be studied enough and does not show practical designing methods of the robot system in contrast to the model-based control method. Therefore it is important to develop robotic designing methods which are based on the ideas of the dynamics-based control.

In this paper, firstly we analyze the behavior of the passive dynamic walking robot via analogy with Phase locked loop circuit which shows similar behaviors to the robot system, and ascertain the robot’s gaits bifurcation and shows the chaotic behavior on the same condition which causes chaotic behaviors of Phase locked loop circuit. Secondly we ascertain that we can get initial conditions and set-up parameters which cause the desired
gaits of the passive dynamic walking robot via this analogy. And at last, we propose a designing method of the passive dynamic walking robot system via this analogy, and show some examples of the method and its effectiveness.

2. Passive dynamic walking and Phase locked loop circuit

In this chapter, we describe outlines and behaviors of the passive dynamic walking and Phase locked loop circuit.

2.1 Passive Dynamic Walking robot model and its behavior

Fig. 2 shows the walking robot model which we consider. Let the support leg angle be \( \theta_p \), the swing non support leg angle be \( \theta_w \), the slope angle be a parameter \( \alpha \), the hip mass be \( M \), the leg mass and length be \( m \) and \( l \), the length from the hip to the mass center of the leg be \( r \).

Using the Euler-Lagrange approach, the dynamic equation of the robot can be derived as shown below;

\[
M(\theta)\ddot{\theta} + N(\theta, \dot{\theta})\dot{\theta} + g(\theta, \alpha) = u(t)
\]  

(1)

where \( \theta = [\theta_p, \theta_w] \), \( M(\theta) \) is the inertia matrix, \( N(\theta, \dot{\theta}) \) is the centrifugal and Coriolis term, \( g(\theta, \alpha) \) is the gravity term and \( u(t) \) is the torque vector supplied to the robot, and elements of the matrices and the vector are

Figure 2. A compass like biped walking robot model
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\[
M(\theta) = \begin{pmatrix}
ML^2 + mL^2 - 2Lr + r^2 & -mlr \cos(\theta_p - \theta_w) \\
-mlr \cos(\theta_p - \theta_w) & mr^2
\end{pmatrix},
\]

\[
N(\theta, \dot{\theta}) = \begin{pmatrix}
0 & -mlr \theta_p \sin(\theta_p - \theta_w) \\
mlr \theta_p \sin(\theta_p - \theta_w) & 0
\end{pmatrix},
\]

\[
g(\theta, \dot{\theta}) = \begin{pmatrix}
(-ML - 2ml + ml) g \sin(\theta_p + \alpha) \\
mlr g \sin(\theta_w + \alpha)
\end{pmatrix}.
\]

And as linearizing (1) about \([\theta_p, \theta_w, \dot{\theta}_p, \dot{\theta}_w] = [0, 0, 0, 0] \), the equation

\[
M_\theta \dot{\theta} + G_\theta \theta + b = u(t)
\]

can be given. And as applying \(x(t) = [\theta_p, \theta_w, \dot{\theta}_p, \dot{\theta}_w]^T\), the linearized dynamical equation (3)

can be written as below.

\[
\dot{x}(t) = Ax(t) + b + Bu(t)
\]

The matrices and vectors in (3) are described as

\[
A = \begin{pmatrix}
O_{2 \times 2} & I_{2 \times 2} \\
-M_0^{-1} G_0 & O_{2 \times 2}
\end{pmatrix},
\]

\[
b = \begin{pmatrix}
O_{2 \times 1} \\
-M_0^{-1} b_0
\end{pmatrix},
\]

\[
B = \begin{pmatrix}
O_{2 \times 2} \\
M_0^{-1}
\end{pmatrix},
\]

and the elements of these matrices are

\[
-M_0^{-1} G_0 = \begin{pmatrix}
\frac{(1 + 2\mu - \mu\eta)}{1 + \eta - 2\mu\eta + \mu^2} \omega & \frac{\mu}{1 + \eta - 2\mu\eta + \mu^2} \\
\frac{1 + \eta - 2\mu\eta + \mu^2}{1 + \eta - 2\mu\eta + \mu^2} \omega & \frac{1 + \eta - 2\mu\eta + \mu^2}{1 + \eta - 2\mu\eta + \mu^2} \omega
\end{pmatrix},
\]

\[
-M_0^{-1} b_0 = \begin{pmatrix}
\frac{(1 + \mu - \mu\eta)}{1 + \eta - 2\mu\eta + \mu^2} \omega \alpha \\
\frac{1 + \eta - 2\mu\eta + \mu^2}{1 + \eta - 2\mu\eta + \mu^2} \omega \alpha
\end{pmatrix},
\]

where the normalized parameters are supposed as

\[
\mu = \frac{m}{M}, \quad \eta = \frac{r}{L}, \quad \omega = \frac{g}{L}.
\]
Next, we assume that a transition of the support leg and the swing leg occurs instantaneously and the impact of the swing leg with the ground is inelastic without sliding. Then the transition equation at the collision of the swing leg with the ground can be derived as

\[
P_a(\beta)\dot{\beta}^- = P_s(\beta)\dot{\beta}^+
\]  

(4)

by using the angular momentum conservation conditions, where \( \dot{\beta}, \dot{\beta}^+ \) are the pre-impact and post-impact angular velocities. And using the following matrix

\[
T_r = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}, \quad T_LT_s^{-1}T_a = \begin{bmatrix}
r_1 & r_2 \\
r_3 & r_4
\end{bmatrix},
\]

the transition equation can be written as below,

\[
x^+(t) = R(x^-(t)) = \begin{bmatrix}
T_r & 0 \\
T_LT_s^{-1}T_a & 0
\end{bmatrix} x^-(t)
\]

(5)

and the elements of these matrices are

\[
\begin{align*}
    r_1 &= \frac{(1 + \mu - \mu \eta) \cos(2\beta)}{1 + 2\mu - 2\mu \eta + \mu \eta^2 - \mu \cos(2\beta)^2}, \\
    r_2 &= \frac{\mu \eta (-1 + \eta)}{1 + 2\mu - 2\mu \eta + \mu \eta^2 - \mu \cos(2\beta)^2}, \\
    r_3 &= \frac{(1 + 2\mu - 2\mu \eta + \mu \eta^2)(-1 + \eta)}{\eta(1 + 2\mu - 2\mu \eta + \mu \eta^2 - \mu \cos(2\beta)^2)^2}, \\
    r_4 &= \frac{\mu \eta (-1 + \eta) \cos(2\beta)}{1 + 2\mu - 2\mu \eta + \mu \eta^2 - \mu \cos(2\beta)^2}.
\end{align*}
\]

In this paper we use the non-linear model (1) for the simulation, and also use the linearized model (3) to analyze the stability of the robot system after the simulation. Then we show one simulative result of the passive dynamic walking by using the above-mentioned model in Fig.3. Parameters used in this simulation are \( M=10[\text{kg}], m=1[\text{kg}], l=0.3[\text{m}], r=0.15[\text{m}], \omega=0.035[\text{rad}], \) and the initial conditions used in this simulation are \( x(0) = [-0.26447, 0.26447, 1.57618, 0.87946]^T. \) As shown in Fig.3, when the appropriate initial conditions and operation parameters are given, the simulation model begins and continues to walk at a single periodic gait which is the leg angle at the collision with the slope.
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2.2 Phase Locked Loop circuit model and its behavior

Phase locked loop (PLL) circuit technology is applied for electrical circuits which need to synchronize two cyclic signals. The circuit is adopted to a lot of area such as radios, television sets, motor speed controllers, digital ICs …etc. Fig.4 shows a basic block diagram of the typical PLL circuit and (6) shows the basic equation of the circuit (Endo & Chua, 1988).

\[
\dot{\phi}(t) + \frac{1}{\tau_1} \left[ 1 + K_c \tau_2 \cos \phi(t) \right] \dot{\phi}(t) + \frac{K_v}{\tau_1} \sin \phi(t) = \frac{1}{\tau_1} \Delta \omega
\]  

PLL circuit has an inner oscillating circuit VCO, and the circuit generates an output signal which is according to the integral value of the phase difference between the input and the output signal. And PLL circuit also has the phase compensation filter \( F(s) \) and proportional gains \( K_P \) and \( K_V \) to minimize the phase difference. The circuit can synchronize the output signal \( V_{out}(t) \) to the input signal \( V_{in}(t) \) when the frequency deviation \( \Delta \omega \) is smaller than a certain value which is referred to as Pull-in range, and the behavior is also referred to as Pull-in behavior. And after synchronizing two signals, PLL circuit can keep synchronizing when \( \Delta \omega \) is less than a certain value which is referred to as Lock-range and the behavior is also referred to as Lock behavior. Generally the value of Lock-range is larger than the value of Pull-in range.

It is noteworthy to remember that once PLL circuit synchronizes the output signal to the input signal, the circuit keeps synchronizing although \( \Delta \omega \) gets larger by degree during \( \Delta \omega \) is less than Lock-range. Then the chaotic behavior such as the bifurcation of the phase \( \phi(t) \) generally appears when the frequency deviation \( \Delta \omega \) becomes larger by degrees in the area between Pull-in range and Lock-range. Fig.5 shows the relation of the two values.
2.3 Analogous behaviors between the passive dynamic walking and PLL circuit

As described in section 2.2, when PLL circuit is active and the value of the frequency deviation is between Pull-in range and Lock range, the phase difference of the circuit bifurcates. On the other hand, it is well-known that the gait of the passive dynamic walking robot also bifurcates according to the changes of the slope angle, mass,...etc. (Goswani et al, 1996) (Garcia et al, 1998). So firstly imitating the example of PLL circuit's Lock behavior and assuming the slope angle $\alpha$ as the frequency deviation, we try to change the slope angle by slow degree and continuously during the robot's walking as shown in Fig.6 (left), and analyze its behavior by the computer simulation (Iribe & Osuka, 2006-1). The parameters used in the simulation are $m=1$kg, $M=10$kg, $l=0.3$m, $r=0.15$m, and the initial value of the slope angle $\alpha_{\text{start}} = 0.035$ rad. Then secondly, we also assume the mass ratio $\mu$ as the frequency deviation and try to change the value just the same as slope angle as shown in Fig.6 (right) (Iribe & Osuka, 2006-3). The simulative results are shown in Fig.7 to Fig.10.
Fig. 7 and Fig. 8 show the relation between the gaits and the slope angle or the mass-ratio, and Fig. 9 and Fig. 10 show the actual trajectories of the changing gaits. The passive dynamic walking robot walks at the single periodic gait when the last slope angle $\alpha_{\text{end}}$ is less than 0.065 rad, and we can also find that the changes of the walking period are very small. Then the robot's gait bifurcates when the last slope angle $\alpha_{\text{end}}$ is between 0.065 and 0.09 rad, and then the robot falls down when the last slope angle $\alpha_{\text{end}}$ is larger than 0.09 rad. And same as the slope angle case, the robot keeps walking by the single periodic gait when the last mass-ratio value $\mu_{\text{end}}$ is between approximately 0.05 and 0.32 when the slope angle is 0.063 rad. Differently from the case of the slope angle's change, the change of the walking period is large. Then the robot's gait bifurcates when the last mass-ratio value $\mu_{\text{end}}$ is less than 0.05 and between 0.032 and 0.65. These behaviors which try to keep the robot walking are similar to the Lock behaviors of PLL circuits.

Figure 6. Experimental condition of the slope (left) and the mass-ratio (right) change

Figure 7. The relation between the slope angle and the gait
Every gaits can be changed by the change of the mass ratio continuously.

Figure 8. The relation between the mass-ratio and the gait

Furthermore, by changing the slope angle or the mass ratio continuously, we can get several initial-conditions (the sets of the vector $x(t)$) and set-up parameters (slope angles and mass ratios) which cause the desired gaits, large or small, bifurcated or non-bifurcated. The fact means that we can get set-up parameters and initial conditions for the desired gait easily if we have one initial condition for the passive dynamic walking at a single periodic gait. Fig.11 shows the concept of the fact.

Figure 11. The figure shows that the initial condition sets and the set-up parameters for any kind of the gaits are connected each other and continuously.
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Figure 9. Left side figures show the time-series changes of the gait according to the slope angle change, and right side figures show the time-series transitions of the leg angles \( \theta_s \) and \( \theta_p \). These figures describe that the gait changes continuously according to the change of the slope angle.
Figure 10. Left side figures show the time-series changes of the gait according to the mass-ratio change, and right side figures show the time-series transitions of the leg angles $\theta_s$ and $\theta_p$. These figures describe that the gait changes continuously according to the change of the mass-ratio.

(a) Time-series trajectories of the gait (left) and leg angles (right) when the mass-ratio changes from 0.1 to 0.25 rad.

(b) Time-series trajectories of the gait (left) and leg angles (right) when the mass-ratio changes from 0.1 to 0.55 rad.

(c) Time-series trajectories of the gait (left) and leg angles (right) when the mass-ratio changes from 0.1 rad to 0.65 rad.
3. A designing method of the passive dynamic walking robot inspired via the analogy with Phase locked loop circuit

In this chapter we propose a new designing method of the passive dynamic walking robot. The proposed method provides the procedure for fixing the physical parameters such as the length and the mass of the legs and hip, and for stabilizing the robot’s walking gaits.

3.1 Fixing the physical parameters of the PDW robot

From the results of the previous section, we can find that the gait of the passive dynamic walking robot bifurcates according to the increase of the slope angle $\alpha$ and the mass ratio $\mu$ which are referred to as the frequency deviation of PLL circuit. In this section firstly we try to analyze the relation between the parameters $\alpha$, $\mu$ and the initial conditions $x(0)$, and then we propose one procedure to fix the physical parameters such as $\alpha$ and $\mu$ via the analysis result.

The result of the analysis by computer simulations shows that the areas of the initial conditions $x(0)$ for any kind of the gait are fixed by the parameters $\alpha$ and $\mu$ as shown in Fig.12. According to the Fig.7 and Fig.8, the gaits become larger where $\mu$ is small and $\alpha$ is large, and the initial condition area is in the area of upper left of Fig.12. And the area where the parameters $\mu$ and $\alpha$ become larger, the walking gait starts to bifurcate.

![Figure 12. The diagram shows the relation among the parameters $\alpha$, $\mu$, and the kind of the gaits. The parameters in the blue square area cause the single periodic gait; the parameters in the magenta circle area cause the double periodic gait; the parameters in the red asterisk area causes quad periodic gait; the parameters in the green cross area causes multi periodic gait; and in the external area the robot can not keep walking](image-url)
Figure 13. The figure shows that the leg length becomes long as increasing the mass-ratio $\mu$. Suppose the mass density of the leg is constant, the changes of the mass (or the mass-ratio) cause the changes of the leg length of the robot in proportion to the rate of the mass (or the mass-ratio) change.

Figure 14. The diagrams show the simulative results of changing the mass-ratio $\mu$ during the passive dynamic walking.
So if we hope to realize the large and single periodic gait which means high speed and stable walking, we shall select the slope angle $\alpha$ which is larger than 0.06 rad and also select the mass-ratio $\mu$ which is less than 0.1 in the area of blue box-shaped points shown in Fig.12. And if we hope to realize larger gaits and don't mind the gaits' bifurcation, we can select the parameter sets in the higher right area which cause the bifurcated gait. Then we apply the properties which we ascertained by the analogy with the Lock behavior of PLL circuit to fix the physical parameters of the passive dynamic walking robot. Suppose that the mass density of the robot's legs is constant, the legs become longer in proportion to the increase of the mass. So the increase of the leg's mass means the increase of the leg length and the mass-ratio of the passive dynamic walking robot, and then we are able to fix the leg length and the mass of the robot. As shown in Fig.13, during the passive dynamic walking robot walking, we increase the mass-ratio $\mu$ to change the leg length of the robot. The simulative result is shown in Fig.14.

The third diagram shows the changes of the mass-ratio $\mu$ which we operate. The first diagram shows the transitions of the robot's shape while walking, the second diagram shows the changes of the gait, and the last diagram shows the changes of the walking period according to the changes of the mass-ratio $\mu$. These diagrams show that the robot's shape, mass, and walking state can be changed continuously and stably when the changing rate of the mass-ratio $\mu$ is well-chosen. And then the result shows that we can design the passive dynamic walking robot shown in Fig. 2 physically and can also get the initial condition for the changed robot's walking, when we have only one initial condition for the passive dynamic robot walking.

### 3.2 A stabilization method for the bifurcated gait

When we hope to make the passive dynamic walking robot walking fast, it is the easy way to increase the slope angle $\alpha$ from the simulative results shown in Fig.9. However the increase of the slope angle $\alpha$ causes the bifurcation of the gait as shown in Fig. 7 and Fig.12. The bifurcated gait causes several modes of the vibration at the leg collision with the ground, so it is important and valuable to prevent the gait bifurcation during the robot walking. And as described later, the bifurcated gait shows unstable state by the analysis via the linearized Poincare map (Sugimoto & Osuka, 2005). So the method of the stabilization for the bifurcated gait is needed.

On the other hand, in the case of designing PLL circuit, phase compensation filters which are equal to the derivative compensation are adopted to stabilize its system and to improve the transient response generally. So we try to improve the stability of the passive dynamic walking robot's behavior by the designing method inspired by PLL circuit. And so we apply the same idea as the velocity feedback like control method using the viscous friction of the hip joint (Iribe & Osuka, 2006-3). Practically the torque term caused by the viscous friction is applied to (1) when the transition of angle $\alpha$ and mass-ratio $\mu$ occurs. The parameter D is the viscous friction coefficient of the hip joint. The dynamical equation is shown as below.

$$ M(\theta) \ddot{\theta} + \left( N(\theta, \dot{\theta}) + D \right) \dot{\theta} + g(\theta, \alpha) = 0 $$

(7)

The simulative result which shows the effectiveness of the compensation method is shown in Fig.15. The slope angle $\alpha$ is fixed at 0.08rad and the value of D is increased by degree in
this simulation. As increasing the value of $D$, the absolute maximum eigen value of the Poincare map becomes less than 1.0, and the bifurcated gait converges.

Figure 15. The diagram shows the effectiveness of the viscous friction.

Then, in order to confirm the effectiveness of the method, we show the next simulative result in Fig. 16. Same as the procedure in Fig. 9, the slope angle $\alpha$ is increased from 0.06 to 0.08 rad during the robot walking. Then the slope angle begins to change, the parameter $D$ is increased from 0.0 to 0.008 Ns/rad. Fig. 16 shows the effectiveness of the compensation method.

From this simulative result the passive dynamic walking robot keeps single periodic gait with the proposed compensation method as the gait becomes larger. And analyzing the eigen values of the Poincare map of this system, the absolute maximum of the eigen value becomes less than 1.0 with this compensation method. This result shows the effectiveness of the method for designing the gait of the passive dynamic walking robot.
This method seems to be a kind of the Delayed Feedback Control (DFC) reported in (Pyragas, 1992), (Sugimoto & Osuka, 2004) which is the effective control method for the non-linear system which shows the chaotic behavior.

And in addition, here, we describe the linearized Poincare map which we use for analyzing the behavior of the passive dynamic walking. The linearized Poincare map $P_k$ around the $x^*$ is given by the following equation (Sugimoto & Osuka, 2005):

$$P_k = S \ e^{A_* \tau} \ R_d$$  \hspace{1cm} (8)

where

$$v_\tau = A \ x_\tau^- + b, \ S = I - \frac{v_\tau \ C}{C \ v_\tau}, \ R_d = \left. \frac{\partial R(x)}{\partial x} \right|_{x=x^-}$$  \hspace{1cm} (9)

and the details in (8) and (9) are as follows:

- $x_\tau$: perturbation value given by $x_\tau = x_\tau^* - x^*$
- $C$: geometric condition (jump condition) as $C = [1, 1, 0, 0]$
- $S$: state constraint at the collision
- $e^{A_* \tau}$: state transition from last collision to the next collision
- $R_d$: state transition at the collision

The matrix $A$ and vector $v'$ are described in (3). And as applying this equation we can analyze the gait’s stability of the passive dynamic walking robot.

### 4. Conclusion

In this paper we ascertained the analogous behavior between Phase locked loop circuit and the passive dynamic walking robot, and showed the effective property to get set-up parameters and initial conditions. Then we proposed one method to fix the physical parameters and conditions which cause the desired walking gaits, and at last we showed the compensation method which is effective to stabilize the bifurcated gait of the robot. The proposed method which is inspired by the analogy with Phase locked loop circuit is probably valuable for designing the passive dynamic walking robot before beginning the actual prototyping.

For the future work, we try to investigate the characteristic behavior of the passive dynamic walking robot which is similar to the Lock behavior of Phase locked loop circuit in detail. The behavior seems to be a kind of property of the limit cycle, so we may describe the behavior mathematically.

Then we try to increase the degree of freedom of the passive dynamic walking robot and also try to simulate if the same behavior appears or not when the degree of freedom is increased.

And then, the actual experiment by means of the real prototype is needed to verify the proposed method in this paper.
5. References


Nature has always been a source of inspiration and ideas for the robotics community. New solutions and technologies are required and hence this book is coming out to address and deal with the main challenges facing walking and climbing robots, and contributes with innovative solutions, designs, technologies and techniques. This book reports on the state of the art research and development findings and results. The content of the book has been structured into 5 technical research sections with total of 30 chapters written by well recognized researchers worldwide.

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