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Hybrid Energy Storage and Applications Based on High Power Pulse Transformer Charging

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1. Introduction

1.1. HES based on pulse transformer charging

In the fields of electrical discipline, power electronics and pulsed power technology, the common used modes of energy transferring and energy storage include mechanical energy storage (MES), chemical energy storage (CHES), capacitive energy storage (CES), inductive energy storage (IES) and the hybrid energy storage (HES) [1-3]. The MES and CHES are important ways for energy storage employed by people since the early times. The MES transfers mechanical energy to pulse electromagnetic energy, and the typical MES devices include the generator for electricity. The CHES devices, such as batteries, transfer the chemical energy to electrical energy. The energy storage modes aforementioned usually combine with each other to form an HES mode. In our daily life, the MES and CHES usually need the help of other modes to deliver or transfer energy to drive the terminal loads. As a result, CES, IES and HES become the most important common used energy storage modes for users. So, these three energy storage modes are analyzed in detail as the central topics in this chapter.

The CES is an energy storage mode employing capacitors to store electrical energy [3-5]. As Fig. 1(a) shows, \( C_0 \) is the energy storage component in CES, and the load of \( C_0 \) can be inductors, capacitors and resistors respectively. Define the permittivity of dielectric in capacitor \( C_0 \) as \( \varepsilon \), the electric field intensity of the stored electrical energy in \( C_0 \) as \( E \). The energy density \( W_E \) of CES is as

\[
W_E = \frac{1}{2} \varepsilon E^2 .
\]  

(1)

Usually, \( W_E \) which is restricted to \( \varepsilon \) and the breakdown electric field intensity of \( C_0 \) is about \( 10^4 \sim 10^5 \) J/m\(^3\). The traditional Marx generators are in the CES mode [4-5].
The IES is another energy storage mode using inductive coils to generate magnetic fields for energy storage. As shown in Fig. 1(b), the basic IES cell needs matched operations of the opening switch \((S_{\text{open}})\) and the closing switch \((S_{\text{close}})\) [6-7], while \(L_0\) is as the energy storage component. When the charging current of \(L_0\) reaches its peak, \(S_{\text{open}}\) becomes open and \(S_{\text{close}}\) becomes closed at the same time. As the instantaneously induced voltage on \(L_0\) grows fast, the previously stored magnetic energy in the magnetic field is delivered fast to the load through \(S_{\text{close}}\). The load of \(L_0\) also can respectively be inductors, capacitors and resistors. The explosive magnetic flux compression generator is a kind of typical IES device [7]. The coil winding of pulse transformer which has been used in Tokamak facility is another kind of important IES device [8]. Define the permeability of the medium inside the coil windings as \(\mu\), the magnetic induction intensity of the stored magnetic energy as \(B\). The energy density \(W_B\) of IES is as

\[
W_B = \frac{1}{2} \frac{B^2}{\mu} \quad \text{(2)}
\]

Usually, \(W_B\) restricted by \(\mu\) and \(B\) is about \(10^7 \frac{\text{J}}{\text{m}^3}\). IES has many advanced qualities such as high density of energy storage, compactness, light weight and small volume in contrast to CES. However, disadvantages of IES are also obvious, such as requirement of high power opening switches, low efficiency of energy transferring and disability of repetitive operations.

![Figure 1. Schematics of three kinds of common-used energy storage modes. (a) Capacitive energy storage mode; (b) Inductive energy storage mode; (c) Typical hybrid energy storage mode; (d) Hybrid energy storage based on pulse transformer.](image)

In many applications, CES combining with IES is adopted for energy storage as a mode of HES. Fig. 1(c) shows a typical HES mode based on CES and IES. Firstly, the energy source charges \(C_1\) in CES mode. Secondly, \(S_{\text{close}1}\) closes and the energy stored in \(C_1\) transfers to \(L_0\)
through the resonant circuit in IES mode. Thirdly, the previously closed switch $S_{\text{open}}$ opens, and $S_{\text{close2}}$ closes at the same time. The accumulated magnetic energy in $L_0$ transfers fast to capacitor $C_2$ in CES mode again. Finally, $S_{\text{close3}}$ closes and the energy stored in $C_2$ is delivered to the terminal load. So, in the HES mode shown in Fig. 1(c), the HES cell orderly operates in CES, IES and CES mode to obtain high power pulse energy. Furthermore, the often used HES mode based on CES and IES shown in Fig. 1(d) is a derivative from the mode in Fig. 1(c). In this HES mode, pulse transformer is employed and the transformer windings play as IES components. In Fig. 1(d), if $S_{\text{open}}$ and $S_{\text{close1}}$ operate in order, the HES cell also orderly operates in CES, IES and CES mode. Of course, switch $S_{\text{close1}}$ in Fig. 1(d) also can be ignored in many applications for simplification.

Generally speaking, a system can be called as HES module if two or more than two energy storage modes are included in the system. In this chapter, the centre topics just focus on CES, IES and the HES based on the CES and IES, as they have broad applications in our daily life. The CES and IES both have their own advantages and defects, but the HES mode based on these two achieves those individual advantages at the same time. In applications, a lot of facilities can be simplified as the HES module including two capacitors and a transformer shown in Fig. 2 [9-16]. Switch $S_1$ has ability of closing and opening at different time. This kind of HES module based on transformer charging can orderly operate in CES, IES and CES mode. And it has many improved features for application at the same time, such as high efficiency of energy transferring, high density of energy storage and compactness.

![Figure 2. Schematic of the common used hybrid energy storage mode based on capacitors and pulse transformer](image)

### 1.2. Applications of HES based on pulse transformer charging

The HES based on pulse transformer charging is an important technology for high-voltage boosting, high-power pulse compression, pulse modification, high-power pulse trigger, intense electron beam accelerator and plasma source. The HES cell has broad applications in the fields such as defense, industry, environmental protection, medical care, physics, cell biology and pulsed power technology.

The HES based on pulse transformer charging is an important way for high-power pulse compression. Fig. 3(a) shows a high-power pulse compression facility based on HES in
Nagaoka University of Technology in Japan [9], and its structure is shown in Fig. 3(b). The Blumlein pulse forming line plays as the load capacitor in the HES cell, and two magnetic switches respectively control the energy transferring. The pulse compression system can compress the low voltage pulse from millisecond range to form high voltage pulse at 50ns/480kV range.

![Figure 3](image1.png)

**Figure 3.** Typical high power pulse compressor with a transformer-based HES module. (a) The pulse compressor system; (b) the diagram and schematic of the pulse compressor system

The HES based on pulse transformer charging is an important way for high-power pulse trigger. Fig. 4(a) shows a solid state pulse trigger with semiconductor opening switches (SOS) in the Institute of Electrophysics Russian Academy of Science [10-11]. Fig. 4(b) presents the schematic of the pulse trigger, which shows a typical HES mode based on pulse transformer charging. SOS switch and IGBT are employed as the switches controlling energy transferring. The pulse trigger delivers high-voltage trigger pulse with pulse width at 70ns and voltage ranging from 20 to 80kV under the 100Hz repetition. And the average power delivered is about 50kW.

![Figure 4](image2.png)

**Figure 4.** Typical high-voltage narrow pulse trigger with the transformer-based HES module. (a) The pulse trigger with the SOS switches; (b) The schematic of the high-power pulse trigger system
The HES cell based on pulse transformer charging is also an important component in intense electron beam accelerator for high-power pulse electron beams which are used in the fields of high-power microwave, plasma, high-power laser and inertial fusion energy (IFE). Fig. 5(a) shows the “Sinus” type accelerator in Russia [12], and it also corresponds to the HES mode based on transformer charging shown in Fig. 2. The pulse transformer of the accelerator is Tesla transformer with opened magnetic core, while spark gap switch controls energy transferring. The accelerator has been used to drive microwave oscillator for high-power microwave. Fig. 5(b) presents a high-power KrF laser system in Naval Research Laboratory of the U. S. A., and the important energy storage components in the system just form an HES cell based on transformer charging [13-14]. The HES cell drives the diode for pulse electron beams to pump the laser, and the laser system delivers pulse laser with peak power at 5GW/100ns to the IFE facility.

Figure 5. Typical intense electron beam accelerator with the transformer-based HES module. (a) The pulse electron beam accelerator based on HES for high-power microwave application in Russia; (b) The pulse electron beam accelerator based on HES for high-power laser application in Naval Research Laboratory, the U. S. A.

The HES based on pulse transformer charging also have important applications in ultra-wideband (UWB) electromagnetic radiation and X-ray radiography. Fig. 6 shows an ultra-wideband pulse generator based HES mode in Loughborough University of the U. K. [15]. The air-core Tesla transformer charges the pulse forming line (PFL) up to 500kV, and spark gap switch controls the energy transferring form the PFL to antenna. The “RADAN” series pulse generators shown in Fig. 7 are portable repetitive high-power pulsers made in Russia for X-ray radiography [16]. The “RADAN” pulser which consists of Tesla transformer and PFL are also based on the HES mode shown in Fig. 2. The controlling switches are thyristors and spark gap.

Besides, the HES cell is also used in shockwave generator [17], dielectric barrier discharge [18], industrial exhaust processing [19], material surface treatment [20], ozone production [21], food sterilization [22], cell treatment and cell mutation [23].
2. Parametric analysis of pulse transformer with closed magnetic core in HES

Capacitor and inductor are basic energy storage components for CES and IES respectively, and pulse transformer charging is important to the HES mode shown in Fig. 2. So, it is essential to analyze the characteristic parameters of the common used high-power pulse transformer, and provide theoretical instructions for better understanding of the HES based on transformer charging.
There are many kinds of standards for categorizing the common used pulse transformers. From the perspective of magnetic core, pulse transformers can be divided into two types, such as the magnetic-core transformer [24-25] and the air-core transformer [26]. In view of the geometric structures of windings, the pulse transformer can be divided to many types, such as pulse transformer with closed magnetic core, solenoid-winding transformer, curled spiral strip transformer [26], the cone-winding Tesla transformer [16, 27], and so on. The transformer with magnetic core is preferred in many applications due to its advantages such as low leakage inductance, high coupling coefficient, high step-up ratio and high efficiency of energy transferring. Russian researchers produced a kind of Tesla transformer with cone-like windings and opened magnetic core, and the transformer with high coupling coefficient can deliver high voltage at MV range in repetitive operations [27]. Usually, pulse transformer with closed magnetic core, as shown in Fig.8, is the typical common used transformer which has larger coupling coefficient than that of Tesla transformer. The magnetic core can be made of ferrite, electrotechnical steel, iron-based amorphous alloy, nano-crystallization alloy, and so on. The magnetic core is also conductive so that the core needs to be enclosed by an insulated capsule to keep insulation from transformer windings.

Paper [28] presents a method for Calculation on leakage inductance and mutual inductance of pulse transformer. In this chapter, the common used pulse transformer with toroidal magnetic core will be analyzed in detail for theoretical reference. And a more convenient and simple method for analysis and calculation will be presented to provide better understanding of pulse transformer [24-25].

The typical geometric structure of pulse transformer with toroidal magnetic core is shown in Fig. 9(a). The transformer consists of closed magnetic core, insulated capsule of the core and transformer windings. The cross section of the core and capsule is shown in Fig. 9(b). Transformer windings are formed by high-voltage withstanding wires curling around the capsule, and turn numbers of the primary and secondary windings are $N_1$ and $N_2$. 

![Figure 8. Common used pulse transformers with closed toroidal magnetic cores](image-url)
respectively. Usually, transformer windings have a layout of only one layer of wires as shown in Fig. 9(a), which corresponds to a simple structure. In other words, this simple structure can be viewed as a single-layer solenoid with a circular symmetric axis in the azimuthal direction. The transformer usually immerses in the transformer oil for good heat sink and insulation.

Figure 9. Typical structure of the pulse transformer with a closed magnetic core and an insulated capsule. (a) Assembly structure of the pulse transformer; (b) Geometric structure of the cross section of the pulse transformer.

Define the geometric parameters in Fig. 9(b) as follows. The height, outer diameter and inner diameter of the closed magnetic core are defined as \( l_m, D_4 \) and \( D_3 \) respectively. The height, outer diameter and inner diameter of the insulated capsule are defined as \( l_0, D_2 \) and \( D_1 \) respectively. The thicknesses of the outer wall, inner wall and side wall of insulated capsule are defined as \( d_1, d_2 \) and \( d_5 \) in order. The distances between the side surfaces of capsule and magnetic core are \( d_3 \) and \( d_4 \) shown in Fig. 9(b). Define diameters of wires of the primary windings and secondary windings as \( d_p \) and \( d_s \) respectively. The intensively wound primary windings with \( N_1 \) turns have a width about \( N_1 d_p \).

2.1. Inductance analysis of pulse transformer windings with closed magnetic core

2.1.1. Calculation of magnetizing inductance

Define the permittivity and permeability of free space as \( \varepsilon_0 \) and \( \mu_0 \), relative permeability of magnetic core as \( \mu_r \), the saturated magnetic induction intensity of core as \( B_s \), residue magnetic induction intensity of core as \( B_r \), and the filling factor of magnetic core as \( K_f \). The cross section area \( S \) of the core is as

\[
S = \frac{(D_4 - D_3)l_m}{2}. \tag{3}
\]

Define the inner and outer circumferences of magnetic core as \( l_1 \) and \( l_2 \), then \( l_1 = \pi D_3 \) and \( l_2 = \pi D_4 \). The primary and secondary windings tightly curl around the insulated capsule in
separated areas in the azimuthal direction. In order to get high step-up ratio, the turn number $N_1$ of primary windings is usually small so that the single-layer layout of primary windings is in common use. Define the current flowing through the primary windings as $i_p$, the total magnetic flux in the magnetic core as $\Phi_0$, and the magnetizing inductance of transformer as $L_\mu$. According to Ampere’s circuital law,

$$\Phi_0 = l_p \mu_0 \mu_r N_1^2 S K_T \ln(l_2/l_1) / (l_2 - l_1)$$

(4)

As $\Phi_0 = L_\mu i_p$, $L_\mu$ is obtained as

$$L_\mu = \frac{\mu_0 \mu_r N_1^2 S K_T \ln(l_2/l_1)}{l_2 - l_1}.$$  

(5)

2.1.2. Leakage inductance of primary windings

The leakage inductances of primary and secondary windings also contribute to the total inductances of windings. The leakage inductance $L_{lp}$ of the primary windings is caused by the leakage magnetic flux outside the magnetic core. If $\mu_r$ of magnetic core is large enough, the solenoid approximation can be used. Through neglecting the leakage flux in the outside space of the primary windings, the leakage magnetic energy mainly exists in two volumes. As Fig.10 shows, the first volume defined as $V_1$ corresponds to the insulated capsule segment only between the primary windings and the magnetic core, and the second volume defined as $V_2$ is occupied by the primary winding wires themselves. The leakage magnetic field in the volume enclosed by transformer windings can be viewed in uniform distribution. The leakage magnetic energy stored in $V_1$ and $V_2$ are as $W_{m1}$ and $W_{m2}$, respectively.

Define the magnetic field intensity generated by $i_p$ from the $N_1$-turn primary windings as $H_p$ in $V_1$. According to Ampere’s circuital law, $H_p = i_p / d_p$. From Fig. 10,
When the magnetic core works in the linear district of its hysteresis loop, the magnetic energy $W_{m1}$ stored in $V_1$ is as

$$W_{m1} = \frac{\mu_0}{2} H_{p1}^2 V_1 = \frac{\mu_0}{2} \left( \frac{i_p}{d_p} \right)^2 V_1.$$  \hfill (7)

In $V_2$, the leakage magnetic field intensity defined as $H_{px}$ can be estimated as

$$H_{px} = \frac{i_p x}{d_p d_p}, \quad 0 \leq x \leq d_p.$$  \hfill (8)

From the geometric structure in Fig. 10, $V_2 = 2N_1d_p^2(\ell_0 + (D_2 - D_1) / 2)$), the leakage magnetic energy $W_{m2}$ stored in $V_2$ is as

$$W_{m2} = \frac{1}{2} \mu_0 \int_0^{d_p} d(V_2H_{px}^2) = \frac{\mu_0 V_2^2}{2} \frac{i_p^2}{3d_p^2}.$$  \hfill (9)

So, the total leakage magnetic energy $W_{mp}$ stored in $V_1$ and $V_2$ is presented as

$$W_{mp} = W_{m1} + W_{m2} = L_p i_p^2 / 2.$$  \hfill (10)

In (10), $L_p$ is the leakage inductance of the primary windings, and $L_p$ can be calculated as

$$L_p = \frac{\mu_0}{3d_p^2} (V_2 + 3V_1).$$  \hfill (11)

### 2.1.3. Leakage inductance of secondary windings

Usually, the simple and typical layout of the secondary windings of transformer is also the single layer structure as shown in Fig. 11(a). The windings are in single-layer layout both at the inner wall and outer wall of insulated capsule. As $D_2$ is much larger than $D_1$, the density of wires at the inner wall is larger than that at the outer wall. However, if the turn number $N_2$ becomes larger enough for higher step-up ratio, the inner wall of capsule can not provide enough space for the single-layer layout of wires while the outer wall still supports the previous layout, as shown in Fig. 11(b). We call this situation as “quasi-single-layer” layout. In the “quasi-single-layer” layout shown in Fig. 11 (b), the wires at the inner wall of capsule is in two-layer layout. After wire 2 curls in the inner layer, wire 3 curls in the outer layer next to wire 2, and wire 4 curls in the inner layer again next to wire 3, then wire 5 curls in the outer layer again next to wire 4, and so on. This kind of special layout has many
advantages, such as minor voltage between adjacent coil turns, uniform voltage distribution between two layers, good insulation property and smaller distributed capacitance of windings.

In this chapter, the single-layer layout and “quasi-single-layer” layout shown in Fig. 11 (a) and (b) respectively are both analyzed to provide reference for HES module. And the multi-layer layout [29] can also be analyzed by the way introduced in this chapter.

![Secondary windings structures of pulse transformer with closed magnetic core.](image)

Figure 11. Secondary windings structures of pulse transformer with closed magnetic core. (a) Single-layer distribution of the secondary windings of transformer; (b) Inter-wound “quasi-single-layer” distribution of the secondary windings

Define the current flowing through the secondary windings as $i_s$, the two volumes storing leakage magnetic energy as $V_a$ and $V_b$, the corresponding leakage magnetic energy as $W_{ma}$ and $W_{mb}$, the total leakage magnetic energy as $W_{ms}$, wire diameter of secondary windings as $d_s$, and the leakage inductance of secondary windings as $L_s$.

Firstly, the single-layer layout shown in Fig. 11 (a) is going to be analyzed. The analytical model is similar to the model analyzed in Fig. 10. If $(D_2-D_1)<<D_1$, the length of leakage magnetic pass enclosed by the secondary windings is as $l_{ms} = N_s d_s (D_2 + D_1)/2(D_1 - d_s)$. The leakage magnetic field intensity defined as $H_s$ in $V_a$ is presented as $H_s=\mu_0 N_s i_s/l_{ms}$. $V_a$ and $W_{ma}$ can be estimated as

$$V_a = \frac{N_s d_s}{4(D_1 - d_s)}[l_0(D_2^2 - D_1^2) - l_n \pi(D_4^2 - D_3^2)]$$

$$W_{ma} = \frac{\mu_0 N_s^2 i_s^2}{2 l_{ms}^2} V_a$$

In volume $V_b$ which is occupied by the secondary winding wires themselves, $W_{mb}$ can be estimated as

$$W_{mb} = \frac{1}{2} \mu_0 \int_0^{d_s} \left(\frac{H_s x}{d_s}\right)^2 l_{ms} (2l_0 + 4d_s + D_2 - D_1) dx = \frac{\mu_0 d_s N_s^2 i_s^2}{3 l_{ms}^2} [l_0 + 2d_s + (D_2 - D_1)/2].$$
In view of that \( W_{ms} = W_{ma} + W_{mb} = L_{ps} i_s^2 / 2 \), the leakage inductance of single-layer layout of the secondary windings is as

\[
L_{ls} = \frac{\mu_0 N_s^2 V_a}{l_{ms}^2} + \frac{2\mu_0 d_s N_s^2}{3l_{ms}} \left[ l_0 + 2d_s + (D_2 - D_1) / 2 \right] \tag{14}
\]

As to the “quasi-single-layer” layout shown in Fig. 11 (b), it also can be analyzed by calculating the leakage magnetic energy firstly. Under this condition, the length of leakage magnetic pass enclosed by the secondary windings is revised as \( l_{ms} \approx N_2 d_s (D_2 + D_1) / 4(D_1 - d_s) \). The leakage magnetic energy \( W_{ma} \) and \( W_{mb} \) can be estimated as

\[
W_{ma} = \frac{\mu_0 N_s^2 d_s}{2l_{ms}^2} \sum_i \left\{ \left( \frac{H_a}{d_s} \right)^2 \pi (D_2 + d_s)(l_0 + 2d_s) + \pi (D_1 - d_s)(l_0 + 2d_s) \right. \\
\left. + \pi (D_1 - 3d_s)(l_0 + 2d_s) + \pi (D_2^2 - D_1^2) / 2 \right\} \tag{15}
\]

\[
W_{mb} = \frac{\mu_0 \pi N_s^2 d_s}{6d_s^2} \left[ (l_0 + 2d_s)(D_2 + 2D_1 - 3d_s) + (D_2^2 - D_1^2) / 2 \right]
\]

Finally, the leakage inductance of the “quasi-single-layer” layout is obtained by the same way of (14) as

\[
L_{ls} = \frac{\mu_0 N_s^2}{l_{ms}^2} V_a + \frac{\mu_0 \pi N_s^2 d_s}{2d_s^2} \left[ (l_0 + 2d_s)(D_2 + 2D_1 - 3d_s) + (D_2^2 - D_1^2) / 2 \right] \tag{16}
\]

### 2.1.4. The winding inductances of pulse transformer

Define the total inductances of primary windings and secondary windings as \( L_1 \) and \( L_2 \) respectively, the mutual inductance of the primary and secondary windings as \( M \), and the effective coupling coefficient of transformer as \( K_{eff} \). From (5), (11), (14) or (16),

\[
\begin{align*}
L_1 &= L_{\mu} + L_{ps} \\
L_2 &= L_{\mu}(N_2 / N_1)^2 + L_{ss}
\end{align*} \tag{17}
\]

When \( \mu >> 1 \), \( M \) and \( K_{eff} \) are presented as

\[
\begin{align*}
M &= L_{\mu} N_2 / N_1 \\
K_{eff} &= \frac{M}{\sqrt{L_1 L_2}} = \sqrt{1 - \frac{L_{ps} + L_{ss}(N_1 / N_2)^2}{L_{\mu}}}
\end{align*} \tag{18}
\]
2.2. Distributed capacitance analysis of pulse transformer windings

The distributed capacitances of pulse transformer include the distributed capacitances to ground [30], capacitance between adjacent turns or layers of windings [29-32], and capacitance between the primary and secondary windings [32-33]. It is very difficult to accurately calculate every distributed capacitance. Even if we can do it, the results are not liable to be analyzed so that the referential value is discounted. Under some reasonable approximations, lumped capacitances can be used to substitute the corresponding distributed capacitances for simplification, and more useful and instructive results can be obtained [29]. Of course, the electromagnetic dispersion theory can be used to analyze the lumped inductance and lumped capacitance of the single-layer solenoid under different complicated boundary conditions [34-35]. In this section, an easier way is introduced to analyze and estimate the lumped capacitances of transformer windings.

2.2.1. Distributed capacitance analysis of single-layer transformer windings

In the single-layer layout of transformer windings shown in Fig. 11(a), the equivalent schematic of transformer with distributed capacitances is shown in Fig. 12. $C_{Dpi}$ is the distributed capacitance between two adjacent coil turns of primary windings, and $C_{Dsi}$ is the counterpart capacitance of the secondary windings. $C_{psi}$ is the distributed capacitance between primary and secondary windings. Common transformers have distributed capacitances to the ground, but this capacitive effect can be ignored if the distance between transformer and ground is large. If the primary windings and secondary windings are viewed as two totalities, the lumped parameters $C_{Dp}$, $C_{Ds}$, and $C_{Ps}$ can be used to substitute the “sum effects” of $C_{Dpi}$, $C_{Dsi}$, and $C_{psi}$ in order, respectively. And the lumped schematic of the pulse transformer is also shown in Fig. 12. $C_{ps}$ decreases when the distance between primary and secondary windings increases. In order to retain good insulation for high-power pulse transformer, this distance is usually large so that $C_{ps}$ also can be ignored. At last, only the lumped capacitances, such as $C_{Dp}$ and $C_{Ds}$, have strong effects on pulse transformer.

![Figure 12. The distributed capacitances of single-layer wire-wound pulse transformer and the equivalent schematic with lumped parameters](image-url)
In the single-layer layout shown in Fig. 11(a), define the lengths of single coil turn in primary and secondary windings as \( l_{s1} \) and \( l_{s2} \) respectively, the face-to-face areas between two adjacent coil turns in primary and secondary windings as \( S_{w1} \) and \( S_{w2} \) respectively, and the distances between two adjacent coil turns in primary and secondary windings as \( \Delta d_p \) and \( \Delta d_s \) respectively. According to the geometric structures shown in Fig. 10 and Fig. 11(a), 

\[
l_{s1} = 2l_0 + 4d_p + D_2 - D_1, \quad l_{s2} = 2l_0 + 4d_s + D_2 - D_1,
\]

\[
S_{w1} = d_pl_{s1} \quad \text{and} \quad S_{w2} = d_sl_{s2}.
\]

Because the coil windings distribute as a sector, \( \Delta d_p \) and \( \Delta d_s \) both increase when the distance from the centre point of sector increases in the radial direction. \( \Delta d_p \) and \( \Delta d_s \) can be estimated as

\[
\Delta d_p(r) = \frac{2N_d d_p r}{(N_1 - 1)(D_1 - d_p)}, \quad \Delta d_s(r) = \frac{2N_s d_s r}{(N_2 - 1)(D_1 - d_s)}, \quad \frac{D_1}{2} < r < \frac{D_2}{2},
\]

If the relative permittivity of the dielectric between adjacent coil turns is \( \varepsilon_r \), \( C_{Dpi} \) and \( C_{Dis} \) can be estimated as

\[
C_{Dpi} = \frac{D_2}{2} \int_0^{\frac{D_1}{2}} \frac{2N_d d_p}{r^2} d_r, \quad C_{Dis} = \frac{D_2}{2} \int_0^{\frac{D_1}{2}} \frac{2N_s d_s}{r^2} d_r.
\]

Actually, the whole long coil wire which forms the primary or secondary windings of transformer can be viewed as a totality. The distributed capacitances between adjacent turns are just formed by the front surface and the back surface of the wire totality itself. In view of that, lumped capacitances \( C_{Dp} \) and \( C_{Ds} \) can be used to describe the total distributed capacitive effect. As a result, \( C_{Dp} \) and \( C_{Ds} \) are calculated as

\[
C_{Dp} = \left\{ \begin{array}{l}
(N_1 - 1) l_{s1} \int_0^{\frac{D_1}{2}} \frac{2N_d d_p}{r^2} d_r = \frac{\varepsilon_0 \varepsilon_r l_{s1}(N_1 - 1)^2 (D_1 - d_p)^2}{2N_1 D_1 D_2} \\
(N_1 - 1) C_{Dpi} \\
\end{array} \right. \]

\[
C_{Ds} = \left\{ \begin{array}{l}
(N_2 - 1) l_{s2} \int_0^{\frac{D_1}{2}} \frac{2N_s d_s}{r^2} d_r = \frac{\varepsilon_0 \varepsilon_r l_{s2}(N_2 - 1)^2 (D_1 - d_s)^2}{2N_2 D_1 D_2} \\
(N_2 - 1) C_{Dis} \\
\end{array} \right. \]

From (21), \( C_{Dp} \) or \( C_{Ds} \) is proportional to the wire length \( l_{s1} \) or \( l_{s2} \), while larger turn number and smaller distance between adjacent coil turns both cause larger \( C_{Dp} \) or \( C_{Ds} \).

2.2.2. Distributed capacitance analysis of inter-wound “quasi-single-layer” windings

Usually, large turn number \( N_2 \) corresponds to the “quasi-single-layer” layout of wires shown in Fig. 13(a). In this situation, distributed capacitances between the two layers of
wires at the inner wall of capsule obviously exist. Of course, lumped capacitance $C_{ls}$ can be used to describe the capacitive effect when the two layers are viewed as two totalities, as shown in Fig. 13(b). Define $C_{Ds1}$ and $C_{Ds2}$ as the lumped capacitances between adjacent coil turns of these two totalities, and $C_{Ds}$ is the sum when $C_{Ds1}$ and $C_{Ds2}$ are in series. As a result, the lumped capacitances which have strong effects on pulse transformer are $C_{Ds}$, $C_{Ds1}$, $C_{Ds2}$ and $C_{ls}$.

Figure 13. The distributed capacitances of “quasi-single-layer” pulse transformer and the equivalent schematic with lumped parameters. (a) Double-layer inside distribution of the secondary windings; (b) Equivalent circuit of transformer with distributed capacitances and lumped parameters

If the coil turns are tightly wound, the average distance between two adjacent coil turns is $d_s$. The inner layer and outer layer at the inner wall of capsule have coil numbers as $1+N_2/2$ and $N_2/2-1$ respectively. According to the same way for (21), $C_{Ds1}$ and $C_{Ds2}$ are obtained as

$$
C_{Ds1} = \frac{\varepsilon_0 \varepsilon_r l_{s2} (N_2 / 2 + 1)^2 (D_1 - d_s)(D_2 - D_1)}{(N_2 / 2)D_1D_2} \\
C_{Ds2} = \frac{\varepsilon_0 \varepsilon_r (l_{s2} + 3d_s)(N_2 / 2 - 1)^2 (D_1 - d_s)(D_2 - D_1)}{(N_2 / 2)D_1D_2}
$$

(22)

The non-adjacent coil turns have large distance so that the capacitance effects are shielded by adjacent coil turns. In the azimuthal direction of the inner layer wires, small angle $d\theta$ corresponds to the azimuthal width of wires as $dl$ and distributed capacitance as $dC_{ls}$. Then, $dC_{ls} = \frac{\varepsilon_0 \varepsilon_r (D_2 - D_1 + 5d_s + 2l_0)}{2d_s} dl$. If the voltage between the $(n-1)$th and $n$th turn of coil ($n \leq N_2$) is $\Delta U_0$, the inter-wound method of the two layers aforementioned retains the voltage between two layers at about $2\Delta U_0$. So, the electrical energy $W_{ls}$ stored in $C_{ls}$ between the two layers is as

$$
W_{ls} = \frac{1}{2} \int_0^{2\Delta U_0} dC_{ls} = \Delta U_0^2 \varepsilon_0 \varepsilon_r N_2 (D_2 - D_1 + 5d_s + 2l_0) / 2.
$$

(23)
In view of that $W_{Ls} = C_{Ls}(2\Delta U_0)^2/2$, $C_{Ls}$ can be calculated as

$$C_{Ls} = \frac{\varepsilon_r \varepsilon_a (D_2 - D_1 + 5d_s + 2l_0)N_2}{4}.$$  \hspace{1cm} (24)

According to the equivalent lumped schematic in Fig. 13(b), the total lumped capacitance $C_{Ds}$ can be estimated as

$$C_{Ds} = \frac{1}{\frac{1}{C_{Dn1}} + \frac{1}{C_{Dn2}} + C_{Ls}}.$$  \hspace{1cm} (25)

### 2.3. Dynamic resistance of transformer windings

Parasitic resistance and junction resistance of transformer windings cause loss in HES cell. Define the resistivity of winding wires under room temperature ($20^\circ$C) as $\rho_0$, the work temperature as $T_w$, resistivity of winding wires under $T_w$ as $\rho(T_w)$, radius of the conductive section of wire as $r_w$, total wire length as $l_w$, and the static parasitic resistance of winding wires as $R_{w0}$. The empirical estimation for $R_{w0}$ is as

$$R_{w0} = \rho(T_w) \frac{l_w}{\pi r_w^2} = \rho_0 [1 + 0.004(T_w - 20)] \frac{l_w}{\pi r_w^2}.$$  \hspace{1cm} (26)

When the working frequency $f$ is high, the “skin effect” of current flowing through the wire cross-section becomes obvious, which has great effects on $R_{w0}$. Define the depth of “skin effect” as $\Delta d_w$, and the dynamic parasitic resistance of winding wires as $R_w(f, T_w)$. As $\Delta d_w = (\rho / \pi f \mu_0)^{0.5}$, $R_w(f, T_w)$ is presented as

$$R_w(f, T_w) = \begin{cases} \frac{l_w}{(2r_w - \rho(T_w)/\pi f \mu_0)^{1/2}} \frac{\pi}{\sqrt{f \mu_0 \rho(T_w)}}, & r_w > \Delta d_w \\ R_{w0} = \rho(T_w) \frac{l_w}{\pi r_w^2}, & r_w \leq \Delta d_w \end{cases}.$$  \hspace{1cm} (27)

### 3. Pulse response analysis of high power pulse transformer in HES

In HES cell based on pulse transformer charging, the high-frequency pulse response characteristics of transformer show great effects on the energy transferring and energy storage. Pulse response and frequency response of pulse transformer are very important issues. The distributed capacitances, leakage inductances and magnetizing inductance have great effects on the response pulse of transformer with closed magnetic core [36-39]. In this Section, important topics such as the frequency response and pulse response characteristics to square pulse, are discussed through analyzing the pulse transformer with closed magnetic core.
3.1. Frequency-response analysis of pulse transformer with closed magnetic core

The equivalent schematic of ideal pulse transformer circuit is shown in Fig. 14(a). \( L_{lp} \) and \( L_{ls} \) are the leakage inductances of primary and secondary windings of transformer calculated in (11), (14) and (16). Lumped capacitances \( C_{ps}, C_{DP} \) and \( C_{DS} \) represent the “total effect” of the distributed capacitances of transformer, while \( C_{DP} \) and \( C_{DS} \) are calculated in (21) and (25). \( L_{\mu} \) is the magnetizing inductance of pulse transformer calculated in (5). Define the sum of wire resistance of primary windings and the junction resistance in primary circuit as \( R_p \), the counterpart resistance in secondary circuit as \( R_s \), load resistance as \( R_L \), the equivalent loss resistance of magnetic core as \( R_c \), and the sinusoidal/square pulse source as \( U_1 \).

![Figure 14. Equivalent schematics of pulse transformer based on magnetic core with a square pulse source and a load resistor. (a) Equivalent schematic of pulse transformer with all the distributed parameters; (b) Simplified schematic of pulse transformer when the secondary circuit is equated into the primary circuit.](image)

Figure 14. Equivalent schematics of pulse transformer based on magnetic core with a square pulse source and a load resistor. (a) Equivalent schematic of pulse transformer with all the distributed parameters; (b) Simplified schematic of pulse transformer when the secondary circuit is equated into the primary circuit.

Usually, \( C_{ps} \) is so small that it can be ignored due to the enough insulation distance between the primary and secondary windings. In order to simplify the transformer circuit in Fig. 14(a), the parameters in the secondary circuit such as \( C_{Ds}, L_{ls}, R_s \) and \( R_L \) can be equated into the primary circuit as \( C_{Ds0}, L_{ls0}, R_{s0} \) and \( R_{L0} \), respectively. And the equating law is as

\[
\begin{align*}
L_{ls0} &= L_{ls} \left( \frac{N_1}{N_2} \right)^2, \\
C_{Ds0} &= C_{Ds} \left( \frac{N_2}{N_1} \right)^2, \\
R_{s0} &= R_s \left( \frac{N_1}{N_2} \right)^2, \\
R_{L0} &= R_L \left( \frac{N_1}{N_2} \right)^2.
\end{align*}
\tag{28}
\]

3.1.1. Low-frequency response characteristics

Define the frequency and angular frequency of the pulse source as \( f \) and \( \omega_0 \). When the transformer responds to low-frequency pulse signal (\( f<10^3 \) Hz), Fig. 14(b) can also be simplified. In Fig. 14(b), \( C_{DP} \) is in parallel with \( C_{Ds0} \) and the parallel combination capacitance of these two is about \( 10^{-6}-10^{-9}F \) so that the reactance can reach \( 10k\Omega-1M\Omega \). Meanwhile, the reactance of \( L_{\mu} \) is small. As a result, \( C_{DP} \) and \( C_{Ds0} \) can also be ignored. Reactances of \( L_{ls0} \) and
$L_p$ ($10^{-7}$H) are also small under the low-frequency condition, and they also can be ignored. Usually, the resistivity of magnetic core is much larger than common conductors to restrict eddy current. In view of that $R_{so} << R_{L0} << R_c$, the combination of $R_{so}$, $R_{L0}$ and $R_c$ can be substituted by $R_0$. Furthermore, $R_0 \approx R_{L0}$. Finally, the equivalent schematic of pulse transformer under low-frequency condition is shown in Fig.15(a).

![Figure 15](image)

**Figure 15.** Simplified schematic and analytical result of transformer for low-frequency pulse response. (a) Equivalent schematic of pulse transformer under the condition of low frequency; (b) Low-frequency response results of an example of transformer.

In Fig. 15(a), $L_\mu$ and $R_0$ are in parallel, and then in series with $R_P$ which is at m$\Omega$ range. $R_0$ is usually very small due to the equating process from (28). When $\omega_0$ of the pulse source increases, reactance of $L_\mu$ also increases so that $\omega_0 L_\mu >> R_0$. In this case, the $L_\mu$ branch gets close to opening, and an ideal voltage divider is formed only consisting of $R_P$ and $R_0$. At last, the pulse source $U_1$ is delivered to the load $R_0$ without any deformations. And the response voltage pulse signal $U_2$ of transformer on the load resistor is as

$$U_2 = U_1 \frac{R_0}{R_0 + R_P}.$$  \hspace{1cm} (29)  

When $R_P << R_0$, $U_1 = U_2$ which means the source voltage completely transfers to the load resistor. On the other hand, if $\omega_0 L_\mu << R_0$, $L_\mu$ shares the current from the pulse source so that the current flowing through $R_0$ gets close to 0. In this situation, the pulse transformer is not able to respond to the low-frequency pulse signal $U_1$.

An example is provided as follows to demonstrate the analysis above. In many measurements, coaxial cables and oscilloscope are used, and the corresponding terminal impedance is about $R_L=50\Omega$. So, the $R_0$ may be at m$\Omega$ range when it is equated to the primary circuit. Select conditions as follows: $R_p=0.09\Omega$, $L_\mu=12.6\mu$H, and $U_1$ is the periodical sinusoidal voltage pulse with amplitude at 1V. The low-frequency response curve of pulse transformer is obtained from Pspice simulation on frequency scanning, as Fig. 15(b) shows. When $f$ of $U_1$ is larger than the second inflexion frequency (100Hz), response signal $U_2$ is large and stable. However, when $f$ is less than the first inflexion frequency (10Hz), response signal $U_2$ gets close to 0. And the cut-off frequency $f_L$ is about 10Hz.
The conclusion is that low-frequency response capability of pulse transformer is mainly determined by $L_{\mu}$, and the response capability can be improved through increasing $L_{\mu}$ calculated in (5).

### 3.1.2. High-frequency response characteristics

When the transformer responds to high-frequency pulse signal ($f > 10^6$ Hz), conditions “$\omega_0L_{\mu} >> R_0$” and “$\omega_0L_{\mu} >> R_p$” are satisfied so that the branch of $L_{\mu}$ seems open. In Fig. 14(b), the combination effect of $R_0$, $L_{0}$ and $R_{L}$ still can be substituted by $R_0$. Substitute $L_p$ and $L_{s0}$ by $L_{l}$ and combine $C_{D0}$ and $C_{Dp}$ as $C_{D}$. The simplified schematic of pulse transformer for high-frequency response is shown in Fig. 16(a).

In Fig. 16(a), when $\omega_0$ of pulse source increases, reactance of $L_{l}$ increases while reactance of $C_{D}$ decreases. If $\omega_0$ is large enough, $\omega_0L_{l} >> R_0 >> 1/(\omega_0C_{D})$ and the response signal $U_2$ gets close to 0. On the other hand, condition $1/(\omega_0C_{D}) \geq R_0$ is satisfied when $\omega_0$ decreases. The pulse current mainly flows through the load resistor $R_0$, and the good response of transformer is obtained. Especially, when $\omega_0L_{l} << R_p$, $L_{l}$ also can be ignored. Under this situation, $R_p$ is in series with $R_0$ again, and the response signal $U_2$ which corresponds to the best response still conforms to (29).

Select the amplitude of the periodical pulse signal $U_1$ at 1V. If $R_p$, $L_{l}$ and $C_{D}$ are at ranges of m$\Omega$, 0.1$\mu$H and pF respectively, the high-frequency response curve of transformer is also obtained as shown in Fig.16(b) from Pspice simulation. When $f$ is less than the first inflexion frequency (about 300kHz), response signal $U_2$ is stable. When $f$ is larger than the second inflexion frequency (about 10MHz), response signal $U_2$ gets close to 0. And the cut-off frequency $f_H$ is about 10MHz.

![Figure 16](image)

**Figure 16.** Simplified schematic and analytical result of transformer for high-frequency pulse response. (a) Equivalent schematic of pulse transformer under the condition of high frequency; (b) High-frequency response results of an example of transformer

The conclusion is that high-frequency response characteristics of transformer are mainly determined by distributed capacitance $C_{D}$ and leakage inductance $L_{l}$. The high-frequency
response characteristics can be obviously improved through restricting $C_D$ and $L_L$, or increasing $L_{\mu}$.

### 3.2. Square pulse response of pulse transformer with closed magnetic core

In Fig. 14(b), $R_{s0} \ll R_{L0} \ll R_c$, and the combination effect of $R_{s0}$, $R_{L0}$ and $R_c$ can be substituted by $R_0$. Combine $C_{D0}$ with $C_{Dp}$ as $C_D$. The simplified schematic of pulse transformer circuit for square pulse response is shown in Fig. 17. $U_1$ and $U_2$ represent the square voltage pulse source and the response voltage signal on the load respectively. The total current from the pulse source is $i(t)$, while the branch currents flowing through $R_0$, $C_D$ and $L_{\mu}$ are as $i_1$, $i_2$ and $i_3$ respectively.

![Image of Figure 17](image)

**Figure 17.** Equivalent schematic of transformer for square pulse response

#### 3.2.1. Response to the front edge of square pulse

Usually, $L_{\mu}$ ranges from $10^{-6} \text{H}$ up to more than $10^{-5} \text{H}$, and the square pulse has front edge and back edge both at 100ns~1µs range. So, when the fast front edge and back edge of square pulse appear, reactance of $L_{\mu}$ is much larger than the equated load resistor $R_0$. Under this condition, $i_3$ is so small that the effect of $L_{\mu}$ on the front edge response can be ignored.

Define the voltage of $C_D$ as $U_c(t)$. As aforementioned, $L_{\mu}$ has little effect on the response to the front edge of square pulse. Through Ignoring the $L_{\mu}$ branch, the circuit equations are presented in (30) with initial conditions as $i(0)=0$, $i_1(0)=0$ and $U_c(0)=0$.

\[
\begin{align*}
U_1(t) &= i(t)R_p + L_{lp}di(t) / dt + L_{l0}di_1(t) / dt + i_1(t)R_0 \\
L_{l0}di_1(t) / dt + i_1(t)R_0 &= \int i_2(t)dt / C_D \\
i(t) &= i_1(t) + i_2(t)
\end{align*}
\]  \( (30) \)

If the factor for Laplace transformation is as $p$, the transformed forms of $U_1(t)$ and $i(t)$ are defined as $U_1(p)$ and $I(p)$. Firstly, four constants such as $\alpha$, $\beta$, $\gamma$ and $\lambda$ are defined as
Define the amplitude and pulse duration of square voltage pulse source as $U_s$ and $T_0$ respectively. $U_1(t)$ is as

$$U_1(t) = \begin{cases} 0, & t < 0 \text{ or } t \geq T_0, \\ U_s, & 0 \leq t < T_0. \end{cases}$$

Equations (30) can be solved by Laplace transformation and convolution, and there are three states of solutions such as the over damping state, the critical damping state and the under dumping state. In the transformer circuit, the resistors are always small so that the under dumping state usually appears. Actually, the under dumping state is the most important state which corresponds to the practice. In this section, the centre topic focuses on the under dumping state of the circuit.

Define constants $a$, $b$, $\omega$, $\xi$ ($a, b < 0; \omega > 0$), $A_1$, $A_2$ and $A_3$ as (33).

$$A_1 = \frac{1}{(a-b)^2 + \omega^2}, \quad A_2 = \frac{-1}{(a-b)^2 + \omega^2}, \quad A_3 = \frac{2b-a}{(a-b)^2 + \omega^2}.$$

$$a = \frac{1}{\xi^3 - \xi^{-3}} (3\beta - \alpha^2) / 9 - \alpha / 3$$

$$b = \frac{1}{\xi^3} (3\beta - \alpha^2) / 18 - \alpha / 3 - \xi^{-3} / 2$$

$$\omega = \sqrt{5[(3\beta - \alpha^2)\xi^3 / 9 + \xi^{-3}]] / 2}$$

$$\xi = \sqrt{\beta^2 + \alpha^2 \gamma - \alpha^2 \beta^2 / 27 - \alpha \beta \gamma / 108 - \alpha^2 \beta^2 / 6 + \gamma^2 / 4 - 3 \gamma - \alpha \beta - \alpha^3 / 27}$$

The under dumping state solution of (30) is as

$$U_2(t) = \begin{cases} 0, & t < 0, \\ \lambda R_0 U_s[A_1 \exp(at) + (A_2 b + A_3) \sin(\omega t) / \omega \exp(bt)], & 0 < t \leq T_0, \\ \lambda R_0 A_1 \exp(at) + (A_2 b + A_3) \sin(\omega t - T_0) / \omega \exp(bt) - \lambda R_0 A_1 \exp(at - T_0) + (A_2 b + A_3) \sin(\omega t - T_0) / \omega \exp(bt - T_0)], & T_0 < t. \end{cases}$$

The load current $i(t) = U_2(t) / R_0$. From (34), response voltage pulse $U_2(t)$ on load consists of an exponential damping term and a resonant damping term. The resonant damping term which has main effects on the front edge of pulse contributes to the high-frequency resonance at the front edge. Constant $a$ defined in (33) is the damping factor of the pulse.
droop of square pulse $U_2(t)$, $b$ is the damping factor of the resonant damping term, and $\omega$ is the resonant angular frequency. Substitute $\lambda R_0 U_s$ by $U_0$, and define two functions $f_1(t)$ and $f_2(t)$ as

$$f_1(t) = \begin{cases} 
0, & t \leq 0 \\
U_0 [A_2 \cos(\omega t) + (A_2 b + A_3) \sin(\omega t) / \omega] \exp(bt), & 0 < t < T_0 \\
U_0 [A_2 \cos(\omega t) + (A_2 b + A_3) \sin(\omega t) / \omega] \exp(bt) - U_0 [A_2 \cos(\omega t - T_0) + (A_2 b + A_3) \sin(\omega t - T_0) / \omega] \exp[b(t - T_0)], & t > T_0 
\end{cases}$$  \hspace{1cm} (35)

$$f_2(t) = U_0 [A_2 \cos(\omega t) + (A_2 b + A_3) \sin(\omega t) / \omega]$$

$f_1(t)$ is just the resonant damping term divided from (34), while $f_2(t)$ is the pure resonant signal divided from $f_1(t)$. If pulse width $T_0=5\mu$s, the three signals $U_2(t)$, $f_1(t)$ and $f_2(t)$ are plotted as Curve 1, Curve 2 and Curve 3 in Fig. 18 respectively. In the abscissa of Fig. 18, the section when $t < 0$ corresponds to the period before the time when the square pulse appears. Obviously, Curve 1 - Curve 3 all have high-frequency resonances with the same angular $\omega$. The resonances of Curve 1 and Curve 2 at the front edge are in superposition. Under the under dumping state of the circuit, the rise time $t_r$ of the response signal is about half of a resonant period as (36).

$$t_r = \pi / \omega.$$  \hspace{1cm} (36)

From (33) and (36), the rise time of the response signal $U_2(t)$ is determined by the parasitic inductance, leakage inductances ($L_{lp}$ and $L_{ls0}$) and distributed capacitance $C_D$. The rise time $t_r$ of the front edge can be minimized through increasing the resonant angular frequency $\omega$. In the essence, the high-frequency “L-C-R” resonance is generated by the leakage inductances and distributed capacitance in the circuit.

Figure 18. Typical pulse response waveforms of pulse transformer to the front edge of square pulse
The conclusion is that the rise time of the front edge of response pulse can be improved by minimizing the capacitance \( C_D \) and leakage inductance \( L_{lp} \) and \( L_{ls0} \) of the transformer. The waveform of the response voltage signal can be improved through increasing the damping resistor of the circuit in a proper range.

### 3.2.2. Pulse droop analysis of transformer response

In Fig. 17, when the front edge of pulse is over, \( U_c(t) \) of \( C_D \) and the currents flowing through \( L_{lp} \) and \( L_{ls0} \) all become stable. And these parameters have little effects on the response to the flat top of square pulse. During this period, load voltage signal \( U_2(t) \) is mainly determined by \( L_\mu \). So, the simplified schematic from Fig. 17 is shown as Fig. 19 (a). The circuit equations are as

\[
\begin{align*}
U_1(t) &= i(t)R_p + i_3(t)R_0 \\
U_2(t) &= i_1(t)R_0 = L_\mu \frac{di_3(t)}{dt} \\
i(t) &= i_1(t) + i_3(t)
\end{align*}
\]  

(37)

The initial conditions are as \( i_3(0)=0 \) and \( U_2(0)=R_0U_s/(R_0+R_p) \). The load voltage \( U_2(t) \) is obtained as (38) through solving equations in (37).

\[
U_2(t) = \frac{R_0U_s}{R_p+R_0} \exp\left(-\frac{t}{\tau}\right), \quad \tau = \frac{L_\mu (R_p+R_0)}{R_pR_0}, \quad 0 < t < T_0.
\]  

(38)

In (38), \( \tau \) is the constant time factor of the pulse droop. When \( L_\mu \) increases which leads to an increment of \( \tau \), the pulse droop effect is weakened and the pulse top becomes flat. If \( U_{20}=R_0U_s/(R_p+R_0) \), the response signal to the flat top of square pulse is shown in Fig. 19(b). When pulse duration \( T_0 \) is short at \( \mu \)s range, the pulse droop effect \((0<t<T_0)\) of \( U_2(t) \) is not obvious at all. However, when \( T_0 \) ranges from 0.1ms to several milliseconds, time factor \( \tau \) has great effect on the flat top of \( U_2(t) \), and the pulse droop effect of the response signal is so obvious that \( U_2(t) \) becomes an triangular wave.

**Figure 19.** Schematic and response pulse of transformer to the flat-top of square pulse. (a) Equivalent schematic of transformer for flat top response of square pulse; (b) The pulse droop of the response pulse of transformer
3.2.3. Response to the back edge of square pulse

When the flat top of square pulse is over, all the reactive components in Fig. 17 have stored certain amount of electrical or magnetic energy. Though the main pulse of the response signal is over, the stored energy starts to deliver to the load through the circuit. As a result, high-frequency resonance is generated again which has a few differences from the resonance at the front edge of pulse. In Fig. 17, $U_1$ and $R_p$ have no effects on the pulse tail response when the main pulse is over. $C_D$ which was charged plays as the voltage source. Combine $L_{sp}$ and $L_{so}$ as $L_s$. The equivalent schematic for pulse tail response of transformer is shown in Fig. 20.

![Figure 20. Equivalent schematic for back edge response of transformer to square pulse](image)

The circuit equations are presented in (39) with initial condition as $U_C(0)=U_0$.

\[
\begin{aligned}
U_{c0} - \int i(t) dt / C_D &= L_i di(t) / dt + i_1(t) R_0 \\
U_2(t) &= i_1(t) R_0 = L_\mu di_3(t) / dt \\
i(t) &= i_1(t) + i_3(t)
\end{aligned}
\]  
(39)

$i_1(0)$ and $i_3(0)$ are determined by the final state of the pulse droop period. There are also three kinds of solutions, however the under dumping solution usually corresponds to the real practices. So, this situation is analyzed as the centre topic in this section. Define six constants $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2$ and $\gamma_2$ as (40).

\[
\begin{aligned}
\alpha_1 &= \frac{R_0}{L_\mu} + \frac{R_0}{L_i}, \quad \beta_1 = \frac{1}{L_i C_D}, \quad \gamma_1 = \frac{R_0}{L_i L_\mu C_D} \\
\alpha_2 &= R_0[i(0)-i_2(0)], \quad \beta_2 = \frac{R_0 U_0}{L_i}, \quad \gamma_2 = -\frac{R_0 i_3(0)}{L_i C_D}
\end{aligned}
\]  
(40)

The under dumping solution of (39) is calculated as
In (41), $B_1$, $B_2$ and $B_3$ are three coefficients while $a_1$, $b_1$, $\omega_s$ and $\xi_1$ are another four constants as

$$\begin{align*}
B_1 &= \frac{\alpha_2 a_1^2 + \beta_2 a_1 + \gamma_2}{(a_1 - b_1)^2 + \omega_s^2}, \\
B_2 &= \alpha_2 - \frac{\alpha_2 a_1^2 + \beta_2 a_1 + \gamma_2}{(a_1 - b_1)^2 + \omega_s^2}, \\
B_3 &= \frac{\alpha_2 a_1^2 + \beta_2 a_1 + \gamma_2}{a_1[(a_1 - b_1)^2 + \omega_s^2]} - \frac{\gamma_2}{a_1}.
\end{align*}$$

(42)

The responses to front edge and back edge of square pulse have differences in essence, as the exciting sources are different. Define functions $f_3(t)$ and $f_4(t)$ as (43), according to (41).

$$\begin{align*}
U_2(t) &= B_1 \exp(a_1 t) + \exp(b_1 t)[B_2 \cos(\omega_s t) + B_3 \frac{b_1 + B_3}{\omega_s} \sin(\omega_s t)].
\end{align*}$$

(41)

According to (40) and (42), $R_0$ has effects on the damping factors of $f_3(t)$ and $f_4(t)$. The resonant frequency is mainly determined by leakage inductance, magnetizing inductance and distributed capacitance of transformer.

Fig. 21(b) shows an impression of the effect of $L_{\mu}$ on the tail of response signal. When $L_{\mu}$ changes from 0.1µH to 1mH while other parameters retain the same, the resonant
waveforms with the same frequency do not have large changes. So, the conclusion is that, $t_d$ and the resonant angular frequency $\omega_s$ are not mainly determined by $L_\mu$. Fig. 21(c) shows the effect of leakage inductances of transformer on the pulse tail of response signal. When $L_l$ is small at 10nH range, the back edge of pulse (Curve 2) is good as which of standard square pulse. When $L_l$ increases from 0.01µH to 1µH range, the resonances become fierce with large amplitudes. If $L_l$ increases to 10µH range, the previous under damping mode has a transition close to the critical damping mode (Curve 4). The fall time $t_d$ of the pulse tail increases obviously. Fig. 21(d) shows the effect of distributed capacitances of transformer on the pulse tail of response signal. The effect of $C_D$ obeys similar laws obtained from $L_\mu$. So, the conclusion is that the pulse tail of the response signal can be improved by a large extent through minimizing the leakage inductances and distributed capacitances of transformer windings. Paper [24] demonstrated the analysis above in experiments.

**Figure 21.** Typical back tail response signals of pulse transformer to the square pulse.
(a) The typical back edge response signals to square pulse in theory; (b) Effects of magnetizing inductance on the back edge response of transformer; (b) Effects of leakage inductance on the back edge response of transformer; (c) Effects of distributed capacitance on the back edge response of transformer;
4. Analysis of energy transferring in HES based on pulse transformer charging

As an important IES component, the pulse transformer is analyzed and the pulse response characteristics are also discussed in detail. The analytical theory aforementioned is the base for HES analysis based on pulse transformer charging in this section. In Fig. 2, the HES module based on capacitors and transformer operates in three courses, such as the CES course, the IES course and the CES course. Actually, the IES course and the latter CES course occur almost at the same time. The pulse transformer plays a role on energy transferring. There are many kinds of options for the controlling switch ($S_1$) of $C_1$, such as mechanical switch, vacuum trigger switch, spark gap switch, thyristor, IBGT, thyatron, photo-conductive switch, and so on. $S_1$ has double functions including opening and closing. $S_1$ ensures the single direction of HES energy transferring, from $C_1$ and transformer to $C_2$. In this section, the energy transferring characteristics of HES mode based on transformer charging is analyzed in detail.

The pulse signals in the HES module are resonant signals. According to the analyses from Fig. 15 and Fig. 16, the common used pulse transformer shown in Fig. 9(a) has good frequency response capability in the band ranging from several hundred Hz to several MHz. Moreover, $C_1$ and $C_2$ in HES module are far larger than the distributed capacitances of pulse transformer. So, the distributed capacitances can be ignored in HES cell. In the practical HES module, many other parameters should be considered, such as the junction inductance, parasitic inductance of wires, parasitic inductance of switch, parasitic resistance of wires, parasitic resistance of switch, and so on. These parameters can be concluded into two types as the parasitic inductance and parasitic resistance. As a result, the equivalent schematic of the HES module is shown in Fig. 22(a).

![Figure 22](image)

Figure 22. The basic hybrid energy storage (HES) system based on a source capacitor, a pulse transformer and a load capacitor. (a) Typical schematic of the transformer-based HES module; (b) Simplified schematic when the secondary circuit is equated into the primary circuit.

In Fig. 22(a), $C_1$ and $C_2$ represent the primary energy-storage capacitor and load capacitor respectively. $L_{p_1}$ and $L_{s_1}$ represent the parasitic inductances in the primary circuit and secondary circuit, while $R_p$ and $R_s$ stand for the parasitic resistances in the primary circuit and secondary circuit respectively. $L_1$, $L_2$ and $M$ of transformer are defined in (17) and (18). $i_p(t)$ and $i_s(t)$ represent the current in the primary and secondary circuit. The pulse...
The transformer with closed magnetic core has the largest effective coupling coefficient (close to 1) in contrast to Tesla transformer and air-core transformer. Under the condition of large coupling coefficient, the transformer in Fig. 22(a) can be decomposed as Fig. 22(b) shows. $L_\mu$, $L_{lp}$ and $L_{ls}$ are defined in (5), (11) and (14), respectively. Define the turns ratio of transformer as $n_s=(N_2/N_1)$. $C_2$, $L_{ls}$, $R_s$ and $i_s$ in the secondary circuit also can be equated into the primary circuit as $C_2$, $L_{ls}$, $R_s$ and $i_s$. The equating law are as

$$
\begin{align*}
C_2 & = C_2 n_s^2, \\
L_{ls} & = L_{ls} / n_s^2, \\
R_s & = R_s / n_s^2, \\
i_s & = i_s n_s.
\end{align*}
$$

The initial voltage of $C_1$ and $C_2$ are as $U_0$ and $0$ respectively.

The voltages of $C_1$ and $C_2$ are $U_{c1}(t)$ and $U_{c2}(t)$, respectively. According to Fig. 22(a), the circuit equations of HES module are as

$$
\begin{align*}
U_0 - \int i_p(t) \frac{dt}{C_1} &= R_p i_p(t) + (L_{\mu} + L_{lp} + L_{pl}) \frac{di_p(t)}{dt} - M \frac{di_s(t)}{dt}, \\
M \frac{di_s(t)}{dt} &= (L_{\mu} n_s^2 + L_{ls} + L_{sl}) \frac{di_s(t)}{dt} + R_s i_s(t) + \int i_s(t) \frac{dt}{C_2}.
\end{align*}
$$

(45)

In view of Fig. 22(b), the circuit equations of HES module can also be established as

$$
\begin{align*}
(L_{\mu} + L_{lp} + L_{pl}) \frac{d^2 i_p}{dt^2} + R_p \frac{di_p}{dt} + \frac{i_p}{C_1} &= L_{\mu} \frac{d^2 i_s}{dt^2}, \\
(L_{\mu} + L_{ls} + L_{sl}) \frac{d^2 i_s}{dt^2} + R_s \frac{di_s}{dt} + \frac{i_s}{C_2} &= L_{\mu} \frac{d^2 i_p}{dt^2}.
\end{align*}
$$

(46)

The initial conditions are as $i_p(0)=0$, $i_s(0)=0$, $U_{c1}(0)=U_0$ and $U_{c2}(0)=0$. In view of that $i_p(t)=-C_1 \frac{dU_{c1}(t)}{dt}$ and $i_s(t)=-C_2 \frac{dU_{c2}(t)}{dt}$, Equations in (45) can be simplified as

$$
\begin{align*}
\frac{d^2 U_{c1}(t)}{dt^2} + 2\alpha_p \frac{dU_{c1}(t)}{dt} + \omega_p^2 U_{c1} - k_p \frac{d^2 U_{c2}(t)}{dt^2} &= 0, \\
\frac{d^2 U_{c2}(t)}{dt^2} + 2\alpha_s \frac{dU_{c2}(t)}{dt} + \omega_s^2 U_{c2} - k_s \frac{d^2 U_{c1}(t)}{dt^2} &= 0.
\end{align*}
$$

(47)

In (47), $\omega_p$ and $\omega_s$ are defined as the resonant angular frequencies in primary and secondary circuits, while $k_p$ and $k_s$ are defined as the coupling coefficients of the primary and secondary circuits respectively. These parameters are presented as

$$
\begin{align*}
\omega_p^2 &= 1/[(L_1 + L_{pl})C_1], \\
\omega_s^2 &= 1/[(L_2 + L_{sl})C_2], \\
\alpha_p &= R_p / 2(L_1 + L_{pl}), \\
\alpha_s &= R_s / [2C_2(L_2 + L_{sl})], \\
k_p &= MC_2 / C_1(L_1 + L_{pl}), \\
k_s &= MC_1 / C_2(L_2 + L_{sl}).
\end{align*}
$$

(48)

Define the effective coupling coefficient of the HES module based on transformer charging as $k$, and the quality factors of the primary and secondary circuits as $Q_1$ and $Q_2$ respectively. $k$, $Q_1$ and $Q_2$ are presented as
Equations (47) have general forms of solution as $U_{c1}(t) = D_1 e^{x t}$ and $U_{c2}(t) = D_2 e^{x t}$. Through substituting the general solutions into (47), linear algebra equations of the coefficients $D_1$ and $D_2$ are obtained. The characteristic equation of the linear algebra equations obtained is calculated as

$$ (1 - k^2) x^4 + 2(\alpha_p + \alpha_s) x^3 + (\alpha_p^2 + 4\alpha_p \alpha_s + \alpha_s^2) x^2 + 2(\alpha_p \omega_p^2 + \alpha_s \omega_s^2) x + \omega_p^2 \omega_s^2 = 0. \quad (50) $$

$x$ in the characteristic equation (50) represents the characteristic solution. As a result, $x$, $D_1$ and $D_2$ should be calculated before the calculations of $U_{c1}(t)$ and $U_{c2}(t)$. Obviously, the characteristic solution $x$ can be obtained through the solution formula of algebra equation (50), but $x$ will be too complicated to provided any useful information. In order to reveal the characteristics of the HES module in a more informative way, two methods are introduced to solve the characteristic equation (50) in this section.

### 4.1. The lossless method

The first method employs lossless approximation. That’s to say, the parasitic resistances in the HES module are so small that they can be ignored. So, the HES module has no loss. Actually in many practices, the “no loss” approximation is reasonable. As a result, equation (50) can be simplified as

$$ (1 - k^2) x^4 + (\alpha_p^2 + \alpha_s^2) x^2 + \omega_p^2 \omega_s^2 = 0. \quad (51) $$

In (51), it is easy to get the two independent characteristic solutions defined as $x_{\pm}$. $U_{c1}(t) = D_{1e^{x_{\pm} t}}$ and $U_{c2}(t) = D_{2e^{x_{\pm} t}}$ can also be calculated combining with the initial circuit conditions. Finally, the most important four characteristic parameters such as $U_{c1}(t)$, $U_{c2}(t)$, $i_p(t)$ and $i_s(t)$, are all obtained as

$$ \left\{ \begin{array}{l}
U_{c1}(t) = \frac{(1+T)L_{\mu} - L_{\Sigma}}{(1+T)^2 L_{\mu} - TL_{\Sigma}} U_0 \left[ \frac{(1+T)L_{\mu}}{(1+T)L_{\mu} - L_{\Sigma}} \cos(\omega_s t) + T \cos(\omega_s t) \right] \\
U_{c2}(t) = \frac{(1+T)L_{\mu} - L_{\Sigma}}{(1+T)^2 L_{\mu} - TL_{\Sigma}} \sqrt{\frac{L_1 + L_{pl}}{L_2 + L_{sl}}} C_1 U_0 \left[ \cos(\omega_s t) - \cos(\omega_s t) \right]
\end{array} \right. \quad (52)
$$

$$ \left\{ \begin{array}{l}
i_p(t) = \frac{(1+T)L_{\mu} - L_{\Sigma}}{(1+T)^2 L_{\mu} - TL_{\Sigma}} C_1 U_0 \left[ \frac{(1+T)L_{\mu}}{(1+T)L_{\mu} - L_{\Sigma}} \sin(\omega_s t) + T \omega_s \sin(\omega_s t) \right] \\
i_s(t) = \frac{(1+T)L_{\mu} - L_{\Sigma}}{(1+T)^2 L_{\mu} - TL_{\Sigma}} \sqrt{\frac{L_1 + L_{pl}}{L_2 + L_{sl}}} \frac{C_1 U_0}{k} \left[ \omega_s \sin(\omega_s t) - \omega_s \sin(\omega_s t) \right]
\end{array} \right. \quad (52)
In (52), \( L \Sigma \) represents the sum of the parasitic inductances and leakage inductances, while \( \omega^+ \) and \( \omega^- \) stand for the two resonant angular frequencies existing in the HES module (\( \omega^+ > \omega^- \)). Parameters such as \( T \), \( L \Sigma \), \( \omega^+ \), and \( \omega^- \) are as follows:

\[
\begin{align*}
T & = \frac{\omega^+_2 + \omega^-_2}{\omega^+_p}, \quad L \Sigma = L_{pl} + L_{lp} + (L_{ls} + L_{ld}) / n_s^2 \\
\omega^+_p & = \frac{1 + T}{L \Sigma C_1}, \quad \omega^- = \frac{T}{(1 + T)L \mu C_1}.
\end{align*}
\]

(53)

In (52), the voltages of energy storage capacitors have phase displacements in contrast to the currents. All of the voltage and current functions have two resonant angular frequencies as \( \omega^+ \) and \( \omega^- \) at the same time, which demonstrates that the HES module based on transformer with closed magnetic core is a kind of double resonant module. The input and output characteristics and the energy transferring are all determined by (52).

### 4.2. The “little disturbance” method

The “little disturbance” method was introduced to analyze the Tesla transformer with open core by S. D. Korovin in the Institute of High-Current Electronics (IHCE), Tomsk, Russia. Tesla transformer with open core has a different energy storage mode in contrast to the transformer with closed magnetic core. Tesla transformer mainly stores magnetic energy in the air gaps of the open core, while transformer with closed core stores magnetic energy in the magnetic core. So, the calculations for parameters of these two kinds of transformer are also different. However, the idea of “little disturbance” is still a useful reference for pulse transformer with closed core [24-25]. So, the “little disturbance” method is introduced to analyze the pulse transformer with closed magnetic core for HES module.

The “little disturbance” method employs two little disturbance functions \( \Delta x_s \) to rectify the characteristic equation (50) or (51). That’s to say, the previous characteristic solutions \( x_s \) are substituted by \( x_s + \Delta x_s \). In HES module, the parasitic resistances which cause the energy loss still exist, though they are very small. So, the parasitic resistances also should be considered. Define \( j \) as unit of imaginary number, and variable \( x_j \) as \( -jx / \omega s \). Equation (50) can be simplified as

\[
(x_j^2 - \frac{j}{1 - \frac{1}{Q_1}} x_j - \frac{1}{Q_2}) x_j^2 - \frac{j}{Q_2} x_j - 1 = k^2 x_j^4.
\]

(54)

Through substituting \( x_j \) by \( x_s + \Delta x_s \) in (54), the characteristic equation of \( \Delta x_s \) can be obtained. If the altitude variables are ignored, the solutions of the characteristic equation of \( \Delta x_s \) are presented as
\[
\begin{aligned}
\dot{x}_+ &= \left(\frac{(1+T)L}{2\pi L}\right)^{\frac{1}{2}}, \quad \dot{x}_- = \left(\frac{1}{1+T}\right)^{\frac{1}{2}}, \\
\Delta x_\pm &= \frac{1}{2} \left( x_\pm^2 - 1 \right) + \frac{1}{Q_2} \left( x_\pm^2 - 1 \right). 
\end{aligned}
\] (55)

The solutions of (50) are as
\[
x = j\omega_0 \omega_s = j(x_+ \Delta x_\pm \omega_s). 
\]

\[
\Delta x_\pm = \frac{j \alpha^2 Q_1}{2x_\pm^2 (1-k^2) - 1 - \frac{1}{\alpha}} 
\] (55)

Define two effective quality factors of the double resonant circuit of HES module as
\[
Q_{\text{eff}}^+ = \frac{\omega_+}{2|\Delta \omega_+|}, \quad Q_{\text{eff}}^- = \frac{\omega_-}{2|\Delta \omega_-|}. 
\] (57)

\[
\begin{aligned}
U_{c1}(t) &= G_1 e^{-\beta_+ t} \left[ \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^+} + G_2 e^{-\beta_- t} \left[ \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^-} \right] \right] \\
U_{c2}(t) &= G_3 e^{-\beta_- t} \left[ \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^-} - \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^+} \right] \\
i_p(t) &= -C_1 \left[ G_1 e^{-\beta_+ t} \left[ -\beta_+ \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^+} \right] + \omega_+ \left( \frac{\cos(\omega_s t)}{2Q_{\text{eff}}^+} - \sin(\omega_s t) \right) \right] + \\
G_2 e^{-\beta_- t} \left[ -\beta_- \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^-} + \omega_- \left( \frac{\cos(\omega_s t)}{2Q_{\text{eff}}^-} - \sin(\omega_s t) \right) \right] \\
i_s(t) &= -C_2 G_3 \left[ -\beta_+ e^{-\beta_+ t} \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^+} - \omega_+ e^{-\beta_+ t} \left( \frac{\cos(\omega_s t)}{2Q_{\text{eff}}^+} - \sin(\omega_s t) \right) \right] + \\
\beta_+ e^{-\beta_+ t} \cos(\omega_s t) + \frac{\sin(\omega_s t)}{2Q_{\text{eff}}^+} - \omega_+ e^{-\beta_+ t} \left( \frac{\cos(\omega_s t)}{2Q_{\text{eff}}^+} - \sin(\omega_s t) \right) \right]. 
\end{aligned}
\] (58)

In (57), \(\rho_1\) represents the characteristic impedance of the resonant circuit, and \(\rho_1 = \sqrt{L_1(1+T)/C_1}\). According to (55), the general solutions of (49) \((U_{c1}(t)=D_1e^{\alpha_1 t}\) and \(U_{c2}(t)=D_2e^{\alpha_2 t})\) are clarified. When the initial circuit conditions are considered, the important four characteristic parameters such as \(U_{c1}(t), U_{c2}(t), i_p(t)\) and \(i_s(t)\) are obtained as (58). In (58), \(\beta = |\Delta \omega_+| = |\Delta \omega_-| \omega_0\), coefficients such as \(G_1, G_2\) and \(G_3\) are defined as
\[
G_1 = \frac{x_+^2(x_+^2 - 1)}{x_+^2 - x_-^2} U_0, \quad G_2 = \frac{x_+^2 x_T}{x_+^2 - x_-^2} U_0, \quad G_3 = \frac{x_+^2 x_T}{x_+^2 - x_-^2} \sqrt{\frac{L_1 + L_{pl}}{L_2 + L_{sl}}} C_1 U_0. 
\] (59)
From (58), all of the voltage and current functions have two resonant frequencies. In many situations of practice, the terms in (58) which include \( \cos(\omega t) \) and \( \sin(\omega t) \) can be ignored, as \( \omega_+ \gg \omega_- \) and \( \beta_+ \gg \beta_- \). The resonant currents \( i_p(t) \) and \( i_s(t) \) in primary and secondary circuit are almost in synchronization as shown in Fig. 23, and their resonant phases are almost the same. The first extremum point of \( U_{c2}(t) \) defined as \( (t_m, U_{c2}(t_m)) \) corresponds to the maximum charge voltage and peak charge time of \( C_2 \). Of course, \( t_m \) also corresponds to the time of minimum voltage on \( C_1 \). That’s to say, \( t_m \) is a critical time point which corresponds to maximum energy transferring. As \( i_s(t) = -C_2 dU_{c2}(t)/dt \), \( i_s(t) \) gets close to 0 when \( t = t_m \). If \( \omega_+ \gg \omega_- \), the maximum charge voltage and peak charge time of \( C_2 \) are calculated as

\[
\begin{align*}
  t_m &= \frac{\pi}{\omega_+} = \pi \left( \frac{L_2 C_1}{1 + T} \right)^{\frac{1}{2}} \\
  U_{c2}(t_m) &= -G \left( 1 + \exp(-\frac{\pi}{2Q_{off}}) \right)
\end{align*}
\]

(60)

Obviously, when the switch of \( C_1 \) in Fig. 2 opens while the switch of \( C_2 \) closes both at \( t_m \), the energy stored in \( C_2 \) reaches its maximum, and the energy delivered to the terminal load also reaches the maximum. This situation corresponds to the largest efficiency of energy transferring of the HES module. Of course, if the switch in Fig. 22(a) is closed all the time, the HES module acts in line with the law shown in (58). The energy stored in \( C_1 \) is transferred to transformer and capacitor \( C_2 \), then the energy is recycled from \( C_2 \) and transformer to \( C_1 \) excluding the loss, and then the aforementioned courses operate repetitively. Finally, all of the energy stored in \( C_1 \) becomes loss energy on the parasitic resistors, and the resonances in the HES module die down.

\[\text{Figure 23. Typical theoretical waveforms of the output parameters of HES module based on pulse transformer charging, according to the “little disturbance” method}\]
Under the condition $\omega_+ \gg \omega_-$, the peak time and the peak current of $i_p(t)$ are calculated as

$$
\begin{align*}
t_{m1} & \approx \frac{\pi}{2\omega_+} = \frac{\pi}{2} \left( \frac{L_2 C_1}{1 + T} \right)^{1/2}, \\
i_p(t_{m1}) & = G_i C_i \omega_+ (1 + \frac{1}{4Q_{eff+}}) \exp\left(-\frac{\pi}{4Q_{eff+}}\right).
\end{align*}
$$

(61)

Usually, semiconductor switch such as thyristor or IGBT is used as the controlling switch of $C_1$. However, these switches are sensitive to the parameters of the circuit such as the peak current, peak voltage, and the raising ratios of current and voltage. The raising ratio of $U_{c1}(t)$ and $i_p(t)$ ($dU_{c1}(t)/dt$ and $i_p(t)/dt$) can also be calculated from (58), which provides theoretical instructions for option of semiconductor switch in the HES module.

Actually, the efficiency of energy transferring is also determined by the charge time of $C_2$ in practice. Define the charge time of $C_2$ as $t_c$, the maximum efficiency of energy transferring on $C_2$ as $\eta_a$, and the efficiency of energy transferring in practice as $\eta_e$. If the core loss of transformer is very small, the efficiencies of HES module based on pulse transformer charging are as

$$
\eta_a = \frac{1}{2} \frac{C_2 U_0^2(t_{m})}{C_2 U_0^2} = \frac{C_1}{k^2 C_2} \frac{L_1 + L_{pl}}{L_2 + L_{sl}} \frac{(1 + T)L_{\mu} - L_{\Sigma_{2}}}{(1 + T)^2 L_{\mu} - TL_{\Sigma_{2}}} \leq \eta_a.
$$

(62)

Actually, $t_c$ corresponds to the time when $S_2$ closes in Fig. 2.

5. Magnetic saturation of pulse transformer and loss analysis of HES

5.1. Magnetic saturation of pulse transformer with closed magnetic core

Transformers with magnetic core share a communal problem of magnetic saturation of core. The pulse transformer with closed magnetic core consists of the primary windings ($N_1$ turns) and the secondary windings ($N_2$ turns), and it works in accordance with the hysteresis loop shown in Fig.24. Define the induced voltage of primary windings of transformer as $U_p(t)$, and the primary current as $i_p(t)$. If the input voltage $U_p(t)$ increases, the magnetizing current in primary windings also increases, leading to an increment of the magnetic induction intensity $B$ generated by $i_p(t)$. When $B$ increases to the level of the saturation magnetic induction intensity $B_s$, dB/dH at the working point $(H_0, B_0)$ decreases to 0 and the relative permeability $\mu_r$ of magnetic core decreases to 1. Under this condition, magnetic characteristics of the core deteriorate and magnetic saturation occurs. Once the magnetic saturation occurs, the transformer is not able to transfer voltage and energy. So, it’s an important issue for a stable transformer to improve the saturation characteristics of magnetic core and keep the input voltage $U_p(t)$ at a high level simultaneously.
The total magnetic flux in the magnetic core is $\Phi_0$ defined in (4). According to Faraday’s law, $U_p(t) = \frac{d\Phi_0}{dt}$. Define the allowed maximum increment of $B$ in the hysteresis loop as $\Delta B_{\text{max}}$, and the corresponding maximum increment of $\Phi_0$ as $\Delta \Phi$. Obviously, $\Delta B_{\text{max}} = B_s - (-B_r)$ and $\Delta \Phi = N_1 \Delta B_{\text{max}} S K_T$, while parameters such as $B_r$, $S$ and $K_T$ are defined before (3). So, the relation between $S$ and the voltage second product of core is presented as

$$S = \int_0^t U_p(t) dt = \int_0^t U_s(t) dt = \frac{N_1 \Delta B_{\text{max}}}{N_2 \Delta B_{\text{max}} K_T}.$$

(63)

As parameters such as $\Delta B_{\text{max}}$, $N_1$, $N_2$, $S$ and $K_T$ are unchangeable and definite in an already produced transformer, the charge time $t_c$ defined in (62) can not be long at random. Otherwise, $\int_0^t U_p(t) dt > N_1 \Delta B_{\text{max}} K_T S$, the core saturates and the transformer is not able to transfer energy. That’s to say, (63) just corresponds to the allowed maximum charge time without saturation. If the allowed maximum charge time is defined as $t_s$, $\int_0^{t_s} U_p(t) dt = N_1 \Delta B_{\text{max}} K_T S$.

According to (63), some methods are obtained to avoid saturation of core as follows. Firstly, $\Delta B_{\text{max}}$ and $K_T$ of the magnetic material should be as large as possible. Secondly, the cross-
section area of core should be large enough. Thirdly, the turn number of transformer windings \((N_1)\) should be enhanced. Fourthly, the charge time \(t_c\) of transformer should be restricted effectively. Lastly, the input voltage \(U_p(t)\) of transformer should decrease to a proper range.

Generally speaking, it is quite difficult to increase \(\Delta B_{\text{max}}\) and \(K_T\). The increment of \(N_1\) leads to decrement of the step-up ratio of transformer. And the decrement of \(U_p(t)\) leads to low voltage output from the secondary windings. As a result, the common used methods to avoid saturation of core include the increasing of \(S\) and decreasing the charge time \(t_c\) through proper circuit designing. Finally, the minimum cross-section design \((S_{\text{min}})\) of magnetic core in transformer should follow the instruction as shown in (64).

\[
S \geq S_{\text{min}} = \frac{\int_0^t U_{p}(t)dt}{N_1\Delta B_{\text{max}}K_T} \geq \frac{\int_0^t U_{s}(t)dt}{N_1\Delta B_{\text{max}}K_T} = \frac{\int_0^t U_{s}(t)dt}{N_2\Delta B_{\text{max}}K_T}. \tag{64}
\]

In (64), \(U_p(t)\) and \(U_s(t)\) can be substituted by \(U_{c1}(t)\) and \(U_{c2}(t)\) calculated in (52) or (58). Moreover, small air gaps can be introduced in the cross section of magnetic core to improve the saturation characteristics, which has some common features with the Tesla transformer with opened magnetic core. Reference [40] explained the air-gap method which is at the costs of increasing leakage inductances and decreasing the coupling coefficient.

### 5.2. Loss analysis of HES

The loss is a very important issue to estimate the quality of the energy transferring module. In Fig. 22(a), the main losses in the HES module based on pulse transformer charging include the resistive loss and the loss of magnetic core of transformer. The resistive loss in HES module consists of loss of wire resistance, loss of parasitic resistance of components, loss of switch and loss of leakage conductance of capacitor. Energy of resistive loss corresponds to heat in the components. The wire resistance is estimated in (27), and the switch resistance and leakage conductance of capacitor are provided by the manufacturers. According to the currents calculated in (58), the total resistive loss defined as \(\Delta W_R\) can be estimated conveniently. In this section, the centre topic focuses on the loss of magnetic core of transformer as follows.

#### 5.2.1. Hysteresis loss analysis

In the microscope of the magnetic material, the electrons in the molecules and atoms spin themselves and revolve around the nucleuses at the same time. These two types of movements cause magnetic effects of the material. Every molecule corresponds to its own magnetic dipole, and the magnetic dipole equates to a dipole generated by a hypothetic molecule current. When no external magnetic field exists, large quantities of magnetic dipoles of molecule current are in random distribution. However, when external magnetic
field exists, the external magnetic field has strong effect on these magnetic dipoles in random distribution, and the dipoles turn to the same direction along the direction of external magnetic field. The course is called as magnetizing, in which a macroscopical magnetic dipole of the material is formed. Obviously, magnetizing course of the core consumes energy which comes from capacitor $C_1$ in Fig. 2, and this part of energy corresponds to the hysteresis loss of core defined as $W_{\text{loss1}}$.

Define the electric field intensity, electric displacement vector, magnetic field intensity and magnetic induction intensity in the magnetic core as $\vec{E}$, $\vec{D}$, $\vec{H}$ and $\vec{B}$ respectively. The total energy density of electromagnetic field $W = \int \left( \vec{E} \cdot \partial \vec{D} / \partial t + \vec{H} \cdot \partial \vec{B} / \partial t \right) dt$. As the energy density of electric field is the same as which of magnetic field, the total energy density $W$ in isotropic material can be simplified as

$$W = \left( \vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B} \right) / 2 = \vec{H} \cdot \vec{B} \quad \text{or} \quad dW = H dB. \quad (65)$$

The magnetizing current which corresponds to $W_{\text{loss1}}$ is a small part of the total current $i_p(t)$ in primary windings. Define the magnetizing current as $I_m(t)$, the average length of magnetic pass as $<l_c>$, and the total volume of magnetic core as $V_m$. According to the Ampere’s circuital law and Faraday’s law,

$$H = N_1 I_m(t) / <l_c>, \quad dB = -U_p(t) dt / N_1 S K_t. \quad (66)$$

According to (65) and (66), the hysteresis loss of magnetic core of transformer is obtained as

$$W_{\text{loss1}} = \int_0^t \left( \frac{|U_p(t)| |I_m(t)| V_m}{<l_c> SK_t} \right) dt \quad (67)$$

In some approximate calculations, the loss energy density is equivalent to the area enclosed by the hysteresis loop. If the coercive force of the loop is $H_c$, $W_{\text{loss1}} \approx 2H_c B_s V_m$.

### 5.2.2. Eddy current loss analysis

When transformer works under high-frequency conditions, the high-frequency current in transformer windings induces eddy current in the cross section of magnetic core. Define the eddy current vector as $\vec{j}$, magnetic induction intensity of eddy current as $\vec{B}$, magnetic field intensity of eddy current as $\vec{H}$, magnetic induction intensity of $i_p(t)$ as $\vec{B_0}$, and magnetic field intensity of $i_p(t)$ as $\vec{H_0}$. As shown in Fig. 25, the direction of $\vec{j}$ is just inverse to the direction of $i_p(t)$, so the eddy current field $\vec{B}$ weakens the effect of $\vec{B_0}$. The eddy current heats the core and causes loss of transformer, and it should be eliminated by the largest extent when possible.

In order to avoid eddy current loss, the magnetic core is constructed by piled sheets in the cross section as Fig. 25 shows. Usually, the sheet is covered with a thin layer of insulation.
material to prevent eddy current. However, the high-frequency eddy current has “skin effect”, and the depth of “skin effect” defined as $\delta$ is usually smaller than the thickness $h$ of the sheet. As a result, the eddy current still exists in the cross section of core. Cartesian coordinates are established in the cross section of core as shown in Fig. 25, and the unit vectors are as $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$. To a thin sheet, its length ($\vec{e}_z$) and width ($\vec{e}_y$) are both much larger than the thickness $h$ ($\vec{e}_x$). So, approximation of infinite large dimensions of sheet in $\vec{e}_y$ and $\vec{e}_z$ directions is reasonable. That’s to say, $\partial / \partial y = 0$ and $\partial / \partial z = 0$. The “little disturbance” theory aforementioned before still can be employed to calculate the field $\vec{B}$ generated by eddy current $\vec{j}'$.

![Figure 25. Distribution of eddy current in the cross section of toroidal magnetic core](image)

The total magnetic induction intensity in the core is as $\vec{B} = \vec{B}_0 + \vec{B}' = (\vec{B}_0 - \vec{B})\vec{e}_z$, and $\vec{B}' \ll \vec{B}_0$. $\vec{B}'$ generated by eddy current can be viewed as the variable of “little disturbance”. According to Maxwell equations,
Define the conductivity of the sheet in magnetic core as \( \sigma \). From the second equation in (68), \((-\partial H_z / \partial x) \vec{e}_y = \vec{j} = \sigma \vec{E} \). It demonstrates that infinitesimal conductivity is the key factor to prevent eddy current. When working frequency is \( f \), the depth of “skin effect” of the sheet is calculated as \( \delta = (\pi f \mu \sigma)^{-1/2} \). According to (69), the “little disturbance” field of eddy current in isotropic magnetic material is presented as

\[
\vec{H}(x) = \frac{\sigma x^2}{2} \frac{\partial B_0}{\partial t} \vec{e}_z, \quad \vec{B}(x) = \frac{\mu_0 \mu_r \sigma x^2}{2} \frac{\partial B_0}{\partial t} \vec{e}_z, \quad \left(-\frac{\delta}{2} \leq x \leq \frac{\delta}{2}\right).
\]

Through averaging the field along the thickness direction \( (\vec{e}_z) \) of sheet,

\[
\vec{H} = \frac{\sigma h^2}{24} \frac{\partial B_0}{\partial t}, \quad \vec{B} = \frac{1}{12 \omega} \frac{h^2}{\delta} \frac{\partial B_0}{\partial t}.
\]

As the electric energy and magnetic energy of the eddy current field are almost the same, the eddy current loss defined as \( W_{\text{loss2}} \) is calculated as

\[
W_{\text{loss2}} = \int_0^1 dt \int \int_V \vec{B} \cdot \vec{H} \, dV = \frac{\sigma h^2}{288 \omega} \left(\frac{h}{\delta}\right)^2 V_m \int_0^\delta \left(\frac{\partial B_0}{\partial t}\right)^2 dt
\]

From (72), \( W_{\text{loss2}} \) is proportional to the conductivity \( \sigma \) of the core, and it is also proportional to \((h/\delta)^2\). As a result, \( W_{\text{loss2}} \) can be limited when \( h \ll \delta \).

### 5.2.3. Energy efficiency of the HES module

As to the HES module based on transformer charging shown in Fig. 22(a), the energy loss mainly consists of \( \Delta W_e \), \( W_{\text{loss1}} \) and \( W_{\text{loss2}} \). Total energy provided from \( C_1 \) is as \( \frac{1}{2} C_1 U_0^2 \).

In practice, the energy stored in \( C_1 \) can not be transferred to \( C_2 \) completely, though the loss of the module is excluded. In other words, residue energy defined as \( W_{\text{res}} \) exists in \( C_1 \). Define the allowed maximum efficiency of energy transferring from \( C_1 \) to \( C_2 \) as \( \eta_{\text{max}} \). So, \( \eta_{\text{max}} \) of the HES module is as
\[
\eta_{\text{max}} = \frac{W_0 - (\Delta W_R + W_{\text{loss1}} + W_{\text{loss2}})}{W_0}.
\] (73)

From (62), \(\eta_a, \eta_e\) and \(\eta_{\text{max}}\) have relation as \(\eta_e \leq \eta_a \leq \eta_{\text{max}}\).

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**6. References**


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