Chapter 6

Integrated Reliability Modelling for Wireless Sensor Networks

Qiang Zhou

Additional information is available at the end of the chapter

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1. Introduction

This chapter deals with the problem of reliability modelling with wireless sensor network (WSN) (Akyildiz et al., 2002), which is rapidly becoming a platform for applications including antiterrorism, smart spaces, numerous military sensing and command and control applications, and entertainment. Inherent in these safety-critical applications is a priority and urgency of the information or messages.

There are significant research results on reliability modelling for distributed systems with wired network. (Lin, 1976) approaches a connectivity-based reliability modelling from the perspective of the networks, which consider the node unit and link unit. (Tripath, 1997) proposes task-based reliability modelling by just considering processor unit, and defines a reliability index for a task, but don’t cover the system reliability perspective. (Zhou et al., 2001) approaches the reliability modelling from the perspective of the nodes in which a task involves and uses the reliability matrix with each element as the reliability of a task to evaluate reliability of a distributed system in avionics platform.

However, as far as we know, few attention has been paid by researchers to addressing reliability modelling on WSNs. Especially for safety-critical application, the reliability is influenced mainly and directly by not only the connectivity of the network topology but also the (such as energy-/time-) efficiency of the system. (Feng & Kumar, 2004) researches the connectivity reliability of wireless networks, but don’t consider the reliability modelling. (Xing & Shrestha, 2006) considers the problem of reliability modelling and analysis of hierarchical clustered wireless sensor networks (WSN), proposes reliability measures that integrate the conventional connectivity-based network reliability with the sensing coverage measure indicating the quality of service (QoS) of the WSN. Both work above research reliability problem of WSN just from view of connectivity and coverage, don’t introduce the efficiency of the system. (Xing, 2006; AboElFotoh et al., 2005) consider the efficiency in
reliability modelling for WSN. (Xing, 2006) proposes an integrated modelling on WSN reliability and security. (AboElFotoh et al., 2005) considers the delay-efficiency factor into the reliability modelling, by computing a measure for the reliability and a measure for the message delay between data sources & data sinks in an WSN, respectively. (Silva et al., 2012) proposes a methodology based on an automatic generation of a fault tree to evaluate the reliability and availability of WSNs, when permanent faults occur on network devices. (Johannes et al., 2012) generalizes the expected hop count metric (EHC) into an expected message delay (EMD) that permits arbitrary delay values for both links and devices. Further, it proposes a method based on Augmented Ordered Multivariate Decision Diagram (OMDD-A) that can be used to compute reliability (REL), EHC and EMD for WSN with both device and link failures.

To the best of our knowledge, however, there is no systematical research done to unify energy consume and message delay into reliability modelling for WSN. The work in this chapter differs from the previous work in that it proposes a model of the system and an integrated model of the task which considers energy consume and message delay for the safety-critical application, introduces both the energy factor function and time factor function, and also establishes an integrated reliability model of WSN based on a task. The illustration of modelling suggests that the method studied has a directive influence to both task division and topology selection of WSN system.

The rest of the chapter is organized as follows. The basic node model and network structure of wireless sensor networks are introduced in Section 2. We propose an integrated model of the task which considers energy consume and message delay based on a task, introduce both the energy factor function and time factor function, and also establish an integrated reliability model of WSN based on a task in Section 3 and 4, respectively. In Section 5, we present an illustration of modelling of representative hierarchical cluster topology in WSN. Finally, the chapter is concluded in Section 6.

2. The network structure and node model

2.1. The network structure

WSNs composed of multiple sensor nodes and one sink node. The sensor nodes are usually scattered in a sensor field as in Fig.1. Each of these scattered sensor nodes has the capabilities to collect data and route data back to the sink node. Data are routed back to the sink node by a multi-hop architecture through the sink node as shown in Fig.1.

2.2. Node model

A sensor node is made up of four basic components as shown in Fig. 1: an acquisition unit, a processing unit, a communication unit and a power unit. Acquisition units are usually composed of two subunits: sensors and analog to digital converters (ADCs). The analog signals produced by the sensors based on the observed phenomenon are converted to digital signals by the ADC, and then fed into the processing unit. The processing unit, which is
generally associated with a small storage unit, manages the procedures that make the sensor
node collaborate with the other nodes to carry out the assigned sensing tasks. A
communication unit connects the node to the network. One of the most important
components of a sensor node is the power unit. Power units may be supported by a power
scavenging unit such as solar cells. There are also other subunits, which are application
dependent.

Figure 1. Wireless sensor node model and network structure

Data interchanges between nodes are generally supported by processing units and
communication units. Because those acquisition units can only have effect in source nodes,
instead of taking effect on data relay, we suppose the acquisition unit works well. Besides,
we also suppose the power unit works well.

3. The network task models

WSN can support a serial of data interchanges to satisfy the functional need for some
applications. Moreover, the process of every such data interchange can be looked as a task
message, which includes message source, message route, message destination and so on. So,
the whole WSN can be functionally looked as a task set.

In this Section, we propose an integrated model of the task which considers energy consume
and message delay for the safety-critical application, and introduce both the energy factor
function and time factor function.

3.1. Task model

[definition A]: A task in WSN can be characterized as $\tau_{S,D,E,T}$, where
• $S$ : represents a source node;
• $D$ : represents a destination node;
• $E$ : represents the maximum value of all energy consumer to transmit the task message from one node to another in a single-hop way through the route;
• $T$ : represents the amount of time to transmit the task message from the source node to the destination one.

3.2. Route set of a task

[definition B]: when a task $\tau_{S,D,E,T}$ is executed, task message can bypass several node orderly, so the ordered node set composes a route of the task $\tau_{S,D,E,T}$, which characterized as $r_{S,D}$. Where, $r_{S,D}$ just consider the connectivity of the task instead of energy/time constraint. Generally speaking, there maybe more than one route for a task in WSN, so we characterize the number of the route of the task $\tau_{S,D,E,T}$ as $k_{S,D}$.

3.2.1. Energy available route

The wireless sensor network (WSN) system is a kind of system in which the consumer energy of a task is strictly bounded. So, an energy factor function is defined to decide whether the energy is available for a route.

• Energy factor function

Suppose that task $\tau_{S,D,E,T}$ has one route $r_{i}^{j}$, which has the ordered nodes as follows $n_{j_{1}}^{i}, n_{j_{2}}^{i}, ..., n_{j_{h}}^{i}$. Where $h$ represents the number of nodes bypassed by task message. We denote $E_{n_{j_{1}}^{i}, n_{j_{2}}^{i}}$ as the consumer energy for message transmission from node $n_{j_{1}}^{i}$ to node $n_{j_{2}}^{i}$ of route $r_{i}^{j}$, and denote $E_{n_{j_{i}}^{i}}$ as the available power in node $n_{j_{i}}^{i}$, that means $E_{n_{j_{i}}^{i}}$ is the remaining energy of $n_{j_{i}}^{i}$ (Sun et al., 2005). Then, the energy factor function can be defined as follows:

$$F(\varepsilon_{i}) = \begin{cases} 
1 & E_{n_{j_{1}}^{i}, n_{j_{2}}^{i}} \leq E_{n_{j_{i}}^{i}} \quad (j = 1, 2, ..., h - 1) \\
0 & \text{other case}
\end{cases}$$  \hspace{1cm} (1)

The expression (1) indicates that route $r_{i}^{j}$ is an energy available route only if that the available power $E_{n_{j_{i}}^{i}}$ in each node is no less than $E_{n_{j_{1}}^{i}, n_{j_{2}}^{i}}$. Specifically, $\tau_{S,D,E,T}$ can be denoted as $\tau_{S,D,0,T}$, if $E = 0$. This means that the energy constrains can be ignored.

3.2.2. Delay available route

The wireless sensor network (WSN) system is also a kind of system in which the message delay of a task is strictly bounded. So, a time factor function is defined to decide whether the delay is available.
• **Time factor function**

Suppose that task $\tau_{S,D,E,T}$ has one route $r^i_{S,D}$, which is defined as in section 3.2.1. We denote $T_{n'_i,n''_i}$ as the consumer time for message transmission from node $n'_i$ to node $n''_i$ of route $r^i_{S,D}$, and denote $t_i$ as the sum of $T_{n'_i,n''_i}$ for route $r^i_{S,D}$. Then, the *time factor function* can be defined as follows:

$$F(t_i) = \begin{cases} 1 & t_i \leq T \\ 0 & t_i > T \end{cases}$$  \hspace{1cm} (2)

The expression (2) indicates that route $r^i_{S,D}$ is a *delay available route* only if the transmission delay $t_i$ is no more than message deadline $T$. Specifically, $\tau_{S,D,E,T}$ can be denoted as $\tau_{S,D,E,\infty}$, if $T = \infty$. This means that the time constraints can be ignored. In another word, $\tau_{S,D,E,\infty}$ will consider the energy constraints, while $\tau_{S,D,D,0}$ will consider the time constraints. Moreover, $\tau_{S,D,E,T}$ can be denoted as $\tau_{S,D,0,\infty}$ if both the time constraints and the energy constraints are ignored.

### 3.3. Available route set of a task

So, consider both the energy factor and the time factor, if the route $r^i_{S,D}$ can meet both energy constraint and time constraint, it is called an *available route*, denoted as $r^i_{S,D,E,T}$. And then, the *available route set* of $\tau_{S,D,E,T}$ is:

$$R_{S,D,E,T} = \sum_{i=1}^{k_{S,D}} r^i_{S,D,E,T} = \sum_{i=1}^{k_{S,D}} F(\varepsilon_i) \cdot F(t_i) \cdot r^i_{S,D}$$  \hspace{1cm} (3)

Where the route $r^i_{S,D,E,T}$ of the task $\tau_{S,D,E,T}$ composed of a processing unit set and a communication unit set, which are denoted as $p^i_{S,D,E,T}$ and $c^i_{S,D,E,T}$, respectively.

$$p^i_{S,D,E,T} = \{ p^i_{S,D,E,T} | p^i_{S,D,E,T} \in r^i_{S,D,E,T} \}$$

$$c^i_{S,D,E,T} = \{ c^i_{S,D,E,T} | c^i_{S,D,E,T} \in r^i_{S,D,E,T} \}$$

Therefore, the processing unit set and the communication unit set of $\tau_{S,D,E,T}$ can be denoted as, respectively

$$P_{S,D,E,T} = \bigcup_{i=1}^{k_{S,D}} p^i_{S,D,E,T}$$

$$C_{S,D,E,T} = \bigcup_{i=1}^{k_{S,D}} c^i_{S,D,E,T}$$
4. Reliability model

In this Section, we establish an integrated reliability model of WSN based on task.

4.1. Assumptions

Given the WSN nodes has a number size of $M$.

1. We take the assumption that the occurrence of component failures is independent, components either work or fail.
2. Assume the variable $m$ is the number of tasks; therefore, the expression of the system task set is $\Gamma = \{\tau_{S,D,E,T}^k | k = 1, \ldots, m\}$.

4.2. Task reliability

In WSNs, task $\tau_{S,D,E,T}$ can have more than one route. The reliability of the route $r_{S,D,E,T}^i$ of task $\tau_{S,D,E,T}$ is equivalent to the probability of the processing units $p_{S,D,E,T}^i$ and communication units $c_{S,D,E,T}^i$ working properly. That is:

$$R_{S,D,E,T}^i = Pr\{r_{S,D,E,T}^i\} = Pr\left(p_{S,D,E,T}^i\right) \cdot Pr\left(c_{S,D,E,T}^i\right)$$  \hspace{1cm} (8)

Where, $Pr\{\bullet\}$ denotes the probability of the object’ s working properly in above bracket.

Task reliability is equivalent to the probability that there exists at least one path among Task paths, which is

$$R_{S,D,E,T} = Pr\left(\bigcup_{i=1}^{k_{S,D}} r_{S,D,E,T}^i\right)$$  \hspace{1cm} (9)

According to the formula of probability for incompatible event (Zhang et al., 1997), we have

$$R_{S,D,E,T} = Pr\left(\bigcup_{i=1}^{k_{S,D}} r_{S,D,E,T}^i\right) = \sum_{i=1}^{k_{S,D}} Pr\left(r_{S,D,E,T}^i\right) - \sum_{i<j=2}^{k_{S,D}} Pr\left(r_{S,D,E,T}^i \cap r_{S,D,E,T}^j\right) + \sum_{i<j<k=3}^{k_{S,D}} Pr\left(r_{S,D,E,T}^i \cap r_{S,D,E,T}^j \cap r_{S,D,E,T}^k\right) + \cdots + (-1)^{k_{S,D}-1} \cdot Pr\left(\bigcap_{i=1}^{k_{S,D}} r_{S,D,E,T}^i\right)$$ \hspace{1cm} (10)

5. Examples

In this Section, we present an illustration of modelling of representative hierarchical cluster topology in WSN, as shown in Figure 2.
WSN can offer unprecedented flexibility in the choice of network topology to match the mission requirements, and a large number of network topology architectures have been proposed for WSNs (Tilak et al., 2002; Edgar et al., 2003), and a topology solution that is efficient for one architecture is likely not to be the best for another, as different network architectures exhibit different communication patterns. Therefore, the topology selection and reliability evaluation are important issues for distributed WSN.

Presently, mesh and hierarchical clustered topology have emerged as the choice topologies for sensor networks. To decrease communication traffic and communication frequency, to ensure scalability and fault tolerance, and to manage the large number of sensors, WSN use the clustered hierarchical architecture (Heinzelman et al., 2000; Tubaishat & Madria, 2003).

5.1. Hierarchical clustered topology

Figure 2 shows an example hierarchical clustered structure with nodes organized into different layers (Banerjee & Khuller, 2001; Tubaishat et al., 2003; Kim, 2010). All the sensor nodes in the network are joined at the lowest layer. The cluster heads in layer-0 are arranged into clusters in layer-1 and a cluster head is assigned for each cluster at this layer. The process is repeated for each layer until the highest layer in the architecture is reached. The hierarchical scheme forms a tree structure for routing with the sink node as the root of the tree (Callaway, 2004). Whenever a sensor node needs to send a message to the sink or another sensor node, it sends the message to its cluster head. The message is routed progressively to the immediately higher-level cluster heads, each of which forms a more detailed segment of the route, until it reaches the cluster head that has the routing information about the destination node. The message is then routed progressively to lower-level cluster heads until it reaches the destination node.

Base on the reliability modelling method above, we analyze the reliability of the hierarchical cluster structure of WSNs. The analysis includes two parts, 1) all cluster heads from \( L_1 \) to \( L_n \); 2) the lowest cluster \( L_n \).

Suppose a task \( \tau_{i,\text{sink}} \), where sink represents the destination node, and \( i \) represents the source node which belongs to the lowest cluster \( L_n \). Based on the characteristic of hierarchical cluster structure, the routes of \( \tau_{i,\text{sink}} \) have the ordered cluster layers as \( (L_n \rightarrow L_{n-1} \rightarrow \ldots \rightarrow L_1 \rightarrow \text{sink}) \) which is indicated in Table 1. Moreover, the cluster heads in each layer forms a tree structure for routing with the sink node as the root of the tree. Correspondingly, the part of the ordered cluster heads in a route of the task \( \tau_{i,\text{sink}} \) is always \( (h_{n-1}, \ldots, h_1, \text{sink}) \). By contrary, the part of the lowest cluster in the route may have multiple sub-routes such as \( (1, 2, 3, 6, h_n) \), \( (1, 2, 5, 8, h_n) \), \( (1, 4, 5, 6, h_n) \), and etc. (see Table 1 and Fig.2.)

So, the reliability of task \( \tau_{i,\text{sink}} \) can be presented as

\[
R_{\tau_{i,\text{sink}}} = R_{\tau_{h(n-1),\text{sink}}} \bigcap R_{\tau_{h(n)}}
\]

(11)

Where \( n \) represents the cluster depth; \( h(w) = h_w \ (w = 1, \ldots, n) \);
$R_{r_{h(n-1),sink}}$ represents the reliability of the part of the ordered cluster heads in the route; $R_{r_{h(n)}}$ represents the reliability of the part of the lowest cluster in the route.

Therefore, by (8), we can deduce the following expression

$$R_{r_{h(n-1),sink}} = Pr\left(P_{r_{h(n-1),sink}}\right) \cdot Pr\left(C_{r_{h(n-1),sink}}\right)$$

(12)

---

**Figure 2.** Hierarchical cluster structure of WSNs

<table>
<thead>
<tr>
<th>No.</th>
<th>Route</th>
<th>Clusters to route</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,2,3,6, h_n, h_{n-1},..., h_1,sink</td>
<td>L_n-&gt;L_{n-1}-&gt;...-&gt;L_1-&gt;sink</td>
</tr>
<tr>
<td>2</td>
<td>1,2,5,8, h_n, h_{n-1},..., h_1,sink</td>
<td>L_n-&gt;L_{n-1}-&gt;...-&gt;L_1-&gt;sink</td>
</tr>
<tr>
<td>3</td>
<td>1,4,5,6, h_n, h_{n-1},..., h_1,sink</td>
<td>L_n-&gt;L_{n-1}-&gt;...-&gt;L_1-&gt;sink</td>
</tr>
<tr>
<td>...</td>
<td>Other route</td>
<td>L_n-&gt;L_{n-1}-&gt;...-&gt;L_1-&gt;sink</td>
</tr>
</tbody>
</table>

**Table 1.** Routes of $\tau_{i,sink}$ ($i=1$) and its clusters
5.2. Reliability analysis

Example: Suppose a task set $\Gamma = \{\tau_{1, \text{sink}, 2, 16}, \tau_{3, \text{sink}, 2, 16}, \tau_{7, \text{sink}, 2, 16} \}$ in Fig.2, we will analyze the reliability of every task in $\Gamma$ by using the reliability modelling method proposed in the section above. And the analysis content includes:

- To analyze the routes of every task in $\Gamma$ in the lowest cluster.
- To analyze the constrained energy/time influence on task reliability/system reliability.

Settings and assumptions:
- The cluster depth $n = 5$;
- For the part of the ordered cluster heads in the route, it is supposed that:
  \[ E_{h(n),h(n-1)} = E_{h(n-1),h(n-2)} = \ldots = E_{h(1),\text{sink}} = 2; \]
  \[ E_{h(n)} = E_{h(n-1)} = \ldots = E_{h(1)} = E_{\text{sink}} = 2; \]
  \[ T_{h(n),h(n-1)} = \ldots = T_{h(2),h(1)} = T_{h(1),\text{sink}} = 2. \]
  Therefore, by (12), we can get the sub-task $\Gamma' = \{\tau_{1,h(n),2,8}, \tau_{3,h(n),2,8}, \tau_{7,h(n),2,8} \}$ corresponding to the lowest cluster.
- For the part of the lowest cluster in the route, as in Fig.3, it is supposed that:
  \[ E_1 = E_3 = E_4 = E_5 = E_6 = E_8 = 2; \]
  \[ E_2 = E_7 = 1; \]
  \[ E_{2,3} = E_{2,5} = E_{5,6} = E_{7,8} = 1.5; \]
  \[ E_{1,2} = E_{1,4} = E_{4,7} = E_{5,8} = E_{4,5} = E_{3,6} = E_{8,h(n)} = E_{6,h(n)} = 2; \]
  \[ T_{7,8} = 1; T_{5,8} = T_{5,6} = T_{3,6} = 3; \]
  \[ T_{1,2} = T_{2,3} = T_{1,4} = T_{2,5} = T_{4,7} = T_{6,h(n)} = T_{8,h(n)} = 2. \]
- All processing units and communication units have the same failure rate, in order to reflect the influence of the failure rate to the system efficiently, the range of the failure rate is $[1e-1, 1e-3]$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Nodes in the lowest cluster and its energy/time setting}
\end{figure}
Reliability analysis:

According to both the reliability modeling method and the routing scheme (see Fig.2) in the lowest cluster, we firstly obtain all the routes of each task, then obtain both the processing unit set and the communication unit set of those routes, after that search the energy available route and the time available route of each task, and finally analyze the reliability index of each task.

Table 2 shows that multiple routes exist when considering the connectivity of the task $\tau_{1,t(n),2,8}$; that only route 4 and route 5 can meet energy constraint, so they are energy available routes; and that only route 6 can meet time constraint, so it is a delay available route. Further more, there no route can meet both energy constraint and time constraint, so there no available route of the task.

In the same way, table 3 and table 4 show the route and its availability of $\tau_{3,t(n),2,8}$ and $\tau_{7,t(n),2,8}$, respectively.

By (8),(9),and (10), we can deduce the reliability index of these three tasks. Taking $\tau_{3,t(n),2,8}$ as an example, when just considering the energy available route of route 1 and route 2, its reliability index can be calculated as

$$R_{\tau_{3,t(n),2,8}} = \Pr \left( \bigcup_{i=1}^{2} r_{3,t(n),2,\infty}^{i} \right) = \Pr \left( r_{3,t(n),2,\infty}^{1} \right) + \Pr \left( r_{3,t(n),2,\infty}^{2} \right) - \Pr \left( r_{3,t(n),2,\infty}^{1} \cap r_{3,t(n),2,\infty}^{2} \right) = \Pr \left( \bigcup_{i=1}^{2} r_{3,t(n),2,\infty}^{i} \right) \cdot \Pr \left( C_{3,t(n),2,\infty}^{1} \right) + \Pr \left( P_{3,t(n),2,\infty}^{2} \right) \cdot \Pr \left( C_{3,t(n),2,\infty}^{2} \right) -$$

$$- \Pr \left( r_{3,t(n),2,\infty}^{1} \cap r_{3,t(n),2,\infty}^{2} \right) \cdot \Pr \left( C_{3,t(n),2,\infty}^{1} \cap C_{3,t(n),2,\infty}^{2} \right)$$

And when considering the (energy and time) available route of route 1, its reliability index can be calculated as

$$R_{\tau_{3,t(n),2,8}} = \Pr \left( r_{3,t(n),2,\infty}^{1} \right) = \Pr \left( P_{3,t(n),2,\infty}^{1} \cap C_{3,t(n),2,\infty}^{1} \right)$$

Table 2. Routes of $\tau_{1,t(n),2,8}$ and its processing/communication unit set

<table>
<thead>
<tr>
<th>No.</th>
<th>route</th>
<th>Processing unit set</th>
<th>Communication unit set</th>
<th>$f(\varepsilon)$</th>
<th>$f(t)$</th>
<th>Available route?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,2,3,6,$h_{n}$</td>
<td>$p_{1},p_{2},p_{3},p_{6},p_{h(n)}$</td>
<td>$c_{1},c_{2},c_{3},c_{6},c_{h(n)}$</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>2</td>
<td>1,2,5,6,$h_{n}$</td>
<td>$p_{1},p_{2},p_{5},p_{6},p_{h(n)}$</td>
<td>$c_{1},c_{2},c_{5},c_{6},c_{h(n)}$</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>1,2,5,8,$h_{n}$</td>
<td>$p_{1},p_{2},p_{5},p_{8},p_{h(n)}$</td>
<td>$c_{1},c_{2},c_{5},c_{h(n)}$</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>1,4,5,8,$h_{n}$</td>
<td>$p_{1},p_{4},p_{5},p_{8},p_{h(n)}$</td>
<td>$c_{1},c_{4},c_{5},c_{8},c_{h(n)}$</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>5</td>
<td>1,4,5,6,$h_{n}$</td>
<td>$p_{1},p_{4},p_{5},p_{6},p_{h(n)}$</td>
<td>$c_{1},c_{4},c_{5},c_{6},c_{h(n)}$</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>6</td>
<td>1,4,7,8,$h_{n}$</td>
<td>$p_{1},p_{4},p_{7},p_{8},p_{h(n)}$</td>
<td>$c_{1},c_{4},c_{7},c_{8},c_{h(n)}$</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>…</td>
<td>Other route</td>
<td>…</td>
<td>…</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
</tbody>
</table>
Correspondingly, by (11), (12), (13), and (14), the reliability index of task $\tau_{3, \text{sink}, 2, 16}$ can be deduced. In the same way, the reliability index of each task in $\Gamma$ and $\Gamma'$ can be obtained.

Table 3. Routes of $\tau_{3, h(n), 2, 8}$ and its processing/communication unit set

<table>
<thead>
<tr>
<th>No.</th>
<th>route</th>
<th>Processing unit set</th>
<th>Communication unit set</th>
<th>$f(\varepsilon_i)$</th>
<th>$f(t_i)$</th>
<th>Available route?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3, 6, h_n$</td>
<td>$p_3, p_6, p_{h(n)}$</td>
<td>$c_3, c_6, c_{h(n)}$</td>
<td>1</td>
<td>1</td>
<td>Y</td>
</tr>
<tr>
<td>2</td>
<td>$3, 6, 5, 8, h_n$</td>
<td>$p_3, p_6, p_5, p_8, p_{h(n)}$</td>
<td>$c_3, c_6, c_8, c_{h(n)}$</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>$3, 6, 5, 4, 7, 8, h_n$</td>
<td>$p_3, p_6, p_5, p_4, p_7, p_8, p_{h(n)}$</td>
<td>$c_3, c_6, c_5, c_4, c_7, c_8, c_{h(n)}$</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>$3, 6, 5, 2, 1, 4, 7, 8, h_n$</td>
<td>$p_3, p_6, p_5, p_2, p_1, p_4, p_7, p_8, p_{h(n)}$</td>
<td>$c_3, c_6, c_5, c_2, c_1, c_4, c_7, c_8, c_{h(n)}$</td>
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<tr>
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<td>$p_3, p_2, p_5, p_6, p_{h(n)}$</td>
<td>$c_3, c_2, c_5, c_6, c_{h(n)}$</td>
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<td>0</td>
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</tr>
<tr>
<td>6</td>
<td>$3, 2, 5, 8, h_n$</td>
<td>$p_3, p_2, p_5, p_8, p_{h(n)}$</td>
<td>$c_3, c_2, c_5, c_8, c_{h(n)}$</td>
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<td>0</td>
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</tr>
<tr>
<td>7</td>
<td>$3, 2, 5, 4, 7, 8, h_n$</td>
<td>$p_3, p_2, p_5, p_4, p_7, p_8, p_{h(n)}$</td>
<td>$c_3, c_2, c_5, c_4, c_7, c_8, c_{h(n)}$</td>
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<tr>
<td>8</td>
<td>$3, 2, 1, 4, 7, 8, h_n$</td>
<td>$p_3, p_2, p_1, p_4, p_7, p_8, p_{h(n)}$</td>
<td>$c_3, c_2, c_1, c_4, c_7, c_8, c_{h(n)}$</td>
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</tr>
<tr>
<td>9</td>
<td>$3, 2, 1, 4, 5, 8, h_n$</td>
<td>$p_3, p_2, p_1, p_4, p_5, p_8, p_{h(n)}$</td>
<td>$c_3, c_2, c_1, c_4, c_5, c_8, c_{h(n)}$</td>
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</tr>
<tr>
<td>10</td>
<td>$3, 2, 1, 4, 5, 6, h_n$</td>
<td>$p_3, p_2, p_1, p_4, p_5, p_6, p_{h(n)}$</td>
<td>$c_3, c_2, c_1, c_4, c_5, c_6, c_{h(n)}$</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 4. Routes of $\tau_{7, h(n), 2, 8}$ and its processing/communication unit set

Correspondingly, by (11), (12), (13), and (14), the reliability index of task $\tau_{7, \text{sink}, 2, 16}$ can be deduced. In the same way, the reliability index of each task in $\Gamma'$ and $\Gamma'$ can be obtained.
5.3. Results

Figure 4 to figure 8 show the reliability index of the tasks vs unit fail rate denoted as \( e \). And we have that \( e \) equals to \( (1-r) \), where \( r \) denote the reliability index of the unit.

![Figure 4](image1.png)

**Figure 4.** The energy/time constrains’ influence on the reliability index of \( \tau_{L,h(5),2,8} \)

![Figure 5](image2.png)

**Figure 5.** The energy/time constrains’ influence on the reliability index of \( \tau_{3,h(5),2,8} \)
Several results can be obtained directly from these figures, which are as follows:

i. the reliability index of each task decreases with the growth of the fail rate of the processing/communication unit. That means that the reliability index can be increased by improving the reliability index of the unit.

ii. As shown in Figs. 7 and 8, when ignoring the (energy and time) constrains, task $\tau_{7,h(5),2,8}$ or $\tau_{3,sink,2,16}$ has the same reliability index as task $\tau_{7,h(5),2,8}$ or $\tau_{3,sink,2,16}$, respectively. That is concordant with their topology symmetry.

iii. When just considering the energy constrains ($\tau_{S, D, E, \infty}$),
   a. The number of the energy available route of Task $\tau_{1,h(n),2,8}$ or $\tau_{3,h(n),2,8}$ is only two. As shown in Figs. 4 and 5, the reliability index of these two tasks has decreased compared with $\tau_{1,h(5),2,\infty}$ or $\tau_{3,h(n),2,\infty}$, respectively. It means that the energy constrains will have a certain extent negative effect on the execution of a task.
   b. There no energy available route exists for task $\tau_{7,h(5),2,8}$, as shown in Fig. 6, therefore, the reliability index of the task is zero. It means that the energy constrains will have a vital effect on the execution of a task.

iv. When just considering the time constrains ($\tau_{S, D, 0, T}$),
   a. The number of the time available route of Task $\tau_{1,h(n),2,8}$, $\tau_{3,h(n),2,8}$, or $\tau_{3,h(n),2,8}$ is only one. As shown in Figs. 4, 5, and 6, the reliability index of these three tasks has decreased compared with $\tau_{1,h(5),0,8}$, $\tau_{3,h(5),0,8}$ or $\tau_{7,h(5),0,8}$, respectively. It means that the time constrains will have a certain extent negative effect on the execution of a task.
Figure 7. The energy/time constrains’ influence on the reliability index of $\tau_{1,h(5)}$, $\tau_{3,h(5)}$, and $\tau_{7,h(5)}$.

Figure 8. The energy/time constrains’ influence on the reliability index of $\tau_{1,h(5),2,8}$, $\tau_{3,h(5),2,8}$, and $\tau_{7,h(5),2,8}$.
v. When considering both the energy constrains and the time constrains \( (\tau_{S,D,E,T}) \),
   a. The number of the available route of Task \( \tau_{3,h(n),2,8} \) is only one. As shown in Fig. 5,
      the reliability index of the task has decreased compared with \( \tau_{3,h(n),2,\infty} \). It means
      that both the energy constrains and the time constrains will have a certain extent
      negative effect on the execution of a task.
   b. There no available route exists for task \( T_{1,h(5),2,8} \) or \( T_{7,h(5),2,8} \), as shown in Figs. 4 and
      6, therefore, the reliability index of these two tasks is zero. It means that both the
      energy constrains and the time constrains will have a vital effect on the execution
      of a task.

6. Conclusion

The work in this chapter carries on systematically research on unifying energy consume and
message delay into reliability modelling for WSN, proposes an integrated model of the task
which consider energy consume and message delay for the safety-critical application,
introduces both the energy factor function and time factor function, and also establishes an
integrated reliability model of WSN based on a task.

The illustration of modelling suggests that the method studied has a directive influence to
both task division and topology selection in the phase of system design of WSN system.

Based on this work, future directions can cover several research issues in WSNs:

- To implement reliability validation and optimization for the complicated topologies in
  WSNs;
- To analyze the reliability index for some kind of topology, search out the key unit
  (processing unit, or communication unit, or node), and research the redundant scheme
  for the unit.
- To expand the reliability model to consider more factors (for example, safety or security
  or buffer limit) than the energy and the time in order to meet the multiple QoS
  requirements for the safety critical application in WSNs.

Author details

Qiang Zhou
School of Electronic Information Engineering, Beihang University, China

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