# 14

# Time Measurement and Earth Rotation

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## 1. Introduction

This chapter covers some issues of measuring, storage and reproduction of a precise time scale and reference frequencies, as well as techniques and methods for providing consumers with this scale and Earth rotation parameters (ERP) data.

The first paragraph contains historical information on development of time measurement and storage instruments. It reviews development stages of national and world time services, their duties and goals within the scope of an assurance of time and frequencies measurement and reproduction uniformity.

The second paragraph represents a brief classification of time measurement systems basing on different natural periodic process, which are used as a standard time unit. Possibilities of use a pulsar time scale for long-time storage of high-stable time intervals and synchronization of group time and frequencies references are also reviewed in this paragraph.

The third paragraph states the main objectives of national time and frequencies services, describes a drawing scheme for a group time scale, its generation algorithms and a structure of a measurement instruments complex for time and frequency storage and reproduction.

The fourth paragraph studies methods for synchronization of reference clocks, which are distant from each other making a direct connection between them impossible. Different types of radio and wire communications systems used as solutions for providing various consumers with standard time signal are also described briefly. For example, there are following such consumers: communications and telecommunications, information technologies computer systems and computer servers, space and ground-based navigation, space geodesy, geodynamics, transport management systems, etc. Some synchronization methods based on the use of GPS/GLONASS satellite groups are presented.

The fifth paragraph covers algorithmic and software tools, which allow to evaluate the accuracy of solving of time and coordinates support problems by means of a simulation modelling. In particular, a brief description of the simulation model of the quantum clock instability developed by the authors and used in a software simulator of the measurement data, received from a network of non-query measuring stations through the GLONASS and GPS navigation satellites is given in this paragraph.

A comparative analysis based on the main metrological characteristic of the modern methods of ERP determination and prediction is conducted in the sixth paragraph. The ERP prediction accuracy requirements for the time intervals of the coordinate and ephemeris support of GNSS operation are given within the scope of metrological support issues of coordinate and navigational determinations for GPS and GLONASS GNSS. The basic principles and approaches used by authors for the development of new high-precision ERP prediction method are set forth.

## 2. History of information

Need in time measurement arose about 5000 years ago, around a transition from nomadic to sedentary lifestyle, farming and cattle breeding. In particular, need in observance of optimum time for agricultural work and coordination of collective actions caused invention of different methods and devices for long and short time period measurement that were based either on uniform or periodic natural processes. Diurnal solar motion started to be used for time measurement in Ancient Egypt about 3500 B. C. Obelisks of a certain height ("gnomons") were installed in strategic locations to mark noon with the shortest shadow. Two thousand years later the Egyptians invented sun-dial, "face" of which was divided into 10 parts. By the beginning of the Common Era in Mediterranean industrialized countries more than 30 kinds of sun-dials were used. (Mihal, 1983). In order to measure short time periods, such uniform natural processes were used as flow of liquid or light sand through a narrow hole. Sand and water clocks based on this principle were widespread in the ancient world and sometimes are still used nowadays.

First stellar work for the purpose of time measurement began about 600 years B. C. by means of the astronomical tool "merkhet" (http://www.infoniac.ru). After mechanical spring-actuated and pendulum clocks were invented in the early XIV century A. D. in Europe, precision of time period measurements considerably improved. Pendulum clock constructional design was improved by Galileo, Huygens and Hooke. On the basis of their work in the XVIII century wrist watch was made, it was 5 seconds in error and its manufacture started in the XIX century. (Pipunyrov, 1982).

The XX century was characterized by huge scientific and technical progress in terms of mastering new time measuring methods and devices. First in 1920 by Short, and then in 1955 by Fedchenko the best astronomical pendulum clock constructional designs were developed, they were 2ms/per day and 0.2 ms/per day in error respectively. (Bakulin & Blinov, 1977). Quartz-crystal clock, invented by Morrison in 1927 and based on the piezoelectric effect, opened up new possibilities for improvement of time measurement and time keeping. By the mid-1950s time scale, determined by thoroughly investigated quartz crystal clock, started to serve as an independent time standard, more stable than diurnal rotation of the Earth. As a result, the possibility to detect instability of earth time scale first appeared, as well as the possibility to detect nonuniformity of the Earth's rotation. But quartz crystal clock grave disadvantage was that they could not maintain stability over long time periods because of quartz ageing effect (Great Russian Encyclopedic Dictionary [GRED], 2003).

A complete swing-round in terms of time keeping occurred after atomic and molecular frequency standards were invented. In 1955 the Englishman Essen developed the first atomic frequency (time) standard by means of a cesium beam. From that moment on atomic

second, equivalent to a time span over which a cooled caesium atom emits 9192631770 radiation periods, is considered to be the world time and frequency standard. Atomic time standard is free from both diurnal and secular variations, does not have ageing effect and is characterized by sufficient determinateness, precision and reproducibility. With the use of atomic frequency standards, time standards independent from the Earth's rotation were developed, they are characterized by exceptionally high stability over long time periods. It enabled to fulfil the task of large distance measurement metrological assurance. Precise time measurements are also required to fulfil different tasks of navigation, telecommunications, terrestrial and extraterrestrial navigation, geodesics and geodynamics. They are also used in traffic control systems, information technologies and other spheres. A series of basic researches – aimed at a more precise definition of principal laws of nature related to extension of knowledge concerning Macrocosm, Aerospace, Earth, Microcosm – require such a precise standard that approaches to a limit determined by fundamental physics laws. Thus far the highest level of relative precision of playback of atomic second of the order of $3 \cdot 10^{-16}$ was reached by the American National Institute of Standards and Technology (NIST) by means of caesium fountain clock. In the years to come, a new generation of frequency standards that radiate frequencies not in microwave but in optical spectrum is expected. Optical clock is developed especially actively in Japan. For example, Hidetoshi Katori's group of Tokyo University has already reached precision of the order of $10^{-15}$ by means of an experimental model of strontium optical clock, and theoretically they can maintain precision of playback of time and frequency units at $10^{-17} \div 10^{-18}$ (Hall, 2006).

## 3. Time measurement systems

Time measurement is one of the most important tasks of modern metrology, astrometry and physics. In order to fulfil this task, it is necessary:

1. to determine time measurement units and time keeping systems;
2. to have at one's disposal reasonably designed time-interval recorders;
3. to control operation of these recorders either occasionally or constantly.

It is easy to use periodic processes based on natural life cycles and characterized by high stability over long time periods as standard time units. Among all processes used for determining time units there are three basic ones:

1. the Earth's axial rotation;
2. the Earth's circuit of the Sun;
3. emission (absorption) of electromagnetic waves by atoms and molecules of certain substances under certain conditions.

Thanks to these natural processes, three independent time scales were developed; these are universal (astronomical), ephemeris and atomic time scales. Universal or worldwide time scale was so called as it is directly connected to the nychtemeral cycle that is the main cycle of people's lives. In terms of a type of a process that forms a universal time scale, there are three universal time systems.

1. $UT0$ is universal time registered directly as a result of astronomical observations. Technically, it is not universal, as it depends on where on the Earth the observatory is located.

2. $UT1$ is universal time which was corrected as a result of change of longitudes of observatories under the influence of movement of poles, it is calculated according to the formula:

$$UT1 = UT + \Delta\lambda\,, \tag{1}$$

where $\Delta\lambda$ is a correction to the observatory longitude, for movement of the North Pole.

3. $UT2$ is universal time, which also takes into account seasonal variations in the Earth's rotation speed. $UT2$ is the most uniform time, which can be received from astronomical observations of diurnal rotation of the Earth. $UT2$ and $UT1$ time scales differ in the value of seasonal variations $\Delta T_s$ :

$$UT2 = UT1 + \Delta T_s\,. \tag{2}$$

The ephemeris time scale was introduced in 1952 by the decision of Paris international conference on fundamental astronomical constants. Ephemeris time ( $ET$ ) is an independent variable of celestial mechanics equations and is defined as difference between observed and calculated, according to celestial-mechanical theories, coordinates of the Moon, the Sun and planets. As $ET$ standard a second, equal to 1/31556925,9747 part of a tropical year, was taken. Uniformity of $ET$ scale was influenced by observation errors and uncertainties of adopted theories of the Moon's and planetary movement (Bakulin & Blinov, 1977).

In 1967 by the decision of the XII General Conference of Weights and Measures the atomic time scale $TAI$ was introduced, the scale unit is equal to a time span, in which the caesium atom goes through 9192631770 emission periods. The atomic scale is characterized by the highest stability of all mentioned ones and that is why it is the main time standard in the world. It is connected to the ephemeris time scale by the formula:

$$ET = TA1 + 32{,}18\,c \tag{3}$$

Universal Coordinated Time scale $UTC$ is a combined one; it is based on the atomic time scale, indications of which are corrected taking into account data concerning the Earth's rotation. From time to time $UTC$ indications are corrected to 1 second, so that difference between $UT1 - UTC$ does not exceed 0,9 second in modulus. $UTC$ signals are transmitted in broadcasting networks and used in daily life. The $UTC$ advantages are high uniformity that is characteristic for atomic time, and connection to natural processes (sunrise, sunset) that is characteristic for solar time. Some countries form and maintain their own $UTC$ scale. For example, in the U.S. the $UTC$ (USNO) scale is based on the assembly of (about 50) caesium standards, its indications are not more than 50 ns in error compared to the international standard $UTC$. In Russia the national scale $UTC$ (SU) is maintained which is offset by +3 hours from $UTC$ (Belotserkovsky & Kaufman, 1972).

The possibility of independent pulsar time scale formation has lately been the subject of wide speculation. Pulsars are speed-up neutron stars that represent sources of high-stable radio pulses with frequencies of the order of 1-1000 Hz. In view of this property, pulsars are regarded as potential time keepers, located in extraterrestrial environment. This remarkable property of pulsars is especially important in view of creation and development around the globe of Global Positioning Systems that are based on metric unity of four-dimensional

space-time continuum. (GRED, 2003). But there are natural factors that influence radio-frequency radiation stability of pulsars and are related to their inherent properties and difference in conditions of signal transmission in interstellar medium. So, for actual use of pulsars as time keepers it is necessary to minimize accidental variations and systematic deviations of observed radio frequencies. This is achieved by formation of a pulsar time group scale and application of parameterization methods. In the long term integration of pulsar time with a group atomic keeper for the purpose of atomic time scale correction will enable to carry out comparative measurement and determine corrections to bring indications of all atomic keepers in line with high-stable pulsar time scale, using it as the key standard. In European Radio Astronomical Observatories systematic observations of 25 pulsars are carried out. In the U. S. observation of an assembly of 25 pulsars are carried out as part of the NANOGrav program (North American Nanohertz Observatory for Gravitational Waves) by means of the largest radio telescopes: Arecibo (300m) - 13 MP, RT-100 etc. (Ilyasov and others, 2010).

## 4. Facilities and methods of time unit and frequency playback in time services

### 4.1 Basic tasks of time services

Basic tasks of exact time services are determination, storage and transmission of standard time signals, as well as determination and forecasting of the Earth's rotation parameters (ERP). There are primary and secondary time standards and frequencies, as well as reference standards, in some countries. A national primary time and frequency standard (NTFS) is designed for ensuring unity of measurements to a required degree of precision, carried out by time services of the country. This is achieved by a number of activities, and the main ones are as follows:

- playback and storage of national time scales;
- coordination of activities that are carried out in centres for metrology services and are aimed at ensuring operation of secondary time and frequency standards;
- comparison of NTFS clocks' indications with standards of other countries;
- control of standard time and frequency signals' (STFS) transmission by means of different technical devices;
- collection and processing of time-and-frequency information provided by domestic time services;
- delivery of necessary reference information to interested organizations and STFS consumers by way of real-time data dissemination and of official bulletins' issue with results of NTFS application;
- Storage of time and frequency units' sizes and maintaining of the coordinated time scale $UTC$, that is brought to the maximum in line with units and a coordinated time scale of national standards.

Dissemination of standard time and frequency is carried out by transmission of time and frequency signals over broadcast channels, by means of GNSS, as well as on the Internet. One of the main tasks of a time service is ensuring the possibility of signal reception anytime anywhere in the required area. Time precision in case of radio transmission is about 1 ms, transmission over television channels - 10 μS, over global positioning systems GPS/GLONASS - 10 ns, on the Internet – from 10 ms to 10 ns.

### 4.2 Scheme of group time scale formation and storage

Scheme of exact time units' formation with transmission to consumers over broadcast channels and the Internet is given in Figure 1 by the example of Novosibirsk time service.

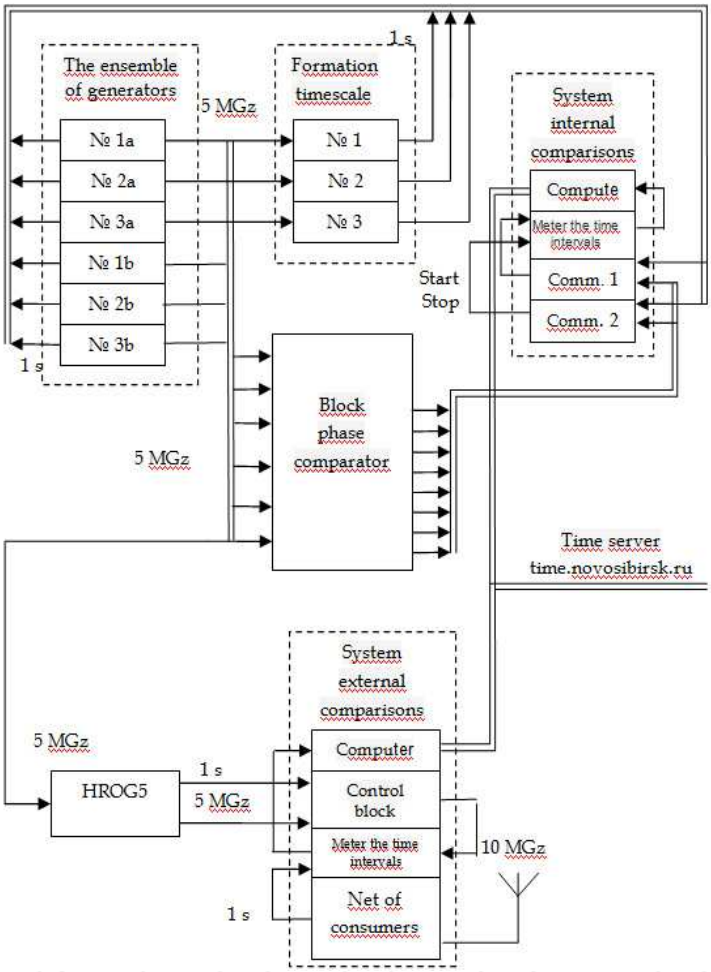


Fig. 1. Block scheme of exact time scale formation equipment

A Standard's atomic time scale is realized by a group keeper which includes at least four hydrogen standards, one of which is selected as a key one. In order to calculate the scale, results of mutual comparison of the Standard keepers' frequency and time scales are used, this is internal comparison. The key keeper is compared to the national standard over communication channels. The results serve as the basis for forecast of frequency variations of the Standard's key keeper. Hereafter, the forecast data are used only in view of routine intercomparison of keepers' time scales. In practical work maintaining of the Standard's atomic scale comes down to determination of time corrections for a key hydrogen keeper.

Frequency and time group keeper is designed for generation of sinusoidal signal with standard frequency of 5 MHz.

Group keeper equipment includes:

- Six hydrogen frequency and time standards of Ч1-75A type;
- Time scale formation system (TSFS), ensuring formation of a group high-precision, high-stable sinusoidal signal with frequency of 5 MHz ;
- a block of phase comparators, which compare signals from each pair of hydrogen standards for time scale correction according to internal comparisons;
- signal switching systems that form control signals for a meter of time intervals;
- time intervals' meters ChZ-64, SR-620, by means of which internal comparisons of differences between scales and individual hydrogen standards are carried out;
- internal comparisons' electronic system that is used to control equipment and transmission of the formed time scale to net of consumers.

Basic metrology characteristics:

- Range of standard frequencies: 1 Hz, 5 MHz, 10 MHz, 100 MHz, 2.048 MHz;
- fractional error of reproduction of a unit of frequency in the form of root-mean-square error measurements' results under condition of daily comparisons with the Russian National time and frequency standard does not exceed 5•10-13;
- difference between time scale *UTC* (NOV) and time scale *UTC* (SU) of Russia does not exceed ± 100 nc;
- Mean square fractional two-sample deviation (Alan variations) of the group keeper frequency in intervals size of which is from 1s to 1 day are given in Table 1.

| Item No. | $\tau_m$ | $\varepsilon_m$ |
|---|---|---|
| 1 | 1 sec. | $3{\cdot}10^{-13}$ |
| 2 | 10 sec. | $5{\cdot}10^{-14}$ |
| 3 | 100 sec. | $1{\cdot}10^{-14}$ |
| 4 | 1 hour | $5{\cdot}10^{-14}$ |
| 5 | 1 day | $1{\cdot}10^{-14}$ |

Table 1. Alan variations of the group time keeper frequency

Transmission of exact time signals to consumers is carried out on the INTERNET by means of NTP protocol which ensures that error related to UTC (NOV) scale is not more than several milliseconds. At the same time, time and frequency signals transmitted by radio stations of short-wave, medium-wave and long-wave range and global positioning systems GLONASS and GPS are controlled.

### 4.3 Algorithms of group time and frequency scale formation

Grouping of keepers ensures high precision and reliability of autonomous storage and playback of group time and frequency scales in intervals of long duration. This is achieved by application of the corresponding algorithms of processing of results of internal and external comparisons of group keeper's time scales. Principle of algorithms is in parameter

estimation of individual keepers according to results of scale and frequency measurements within a group and drift compensation from group time scale nominal value. Changes of points of time of keepers' time scales $T_i(k)$ and frequency scales $f_i(k)$, that form a group, are represented by mathematical models of the kind:

$$T_i(k+1) = T_i(k) + \int_{t_k}^{t_{k+1}} f(\tau)\cdot d\tau,\ T_i(0) = T_{i0}, \tag{4}$$

$$f_i(k+1) = f_i(k) + d_i(k)\cdot[t_{k+1} - t_k] + w(k+1),\ f_i(0) = f_{i0}. \tag{5}$$

Within a group of keepers, differences between points of scales $z_{0i}(k) = T_0(k) - T_i(k) + v_{0i}(k)$ and differences between frequencies $z_{f0i}(k) = f_0(k) - f_i(k) + v_{f0i}(k)$ with reference to the $T_0(k)$ scale and the $f_0(k)$ frequency of the key keeper which has the best characteristics in the group are measured. Estimated time scale point $\hat{T}_0(0)$ and estimated frequency $\hat{f}_0(0)$ of the key keeper for the point of time $t_0$, received as a result of comparison of the key keeper with time and frequency standard, are assumed as initial conditions for group scale formation.

Algorithm of group keeper's time scale formation includes forecast calculation of group keepers frequencies:

$$\vec{f}_i(k+1) = \hat{f}_i(k) + \hat{d}_i(k)\cdot[t_{k+1} - t_k] \tag{6}$$

according to calculated estimations of frequencies $\hat{f}_i(k) = \hat{f}_0(k) - z_{f0i}(k)$ and calculation of an average estimation of the key keeper frequency for the ensemble of keepers:

$$\hat{f}_0(k+1) = \frac{1}{n-1}\sum_{i=1}^{n-1}\beta_i\cdot\left[\vec{f}_i(k+1) - z_{f0i}(k+1)\right]. \tag{7}$$

The received estimations of frequencies $\hat{f}_i(k)$ and time scale estimations $\hat{T}_i(k)$, calculated according to equations $\hat{T}_i(k) = \hat{T}_0(k) - z_{0i}(k)$, are used for calculation of time scales forecast :

$$\vec{T}_i(k+1) = \hat{T}_i(k) + \hat{f}_i(k)\cdot[t_{k+1} - t_k],\ \hat{T}_i(0) = \hat{T}_{i0}. \tag{8}$$

On the basis of these forecasts and results of scale measurements $z_{0i}(k+1)$ an average estimation of the key keeper's scale for the ensemble of keepers is calculated:

$$\hat{T}_0(k+1) = \frac{1}{n-1}\sum_{i=1}^{n-1}\gamma_i\cdot\left[\vec{T}_i(k+1) - z_{0i}(k+1)\right]. \tag{9}$$

The given algorithms are used in practical work for formation of main time and frequency standards scales.

## 5. Synchronization of spaced-apart clocks

### 5.1 General information

Time scale synchronization implies correction of electronic clock that is correction of its indications and of reference generator frequency. After synchronization is carried out, for

some finite time interval a period of an event at the scale being formed coincides with the required precision with a period of the same event at the standard scale. As time goes, coincidence precision decreases as a result of different disturbing influences on the electronic clock; that is why spaced-apart clocks (comparison of clocks) should be synchronized regularly.

Until recently the most widespread methods of exact time scale synchronization were those that used radio signals of broadcasting stations having transmitted information concerning points and intervals of time at different frequencies. In order to transmit this information to a consumer, different codes and train of impulses can be used; the most widely known among them is the method of 6 points. A transmitting station pings 6 second signals (points) and one extended signal (hyphen) at the beginning of a minute within a system of universal coordinated time $UTC$, and according to the latter by means of receiving equipment time services clocks are synchronized. Precision of synchronization over radio signals is limited by conditions of radio-wave propagation in the earth's atmosphere and by instability of signal delay in receiving and transmitting equipment paths. They are used nowadays mostly in ship navigation, where permissible errors of clocks' indications are several dozens and hundreds of milliseconds. Use of television channels, man-made Earth satellites, as well as natural radiation sources, periodic electromagnetic oscillations from outer space (of pulsars) enables to considerably improve precision of transmission of time and frequency units' sizes.

### 5.2 Tools and methods for synchronization of the time and frequency scale standards

One of the main tasks of the coordinated time software is a support and development of a group standard time and frequency which forms a single group time scale with the possibility of prompt access to it. In order to form a unified group scale there can be used various methods of synchronization of standard time and frequency which are remote from each other by the distances of hundreds and thousands of kilometers. Currently the most accurate and commonly used methods are:

- Transportable quantum clock (TQC);
- Double frequency phase receiving equipment of global navigation satellite systems (GNSS);
- Ground and space communications facilities over broadcasting channels;
- Equipment for duplex comparisons over satellite channels TWSTFT.

Equipment complex for duplex comparisons of time scales over TWSTFT channels (Two-way Satellite Time and Frequency Transfer) ensures synchronization of time scales with an error that does not exceed ± 2 ns for any geographically distributed time and frequency standards. High precision of this method is explained by the fact that asymmetry of bi-directional channel delay is much less than delay factor for signal propagation in one direction. According to the data given by (Ryzhkov and others, 2007), in duplex method there is no direct correlation between an error and a measuring channel length.

The highest degree of precision in terms of comparison of time scales up to 1 ns is achieved by TQC method, as time scales are compared in close vicinity by means of direct wire connection. Precision of synchronization in TQC method is caused mostly by instability characteristics of transportable clocks and influence of relativistic effect, which appears

because the clocks being compared are located in different coordinate frames, that move in parallel with each other with variable speed and which are different in gravity field potential values. That is why it is also necessary to take into account corrections for relativistic effect in any other methods of synchronization of scales of spaced-apart clocks. Synchronization of time scales by means of TQC, by one- or two-way radio transmission methods, as well as methods of taking into account corrections for relativistic effect and other natural factors which influence precision of measurements, are described in details in the book (Oduan & Gino, 2002).

Synchronization of clocks in compliance with GNSS signals is based on receiving by aerials of spaced locations of the same navigation signals from radiating aerials located in outer space. GNSS satellites radiate exact time signals by means of atomic clock that is synchronized with system time of central synchronizer.

Receiver clock is also synchronized with system time, and as a result it is possible to calculate distance from satellite to receiving aerial according to a measured difference between radiation time and time of receiving signals from the satellite. In order to determine three spatial coordinates of a consumer, delay factors for radio signal propagation from at least three satellites at the same time are measured; it enables to receive a single-valued solution of photographic intersection. In order to determine difference between onboard clock scale and ground borne clock scale, it is necessary to measure additionally a delay factor from one more satellite. Then system of four equations that has 4 indeterminants (three coordinates and difference between scales) and a single-valued solution can be set up. In practical work many redundant measurements from ground-station network by GNSS spacecrafts constellations are carried out. Number of redundant equations achieves hundreds and thousands. These equations are solved with an ordinary least square method and its versions. Uncertainties of measurements at any distances can be reduced to several nanoseconds by averaging for more than one day. Precision of synchronization in this case is influenced most of all by uncertainties of coordinate assignment of receiving aerials and equipment delays in the receiver. That is why coordinates of receiving aerials of non-requesting measuring stations (NMS) and coordinates of spacecrafts (SC) should be known with an error that does not exceed several dozens of centimetres in the same coordinate system. For measurement of delays navigation signals' simulators, which form signals similar to those that are radiated from the spacecraft, are used. In practical work task of synchronization is reduced to calibration of difference between delays of receivers by means of transportation of a standard receiver. Precision of synchronization is also greatly influenced by instability of clock scales located on board a spacecraft and NMS clock scales. Consequently, desynchronization of clock scales occurs which can be partially taken into account by means of an appropriate mathematical model, which would enable to calculate values of compensating corrections. Precision of calculation of these corrections is influenced by:

- errors of an unpredictable deviation of standard clock scale in time intervals between synchronization sessions;
- errors of measurements of points of standard clock scale and spaced-apart clock NMS scale;
- errors of an unpredictable deviation of NMS clock scale in time intervals between synchronization sessions.

According to researches described in the work (Unoshev, 1983), the highest precision of synchronization of spaced-apart clocks A and B is achieved by application of quasi-synchronous receiving method in locations A and B of signals from one $SC_i$ (method in common view (CV)). This method enables to minimize the listed errors thanks to choice of optimum viewing conditions and application of processing algorithm. Principle of method can be explained by means of analysis of equation of signal propagation delay measurement in the way from SC to consumer equipment (CE) in location A. Let us represent the delay equation as follows:

$$\tau_{iA} = \frac{|\overline{r}_{iA}|}{\upsilon} + \Delta T_A - \Delta T_i + \delta_{UiA} + \delta_{TiA}\ , \tag{10}$$

where

$$|\overline{r}_{iA}| = \sqrt{(x_A - x_i)^2 + (y_A - y_i)^2 + (z_A - z_i)^2} \tag{11}$$

is a geometric range from $SC_i$, to CE in location A;

$\upsilon$ is average radio signal propagation speed from SC to NMS;

$\Delta T_A = T_{RA} - T_C$ is difference between receiving equipment clock scales in location A and system scale $T_C$;

$\Delta T_i = T_i - T_C$ is difference between $SC_i$ clock scale and system scale $T_C$;

$\delta_{UiA}$ is signal delay in ionosphere on the way $SC_i$-A;

$\delta_{TiA}$ is signal delay in troposphere on the way $SC_i$-A.

Let us assume that in locations A and B time keepers are located. At these locations the reception of the same signal from some SCi is carried out at the time points $T_A$ and $T_B$ and differences between receiving equipment scales $T_{RA}$, $T_{RB}$ and time keepers scales are measured:

$$\Delta_A = T_{RA} - T_A\,,\quad \Delta_B = T_{RB} - T_B \tag{12}$$

Then, after data exchange $\Delta_A$ и $\Delta_B$ in locations A and B one can find the second differences:

$$\Delta_{AB} = \Delta_A - \Delta_B = e_\Sigma - (T_A - T_B)\,. \tag{13}$$

These differences contain information concerning discrepancy between keepers scales $T_A$ and $T_B$ and overall synchronization error $e_\Sigma$. The lowest level of synchronization error in the "CV" mode is achieved at the moment SC crosses "traverse plane" (TP) – a plane that crosses the midpoint of segment AB transversely to it.

Now the various consumers set the increasingly higher requirements for time standards synchronization and frequency nominal coincidence. For example, a synchronization of the primary time standards with an accuracy of (100-10) ps and a frequency coincidence of $10^{-15}$ to $10^{-16}$ per day is required for an effective operation of the high-speed digital fibre optic communication lines, which transfer data at the rates of tens or hundreds tera-bits per

second. Only the VLBI technology can fulfil such requirements at long distances 2000 km or more (Finkelshtein, A. (2007).

## 6. Simulation of navigation measurements

### 6.1 General information

It is worth using software simulators of navigation measurements for development of model systems designed for registry of effects of various natural factors on spacecraft (SC) GNSS traffic and for improvement of navigation measurement information processing technology. Such simulators help to solve the task of parameter definition of a SC GNSS traffic mathematical model, as well as the task of some nuisance parameter part definition. They also serve to determine metrological estimations of effects of these parameters on navigation measurements results. In order to solve these tasks, a software simulator should fulfil the following functions:

1. computation of motion of a navigation satellites orbit group under conditions that produce an effect of perturbation on satellites;
2. building of non-requesting measuring stations (NMS) network and observed SC constellation;
3. estimation of a geometric range from stations to a SC;
4. imitation of factors that influence precision of trajectory measurements.

Fulfilment of the first item is a theoretical task related to application of the equation of SC traffic in the Earth's gravitational field for the purpose of determination of the selected SC coordinates expected values at preset time points. The second and the third items set parameters describing conditions and an observing station. The fourth item solves tasks of nuisance factors simulation on the basis of the navigation measurements equation:

$$S_{ui} = D_{2i}\left(X_{KA}\left(t_{2i}\right), X_{ИП}\left(t_{3i}\right)\right) + \delta D_{БФЗ} + \delta D_{TP} + \delta D_{ИОН} + \delta D_F + +\delta D_{ЦМ} + \delta D_{АИП} + \delta D_{РЕЛ} + \delta D_{SI}, \quad (14)$$

where $X_{KA}\left(t_{2i}\right)$ are coordinates of a SC position as of the time of radiation of signal $t_{2i}$;
$X_{ИП}\left(t_{3i}\right)$ are meter vector coordinates as of the time of receiving signal $t_{3i}$;
$D_{2i}\left(X_{KA}\left(t_{2i}\right), X_{ИП}\left(t_{3i}\right)\right)$ is the geometric value of a range to the SC;
$\delta D_{БФЗ}$ is the correction for phase delays of vehicle-borne equipment;
$\delta D_{TP}$ is the correction for tropospheric refraction;
$\delta D_{ИОН}$ is the correction for ionospheric refraction;
$\delta D_F$ is the correction for discrepancy of phases and frequencies of SC generators and the meter;
$\delta D_{ЦМ}$ is the correction for shift of the antenna phase centre with reference to SC centre of mass;
$\delta D_{АИП}$ is the correction for shift of the meter antenna phase centre;
$\delta D_{РЕЛ}$ is the relativistic correction;
$\delta D_{SI}$ is the correction for an instrumentation error.

Selection of a particular mathematical model for imitation of natural factors when you calculate the right side of the equation (14) is determined by requirements to precision of necessary parameters definition. In case of navigation non-requesting measurements the following parameters should be defined: orbital parameters; clock parameters and models

parameters. Orbital parameters include six orbital units, three light pressure scaling factors and empiric speed-up factors, and among the latter ones cyclical factors are the most important. Clock parameters include: current parameters of the clock on board SC and non-requesting measuring stations (NMS). Models parameters include corrections for: radio waves propagation delays in ionosphere and troposphere; non-uniqueness of phase modifications; position of SC centre of mass with reference to phase centres of SC and NMS antennas, instrumental noise, etc. In terms of this chapter's subject analysis of the task of clock current parameters simulation is of the utmost interest.

### 6.2 Models of instability of quantum frequency standards

According to the classical concept of clock rate instability, there are long-term and short-term components of its deviation from the uniform time scale. For example, in the book by (Tryon, 1983) a system of linear stochastic difference equations, describing the process of atomic time deviation in time domain $S(k)$ and frequency domain $q(k)$, as well as of frequency drift $W(k)$ is given:

$$\begin{aligned} S(k+1) &= S(k) + q(k)h + 0.5w(k)h^2 + V_s(k), \\ q(k+1) &= q(k) + w(k)h + V_q(k), \\ w(k+1) &= w(k) + V_w(k), \end{aligned} \tag{15}$$

Where $V_s(k)$, $V_q(k)$, $V_w(k)$ are centred Gaussian processes by type of white noise with spread characteristics: $\sigma_s, \sigma_q, \sigma_w$; $h = t_{k+1} - t_k$ discretization interval of processes.

The given stochastic equations include both regular long-term and short-term stochastic components of atomic clock instabilities. They are used in recurrent procedures by Kalman type for estimation of amounts S(t), q(t), w(t) in tasks of time scales formation of group keepers, in synchronization tasks and so on.

In the book by (Oduan & Gino, 2002) a mathematical model of quantum clock is reviewed, which describes the relation of the noise power spectral density at different frequencies by type of:

$$S_y(f) = \sum_{\alpha=-2}^{2} h_\alpha f^\alpha , \tag{16}$$

where $h_\alpha$ are factors that determine power density of some noise components with frequencies $f^\alpha$.

Depending on integral number value $\alpha$ it is assumed that there are 5 types of noise processes. For example, when $\alpha$ changes from -2 to +2 with interval equal to 1, formula (16) describes respectively: white phase noise; flicker phase noise; white frequency noise and frequency random walk noise. Mean square two-sample dispersion (Alan variations) is connected to noise power $S_y(f)$ by the relation (Oduan & Gino, 2002):

$$\sigma_y^2(\tau) = \int_0^\infty \left|H_A(f)\right|^2 \left|H_f(f)\right|^2 S_y(f)\,df , \tag{17}$$

where $\left|H_A(f)\right|^2 = 2\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}$ is a transfer function square modulus of a frequency digital filter in divergent integral (17).

$\left|H_f(f)\right|^2 = \begin{cases} 1 & \text{для} \quad f < f_h \\ 0 & \text{для} \quad f \geq f_h \end{cases}$ is a transfer function square modulus of a low-frequency filter with cutoff frequency $f_h$ .

Formula for Alan dispersion computation is the result of integration of formula (17) with account of formula (16) and in view of condition: $2\pi f_h \tau >> 1$ ,

$$\sigma_y^2(\tau) = \frac{3h_2 f_h}{4\pi^2\tau^2} + \frac{h_1}{4\pi^2\tau^2}\left[1{,}04 + 3\ln(2\pi f_h\tau)\right] + \frac{h_0}{2\tau} + 2h_{-1}\ln 2 + \frac{2}{3}\pi^2 h_{-2}\tau \,. \tag{18}$$

Factors $h_\alpha$ included into (18) are determined by preset Alan variations' values $\sigma_y^2(\tau_i)$ in the left side of equation. In order to receive a unique definition of all five factors $h_\alpha$ in the right side of equation (18), it is necessary to set up a system of five equations. Five values of Alan variations are determined by statistical processing of results of atomic clock rate measurements during five intervals $\tau_i$ of different sizes. Normally intervals of 1, 10, 100 seconds, 1 hour and 1 day are chosen. As a result of solution of the system of equations set up in this way, five values of factors $h_\alpha$ are determined. By inserting the determined values $h_\alpha$ into equation (18), we receive a model of an observed clock instability, which is further used to make forecast of rate of scale deviation from a nominal value. However, the given classical model is not applicable to description of instabilities of some types of quantum clock. In particular, in terms of interval characteristics of instabilities typical for rubidium frequency standards, solution of the system of equations composed of (18) includes negative values of some factors $h_\alpha$ , which is theoretically impossible . This model works only in case of clock, for which white frequency noise is predominant. The reason of restrictions on application of the classical model to description of instabilities of some kinds of quantum clock is that it does not determine extent of influence of some noise components on actual characteristics of instabilities of an observed clock.

A model of instability of quantum frequency standards (QFS) offered in researches by (Tissen, Tolstikov, 2004, 2011) enables to simulate by means of a computer a random process of QFS scale deviation from a nominal value for all probability distributions. It enables in its turn to check adequacy of application of a particular QFS physical model to describe deviation of the observed scale from the nominal value. In this context such a model can be regarded as software tool for metrological control of QFS physical models that are already worked out and just being developed. The algorithm development is based on the supposition of clock deviation from the nominal value in the form of a random recurrent process:

$$x_i = x_{i-1} + f_0\tau + \delta x_i \,, \tag{19}$$

where $x_i$ is clock rate variation as of the time point $t_i$ ;
$f_0$ is generator rated frequency;
$\tau$ is discretization interval;
$\delta x_i$ is a random variable, determining root-mean-square error of clock deviation.

Variable $\delta x_i$ in each interval is calculated according to the formula:

$$\delta x_i = \int_{t_{i-1}}^{t_i} Y_i dt \,, \tag{20}$$

where $Y_i(t) = \sum_{j=1}^{N} y_j(t)$ is relative variation of generator frequency, composed of $N$ components of frequency variations $y_j = \dfrac{f_0 - f_j(t)}{f_0}$;

$f_0$, $f_j(t)$ is an initial value and a current value of generator frequency;
$i$ is number of time interval;
$j$ is number of component of frequency variation.

In Figure 2 a block scheme which illustrates method of the described time scale formation algorithm in case $N = 6$ is given.

A standard programmable random number generator (RNG) synthesizes 6 random number groups $r_1$, $r_2$,... $r_6$ with preset statistical characteristics of spread in 6 intervals of the following sizes: 1,10,…$10^5$ sec. Time scale formation is the result of superposition of six random number groups generated by RNG. Accumulated groups of clock indications are analyzed according to the statistical Alan method to determine statistical characteristics of the time scale being simulated.

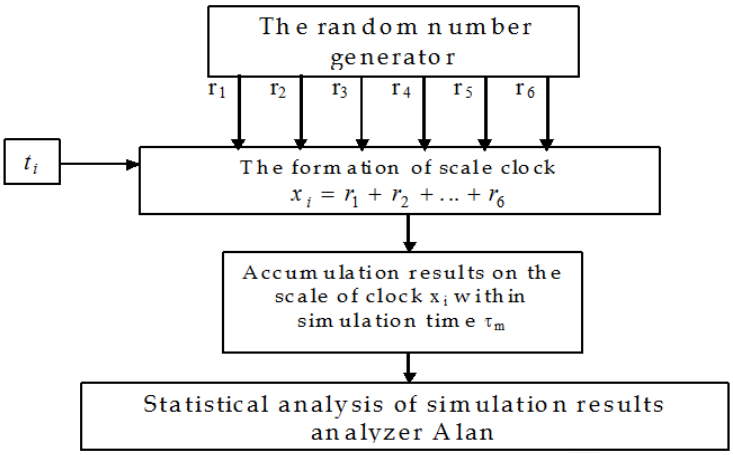


Fig. 2. Block scheme of quantum clock instability model

## 7. Comparison of ERP determination and prediction methods

### 7.1 General information

An almost 200 year long era of precise time determination with conventional astronomical methods ended in 2010. New satellite and radio interferometric techniques with about 2-3-

fold better measurements' resolution have substituted them. However, astrometric observations of stars, moon and planets positions accumulated through over the past centuries are invaluable for an overall picture of the Earth's rotation from the middle of the XVII century to the present. It is impossible to create adequate prediction models of Earth Rotation Parameters (ERP), such as universal time and pole coordinates, without use of these observations over a large enough time periods. A number of applied and fundamental Earth sciences have a need for ERP accurate data and predicted values for different periods. A study of fine structures in Earth rotation speed variations and its poles movements is the most urgent challenges of geodesy, astronomy and geodynamics.

According to a data cited in (Finkelshtein, 2007) the accuracy of ERP in IERS is about 50-60 microseconds of arc for pole coordinates and nutation angles, and about 4-10 microseconds for Universal Time. Achievement of such a high accuracy in comparison to classical methods, where the same estimation was usually made with a tolerance of approximately 1-2 ms, is now possible due to the comprehensive use of new means and techniques of the Measurement Assurance and Earth Rotation Parameters Prediction (MAERPP). The MAERPP Complex functionality includes the following measurement assurance technologies:

- Very Long Baseline Interferometers (VLBI);
- Global Navigation Satellite Systems (GNSS);
- Satellite Laser Ranging Systems (SLR).

Each of the methods for Earth rotation measurement has its own advantages, disadvantages and specific sources of systematic errors. Therefore, combining ERP series obtained by different methods and their combination in a single solution is an effective way to minimize systematic and random errors in ERP definitions.

### 7.2 Definition of Earth rotation parameters means VLBI

The VLBI is the most high-precision and independent ERP determination technique among the ones stated above, because stationary objects of the Universe named quasars are being observed. This method's principle of the geodynamics parameters determination consists in a measurement of the delay time of the same radio signal received by radio telescopes located at a distance from each other. Figure 3 shows the operation principle of the VLBI.

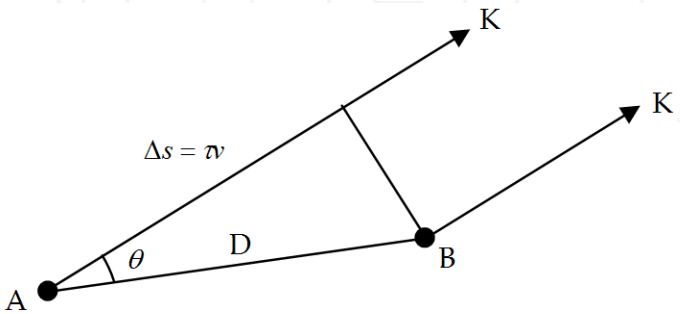


Fig. 3. The measurement principle of the VLBI.

The radio telescopes are installed in the points $A$ and $B$ at the distance D from each other; the interferometer's bases receive the radiation of the same quasar $K$ in the centimetric spectrum synchronously. Processing of radio signals recorded allows determination of the time delay of $\tau$ moments of radio wavefronts' arrival at the radio telescope $A$ relative to the $B$, as well as interference frequency $f$. The difference between the distances from the radio telescopes to the quasar at the moment of observation $\Delta s = \tau\upsilon$, where $\upsilon$ - radio propagation velocity. As a result of coprocessing of multiple values of $\tau$ and $f$ the following parameters can be determined: the difference in geocentric coordinates between the ends of the radio interferometer's base, which is used for determination of the $D$ length and $\Theta$ angle, source's and the Earth's pole coordinates; the Earth's rotation instantaneous velocity; the elements of precession and nutation; the Greenwich apparent time of $\tau$ and $f$ determination moments; and other parameters.

### 7.3 Navigation and laser methods of the ERP determination

By the navigation and satellite laser ranging methods the ERP are being determined jointly with the spacecrafts' orbits parameters subsequent to the results of aggregate measurements from a large number of observation stations. The parameters, which have to be defined more precisely, may also include parameters of the models of atmosphere, Earth's gravity field, solar radiation, etc. The principles and methods of determination of the parameters named above are described quite full in a number of books (Urmaev, 1981), (Duboshin, 1983). However, the descriptions in literature are mostly general in nature. In order to implement them by means of software an additional work on calculation algorithms compilation has to be done first. The algorithm for joint determination of ERP and spacecrafts' orbits parameters, which can be used as a basis for ERP and spacecrafts' orbits parameters software compilation, is shown below.

If the earth referenced coordinates $\bar{X},\bar{Y},\bar{Z}$ of a station tracking the passages of the spacecraft are known and the preliminary approximate values of the $\bar{X}_p,\bar{Y}_p$ pole coordinates and the differences $UT(t) = UT1 - UTC$ are known also, then the station position vector $X,Y,Z$ at the time t in the middle equatorial coordinate system can be calculated by the following formula:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_x(-\varepsilon) R_z(\Delta\Psi) R_x(\varepsilon + \Delta\varepsilon) R_z(-S_\otimes) R\left(\bar{X}_p, \bar{Y}_p\right) \begin{pmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{pmatrix}, \tag{21}$$

where the ERP orientation matrixes are specified as usual:

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}, \; R_z(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; R\left(\bar{X}_p, \bar{Y}_p\right) = \begin{pmatrix} 1 & 0 & -\bar{X}_p \\ 0 & 1 & \bar{Y}_p \\ \bar{X}_p & -\bar{Y}_p & 1 \end{pmatrix};$$

$\varepsilon$ - is the obliquity of the movable equator plane to the plane of the instantaneous ecliptic;
$\Delta\psi$ – is the nutation in longitude;
$\Delta\varepsilon$ -is the nutation in obliquity;
$S_{\oplus}$ is the Greenwich apparent sidereal time calculated at the moment $\left(t+\Delta UT(t)\right)$.

The calculated topocentric distance:

$$\rho_c(t)=\sqrt{(x-X)^2+(y-Y)^2+(z-Z)^2}\ , \tag{22}$$

where $x, y, z$ are the coordinates of the spacecraft in the middle equatorial coordinate system.

Discrepancies $\Delta\rho(t)=\rho_0(t)-\rho_c(t)$ are caused both by pseudorange measurement errors $\rho_0(t)=c(t_{rec}-t_{rad})$ and by errors of an accepted computation model for the $\rho_c(t)$ value, which depends on the spacecraft coordinates $\Delta x, \Delta y, \Delta z$ errors and station coordinates errors $\Delta X, \Delta Y, \Delta Z$, as it follows from the equation (22).

Assuming that these values are small, the equation for discrepancies can be expanded in a Taylor series with the first term of the series only:

$$\Delta\rho_c = A\begin{pmatrix}\Delta x\\ \Delta y\\ \Delta z\end{pmatrix}+B\begin{pmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{pmatrix}, \tag{23}$$

where $A$ and $B$ vectors from the ground station to a satellite and back are as follows:

$$A=\left(\frac{x-X}{\rho_c},\frac{y-Y}{\rho_c},\frac{z-Z}{\rho_c}\right),\ B=-A\ . \tag{24}$$

The transition from the formula (22) to the equation (23) is called linearization.

The spacecraft's coordinates $x, y, z$ have a complex dependence on mean orbital elements in $T_0$ epoch; $\omega$ is the ascending node-perigee angle; $n$ is the mean motion; $\Omega$ *is the* longitude; $i$ *is the* obliquity; $e$ is the eccentricity; $M$ is the mean anomaly.

The coordinates of the station depend on the position of the pole. $X_p, Y_p$.

Since the value $\Delta UT$ belongs to the sidereal time calculation formula $S_{\oplus}\left(t+\Delta UT(t)\right)$, the following substitute can be made: $\Delta UT(t)\rightarrow\Delta S_{\oplus}(t)$ in rad.

Linear relations between the coordinates $X, Y, Z$ and orbital elements of the spacecraft can be represented as a matrix:

$$\begin{pmatrix}\Delta x\\ \Delta y\\ \Delta z\end{pmatrix}=C\begin{pmatrix}\Delta\omega\\ \Delta\Omega\\ \Delta i\\ \Delta e\\ \Delta \mathrm{M}\\ \Delta n\end{pmatrix}, \tag{25}$$

where $C$ is the sensitivity matrix of the spacecraft coordinates to errors of the orbital elements:

$$C = \begin{pmatrix} \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial \Omega} & \frac{\partial x}{\partial i} & \frac{\partial x}{\partial e} & \frac{\partial x}{\partial \mathrm{M}} & \frac{\partial x}{\partial n} \\ \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial \Omega} & \frac{\partial y}{\partial i} & \frac{\partial y}{\partial e} & \frac{\partial y}{\partial \mathrm{M}} & \frac{\partial y}{\partial n} \\ \frac{\partial z}{\partial \omega} & \frac{\partial z}{\partial \Omega} & \frac{\partial z}{\partial i} & \frac{\partial z}{\partial e} & \frac{\partial z}{\partial \mathrm{M}} & \frac{\partial z}{\partial n} \end{pmatrix}. \tag{26}$$

Linear relations for the station coordinates are as follows:

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} = D \cdot \begin{pmatrix} \Delta x_p \\ \Delta y_p \\ S_\oplus \end{pmatrix}, \tag{27}$$

where $D$ is the sensitivity matrix of the spacecraft coordinates to ERP errors:

$$D = \begin{pmatrix} \frac{\partial X}{\partial x_p} & \frac{\partial X}{\partial y_p} & \frac{\partial X}{\partial S_\oplus} \\ \frac{\partial Y}{\partial x_p} & \frac{\partial Y}{\partial y_p} & \frac{\partial Y}{\partial S_\oplus} \\ \frac{\partial Z}{\partial x_p} & \frac{\partial Z}{\partial y_p} & \frac{\partial Z}{\partial S_\oplus} \end{pmatrix}. \tag{28}$$

Since the partial derivatives of along the spacecraft's ascending node longitude $\Omega$ and the sidereal time $S_\oplus$ are calculated as follows:

$$\frac{\partial x}{\partial \Omega} = \frac{\partial X}{\partial S_\oplus} = -y\,;\ \frac{\partial y}{\partial \Omega} = \frac{\partial Y}{\partial S_\oplus} = x\,;\ \frac{\partial z}{\partial \Omega} = \frac{\partial Z}{\partial S_\oplus} = 0\,, \tag{29}$$

the last columns of the matrixes $C$ and $D$ are linearly dependent. Hence it is impossible to determinate $\Delta\Omega$ and $\Delta UT$ at the same time, and therefore the value $\Delta UT(t)$ at small time intervals shall be approximated by the linear function:

$$\Delta UT(t) = \Delta UT(t_0) + \frac{DR(t - t_0)}{86400}\,, \tag{30}$$

where $DR$ (sec.day) is variation length of the day.

Combining the equations (27) and (28) we get:

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} = \begin{pmatrix} \frac{\partial X}{\partial x_p} & \frac{\partial X}{\partial y_p} & \frac{\partial X}{\partial \dot{S}_\oplus} \\ \frac{\partial Y}{\partial x_p} & \frac{\partial Y}{\partial y_p} & \frac{\partial Y}{\partial \dot{S}_\oplus} \\ \frac{\partial Z}{\partial x_p} & \frac{\partial Z}{\partial y_p} & \frac{\partial Z}{\partial \dot{S}_\oplus} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_p \\ \Delta y_p \\ \Delta \dot{S}_\oplus \end{pmatrix} \tag{31}$$

where $\dot{S}_{\oplus} = n_{\oplus} + \Delta\dot{S}_{\oplus}$ ; $n_{\oplus} \approx 0{,}72921 \cdot 10^{-4}$ radian in sec - mean motion;

$\Delta\dot{S}_{\oplus}$ is the correction for the accepted value $n_{\oplus}$ .

$$\bar{D} = \begin{pmatrix} \frac{\partial X}{\partial x_p} = -z\cos S_{\oplus} & \frac{\partial X}{\partial y_p} = -z\sin S_{\oplus} & \frac{\partial X}{\partial \dot{S}_{\oplus}} = -y(t-t_0) \\ \frac{\partial Y}{\partial x_p} = -z\sin S_{\oplus} & \frac{\partial Y}{\partial y_p} = z\cos S_{\oplus} & \frac{\partial Y}{\partial \dot{S}_{\oplus}} = x(t-t_0) \\ \frac{\partial Z}{\partial x_p} = \bar{X} & \frac{\partial Z}{\partial y_p} = -\bar{Y} & \frac{\partial Z}{\partial \dot{S}_{\oplus}} = 0 \end{pmatrix}. \tag{32}$$

Then the following formula will be true for calculation of the $DR$ value:

$$DR = -\frac{2\pi}{n_{\oplus}} \cdot \frac{\Delta\dot{S}_{\oplus}}{n_{\oplus}}, \tag{33}$$

In the above expressions for the partial derivatives (32) $t_0$ is a fixed time, at which the ERP is determined.

Combining the equations (23), (25) and (31) we get the following conditional equation of the least-squares method:

$$\rho_0(t) - \rho_c(t) = A \cdot C \begin{pmatrix} \Delta\omega \\ \Delta\Omega \\ \Delta i \\ \Delta e \\ \Delta M \\ \Delta n \end{pmatrix} + B \cdot \bar{D} \begin{pmatrix} \Delta x_p \\ \Delta y_p \\ \Delta\dot{S}_{\oplus} \end{pmatrix}, \tag{34}$$

in the matrixes $A, B, C, \bar{D}$ all the partial derivatives are known, since they are calculated on the accepted initial values of mean orbital elements at epoch $T_0$: $\omega, \Omega, i, e, M, n$ , Earth Rotation Parameters $x_p, y_p, \Delta UT(t)$ and coordinates of observatories $\bar{X}, \bar{Y}, \bar{Z}$ .

Corrections $\Delta\omega, \Delta\Omega, \Delta i, \Delta e, \Delta M, \Delta n, \Delta x_p, \Delta y_p, \Delta S_{\oplus}$ are calculated on a number of the spacecraft's constellation observations from several stations at different points of time.

N conditional equations can be worked out on the aggregate data obtained. Discrepancies in the conditional equations can be minimized by the least-squares method by working out the M normal equations corresponding to the number of the parameters to be determined:

$$\sum_{i=1}^{N} \frac{\partial\rho_0(t)}{\partial P_k} \Delta\rho(t) = \sum_{j=1}^{M} \left( \sum_{i=1}^{N} \frac{\partial\rho_c(t)}{\partial P_k} \frac{\partial\rho_c(t)}{\partial P_j} \right) \Delta P_j \tag{35}$$

where $k = 1, \ldots M$ .

Calculated from equation (35) corrections for $\Delta P_j$ add a parameter $P_j$ to the initial values and the process is repeated again until the value of discrepancies get below a specified value $\varepsilon$ .

The algorithm of the differential method for joint correction of the ERP series and spacecraft's orbits parameters is applicable both to navigation and to laser ranging measuring instruments. The main difference between the Satellite Laser Ranging (SLR) method and the GNSS technology is that SLR uses not a radio range of wave length, but an optical spectrum for measurements. This reduces atmospheric effects, eliminates uncertainty of the multiline radio signal propagation and provides a potentially higher resolution of measurements. In addition, the laser ranging of satellites allows calculating a change rate of RLS positions coordinates in the global velocity field. Precision geodetic systems are created on the basis of SLR location points, which monitor the geodetic satellites. WGS-84 is the most famous of these, which consists of dozens of points, located on the Earth's surface. The coordinates of these points are determined constantly, that allows to register tectonic processes. Information processing is performed by specialized centres, where it goes through operational communication channels, for example, the Internet. It implements the main advantage of the laser ranging in comparison to the radio positioning i.e. the possibility to determine a displacement of SLR points on the surface and in height during the geophysical researches. Such networks have a zero accuracy grade according to the geodetic classification. They are a basis for development of other levels of engineering networks integrated into the global system WGS-84. However, despite these advantages the SLR means are less influential in the ERP determination compared to the GNSS and VLBI techniques, so they inferior to them in mass and immediacy of the observational data supply.

In order to compare the quality of the different methods for ERP determination let us present data obtained from the bulletin of the Russian main metrology centre State service of time, frequency and IERS's bulletin. Table 2 shows the systematic deviations by the ERP determination from the IERS data, obtained in 2010 in the leading Russian processing centres according to the: Glonass/GPS; Satellite Laser and VLBI within the network of points of the domestic and world stations (Bulletin E-141-144, 2010).

| Techniques | Analysis centre | dUT1 (µas) | X (0,00001") | Y (0,00001") |
|---|---|---|---|---|
| GNSS (Gl/GPS) | SSTF | - 16 | +55 | -43 |
| GNSS (Gl/GPS) | IAA | + 6 | +31 | +52 |
| Satellite Laser | IAA | + 14 | +61 | +9 |
| Satellite Laser | IAC | - | +58 | -19 |
| VLBI - 24 h | SSTF | + 16 | -58 | +36 |
| VLBI - 24 h | IAA | + 9 | +53 | +47 |

Table 2. Systematic deviations by the ERP determination using different techniques in 2010 according to the Russian processing centres.

The following abbreviations are used in the table 2:

SSTF - State service of time, frequency and the Earth rotation parameters determination;
IAA - Institute for applied astronomy of the Russian Academy of Sciences (RAS);
IAC - Information-analytical centre TSNIIMASH.

Table 3 shows root-mean-square errors (RMS) of the ERP determinations obtained in 2010 by IERS according to the result of GPS, VLBI and SLR measurements processing in the leading processing centres (Bulletin B 285, 2011). Since there are more than 30 processing centres in the IERS Bulletins, we will state only the RMS values spread limits for ERP determinations in these centres.

| Techniques | UT1 (μas) | LOD (μas) | X (0.000001") | Y (0.000001") |
|---|---|---|---|---|
| VLBI - 24 h | 8-15 | - | 30-100 | 70-150 |
| VLBI –Intensive | 14-17 | - | - | - |
| Satellite Laser | - | 15-30 | 100-160 | 80-200 |
| GNSS (GPS) | - | 5-30 | 13-45 | 13-40 |

Table 3. Estimated accuracies of the ERP determinations by using different techniques in 2010 according to the IERS data.

Analyzing the data in the Tables 2 and 3 we can conclude that the VLBI technique is the most accurate for the universal time parameter definition and is inferior to the GNSS in accuracy of the pole coordinates determination. The SLR method shows the results with a similar accuracy of the pole coordinates determinations in comparison with VLBI and is also inferior to the GNSS technique. However, the SLR and GNSS techniques have similar results by determination of the length of the day. It should also be noted that the data obtained from the Russian processing centres (see Table 2.) conform to an international standard for ERP determination accuracy level.

### 7.4 ERP prediction

#### 7.4.1 Statement and urgency of the problem

Prediction of the Earth's rotation parameters is an important activity of the International Earth Rotation Service and national services of time and frequency in some progressive countries. The need of ERP prediction is dictated first of all by fast developing space technologies for civil and military areas. The ERP values, calculated on the basis of observations' processing, are related to the time frames when these observations were made. Therefore the ERP data at the current time can get known only on the basis of their precomputed values. The problem is that the precomputed (ephemeris) ERP values belong to the initial conditions by solution of differential equations of the GNSS spacecrafts motion. This is particularly important when it becomes necessary to transfer the GNSS spacecrafts in an autonomous mode. The ERP prediction errors, ultimately, limit the maximum achievable accuracy of the time and coordinate support (TCS), as well as element of exterior orientation (EEO), because all other causes of errors can be minimized by means of new developments and improvements of the measuring instruments, as well as enhancements of physical and mathematical theories of environmental factors' effect on the results of navigation measurements. Besides the handling of the EEO challenges the high-precision ERP predictions are also necessary for solving of the TCS problems of civil and military facilities, which are during a long time located in points, inaccessible for GNSS signals; they are also important for challenges of geophysics and meteorology by building of matching models of

the internal structure of the Earth and global climate change, as well as for geodynamics and geodesy challenges by the more precise definition of the Earth orientation parameters relative to the inertial sidereal reference system and coordinates determination of points relative to the absolute coordinate system.

Actual requirements of the GNSS EEO are based on the fact that the influence of ERP determination and prediction errors on the overall TCS error is insignificant. For example, pursuant to this condition, the universal time prediction error for the specified 15 days period of the GLONASS spacecraft operation in the autonomous mode must not exceed a value close to 1 ms. Unfortunately, despite the high level of modern scientific and technological developments in the field of the ERP prediction, an achievement of such level of accuracy is still a complicated and unsolved problem.

### 7.4.2 Proposed solutions

The proposed methods are based on the presentation of a number of known UT1 observations data at long periods of 100 years or more in form of a superposition of a large number of harmonics. Such an approach contradicts to some extent the currently prevailing point of view that non-periodic trend variations in the Earth's rotation speed influence much more than periodic components of the Earth's rotation irregularity. For example, according to data cited in (Belotserkovskiy, Kaufman, 1972), contribution to the overall universal time prediction error of 1 year due to the unpredictable trend variations is about 99% compared with the contribution due to the imperfection of tidal and seasonal models of the Earth's rotation speed variations. It allows to make an unequivocal conclusion that the only way to improve the reliability and accuracy of the ERP prediction is connected with a solving of the problem and a development of the prediction method most sensible to recent trend variations. According to the authors' investigations, the creation of such model is possible only subject to involvement of as much as possible known data for estimation of the prediction parameters.

### 7.4.3 Structure of the source data

Thanks to astronomical observations data of the moon and planets motion, maintained from the middle of the seventeenth century, we now have an opportunity to reconstruct the picture of the Earth's rotation over the past 350 years at least approximately. The Figure 4 represents a diagram of the $UT1$ parameter variations starting from the 1656 to the present day in the Terrestrial Dynamic Time system $TT$. The diagram was drawn on the basis of the $\Delta T = UT1 - TT$ differences data, represented in (AE SSSR, 1970 and Konstantinov A. I, Fleer A.G., 1971).

Analyzing the diagram in the Figure 4 we can note that the Earth's rotation speed in past centuries varied slightly and rather evenly until the late nineteenth century, when a sharp slowdown occurred. During the past 100 years some periods of Earth's rotation deceleration and acceleration can be traced with specific intervals of 20-30 years. Nowadays the Earth's rotation slowdown is observed, which is likely to be continued during the next 15 to 20 years.

In the Figure 5 taken from (IERS) the dots indicate the Earth's pole wandering in 1996-2000., and the solid line indicate the trajectory of the mean pole from 1890 to 2000.

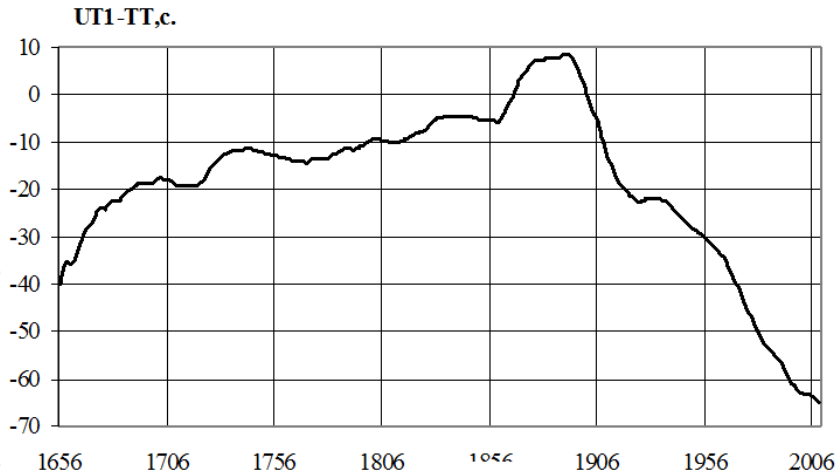


Fig. 4. Earth's rotation variations for the period from 1650 to 2010.

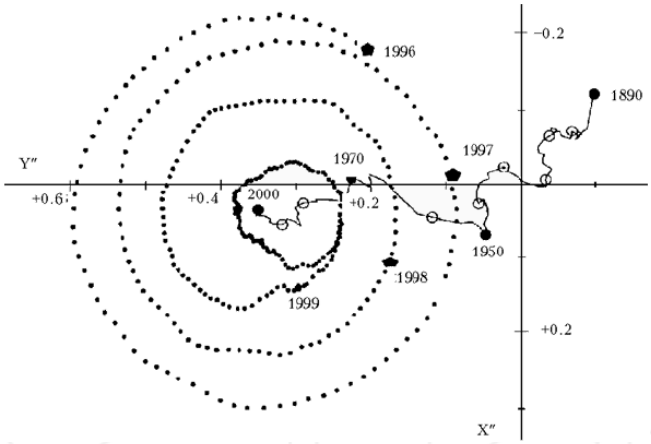


Fig. 5. Wandering of the Earth's North Pole

Analyzing the data, represented in the Figure 5, we can note that during the whole time of observations the mean pole was being displaced at a speed of about 10 cm/year along a complicated zigzag curve mainly in the direction of North America (meridian 290 ° EL).

### 7.5 Overview of the proposed ERP prediction method

The proposed method is based on approximation of periodic and quasi-periodic processes of the Earth's rotation using a polyharmonical mathematical model built on an ERP observations sample for at least 100 years. The calculation technique is based on the sequential adjustment of the original ERP time series by corrections introduction for tidal harmonics (for *dUT*1 series), calculated on the basis of known physical and mathematical theory of the sun and moon motions, as well as harmonics corrections, obtained with

statistical methods (for all ERP series) . Generalized formula for $dUT1$ parameter prediction can be written down as follows:

$$dUT1_{prg,i} = dUT1_{real,0} - dUT1_{pa,0} + dUT_{pa,i} + \Delta T_{pr,i} + \Delta T_{tr,i} + \Delta T_{sz,i}\,, \tag{36}$$

where $dUT1_{prg,i}$ is the prediction of $dUT1$ for the $i$ -day;
$dUT1_{real,0}$ is the known value of $dUT1$ for the reference day;
$dUT1_{pa,0}$ is the prediction of $dUT1$ for the reference day made by means of the autoregression;
$dUT1_{pa,i}$ is the prediction of $dUT1$ for the $i$-day made by means of the autoregression;
$\Delta T_{pr,i}$ , $\Delta T_{tr,i}$ , $\Delta T_{sz,i}$ are the corrections for the lunar and solar tides in the oceans, and for trend and seasonal variations at the $i$ -day.

Tidal harmonics corrections are being calculated in accordance with the method accepted in the IERS (McCarthy, 2003). Consideration of the trend influence corrections by means of the harmonic model composed of harmonics with periods from 1 year to 70 years. Calculation of these harmonics parameters is performed by means of a specially developed for this purpose method of step-by-step summation of intervals of the time series. (Tissen et al., 2009). In order to calculate corrections for seasonal variations of the dUT1 parameter the polyharmonic model shall be used as a sum of a year, half-year wave and an unlimited number of harmonics with periods less than a year. Calculation these harmonics parameters is performed at time intervals of about 15-25 years. The existing methods of dUT1 prediction approximate these variations by a sum of year and half-year harmonics, whose parameters are estimated within the period of 4-6 years. In order to consider short-term stochastic components of the ERP series variations a mathematical model's component in a form of the autoregression shall be used as a restriction to smoothness.

Predictions for the pole coordinates shall be made in a similar way, except that in the equation (36) the influence of the lunar-solar tides shouldn't be taken into account because of their smallness.

### 7.6 Main results

Reports on the results of the ERP predictions obtained using the method developed in SNIIM from 2007 to 2011 were made many times at national conferences in Moscow and St. Petersburg, and at an international conference in Warsaw (Tissen & Tolstikov, 2011) in October 2009 (Tissen et al. , 2009).

Since October 2010 the method is being testing on the basis of the results of SNIIM/SSGA participation in the Earth Orientation Parameters Combination of Prediction Pilot Project (EOPCPPP). Participation in this project presupposes the daily data transfer into the main IERS centres: the Space Research Center of the Polish Academy of Science, Warsaw (eopcppp@cbk.waw.pl) and the United States Naval Observatory (USNO), Washington, District Columbia (eopcppp@maia.usno.navy.mil). Figures 5-7 show evaluations of the RMS predictions for universal time dUT1 and pole coordinates: $x_P$ , $y_P$ at intervals from 1 to 90 days by all EOPCPPP participants. The evaluations above, except for 3 participants, were received during the period of the project from September 2010 to November 2011

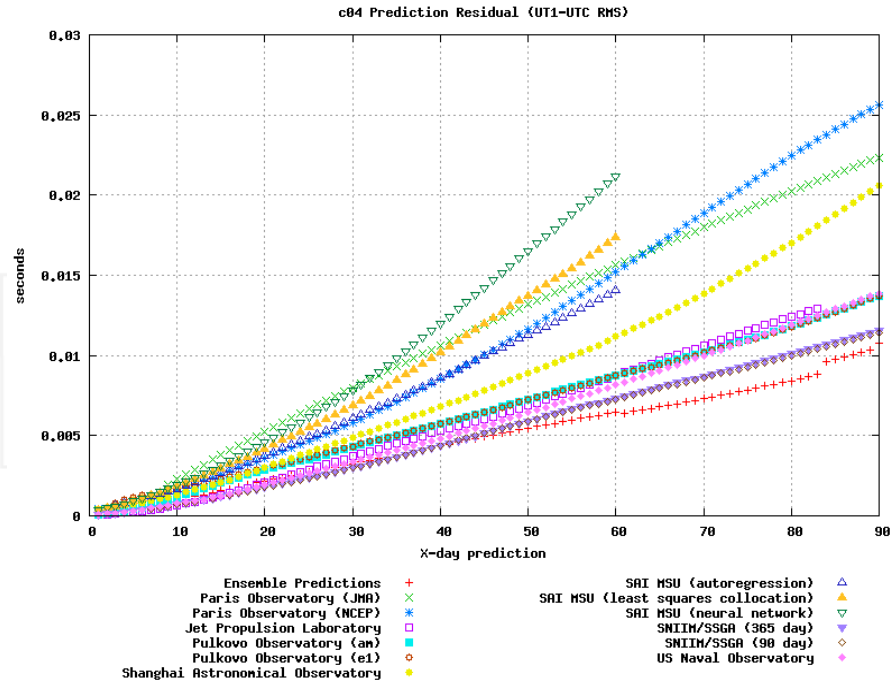


Fig. 6. Comparison of RMS for universal time predictions for 90 days made by EOPCPPP participants from September 2010 to November 2011.

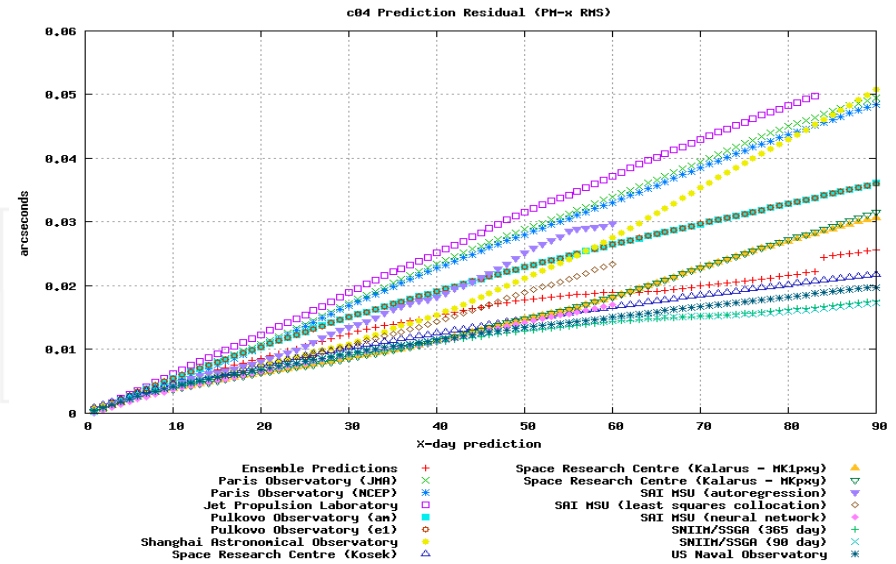


Fig. 7. Comparison of RMS for pole coordinates predictions: $X_p$ and $Y_p$ (top down, respectively) for 90 days made by EOPCPPP participants from September 2010 to November 2011.

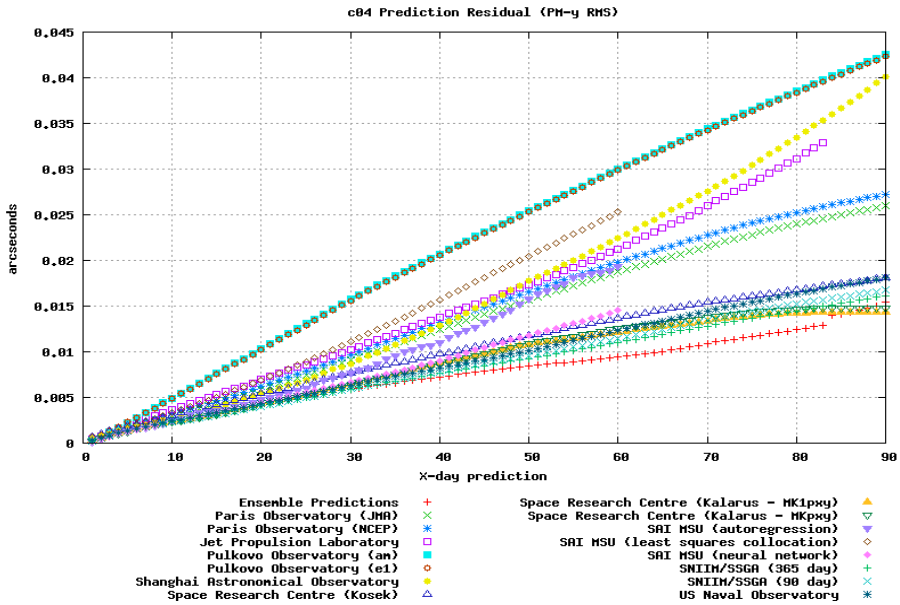


Fig. 8. Comparison of RMS for pole coordinates predictions: $X_p$ and $Y_p$ (top down, respectively) for 90 days made by EOPCPPP participants from September 2010 to November 2011.

Graphic data in the Figures 6-8 demonstrates an advantage of the ERP prediction method of the SNIIM/SSGA in comparison to the methods used by the other EOPCPPP participants, including those with official global suppliers of the ERP predictions used by the IERS (USNO).

During the first 13 months of the pilot project EOPCPPP, which involves more than 10 different methods for ERP prediction, the method used by SNIIM-SSGA showed the best results. In particular, a prediction accuracy of the universal time parameter, which is considered as the most important and difficult to predict one, within the period of less than 30 days was 1.19 times higher by the SNIIM - SSGA in 350 realizations than by the IERS and 1.93 times higher than by SSTF of Russia. Similar results were also obtained in longer intervals of up to 90 days and more. On the IERS website «eopcppp@maia.usno.navy.mil» ERP predictions files from all the EOPCPPP project participants are published daily with their metrological characteristics according to the evaluations of RMS, absolute error (MAE) and standard deviation.

## 8. Conclusions

The issues stated in this chapter and the analyses were made on the basis of materials published in an open literature, as well as the results obtained directly by the authors. The historical information about the development of techniques and methods of time measuring, description of functions and responsibilities of the services of time and description of their workflow by example of the Novosibirsk time and frequency service are given for the familiarization purpose only. This information is necessary for

understanding of the most urgent issues and challenges of time, coordinate and navigational determinations, which successful solution depends largely on the time services' development level.

The simulation model of the quantum clock instability introduced in the fourth section is now used within the scope of the software and mathematical support of GLONASS. It can also be useful for verification of the adequacy of the metrological characteristics of existing and newly developed quantum frequency standards to accepted physical theories and models of their instability. The algorithms from the fifth section for codetermination of the spacecrafts' orbits parameters and ERP on the basis of ground (laser or navigation) measurements introduces the differential method for calculating of the parameters to be determined and can be used as a basis for development of the relevant software.

The sixth section represents comparative assessments of the ERP predictions made by the EOPCPPP project participants, according to which the quality of the SNIIM / SSGA predictions for the period from October 2010 to November 2011 on the majority of statistical evaluations for all ERP was preferable to all others, including the quality of the predictions provided by the USNO observatory. The good results obtained by the SNIIM/SSGA in the EOPCPPP project let us conclude that the right approach was used by the developing of the method. Particularly, the numerical analysis of ERP series of hundred years and more made by us shows that general regular compositions with specific periods of 2.4, 3.6, 4.8, 6.0 multiples of the main Chandler wobbles' period can be observed by the Earth's rotation variations and its poles wanderings. The exposed cross-correlation of the ERP series can be explained by the fact that the Earth's mass redistribution relative to its rotation axis occurs by the pole wandering. Consequently, the total moment of inertia relative to the Earth's rotation axis is changing leading to its angular velocity variations provided that the total angular momentum maintains. The discovered regularities and correlations lead us to the conclusion on the feasibility of creation of a generalized predictive model for the Earth's rotation, which combines all the Earth's angular velocity vector's components into a single consistent calculation pattern.

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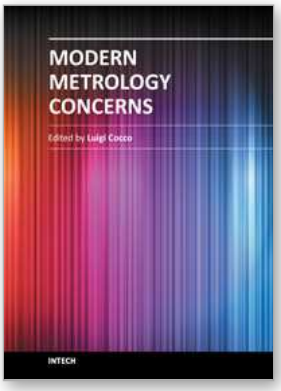


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