Cell Dwell Time and Channel Holding Time Relationship in Mobile Cellular Networks

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1. Introduction

Channel holding time (CHT) is of paramount importance for the analysis and performance evaluation of mobile cellular networks. This time variable allows one to derive other key system parameters such as channel occupancy time, new call blocking probability, and handoff call dropping probability. CHT depends on cellular shape, cell size, user’s mobility patterns, used handoff scheme, and traffic flow characteristics. Traffic flow characteristics are associated with unencumbered service time (UST), while the overall effects of cellular shape, users’ mobility, and handoff scheme are related to cell dwell time (CDT).

For convenience and analytical/computational tractability, the teletraffic analysis of mobile cellular networks has been commonly performed under the unrealistic assumption that CDT and/or CHT follow the negative exponential distribution (Lin et al., 1994; Hong & Rappaport, 1986). However, a plenty of evidences showed that these assumptions are not longer valid (Wang & Fan, 2007; Christensen et al., 2004, Fang, 2001, 2005; Orlik & Rappaport, 1998; Fang & Chlamtac, 1999; Fang et al., 1999; Alfa & Li, 2002; Rahman & Alfa, 2009; Soong & Barria, 2000; Yeo & Jun, 2002; Pattaramalai, et al., 2007). Recent papers have concluded that in order to capture the overall effects of users’ mobility, one needs suitable models for CDT distribution (Lin, 1994; Hong & Rappaport, 1986). In specific, the use of general distributions for modeling this time variable has been highlighted. In this research direction, some authors have used Erlang, gamma, uniform, deterministic, hyper-Erlang, sum of hyper-exponentials, log-normal, Pareto, and Weibull distributions to model the pdf of CDT; see (Wang & Fan, 2007; Fang, 2001, 2005; Orlik & Rappaport, 1998; Fang & Chlamtac, 1999; Fang et al., 1997, 1999; Rahman & Alfa, 2009; Pattaramalai et al., 2007, 2009; Hidata et al., 2002; Thajchayapong & Toguz, 2005; Khan & Zeghlache, 1997; Zeng et al. 2002; Kim & Choi, 2009) and the references therein. Fang in (Fang, 2001)) emphasizes the use of phase-type (PH) distributions for modeling CDT. The reason is twofold. First, PH distributions provide accurate description of the distributions of different time variables in wireless cellular networks, while retaining the underlying Markovian properties of the distribution. Markovian properties are essential in generating tractable queuing models for cellular networks. Second, there have been major advances in fitting PH distributions to real data. Among the PH probability distributions, the use of either Coxian or Hyper-Erlang distributions are of
particular interest because their universality property (i.e., they can be used to approximate any
non-negative distribution arbitrarily close) (Soong & Barria, 2000; Fang, 2001).

Due to the discrepancy and the wide variety of proposed models, it appears mandatory to
investigate the implications of the cell dwell time distribution on channel holding time
characteristics in mobile wireless networks. This is the topic of research of the present
chapter. Let us describe the related work reported in this research direction.

1.1 Previously related work

In (Fang, 2001; Zeng et al. 2002), it is observed that, depending on the variance of CDT, the
mean channel holding time for new calls (CHTn) can be greater than the mean channel
holding time for handoff calls (CHTh). However, in these works, it is neither explained nor
discussed the physical reasons for this observed behavior. This phenomenon (which is
addressed in Section 3.1) and the lack of related published numerical results have motivated
the present chapter.

Most of the previously published papers that have developed mathematical models for the
performance analysis of mobile cellular systems considering general probability distribution
for cell dwell time have either only presented numerical results for the Erlang (Wang & Fan,
2007; Fang et al., 1999; Rahman & Alfa, 2009; Kim & Choi, 2009) or Gamma distributions
with shape parameter greater than one\(^1\) (Yeo & Jun, 2002; Fang, 2005), and/or only for the
CHTh\(^2\) (Fang, 2001; Fang & Chlamtac, 1999), or have not presented numerical results at all
(Fang, 2005; Alfa & Li, 2002; Soong & Barria, 2000). Thus, numerical results both for values
of the coefficient of variation (CoV) of CDT greater than one and/or for the CHTn have been
largely ignored. Exceptions of this are the papers (Orlik & Rappaport, 1998; Fang et al., b,
1997; Pattaramalai, et al., 2009).

On the other hand, probability distribution of CHT has been determined under the
assumption of the staged distributions sum of hyper-exponentials, Erlang, and hyper-Erlang
for the CDT (Orlik & Rappaport, 1998; Soong & Barria, 2000). However, to the best of the
authors’ knowledge, probability distribution of CHT in mobile cellular networks with
neither hyper-exponential nor Coxian distributed CDT has been previously reported in the
literature.

In this Chapter, the statistical relationships among residual cell dwell time (CDTr), CDT,
and CHT for new and handoff calls are revisited and discussed. In particular, under the
assumption that UST is exponentially distributed and CDT is phase-type distributed, a
novel algebraic set of general equations that examine the relationships both between CDT
and CDTr and between CDT and channel holding times are obtained. Also, the condition
upon which the mean CHTn is greater than the mean CHTh is derived. Additionally, novel
mathematical expressions for determining the parameters of the resulting CHT distribution
as functions of the parameters of the CDT distribution are derived for hyper-exponentially
or Coxian distributed CDT.

\(^1\) For the Erlang distribution and for the Gamma distribution with shape parameter greater than one, the
coefficient of variation of its associated random variable is smaller than one.

\(^2\) Also referred as handoff call channel occupancy time.
2. System model

A homogeneous multi-cellular system with omni-directional antennas located at the centre of each cell is assumed; that is, the underlying processes and parameters for all cells within the cellular network are the same, so that all cells are statistically identical. As mobile user moves through the coverage area of a cellular network, several variables can be defined: cell dwell time, residual cell dwell time, channel holding time, among others. These time variables are defined in the next section.

2.1 Definition of time interval variables

In this section the different time interval variables involved in the analytical model of a mobile cellular network are defined.

First, the unencumbered service time per call $x_s$ (also known as the requested call holding time (Alfa and Li, 2002) or call holding duration (Rahman & Alfa, 2009)) is the amount of time that the call would remain in progress if it experiences no forced termination. It has been widely accepted in the literature that the unencumbered service time can adequately be modeled by a negative exponentially distributed random variable (RV) (Lin et al., 1994; Hong & Rappaport, 1986). The RV used to represent this time is $X_s$ and its mean value is $\mathbb{E}(X_s) = 1/\mu$.

Now, cell dwell time or cell residence time $x_d(j)$ is defined as the time interval that a mobile station (MS) spends in the $j$-th (for $j = 0, 1, \ldots$) handed off cell irrespective of whether it is engaged in a call (or session) or not. The random variables (RVs) used to represent this time are $X_d(j)$ (for $j = 0, 1, \ldots$) and are assumed to be independent and identically generally phase-type distributed. For homogeneous cellular systems, this assumption has been widely accepted in the literature (Lin et al., 1994; Hong & Rappaport, 1986; Orlik & Rappaport, 1998; Fang & Chlamtac, 1999; Alfa & Li, 2002; Rahman & Alfa, 2009).

In this Chapter, cell dwell time is modeled as a general phase-type distributed RV with the probability distribution function (pdf) $f_{X_d}(t)$, the cumulative distribution function (CDF) $F_{X_d}(t)$, and the mean $\mathbb{E}(X_d) = 1/\eta$.

The residual cell dwell time $x_r$ is defined as the time interval between the instant that a new call is initiated and the instant that the user is handed off to another cell. Notice that residual cell dwell time is only defined for new calls. The RV used to represent this time is $X_r$. Thus, the probability density function (pdf) of $X_r$, $f_{X_r}(t)$, can be calculated in terms of $X_d$ using the excess life theorem (Lin et al., 1994)

$$f_{X_r}(t) = \frac{1}{\mathbb{E}(X_d)} [1 - F_{X_d}(t)]$$

where $\mathbb{E}(X_d)$ and $F_{X_d}(t)$ are, respectively, the mean value and cumulative probability distribution function (CDF) of $X_d$.

Finally, we define channel holding time as the amount of time that a call holds a channel in a particular cell. In this Chapter we distinguish between channel holding times for handed off (CHTh) and channel holding time for new calls (CHTn). CHTh (CHTn) is represented by the random variable $X_c(h)$ ($X_c^{(N)}$).
3. Mathematical analysis

3.1 Relationship between $X_d$ and $X_r$

The relationship between the probability distributions of CDT and CDTr is determined by the residual life theorem. In Table I some particular typically considered CDT distributions and the corresponding CDTr distributions obtained by applying the residual life theorem are shown.

<table>
<thead>
<tr>
<th>Probability density function of cell dwell time or its Laplace transform.</th>
<th>Probability density function of residual cell dwell time or its Laplace transform.</th>
<th>Parameters of $f_{X_d}(t)$ as a function of the parameters of $f_{X_d}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Exponential $\eta e^{-\eta t}$</td>
<td>Negative Exponential $\eta e^{-\eta t}$</td>
<td>$p_j^{(N)} = \frac{1}{k}$</td>
</tr>
<tr>
<td>Erlang of $k$ order $\frac{\eta^k (k-1)! e^{-\eta t}}{(k-1)!}$</td>
<td>Hyper-Erlang with $k$ stages of 1, 2, ..., and $k$ phases $\sum_{j=1}^{k} p_j^{(N)} \frac{\eta^j (t)^j}{(j-1)!} e^{-\eta t}$</td>
<td>$p_i^{(N)} = \frac{p_i \prod_{j=1}^{n} \lambda_i}{\sum_{j=1}^{i} p_i \prod_{j=1}^{n} \lambda_j}$</td>
</tr>
<tr>
<td>Hyper-exponential of $n$ order $\sum_{i=1}^{n} p_i \lambda_i e^{-\lambda_i t}$</td>
<td>Hyper-exponential of $n$ order $\sum_{i=1}^{n} p_i^{(N)} \lambda_i e^{-\lambda_i t}$</td>
<td>$p_i^{(N)} = \frac{1}{\eta_i} \frac{1}{\sum_{j=1}^{m} \eta_j}$</td>
</tr>
<tr>
<td>Hypo-exponential(^3) of $m$ order $f_{X_d}^*(s) = \prod_{i=1}^{m} \frac{\eta_i}{s + \eta_i}$</td>
<td>Generalized Coxian of $m$ order $f_{X_d}^*(s) = \sum_{i=1}^{m} p_i^{(N)} \prod_{j=1}^{m} \frac{\eta_j}{s + \eta_j}$</td>
<td>$p_i^{(N)} = \frac{1}{\eta_i} \frac{1}{\sum_{j=1}^{m} \eta_j}$</td>
</tr>
<tr>
<td>Hyper-Erlang of common order $(n, m)$ $\sum_{i=1}^{n} \eta_i^{m} (m-1)! e^{-\eta_i t}$</td>
<td>Hyper-Erlang of non common order $\sum_{i=1}^{n} p_i^{(N)} \frac{(\eta_i^{[i-1]} / m^{[i-1]})^z t^{z-1}}{(z-1)!} e^{-\eta_i^{[i-1]} t}$</td>
<td>$p_i^{(N)} = \frac{p_i \prod_{i}^{n} \eta_i}{\sum_{k=1}^{m} p_k \prod_{i}^{n} \eta_i}$</td>
</tr>
<tr>
<td>Constant ${0 ; \delta(t - E(X_d))$ if $t = E(X_d)$ otherwise $}$</td>
<td>Uniform $\frac{1}{E(X_d)}$; $0 \leq t \leq E(X_d)$</td>
<td>$z = \text{mod}(i - 1, m) + 1$</td>
</tr>
<tr>
<td>Coxian of $m$ order $f_{X_d}^*(s) = \sum_{i=1}^{m} p_i^{(N)} \frac{\prod_{j=1}^{m} \eta_j}{s + \eta_j}$</td>
<td>Generalized Coxian of $m$ order $f_{X_d}^*(s) = \sum_{j=1}^{2} p_j^{(N)} \left( \prod_{k=h(j)}^{m} \frac{\eta_j}{s + \eta_j} \right)$</td>
<td>$p_j^{(N)} = \frac{p_j \prod_{i=1}^{m} \eta_k}{\sum_{i=1}^{2} \left( \prod_{k=1}^{m} \eta_k \right) \sum_{i=1}^{m} p_i}$</td>
</tr>
</tbody>
</table>

\(^3\) Also known as Generalized Erlang.
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Probability density function of cell dwell time or its Laplace transform.

| Parameters of $f_{X_a}(t)$ as a function of the parameters of $f_{X_d}(t)$ |
|---|---|---|
| $p_j^{(N)}$ | $\prod_{k=1}^{m} \eta_k \left( \sum_{n=j-h(j)+1}^{j} P_n \right)$ |
| $A$ | $\sum_{i=1}^{m} \prod_{j=1}^{m} \eta_j \left( \sum_{l=1}^{m} P_{l^2+i+2} \right)$ |
| | $\sum_{l=1}^{i-1} P_{l^2+i+2}$ |

Table I. Examples of corresponding distribution for $X_r$ given the distribution of $X_d$.

- **Gamma**
  
  $x^{k-1}e^{-\frac{x}{\theta}} \frac{1}{\Gamma(k)\theta^k}$

- **Weibull**
  
  $k \frac{x}{\lambda}^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \frac{1}{\lambda^\alpha \Gamma\left(1+\frac{1}{k}\right)} e^{-\left(\frac{x}{\lambda}\right)^k}$

- **Pareto**
  
  $\frac{\alpha X_m^\alpha t^\alpha}{t^{\alpha+1}}$; $t > X_m$

The functional relationship between the moments of the residual cell dwell time and the cell residual time was obtained in (Kleinrock, 1975) applying the Laplace transform to the residual life theorem. That is,

$$\mathcal{L}\{f_{X_r}(t)\} = \mathcal{L}\left\{\frac{1}{E[X_a]}\right\} - \mathcal{L}\left\{\frac{1}{E[X_d]}F_{X_d}(t)\right\} \quad (2)$$

This equation can be rewritten as

$$f_{X_r}^{(N)}(s) = \left[\frac{1}{s} \frac{1}{E[X_a]} \right] \left[ 1 - f_{X_d}^{(N)}(s) \right] \quad (3)$$

The $n$-th moment of the residual cell dwell time in terms of the moments of the cell dwell time can be obtained by deriving $n$ times equation (3) with negative argument and substituting $s=0$. Then (Kleinrock, 1975),
The mean residual cell dwell time as function of the moments of cell dwell time can be obtained as (Kleinrock, 1975)

\[ E\{X_r\} = \frac{E\{X_d\} + \text{VAR}(X_d)}{2E(X_d)} \]  

\[ E\{X_d\} \quad \text{and} \quad \text{VAR}(X_d) \] represent the mean and variance of CDT, respectively. Considering this equation and that \( \text{CoV}\{X_d\} \) represents the coefficient of variation of CDT, the condition for which the mean CDTr is greater than the mean CDT (\( E\{X_r\} > E\{X_d\} \)) is given by

\[ \frac{E\{X_d\}}{2} + \frac{\text{VAR}(X_d)}{2E(X_d)} > E\{X_d\} \]

\[ \text{CoV}\{X_d\} > 1 \]  

In this way, the relationship between mean CDT and mean CDTr only depends on the value of the \( \text{CoV} \) of CDT. Thus, the mean CDTr is greater than the mean CDT (i.e., \( E\{X_r\} > E\{X_d\} \)) when the \( \text{CoV} \) of CDT is greater than one. This behavior (i.e., \( E\{X_r\} > E\{X_d\} \)) may seem to be counterintuitive due to the fact that, for a particular realization and by definition, CDTr cannot be greater than CDT\(^4\). This occurs because in such conditions there is a high variability on the cell dwell times in different cells and it is more probable to start new calls on cells where users spent more time. Then, residual cell dwell times tend to be greater than the mean CDT. This phenomenon that may seem to be counterintuitive is now explained and mathematically formulated in this Chapter.

3.2 Channel holding time distribution for handed off and new calls

Channel holding times for handed off and new calls (denoted by \( X^{(h)}_C \) and \( X^{(N)}_C \), respectively) are given by the minimum between UST and CDT or CDTr, respectively. The CDF of the CHTh and CHTn are, respectively, given by

\[ F_{X^{(h)}_C}(t) = 1 - \left[ 1 - F_{X_s}(t) \right] \left[ 1 - F_{X_d}(t) \right] \]  

\[ F_{X^{(N)}_C}(t) = 1 - \left[ 1 - F_{X_s}(t) \right] \left[ 1 - F_{X_r}(t) \right] \]  

Due to the fact that the Laplace transform of the pdf of both UST and CDTr are rational functions, the Laplace transform of the pdf of CHTn can be obtained using the Residue Theorem as follows (Wang & Fan, 2007)

\[ f_{X^{(N)}_C}(s) = f_{X_s}^*(s) + s \sum_{p \epsilon \Omega_{X_S}} \frac{\text{Res}}{\xi - s} \left[ f_{X^{(N)}_C}(\xi) f_{X_s}(s-\xi) \right] \]  

where \( p \epsilon \Omega_{X_S} \) is the set of poles of \( f_{X_s}^*(s) \), and \( f_{X}^*(s) \) is the Laplace transform of pdf \( f_{X}(t) \). A similar expression can be obtained for the Laplace transform of the pdf of the channel holding time for handed off calls by replacing residual cell dwell time \( (X_r) \) by cell dwell time \( (X_d) \).

\(^4\) Note that the beginning of CDTr is randomly chosen within the CDT interval.
Under the condition that UST is general phase type (PH) distributed, the authors of (Alfa & Li, 2002) prove that the CDT is PH distributed if and only if the CHTn is PH distributed or the CHTh is PH distributed.

The probability distributions of CHTn and CHTh for different staged probability distributions of CDT assuming that the UST is exponentially distributed are shown in Table II. The first entry of this table is a well known result. In (Soong & Barria, 2000), it was shown that when CDT has Erlang or hyper-Erlang distribution, channel holding times have the uniform Coxian and hyper-uniform Coxian distribution, respectively. Uniform Coxian is a special case of the Coxian distribution where all the phases have the same parameter (Perros & Khoshgoftaar, 1989). The hyper-uniform Coxian distribution is a mixture of uniform Coxian distributions.

<table>
<thead>
<tr>
<th>pdf of cell dwell time.</th>
<th>pdf of channel holding time for new calls or its Laplace Transform.</th>
<th>pdf of channel holding time for handed off calls or its Laplace Transform.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential (Lin et al., 1994)</td>
<td>Exponential $(\mu + \eta)e^{-(\mu + \eta)t}$</td>
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</tr>
<tr>
<td>Erlang of k-th order (Soong &amp; Barria, 2000)</td>
<td>Uniform Coxian of k-th order $f_{X_c}^*(s) = \sum_{i=1}^{k} p_i^{O(N)} \prod_{j=1}^{i} \frac{\mu + \eta}{s + \mu + \eta}$</td>
<td>Uniform Coxian of k-th order $f_{X_c}^{*(h)}(s) = \sum_{i=1}^{k} p_i^{O(h)} \prod_{j=1}^{i} \frac{\mu + \eta}{s + \mu + \eta}$</td>
</tr>
<tr>
<td>Hyper-Erlang of common order $(n, m)$ (Soong &amp; Barria, 2000)</td>
<td>Hyper-Uniform Coxian $f_{X_c}^*(s) = \sum_{i=1}^{k} p_i^{O(N)} \prod_{j=1}^{i} \frac{\mu + \eta_i}{s + \mu + \eta_i}$ where $z = \text{mod} \left( \frac{i - 1}{m} \right) + 1$, $l = \left\lfloor \frac{i - 1}{m} \right\rfloor + 1$</td>
<td>Hyper-Uniform Coxian $f_{X_c}^{*(h)}(s) = \sum_{i=1}^{k} p_i^{O(h)} \prod_{j=1}^{i} \frac{\mu + \eta_i}{s + \mu + \eta_i}$ where $z = \text{mod} \left( \frac{i - 1}{m} \right) + 1$, $l = \left\lfloor \frac{i - 1}{m} \right\rfloor + 1$</td>
</tr>
<tr>
<td>Hyper-exponential</td>
<td>$\sum_{i=1}^{n} p_i^{(N)} (\mu + \eta_i)e^{-(\mu + \eta_i)t}$</td>
<td>Hyper-exponential $\sum_{i=1}^{n} p_i(\mu + \eta_i)e^{-(\mu + \eta_i)t}$</td>
</tr>
</tbody>
</table>

$^5$ Authors in (Lin et al., 1994) give a condition under which the channel holding time is exponentially distributed, that is, the cell residence time needs to be exponentially distributed.
Next, it is shown that when the UST is exponentially distributed and CDT has hyper-exponential distribution of order \( n \), the distribution of CHTh has also a hyper-exponential distribution of order \( n \). Similarly, when CDT has Coxian distribution of order \( n \), the distribution of CHTn has also a Coxian distribution of order \( n \).

### 3.2.1 Case 1: Hyper-exponentially distributed cell dwell time

Considering that CDT has a hyper-exponential pdf of order \( n \) given by

\[
f_{\mathcal{X}_c}^{(h)}(s) = \sum_{j=1}^{2} P_j^{(h)} \left( \prod_{k=h(j)}^{f(j)-1} \frac{\eta_k}{s + \mu + \eta_k} \right)
\]

where

\[
P_j^{(h)} = \left[ \prod_{i=h(j)}^{f(j)-1} \frac{\eta_i}{\mu + \eta_i} \right] P_j^{(N)} + \sum_{k=f(j)+1}^{m} P_k^{(N)} \left( \frac{\mu}{\mu + \eta_f(j)} \right)
\]

Table II. Examples of corresponding distributions for \( X_c^{(h)} \) and \( X_c^{(N)} \).

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### 3.2.1 Case 1: Hyper-exponentially distributed cell dwell time

Considering that CDT has a hyper-exponential pdf of order \( n \) given by

\[
f_{\mathcal{X}_c}^{(h)}(t) = \sum_{j=1}^{n} P_j \eta_j e^{-\eta_j t}
\]

(10)
For exponentially distributed UST and using (4), the CDF of the CHTh can be expressed as follows

\[
F_{X_c^{(h)}}(t) = 1 - \left[ e^{-\mu t} \right] \sum_{i=1}^{n} P_i e^{-\eta_i t}
\]

This expression corresponds to a hyper-exponential distribution of order \( n \) with phase parameters \( \mu + \eta_i \) and probabilities \( P_i \) of choosing stage \( i \) (for \( i = 1, \ldots, n \)).

As the CDTr is hyper-exponentially distributed when CDT has hyper-exponential distribution, the CHTn is also hyper-exponentially distributed. In this case, the probability of choosing stage \( i \) (for \( i = 1, \ldots, n \)) is given by

\[
P_i^{(N)} = \frac{p_i \prod_{j=i}^{m-1} \eta_j}{\sum_{j=1}^{m} p_j \prod_{i=1}^{m} \eta_j}
\]

3.2.2 Case 2: Coxian distributed cell dwell time

Considering that cell dwell time has an \( m \)-th order Coxian distribution (which diagram of phases is shown in Fig. 1) with Laplace transform of its pdf given by

\[
f_{X_c}(s) = \sum_{j=1}^{m} P_j \prod_{i=1}^{j-1} \eta_i
\]

where

\[
P_j = \alpha_j \prod_{i=1}^{j-1} (1 - \alpha_i)
\]

(1-\( \alpha_i \)) represents the probability of passing from the \( i \)-th phase to the \( (i+1) \)-th one.

![Diagram of phases of the considered Coxian distribution of order \( m \) for modeling cell dwell time.](image)

For exponentially distributed UST and using (9), the Laplace transforms of the pdf of CHTh and CHTn are given by

\[
f_{X_c^{(h)}}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \left[ f_{X_c}^*(s+\mu) \right]
\]

\[
f_{X_c^{(N)}}^*(s) = \frac{\mu}{s+\mu} + \frac{s}{s+\mu} \left[ f_{X_r}^*(s+\mu) \right]
\]
Replacing (13) into (15), it can be written as

\[ f_{X_c}^*(s) = \sum_{j=1}^{m} P_j^{O(h)} \prod_{i=1}^{j} \frac{\eta_i}{(s+\mu+\eta_i)} \]  

(17)

where

\[ P_j^{O(h)} = \left[ \prod_{i=1}^{j-1} \frac{\eta_i}{\mu+\eta_i} \right] \left[ P_j + \sum_{k=j+1}^{m} P_k \left( \frac{\mu}{\mu+\eta_j} \right) \right] \]  

(18)

for \( i = 1, \ldots, m \). Then, CHTh has also a Coxian distribution of order \( m \) but with parameters \((\mu + \eta_i)\), for \( i = 1, \ldots, m \).

On the other hand, the Laplace transform of the residual cell dwell time can be shown to be given by

\[ f_{X_r}(s) = \sum_{j=1}^{\frac{m(m+1)}{2}} P_j^{(N)} \left( \prod_{k=h(j)}^{j} \frac{\eta_k}{s+\eta_k} \right) \]  

(19)

where

\[ P_j^{(N)} = \frac{P_{f(j)} \prod_{k=h(j)}^{m} \eta_k}{\sum_{k=1}^{m} \left( \prod_{k=h(j)}^{m} \eta_k \right) \left( \sum_{l=1}^{m} P_l \right)} \]  

(20)

\[ f(j) = \left\lfloor \frac{1+\sqrt{1+8(j-1)}}{2} \right\rfloor \]  

(21)

\[ h(j) = j - \frac{f(j)(f(j)-1)}{2} \]  

(22)

for \( j = 1, \ldots, m(m+1)/2 \). Substituting (19) into (16), Laplace transform of CHTn can be written as

\[ f_{X_c}^{(N)}(s) = \sum_{j=1}^{\frac{m(m+1)}{2}} P_j^{O(N)} \left( \prod_{k=h(j)}^{j} \frac{\eta_k}{s+\eta_k} \right) \]  

(23)

where

\[ P_j^{O(N)} = \left[ \prod_{i=h(j)}^{f(j)-1} \frac{\eta_i}{\mu+\eta_i} \right] P_j^{(N)} + \sum_{k=f(j)+1}^{m} P_k^{O(N)} \left( \frac{\mu}{\mu+\eta_{f(j)}} \right) \]  

(24)

Equation (23) corresponds to the Laplace transform of a generalized Coxian pdf.

The above analytical results show that CHTh (CHTn) has the same probability distribution as CDT (CDTr) but with different parameters of the phases, probabilities of reaching the absorbing state after each phase, and probabilities of choosing each stage. The detailed derivation of the last entry of Tables I and II (i.e., when cell dwell time has generalized Coxian distribution) is addressed in (Corral-Ruiz et al., a, 2010).

3.3 Relationship between \( X_c^{(h)} \) and \( X_c^{(N)} \)

Using (15) and (16) it is straightforward to show that the mean values of CHTn and CHTh are, respectively, given by
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\[ E\left\{X_c^{(N)}\right\} = \frac{1}{\mu} \left[1 - \frac{\eta}{\mu}\left[1 - f_{X_d}^*(\mu)\right]\right] \]  
\[ E\left\{X_c^{(h)}\right\} = \frac{1}{\mu} \left[1 - f_{X_d}^*(\mu)\right] \]  

At this point, it is important to mention that authors in (Fang, 2001; Zeng et al., 2002) stated that, depending on the variance of CDT, the mean CHTn can be greater than the mean CHTh. However, it was neither explained nor discussed the physical reasons for this observed behavior. This behavior occurs because the residual cell dwell times tend to increase as the variance of cell dwell time increases, as it was explained above.

Using (25) and (26), the condition for which the mean CHTn is greater that the mean CHTh, that is,

\[ E\left\{X_c^{(N)}\right\} > E\left\{X_c^{(h)}\right\} \]  

can be easily found. This condition is given by

\[ f_{X_d}^*(\mu) > \frac{\eta}{\mu + \eta} \]  

Thus, the relationship between the mean new and handoff call channel holding times is determined by the mean values of both CDT and UST and by the Laplace transform of the pdf of CDT evaluated at the inverse of the mean UST.

Finally, in a similar way, the squared coefficient of variation for CHTn and CHTh can be shown to be given, respectively, by

\[ CoV^2\left(X_c^{(N)}\right) = \frac{-4\eta E\left(X_c^{(h)}\right) + 2\left[1 - \frac{d f_{X_d}^*(\mu)}{d\mu}\right]}{\left[E\left(X_c^{(N)}\right)\right]^2} - 1 \]  
\[ CoV^2\left(X_c^{(h)}\right) = \frac{2}{\left(E\left(X_c^{(h)}\right)\right)^2} \left[\frac{d f_{X_d}^*(n)}{d\mu} + E\left(X_c^{(h)}\right)\right] - 1 \]  

It can be shown that the \(n\)-th moments for new and handoff call channel holding times are given, respectively, by

\[ E\left\{\left(X_c^{(N)}\right)^n\right\} = \frac{1}{\mu}\left[nE\left\{\left(X_c^{(N)}\right)^{n-1}\right\} - \eta E\left\{\left(X_c^{(h)}\right)^n\right\}\right] \]  
\[ E\left\{\left(X_c^{(h)}\right)^n\right\} = \frac{n}{\mu} \left((-1)^n \frac{d^n f_{X_d}^*(\mu)}{d\mu^n} + E\left\{\left(X_c^{(h)}\right)^{n-1}\right\}\right) \]  

4. Numerical results and discussion

In this section, numerical results on how the distribution of cell dwell time (CDT) affects the characteristics of channel holding time (CHT) are presented. We use different distributions to model CDT, say, negative-exponential, constant (deterministic), Pareto with shape parameter \(\alpha\) in the range \((1, 2]\) (i.e., when infinite variance is considered), Pareto with \(\alpha>2\)
(i.e., when finite variance is considered), log-normal, gamma, hyper-Erlang of order (2,2), hyper-exponential of order 2, and Coxian of order 2. Three different mobility scenarios for the numerical evaluation are assumed: $E[X_d]=5\cdot E[X_s]$ (low mobility), $E[X_d]=E[X_s]$ (moderate mobility), and $E[X_d]=0.2\cdot E[X_s]$ (high mobility). In the plots of this section we use $E[X_s]=180$ s.

In our numerical results, the effect of CoV and skewness of CDT on CHT characteristics is investigated. In the plots presented in this section, “HC” and “NC” stand for channel holding time for handoff calls (CHTh) and channel holding time for new calls (CHTn), respectively.

**4.1 Cell dwell time distribution completely characterized by its mean value**

Fig. 2 plots the mean value of both CHTn and CHTh versus the mean value of CDT when it is modeled by negative-exponential (EX), constant, and Pareto with $1<\alpha\leq2$ distributions. It is important to remark that all of these distributions are completely characterized by their respective mean values. As expected, Fig. 2 shows that, for the case when CDT is exponentially distributed, mean CHTn is equal to mean CHTh. An interesting observation on the results shown in Fig. 2 is that, irrespective of the mean value of CDT, there exists a significant difference between the mean value of CHTn when CDT is modeled as exponential distributed RV and the corresponding case when it is modeled by a heavy-tailed Pareto distributed RV (this behavior is especially true for the case when $\alpha=1.1$).

Notice, however, that this difference is negligible for the case when $\alpha=2$ and high mobility scenarios (say, $E[X_d]<50$ s) are considered. Similar behaviors are observed if mean CHTh is considered. Consequently, for high mobility scenarios where CDT can be statistically characterized by a Pareto distribution with shape parameter close to 2, the exponential distribution represents a suitable model for the CDT distribution. Fig. 2 also shows that, for

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**Fig. 2. Mean new and handoff call channel holding time for deterministic, negative exponentially, and Pareto distributed CDT against the mean CDT.**
a given value of the mean CDT and considering the case when CDT is Pareto distributed with $\alpha=1.1$ ($\alpha=2$), mean CHTn always is greater (lower) than mean CHTh. This behavior can be explained by the combined effect of the following two facts. First, as $\alpha$ comes closer to 1 (2), the probability that CDT takes higher values increases (decreases). This fact contributes to increase (reduce) the mean CHTh. Second, in general, new calls are more probable to start on cells where users spent more time and, as $\alpha$ comes closer to 1, this probability increases. This fact contributes to increase mean CHTn relative to the mean CHTh. Then, the combined effect is dominated by the first (second) fact as $\alpha$ comes closer to 2 (1). This leads us to the behavior explained above and illustrated in Fig.2. It may be interesting to derive the condition upon which the mean CHTn is greater than the mean CHTh when CDT is heavy-tailed Pareto distributed. This represents a topic of our current research.

### 4.2 Cell dwell time distribution completely characterized by its first two moments

Fig. 3 plots the mean value of both CHTn and CHTh versus the CoV of CDT when it is modeled by Pareto with shape parameter $\alpha>2$, lognormal, and Gamma distributions; all of them with mean value equal to 180 s. It is important to remark that all of these distributions are completely characterized by their respective first two moments. Fig. 3 shows that both mean CHTn and mean CHTh are highly sensitive to the type of distribution of CDT; this fact is especially true for $\text{CoV}>2$. Notice that, for the particular case when $\text{CoV}=0$, the mean values of both CHTn and CHTh are identical to the corresponding values for the case when CDT is deterministic with mean value equals 180 s, as expected. Fig. 3 also shows that, for values of $\text{CoV}$ of CDT greater than 1 (1.2), mean CHTn is greater that mean CHTh when CDT is Gamma (log-normal) distributed. On the other hand, when CDT is Pareto distributed and irrespective of the value of its $\text{CoV}$, CHTh always is greater that mean CHTn. This behavior is mainly due to the heavy-tailed characteristics of the Pareto distribution.

![Fig. 3. Mean new and handoff call channel holding time for gamma, log-normal, and Pareto distributed cell dwell time versus CoV of cell dwell time.](image)

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E(Xd) = 180 seconds

Log-Normal(HC)
Log-Normal(NC)
Gamma(HC)
Gamma(NC)
Pareto(HC)
Pareto(NC)
4.3 Cell dwell time distribution completely characterized by its first three moments

Figs. 4, 5, and 6 (7, 8, and 9) plot the mean value (CoV) of both CHTn and CHTh versus both the CoV and skewness of CDT when it is modeled by hyper-Erlang (2,2), hyper-exponential of order 2, and Coxian of order 2 distributions, respectively. It is important to remark that all of these distributions are completely characterized by their respective first three moments. Results of (Johnson & Taaffe, 1989; Telek & Heindl, 2003) are used to calculate the parameters of these distributions as function of their first three moments. In Figs. 4 to 9, two different values for the mean CDT are considered: 36 s (high mobility scenario) and 900 s (low mobility scenario). From Figs. 2, 5 and 6 the following interesting observation can be extracted. Notice that, for the case when CDT is modeled by either hyper-exponential or Coxian distributions and irrespective of the mean value of CDT, the particular scenario where skewness and CoV of CDT are, respectively, equal to 2 and 1, corresponds to the case when CDT is exponential distributed (in the exponential case mean CHTn and mean CHTh are identical).

Fig. 4. Mean CHTn and mean CHTh for hyper-Erlang distributed CDT versus CoV and skewness of CDT, with the mean CDT as parameter.
Fig. 5. Mean CHTn and mean CHTh for hyper-exponentially distributed CDT versus CoV and skewness of CDT, with the mean CDT as parameter.

Fig. 6. Mean CHTn and mean CHTh for Coxian distributed cell dwell time versus CoV and skewness of cell dwell time, with the mean CDT as parameter.
Fig. 7. CoV of CHTn and CHTh for hyper-Erlang distributed CDT versus CoV and skewness of CDT, with the mean CDT as parameter.

Fig. 8. CoV of CHTn and CHTh for hyper-exponential distributed CDT versus CoV and skewness of CDT, with the mean CDT as parameter.
Fig. 9. CoV of CHTn and CHTh for Coxian distributed CDT versus CoV and skewness of cell dwell time, with the mean cell dwell time as parameter.

On the other hand, Fig. 4 shows that the case when hyper-Erlang distribution with skewness equals 2 and CoV equals 1 is used to model CDT does not strictly correspond to the exponential distribution; however, the exponential model represents a suitable approximation for CDT in this particular case. From Figs. 4 to 9, it is observed that the qualitative behavior of mean and CoV of both CHTn and CHTh is very similar for all the phase-type distributions under study. The small quantitative difference among them is due to moments higher than the third one. Analyzing the impact of moments of CDT higher than the third one on channel holding time characteristics represents a topic of our current research.

From Fig. 10 is observed that the difference among the mean values of CHTn and CHTh is strongly sensitive to the CoV of the CDT, while it is practically insensitive to the skewness of the CDT. This difference is higher for the case when the CDT is modeled as hyper-exponential distributed RV compared with the case when it is modeled as hyper-Erlang distributed RV. Also, it is observed that this difference remains almost constant for the entire range of values of the CoV of the CDT.
Fig. 10. Difference among the mean values of new and handoff call channel holding times for hyper-Erlang and hyper-exponential distributed cell dwell time versus CoV and skewness of cell dwell time, for the moderate-mobility scenario.

Finally, in Fig. 11 the mean channel holding time for new and handoff calls considering the gamma, hyper-Erlang (2,2), hyper-exponential of order 2, and Coxian of order 2 distributions for the cell dwell time are shown for different values of the coefficient of variation. The numerical results shown in Fig. 11 are obtained by equaling the first three moments of the different distributions to those of the gamma distribution. From Fig. 11, it is observed that for the hyper-exponential and Coxian distributions practically the same results are obtained for the mean channel holding time for both new and handoff calls. The differences among the other distributions are due to the fact that they differ on the higher order moments. To show this, the forth standardized moment (i.e., excess kurtosis) of the different distributions is shown in Fig. 12 for different values of the coefficient of variation, equaling the first three moments of the different distributions to those of the gamma distribution. From Fig. 12, it is observed that the hyper-exponential and Coxian distributions practically have the same value of excess kurtosis but this differs for that of the gamma and hyper-Erlang distributions. The gamma distribution shows the more different value of the excess kurtosis and, therefore, for this distribution the more different values of the mean channel holding times in Fig. 11 are obtained. Then, it could be necessary to capture more than three moments, even though the lower order moments dominate in importance. Similar conclusion was drawn in (Gross & Juttijudata, 1997).
Fig. 11. Mean new and handoff call channel holding time for gamma, hyper-exponential (2), hyper-Erlang (2,2) and Coxian (2) distributed cell dwell time versus CoV of cell dwell time.

Fig. 12. Kurtosis of cell dwell time for gamma, hyper-exponential (2), hyper-Erlang (2,2) and Coxian (2) distributed cell dwell time versus CoV of cell dwell time.
5. Conclusions

In this Chapter, under the assumption that unencumbered service time is exponentially distributed, a set of novel general-algebraic equations that examines the relationships between cell dwell time and residual cell dwell time as well as between cell dwell time and new and handoff channel holding times was derived. This work includes relevant new analytical results and insights into the dependence of channel holding time characteristics on the cell dwell time probability distribution. For instance, we found that when cell dwell time is Coxian or hyper-exponentially distributed, channel holding times are also Coxian or hyper-exponentially distributed, respectively. Also, our analytical results showed that the mean and coefficient of variation of the new and handoff call channel times depend on Laplace transform and first derivative of the Laplace transform of the probability density function of cell dwell time evaluated at the inverse of the mean unencumbered service time as well as on the mean of both cell dwell time and unencumbered service time. Additionally, we derive the condition upon which the mean new call channel holding time is greater than the mean handoff call channel holding time. Similarly, the condition upon which the mean residual cell dwell time is greater than the mean cell dwell time was also derived. To the best authors’ knowledge, this phenomenon that may seem to be counterintuitive has been explained and mathematically formulated in this Chapter. We believe that the study presented here is important for planning, designing, dimensioning, and optimizing of mobile cellular networks.

6. References


Cell Dwell Time and Channel Holding Time Relationship in Mobile Cellular Networks


Sinclair B., “Coxian Distributions,” Connexions module: m10854. Available on line: http://cnx.org/content/m10854/latest/


This book will provide a comprehensive technical guide covering fundamentals, recent advances and open issues in wireless communications and networks to the readers. The objective of the book is to serve as a valuable reference for students, educators, scientists, faculty members, researchers, engineers and research strategists in these rapidly evolving fields and to encourage them to actively explore these broad, exciting and rapidly evolving research areas.

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