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1. Introduction

The presence of long-lasting conflicts in situations where reaching an agreement would benefit all parties is a pervasive phenomenon. This paradoxical fact, pointed out by Hicks (1932) in the context of strikes, has especially serious consequences when nations are engaged in war. One of the most common sources of conflict is bargaining over a territory, present in both secessionist movements as in international disputes. This chapter aims to provide a rationale to the "Hicks paradox" in a context where two countries are engaged in open conflict for a territory. Although termination of the conflict (i.e. reaching a negotiated outcome) improves the welfare of both parties, it is shown that when bargainers act rationally, the probability that the conflict persists over time is positive.

There are two main ways in which the inefficiencies associated to delay in bargaining are explained from a rational choice perspective: asymmetric information and dynamic commitment problems. The former invokes the existence of some sort of incomplete information in the bargaining environment. For instance, states might prefer fighting instead of achieving a peaceful settlement when there is uncertainty about the other’s cost of fighting. The latter resorts to dynamic commitment problems whereby the players may face incentives to renege on agreements. Our contribution belongs to the first category.

We propose a game-theoretic approach to understand the relationship between conflict, social welfare and the likelihood that a negotiation process emerges. Although the problem we tackle here is applicable to several bargaining contexts, for expositional purposes we conduct our analysis through the example of an international dispute over territory. In contrast with the standard view of conflict as a bargaining tool (where the threat of war enhances the bargaining power of the parties), we consider conflict as the status quo, and then focus on the strategic elements that influence conflict termination. Our contribution is then very related to the work of Wittman (1979), who envisions the end of war as a rational process.

The key element in our approach is that an inefficient outcome may arise as a consequence of the parties’ ability to make public statements. When engaged in conflict, strong public pronouncements against the other’s territorial claims undermine the possibilities of reaching a mutually beneficial agreement. Why would then the parties make such statements? Our

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1 For a survey of the theoretical work on bargaining and war, see Powell (2002).
3 Other contexts where our analysis would apply are: bargaining between a buyer and a seller, firm-union negotiations, settlements out of court in legal disputes, etc.
view is that countries use public claims as a rational strategy to better their position in a future negotiation process.

The declaration of a position as an irreversible commitment\(^4\) is used as a tactical approach to create a more advantageous bargaining environment. The politician who makes the claim is implicitly committed with the public to fulfill the terms of the pronouncement. Such statements are effective as they have the power to bind oneself. As Schelling (1956) argued:

*When national representatives go to international negotiations knowing that there is a wide range of potential agreement within which the outcome will depend on bargaining, they seem often to create a bargaining position by public statements, statements calculated to arouse a public opinion that permit no concessions to be made. If a binding public opinion can be cultivated, and made evident to the other side, the initial position can thereby be made visibly “final.”* p. 287.

We construct a formal model in which two agents bargain about how to divide the \([0, 1]\) interval (territory). There are two possibilities: either a negotiation process emerges and the interval is divided according to a certain division rule, or the negotiation process does not take place, and the countries must face conflict one more period. In the former case, the territory is divided according to the generalized Nash bargaining solution. If pre-bargaining public statements are “too far” from each other, negotiations don’t start. Statements are represented by numbers belonging to \([0, 1]\). We define a threshold \(T\), whose value is uncertain to the parties, to account for the distance about claims. If such a distance is above \(T\), then neither party is willing to negotiate. Uncertainty about \(T\) reflects the parties’ lack of information about each other’s willingness to make concessions.

Countries are engaged in a game in which extreme claims lower the probability of a negotiated outcome, but they also better their bargaining position in case a pacific settlement is to be reached. The players must then calculate rationally their strategies in the absence of information about the realization of the threshold \(T\). We are posing a situation in which there is a risk-return trade off between aggressive claims and diplomatic efforts (concessions) that increase the likelihood of a negotiated outcome. Faced with this trade-off, the players’ optimal strategy includes making statements that involve in some cases persistence of the conflict.

In order to achieve this result, we first characterize the Bayesian Nash equilibrium vector of public statements. We find that there exists a set of parameters under which the conflict continues with positive probability at equilibrium. Remarkably, this probability turns out to be a non monotone function of the conflict costs. For low enough costs, the probability of conflict is decreasing, then it reaches a minimum at a certain point and when costs go beyond that point, it increases and converges asymptotically to a constant value. From this pattern we can extract some conclusions. First, lowering the conflict costs may increase the probability of conflict if such costs are low enough. Second, when conflict costs are sufficiently high, variations on them have a negligible effect on the likelihood that a negotiation process emerges.

The view of conflict termination presented here complements the usual approach to bargaining and wars, whereby investing resources in military weapons is used as a credible signal by countries wishing to convey certain private information to their opponents\(^5\). In our

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\(^4\) Muthoo (1996) shows how commitments which can be reversed at a cost affect the players’ “share of the cake” at equilibrium.

\(^5\) As in Kennan and Wilson (1993), delay can also be considered as a signal to convey private information credibly.
model, statements are costless\textsuperscript{6}, but they entail consequences that affect the players’ welfare. The players do not send revealing signals, although the strategic process that precedes negotiation is characterized by imperfect information. A threshold whose value is uncertain is introduced to account for all pieces of information that condition the likelihood of a pacific settlement and are not perfectly known to both parties. In this informational environment, we are able to characterize a unique Bayesian Nash equilibrium.

The chapter is organized as follows. Section 2 presents a dynamic model that incorporates the main elements of our analysis. Section 3 characterizes the generalized Nash bargaining solution assumed to prevail if negotiations are held. Section 4 is devoted to compute the statements made by both countries in a symmetric Bayesian Nash equilibrium. Section 5 investigates the welfare consequences of the equilibrium play of the countries, paying special attention to the relationship between the conflict costs and the likelihood of a negotiation process to emerge. Finally, Section 6 concludes.

2. Model

We consider a dynamic infinite horizon model in which two countries\textsuperscript{7}, \(L\) and \(R\), are in conflict over a piece of land of size 1. At any given period, the conflict can be resolved through negotiation, or it can persist at least one more period. Formally, the players face a problem consisting on how to reach an agreement about a division of the interval \([0, 1]\). This situation is repeated every period until an agreement is reached.

At the beginning of each period, countries have the ability to make public statements about their respective territory claims. After the statements are made, two possibilities emerge: Either a negotiation process takes place, or the conflict continues. If the positions publicly announced are “too far” from each other, then no negotiation process starts, and each party \(i = L, H\) faces conflict costs \(H_i\). This situation might occur, for instance, if pre bargaining statements made by country \(L\) are regarded as unacceptable by country \(R\).

The conflict cost is a parameter that influences the countries’ welfare and hence affects their behavior. It reflects the resources lost in the conflict (guns, lives, etc.), or it can even be interpreted as the cost of a delay in reaching a mutually beneficial agreement. Unlike Sanchez-Pagés (2009) and Cramton and Tracy (1992), we consider that countries cannot choose the intensity of the dispute (and hence, cannot affect the conflict costs).

The countries’ utilities after an agreement is reached are linear and given by \(u_L(y) = \theta_L y\) and \(u_R(y) = \theta_R (1 - y)\), where \(y \in [0, 1]\) denotes the piece of land that goes to country \(L\) if the territorial status quo is modified through negotiation, and \(\theta_i\) denotes country’s \(i = L, R\) valuation of the territory gained. It is implicitly assumed that there is no initial owner of the land. Therefore, the status quo refers to a situation in which the share of territory possessed by each country is zero.

We analyze the strategic incentives faced by the conflicting parties before the possibility of negotiations arises. In particular, we consider that countries use the tactical approach of making public statements on \(y\) with the aim of creating a bargaining position. Let \(x_i \in [0, 1]\) be the public statement made by country \(i = L, R\), where \(x_L\) is the territory claim made by \(L\) and \(1 - x_R\) represents the territory claim of country \(R\).

\textsuperscript{6} Croson et. al (2003) show through experiments that cheap talk in bargaining games have real effects, both in the short and in the long run.

\textsuperscript{7} We use the terms “countries”, “players” or “parties” indistinctly throughout the chapter.
We define a threshold $T$, such that if $x_L - x_R > T$, negotiations don’t take place and conflict continues. If $x_L - x_R \leq T$, a bargaining process takes place whose final outcome is assumed to be given by the generalized Nash bargaining solution. In case negotiations don’t start, the conflict is ongoing and each country $i$ faces a cost $H_i$ during the present period. At the beginning of the next period, countries can make again public statements thus opening the possibility of a negotiation in that period. The discount factor between periods is $\delta < 1$.

We denote by $W_i$ the expected utility of country $i = L, R$ at the beginning of period $t$, and usually refer to it as the social welfare. As long as all periods are identical, and we consider an infinite time horizon, the game that starts in period $t$ is identical to the game that starts in period $t + 1$. This allows us to truncate the infinite horizon game and restrict our analysis to the equilibrium strategies of any given period (say $t$). The equilibrium payoff of country $i$ will also be $W_i$ at the beginning of period $t + 1$. Hence, using the discount factor between periods we are able to derive endogenously the stationary equilibrium value of $W_i$. For notational simplicity and wherever there is no risk of confusion, we omit time subscripts throughout the chapter.

It is useful to define the utility achieved by country $i = L, R$, in case the conflict is prolonged for $n$ periods. This utility is given by:

$$d_{in}(H_i, W_i) = -(1 - \frac{\delta^n}{1 - \delta}) H_i + \delta^n W_i.$$ (1)

The utility $d_{in}$ represents the discounted conflict costs during $n$ periods plus the discounted expected utility at the beginning of period $t + n + 1$. Observe that $d_{in}$ is strictly decreasing in $n$. To see this, we just need to check that

$$\frac{\partial d_{in}(H_i, W_i)}{\partial n} = \frac{\delta^n \ln \delta}{1 - \delta} H_i + \delta^n \ln \delta W_i < 0,$$ (2)

since $\delta < 1$ and $\ln \delta < 0$. A quick inspection to Eq. (1) reveals that $d_{in}(H_i, W_i)$ depends negatively on $H_i$ and positively on $W_i$.

A summary of the situation analyzed can be stated as follows: if at the beginning of period $t$ a negotiation process starts, the parties have the possibility of reaching an agreement on the division of land. In the event that negotiations do not start (probably because the public statements are too demanding), the conflict continues one more period and the payoff faced by each country is $d_{in}(H_i, W_i) = -H_i + \delta W_i$, where $\delta W_i$ is the discounted value of the expected utility at the beginning of period $t + 1$. In case negotiations take place, the final outcome obtained corresponds to the Generalized Nash Bargaining solution$^8$. This is a classical axiomatic solution to bargaining problems, which we refer specifically to our context in the next section.

3. Generalized Nash bargaining solution

We denote as $\alpha_L$ and $\alpha_R$ the bargaining power of countries $L$ and $R$, with $\alpha_L + \alpha_R = 1$. If a negotiation process takes place, either an agreement is reached, or negotiations end up with a disagreement that involves returning to conflict again. A bargaining breakdown once the parties have engaged in negotiations is a serious negative outcome. We assume that the

$^8$ See Binmore (1987), Muthoo (1999) and Osborne and Rubinstein (1990) for a detailed discussion of the Nash bargaining solution.
consequence of such a breakdown is to suffer conflict along \( n \) more periods, until the parties in conflict are again ready for dialogue. The number of periods of conflict, \( n \), represents a measure of the disagreement costs once the negotiations are underway.

The pair \((d_{Ln}(H_L, W_L), d_{Rn}(H_R, W_R))\) is the disagreement or threat point\(^9\) in this bargaining problem. The generalized Nash bargaining solution is then given by:

\[
\hat{y}(\alpha_L, \alpha_R) = \arg \max_y \left[ y - d_{Ln}\right]^\alpha_L \left[ \left(1 - y\right) - d_{Rn}\right]^\alpha_R.
\]

Solving the maximization problem stated above yields the following outcome:

\[
\hat{y}(\alpha_L, \alpha_R) = \alpha_L - \alpha_L d_{Rn} + \alpha_R d_{Ln},
\]

(3)

The share of territory that goes to country \( L \), \( \hat{y}(\alpha_L, \alpha_R) \), depends negatively on country \( L \)'s conflict costs \((H_L)\) but positively on country \( R \)'s conflict costs \((H_R)\). The magnitude of such effects increases as the discount factor \( \delta \) approaches one. The reason is that what the bargainers obtain if they fail to reach an agreement is proportional to the conflict costs. Hence, \((\frac{1 - \delta^n}{1 - \delta})\) \( H_i \) can be seen as implicit gains (costs avoided) from reaching an agreement. The higher the costs avoided (i.e., the higher is \( H_i \)) the lower share of land is obtained in the agreement. This effect is reversed when we consider the conflict costs of the opponent country. Moreover, \( \hat{y} \) also depends positively on country \( L \)'s bargaining power \((\alpha_L)\). A similar interpretation can be given to the share of the interval \([0, 1]\) that goes to country \( R \), \(1 - \hat{y}(\alpha_L, \alpha_R)\).

We consider that the bargaining power of country \( i = L, R, \alpha_i \), is given by the relative weight of country \( i \)'s public statement with respect to the sum of both countries’ statements. This assumption highlights the importance of pre-bargaining claims in further negotiation and is central to our analysis. Specifically, we assume that: \( \alpha_L = \frac{x_L}{x_L + 1 - x_R} \), and \( \alpha_R = \frac{1 - x_R}{x_L + 1 - x_R} \). Therefore, the Nash bargaining outcome as a function of the pre-bargaining public statements is given by

\[
y(x_L, x_R) = \frac{x_L}{x_L + 1 - x_R} (1 - d_{Rn}) + \frac{1 - x_R}{x_L + 1 - x_R} d_{Ln}.
\]

(4)

A quick inspection to Eq.(4) reveals that, for any given \( x_R, H_R, H_L \) and \( \delta \), function \( y \) is increasing and concave in \( x_L \). By symmetry, the same occurs to \( 1 - y \) with respect to \( 1 - x_R \).

4. Bayesian Nash equilibrium statements

This section is devoted to analyze the equilibrium values for the pre-bargaining claims made by the countries in conflict. These equilibrium values depend on parameters such as the conflict costs \((H_i)\), the duration of the conflict if no agreement is reached after negotiation \((n)\) and the discount factor \((\delta)\), and they also depend on the stationary value for the social welfare \((W_i)\). In order to obtain closed solutions and simplify calculus, we compute the equilibrium outcome under the assumption that the players are symmetric.

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\(^9\) Observe that the threat point varies as a function of parameters \( \delta, H \) and \( n \). We are only concerned on the effects of such changes on the Nash bargaining solution. To see how the influence of the threat point on the bargaining outcome varies across different type of solutions, see Anbarci et. al (2002).
We assume that threshold $T$ is uncertain. This assumption reflects the parties’ lack of knowledge of each other’s response to any given territory claim. The threshold is uniformly distributed over the interval $[0,1]$, and this distribution is common knowledge. Any given realization of $T$ represents a measure of the ex-ante probability of negotiation abortion. For instance, if $T$ is close to zero, the probability that negotiations don’t take place in this period is close to one. However, the realization of $T$ is not known to the parties before they announce their positions.

The setting described above can be analyzed as a Bayesian game, in which the players’ strategies are the public statements and the payoff of player $i=L,R$, at the beginning of period $t$ is given by $U_i(x_L,x_R) = vu_i[y(x_L,x_R)] + (1-v)d_i$, where $v = 1$ represents a bargaining process leading to outcome $y(x_L,x_R)$, and $v=0$ means that the conflict persists (at least) during period $t$. Provided that $T \sim U[0,1]$, the probability of conflict in period $t$, after the public statements $(x_L,x_R)$ are made, can be expressed as:

$$ p(x_L,x_R) = \Pr(v=0 \mid x_L,x_R) = x_L-x_R. \tag{5} $$

The probability that negotiations are held in period $t$ is then given by $1-p(x_L,x_R)$.

Uncertainty about the exact value of the threshold induces the following strategic situation: more extreme positions tend to favour a better outcome in a negotiation process, but as the players move towards the extremes the probability of aborting the negotiation process increases. Each player faces a trade-off similar to the one faced by the bidders in a first price auction: A lower price is better if one gets the good, but lowering the price also lowers the probability of obtaining the good. Therefore, if a player makes a very tough public statement (to create a stronger bargaining position), negotiations hardly take place. We seek to analyze the Bayesian Nash equilibrium statements that emerge in this context.

The timing of this game is as follows: (i) players simultaneously announce bargaining positions $x_L$ and $x_R$; (ii) the threshold is realized; (iii) each player receives his payoff. We look for the Bayesian Nash equilibrium of this game, defined as a pair of statements $(\hat{x}_L,\hat{x}_R)$ such that

$$ \hat{x}_L = \arg \max_{\{x_L\}} E[U_L(x_L,\hat{x}_R)] \quad \text{and} \quad \hat{x}_R = \arg \max_{\{x_R\}} E[U_R(\hat{x}_L,x_R)], $$

and

$$ E[U_L(x_L,\hat{x}_R)] = p(x_L,\hat{x}_R) d_{L1} + \theta_L [1-p(x_L,\hat{x}_R)] y(x_L,\hat{x}_R), \tag{6} $$

and

$$ E[U_R(\hat{x}_L,x_R)] = p(\hat{x}_L,x_R) d_{R1} + \theta_R [1-p(\hat{x}_L,x_R)] [1-y(\hat{x}_L,x_R)]. \tag{7} $$

The first order condition obtained from maximizing $E[U_L(x_L,\hat{x}_R)]$ with respect to $x_L$ implies the following equation:

$$ \theta_L [1-p(x_L,\hat{x}_R)] \frac{\partial y(x_L,\hat{x}_R)}{\partial x_L} = [\theta_L y(x_L,\hat{x}_R) - d_{L1}] \frac{\partial p(x_L,\hat{x}_R)}{\partial x_L}. \tag{8} $$

If country $L$ reports a higher $x_L$, the probability that negotiations are held decrease, but the piece of territory obtained in case negotiations take place is higher. Therefore, Eq. (8) above represents the equality between the marginal costs of increasing $x_L$ (right hand term) and the marginal benefits of doing so (left hand term). The first order condition $\frac{\partial E[U_R(\hat{x}_L,x_R)]}{\partial x_R} = 0$ admits the same interpretation.

From Eq. (8) it is implicit that $W_i$ is treated as an exogenous parameter. The reason is that $W_i$ represents the welfare at the beginning of the present period $(t)$ and it also represents the...
welfare at the beginning of the next period \((t + 1)\). If changing the statements \(x_L\) and \(x_R\) could change the equilibrium value of \(W_i\), say to \(W'_i \neq W_i\), then the stationary value of the social welfare would be \(W'_i\) throughout all periods. The players, who are aware of the dynamics of the game, take the value of \(W_i\) as given.

Solving Eq. (8) for \(x_L\) we obtain \(x_L(\hat{x}_R)\), the reaction function of country \(L\) to every possible given statement \(\hat{x}_R\). A similar routine leads us to \(x_R(\hat{x}_L)\), the reaction function of country \(R\) to any given \(\hat{x}_L\). In a Bayesian Nash equilibrium, the pair of statements \((\hat{x}_L, \hat{x}_R)\) is such that \(\hat{x}_L = x_L(\hat{x}_R)\) and \(\hat{x}_R = x_R(\hat{x}_L)\).

In order to provide an explicit expression for the pair \((\hat{x}_L, \hat{x}_R)\) we now make some symmetry assumptions. In particular, we assume that both countries are identical in their valuations of territory and in the conflict costs. Namely, \(\theta_L = \theta_R = \theta\) and \(H_L = H_R = H\). In this scenario, we have \(\hat{x}_L = 1 - \hat{x}_R\). Then, it follows that \(W_L = W_R = W\) and hence \(d_{Ln} = d_{Rn} = d_n\). In a symmetric equilibrium we have \(\hat{x}_L = x^*(n, H, W)\) and \(\hat{x}_R = 1 - x^*(n, H, W)\). We use Eq. (8) above to compute

\[
x^*(n, H, W) = \frac{\theta [1 - 2d_n(H, W)]}{2[\theta(1 - d_n(H, W)) - d_1(H, W)]}.
\]

Next we analyze the dependence of \(x^*(.)\) on parameters \(H\) and \(n\). We simplify notation by writing \(x^*\) instead of \(x^*(n, H, W)\) and \(d_n\) instead of \(d_n(H, W)\). Using the expression for \(x^*\) in Eq. (9) we compute

\[
\frac{dx^*}{dH} = \frac{\partial x^*}{\partial d_n} \frac{\partial d_n}{\partial H} + \frac{\partial x^*}{\partial H}
\]

where \(\frac{\partial x^*}{\partial d_n} = -\frac{2\theta^2 + 2d_1}{2\theta(1 - d_1)}\) and \(\frac{\partial d_n}{\partial H} = -\left(\frac{1 - \delta}{1 - d_1}\right) < 0\), and \(\frac{\partial x^*}{\partial H} = \frac{-\theta(1 - 2d_n)}{2[\theta(1 - d_n) - d_1]}\).

The indirect effect reflects the equilibrium reaction of the pre-bargaining claim to an increase in conflict costs through the influence that such costs have on the disagreement point. Under the mild assumption that \(\theta + 2H > 2\delta W\) (i.e. if conflict costs are high enough, and/or the valuation of land is high enough, and/or the discount factor is low enough), the derivative \(\frac{\partial x^*}{\partial d_n}\) is negative. Therefore, the indirect effect is positive provided that \(\frac{\partial d_n}{\partial H} < 0\).

The direct effect operates in the opposite direction. If the conflict costs increase, and the disagreement point \(d_n\) is low enough, then the threat of a conflict dissuades from making aggressive statements (e.g. to claim the entire territory). In general, it cannot be established whether the direct effect predominates or not over the indirect effect. We show later, in a more restricted context, that the response of the equilibrium claims to a change in \(H\) is non monotone. Specifically, \(x^*\) is decreasing if \(H\) is low enough and increasing if \(H\) is high enough.

If the parties disagree in negotiations, the conflict lasts for \(n\) periods. Next we analyze the influence of parameter \(n\) in the behavior of the parties before negotiations. In particular, we compute the sign of \(\frac{dx^*}{dn}\).

\[
\frac{dx^*}{dn} = \frac{\partial x^*}{\partial d_n} \frac{\partial d_n}{\partial n}.
\]

\(\footnote{\text{It is required that } d_n < \frac{1}{\delta}. \text{ Observe that for this inequality to hold it is sufficient that } 1 + 2H > 2\delta W, \text{ provided that } d_n < -H + \delta W \text{ for } n \geq 2.}\)

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From Eq. (2) we know that $\frac{dx}{dn} < 0$. We also know that $\frac{dx^*}{dn} < 0$ whenever $\theta + 2H > 2\delta W$. Hence, we can establish that $\frac{dx}{dn} > 0$, i.e., the share of territory initially claimed is larger if $n$ increases (and so do the costs from disagreement).

The likelihood of being in conflict during period $t$ is also affected by changes in $H$ and $n$. Evaluated at equilibrium, this probability is given by $p^* = p(x^*, 1 - x^*) = 2x^* - 1$. Hence, $\frac{dp^*}{dx} = 2\frac{dx^*}{dx} = 2\frac{dx^*}{dH} > 0$. If the utility obtained in case of disagreement lowers (higher $n$), the probability of conflict increases. This effect becomes apparent in the next section, in which we investigate the relationship between $p^*$ and $H$ under different values of $n$.

Let us now focus our attention on the equilibrium value for the generalized Nash bargaining outcome. First, observe that in our symmetric framework it holds that $\hat{x}_L = 1 - \hat{x}_R = x^*$. Then, the bargaining power of both countries is the same, i.e., $a^*_L = \frac{\hat{x}_1}{\hat{x}_1 - \hat{x}_R} = \frac{x^*}{2x^*} = \frac{1}{2}$, and $a^*_R = \frac{1 - \hat{x}_R}{\hat{x}_L} = \frac{x^*}{2x^*} = \frac{1}{2}$. Plugging these values into Eq. (4) we obtain $y(\hat{x}_L, \hat{x}_R) = \frac{1}{2} (1 - d_n) + \frac{1}{2} d_n = \frac{1}{2}$. Not surprisingly, the Nash bargaining solution evaluated at the equilibrium statements involves that the territory is equally shared between both countries.

It is worth mentioning that the equilibrium statements computed above eventually depend on the equilibrium value for $W$. We derive endogenously the stationary value of $W$ in the next section.

### 5. Social welfare, conflict, and the emergence of negotiations

The purpose of this section is twofold: First, it is devoted to compute the welfare expected from the equilibrium play of the game at the beginning of each period. Secondly, it includes estimations on the way how $x^*$ and $p^*$ respond to $H$ under different scenarios ($n = 1, n = 2$ and $n \to \infty$).

The value for $W$ is implicitly defined in the following expression:

$$ W = E \left\{ U_L [x^*(n, H, W), 1 - x^*(n, H, W)] \right\}. \quad (10) $$

In order to solve Eq. (10) above for $W$, we use the probability of conflict at equilibrium, given by:

$$ p^* = \frac{d_1 - \theta d_n}{\theta (1 - d_n) - d_1}. \quad (11) $$

Taking into account that $y(x^*, 1 - x^*) = \frac{1}{2}$, we rewrite Eq. (10) as:

$$ W = p^* d_1 + (1 - p^*) \frac{1}{2}. \quad (12) $$

Plugging the expression for $p^*$ in Eq. (11) into Eq. (12) we obtain:

$$ W = \frac{(d_1 - 1 - \theta d_n) d_1 + \theta}{\theta (1 - d_n) - d_1}. \quad (13) $$

We just need to substitute $d_1 = -H + \delta W$ and $d_n = -\left(\frac{1 - \delta^2}{1 - \delta}\right) H + \delta W$ in Eq. (13) and solve it for $W$. In order to provide a tractable expression for the social welfare, we assume specific
values for parameters $\theta$ and $\delta$. Namely, $\theta = 1$ and $\delta = 0.5$. Then, we write:

$$W = \frac{(d_1 - d_{n-1}) d_1 + \frac{1}{2}}{1 - d_n - d_1}. \quad (14)$$

Next we discuss how the social welfare varies when $n$ changes. In particular we study the cases where $n = 1$ (case 1), $n = 2$ (case 2), and $n \to \infty$ (case 3). Case 1 corresponds to a situation in which the players receive the payoffs associated with the status quo when they fail to reach an agreement. Cases 2 and 3 should be interpreted as if players had an outside option whose effects imply facing $n$ additional periods of conflict. For each case, we compute both the equilibrium statement $x^*$ and the probability of conflict $p^*$ as a function of the conflict costs $H$.

We obtain that in case 1 the social welfare is constant, but in cases 2 and 3, it is strictly decreasing in $H$. The effect of $H$ on $x^*$ and $p^*$ is non monotonic in cases 2 and 3. For low values of $H$, the statement $x^*$ is decreasing in $H$. It reaches a minimum, and then increases in $H$ and converges asymptotically to a constant value. As long as $p^* = 2x^* - 1$, the response of $p^*$ to changes in $H$ follows a similar pattern.

We can also calculate how long will the conflict last, for different values of $H$. Notice that, if $p^*$ is the probability of conflict in period $t$, the number of periods of conflict (until a negotiation process is undertaken) is a random variable $Y$ that follows a geometric distribution. Specifically, the probability that there are $k$ periods of conflict is given by $Pr(Y = k) = (p^*)^k (1 - p^*)$, and the expected value of $Y$ is $E(Y) = \frac{p^*}{1-p^*}$. Below we compute $p^*$ as a function of $H$ for the cases $n = 1$, $n = 2$ and $n \to \infty$, with $\theta = 1$ and $\delta = 0.5$. Then, we estimate the number of periods the conflict is expected to last as a function of $H$ for each given value of $n$. Moreover, the expected cost of a conflict can also be computed as $E(Y)H$.

5.1 Case 1 ($n = 1$)

In this case, $d_n = d_1 = -H + \delta W$. This means that the utility achieved if negotiations break down is equal to the utility achieved in the case that negotiations do not take place and the conflict situation persists along period $t$. The probability of conflict (Eq. (11)) is zero\(^{11}\) provided that the equilibrium statements are equal to $\frac{1}{2}$ (the reader can check it in Eq. (9)). Then, by Eq. (14) we obtain $W = \frac{1}{2}$.

It is worth to mention that neither the pre-bargaining claims nor the social welfare are affected by the conflict costs $H$. This is due to the fact that such costs exert exactly the same influence on the two possible outcomes (conflict or negotiation) that may arise after $x_L$ and $x_R$ are announced and the threshold $T$ is realized. In the first case (conflict), the payoff for each country is $-H + \delta W$. The Nash bargaining outcome is $y = \frac{1}{2}$, and in case of disagreement the parties would obtain $d_1 = -H + \delta W$. This means that, if the parties broke down negotiations, the disagreement payoffs would equal the cost of suffering conflict during one more period. In equilibrium, both parties claim one half of the territory, a negotiation process emerges with probability one, and each country enjoys a social welfare equal to $\frac{1}{2}$. The territory is equally shared and there is no conflict. The expected number of periods the conflict will last is zero, and the expected cost of the conflict is also zero. As we show below, both the likelihood of a

\(^{11}\) Observe this probability could be greater than zero if we considered $\theta > \frac{d_1}{1-d_1}$. 

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negotiation process to emerge and the social welfare are dependent on the conflict costs when $n \geq 2$.

5.2 Case 2 ($n = 2$)

In this scenario, we have $d_1 = d_2 = -(1 + \delta)H + \delta^2 W$. A conflict in period $t$ involves costs $H$, but the emergence of a negotiation process opens the possibility of ending up with an outcome which is even worse. This outcome occurs if the parties fail to reach an agreement. Then, the payoff to each country is $d_2 < d_1$. Comparing it with the former case, we should expect that now each country claims for more than one half of the territory. Therefore, the probability of conflict must be positive.

From Eq. (14) we write the following expression for the social welfare:

$$W = \frac{(d_1 - d_2 - 1) d_1 + \frac{1}{2}}{1 - d_2 - d_1}. \quad (15)$$

If we substitute $d_1 = -H + \delta W$ and $d_2 = -(1 + \delta)H + \delta^2 W$ and $\delta = 0.5$ into Eq. (15) above, we obtain:

$$W = \frac{(0.25W + 0.5H)(-H + 0.5W) + \frac{1}{4}}{1 + 2.5H - 0.75W} \quad (16)$$

There are two values of $W$ that satisfy Eq. (16), but only one of them satisfies our assumption that $\theta + 2H > 2\delta W$. This value is:

$$W = 1.4286H - 0.57143 \left[ \left( 5H + 8H^2 - \frac{3}{4} \right)^{\frac{1}{2}} - 1 \right]. \quad (17)$$

Notice\(^{12}\) that $\frac{dW}{dH} |_{n=2} < 0$ for all $H \geq \frac{1}{8}$.

The stationary value for the social welfare decreases in response to an increase in the conflict costs. The rationale behind this effect is as follows: If $H$ increases, the disagreement utility becomes lower. This makes a negotiation process be less attractive for the countries, as long as the "bad outcome" of conflict today is not so bad compared to the disagreement outcome (two periods of conflict). As a consequence, the countries are willing to make more aggressive statements. This behavior decreases the probability of negotiations to emerge. Therefore, the social welfare falls to a lower level.

Next we check that the equilibrium statements are higher than $\frac{1}{2}$. For this purpose, we substitute the value for the social welfare in Eq. (16) into Eq. (9) to obtain:

$$x^* = \frac{1}{2} \frac{1 + 3H - 0.5W}{1 + 2.5H - 0.75W} \quad (18)$$

From Eq. (18) it is straightforward that $x^* \in \left( \frac{1}{2}, 1 \right)$ provided that $H > 0$, $W > 0$ and $1 + 2H > W$ for all $H \geq \frac{1}{8}$. The fact that $x^* > \frac{1}{2}$ implies $p^* = 2x^* - 1 > 0$. An important conclusion is that if $n = 2$, and $H \geq \frac{1}{8}$ the probability that countries continue with the conflict instead of initiating a negotiation process is positive. This result is in accordance with

\(^{12}\) A solution for $W$ exists whenever $H / \notin \left( -\frac{3}{4}, \frac{1}{8} \right)$.
Now we compute the equilibrium value of $x^*$ as a function of the conflict costs. By substituting $W$ in Eq. (17) into Eq. (18) we obtain:

$$x^* = \frac{1}{2} \frac{1 + 2.2857H + 0.28572 \left( \left( 5H + 8H^2 - \frac{3}{4} \right)^{\frac{1}{2}} - 1 \right)}{1 + 1.4285H + 0.42857 \left( \left( 5H + 8H^2 - \frac{3}{4} \right)^{\frac{1}{2}} - 1 \right)}$$

(19)

It is easy to check that $x^*$ is strictly decreasing in $H$ across the range $(0.125, 0.5695)$. The minimum value of $x^*$ is 0.5695, taken at $H = 0.57287$. In this case, $p^* = 0.139$, and the conflict is expected to last 0.16144 periods. The expected cost of the conflict is 0.092484. The maximum statement is made when $H = 0.125$. This statement is equal to 0.66667, associated with a probability of conflict of 0.33334. In this case, the expected number of periods of conflict is 0.5, and the expected cost of the conflict is 0.0625. When $H \rightarrow \infty$, we have $x^* \rightarrow 0.58579$. The probability of conflict is 0.17158 and the expected duration of the conflict is 0.20712.

5.3 Case 3 ($n \rightarrow \infty$)

In this third case, we have that $\lim_{n \rightarrow \infty} - \left( \frac{1}{1 - \delta} \right) \left( H + \delta^n W \right) = -\frac{1}{1 - \delta} H$. We are dealing with the somehow extreme situation in which breaking down negotiations involves that conflict will last forever. We substitute $d_n = -\frac{1}{1 - \delta} H$ into Eq. (14) to obtain:

$$W = \frac{0.25W^2 - H^2 + \frac{1}{2}}{1 + 3H - 0.5W}.$$ 

(20)

Solving Eq. (20) for $W$ yields:

$$W = 2H - \frac{2}{3} \left[ 2 \left( \frac{3}{2} H + 3H^2 - \frac{1}{8} \right)^{\frac{1}{2}} - 1 \right].$$

(21)

It is easy to see that $\frac{dW}{dH} \bigg|_{n \rightarrow \infty} < 0$ for all $H \geq \frac{1}{12} \sqrt{15} - \frac{1}{4}$. Moreover, for all $H \geq \frac{1}{8}$ it holds that $\frac{dW}{dH} \bigg|_{n \rightarrow \infty} < \frac{dW}{dH} \bigg|_{n=2} < 0$. This suggests that the magnitude of the social welfare losses provoked by an increase in the conflict costs becomes higher as $n$ grows large. Compared to the two former cases, notice that now the disagreement point does not depend on $W$. Therefore, the effect of an increase in the conflict costs can be easily computed as a variation of magnitude $-\frac{1}{1 - \delta}$ in the disagreement utility.

The equilibrium statements are now:

$$x^* = \frac{1}{2} \frac{1 + 4H}{1 + 3H - 0.5W}.$$ 

(22)

We have that $x^* \in \left( \frac{1}{2}, 1 \right)$ whenever $H > -0.5W$. By substituting $W$ (Eq. (21)) in the latter inequality we find that it holds for values of $H$ such that $6H > 2 \left( \frac{3}{2} H + 3H^2 - \frac{1}{8} \right)^{\frac{1}{2}} - 1$. This
inequality is satisfied by all $H \geq \frac{1}{12} \sqrt{15} - \frac{1}{4} = 0.072749$. Hence, for conflict costs above this value, the probability of conflict is positive, i.e., $p^* > 0$. Now we compute the equilibrium value for $x^*$ by substituting $W$ in Eq. (21) into Eq. (22) to obtain:

$$x^* = \frac{1}{2} \frac{1 + 4H}{1 + 2H + \frac{1}{3} \left( 2 \left( \frac{3}{2} H + 3H^2 - \frac{1}{8} \right) \right)^{\frac{1}{2}} - 1}. \quad (23)$$

The equilibrium statement $x^*$ is above 0.75 for low values of $H$, is decreasing in $H$ across the range $(0.072749, 0.54057)$, and reaches a minimum value of 0.61257 at $H = 0.54057$. For $H > 0.54057$, $x^*$ is strictly increasing and converges asymptotically to 0.63397. The probability of conflict now varies between 0.58744 (when $H = 0.07275$ and $x^* = 0.79372$) and 0.22514 (when $H = 0.54057$ and $x^* = 0.61257$). For $H$ large enough, the probability of conflict is constant and equal to 0.26794. The expected duration of the conflict ranges from 1.4239 periods (for $H = 0.07275$) to 0.29056 periods (for $H = 0.54057$). The expected conflict cost is then 0.10359 if $H = 0.07275$, and 0.15707 if $H = 0.54057$.

In general, when analyzing the effects of variations in $H$ on the probability of conflict, we are able to conclude that, in cases where $n > 1$: (i) $p^*$ achieves a maximum when the conflict costs are minimum; (ii) $p^*$ achieves a minimum for a certain value of $H$; (iii) $p^*$ converges to a constant value as $H \rightarrow \infty$. The pattern of variation of $p^*$ is driven by the way how the equilibrium statement $x^*$ changes in response to $H$.

Therefore, tough public pronouncements are expected to be made when the conflict costs are minimum. From that point on, the claims moderate as $H$ grows large. There is a certain value of $H$ that yields the lowest equilibrium claim (i.e., the highest probability that negotiations emerge). When $H$ is large enough, the claims tend to a constant value.

As the above analysis reveals, favoring the own position with aggressive claims can either be the product of extremely low or extremely high conflict costs. In the first case, the countries find it profitable to take the risk of aborting a negotiation process, as long as the benefits of a higher bargaining power in future negotiations outweigh the costs of facing conflict today. For the second case, it must be considered that failure to reach an agreement in negotiations involves a certain persistence of the conflict (during $n \geq 1$ periods) and then high values of $H$ lower considerably the disagreement utilities. Clearly, the worse outcome is represented by the disagreement point, the less attractive is a negotiation process. Then, strong public pronouncements that lower the probability of negotiations should not come as a surprise.

6. Conclusions

This chapter studies the relationship between social welfare, the cost of conflict and the emergence of a negotiation process. For this purpose, we develop a model in which two countries are bargaining over a fixed size territory. While no agreement is reached, the parties keep engaged in a conflict that entails costs in each period. The countries have the ability to make public statements representing claims over the territory. If the claims are not “too extreme”, a negotiation process emerges that yields as an outcome the generalized Nash bargaining solution, where the bargaining power of each country is conditioned upon the previous claims. The disagreement point of this negotiation process is to face conflict during

\[^{13}\] These results are in line with that of Wittman (1979), who finds that a reduction of hostilities may reduce the probability of a settlement taking place and thus prolong the war.
an indeterminate number of periods (ranging from 1 to infinite). On the other hand, if the public statements are far enough from each other, then no negotiation process emerges and the conflict continues for at least one more period. As a difference with previous work, conflict is not considered as a bargaining instrument. Instead, conflict is the prevailing state, which can only be changed if the parties make diplomatic efforts (concessions, in the form of reasonable claims) to resolve the dispute through a pacific settlement.

In this setting we conclude that, in general, the probability of conflict in period \( t \) is a non-monotonic function of the conflict costs. For low costs it is decreasing, and it increases when costs are above a certain value. At this particular value, the probability of conflict reaches a minimum. A similar pattern is obtained with respect to the pre-bargaining equilibrium claims. The effect of conflict costs on the social welfare is more clear: it decreases monotonically as the conflict costs increase.

These results suggest that the goals of maximizing the social welfare and minimizing the probability of conflict are not equivalent, since both are achieved at different levels of conflict costs. While the welfare is maximum when conflict is costless, the likelihood of a negotiation process to emerge achieves a maximum for positive conflict costs. Moreover, if such costs are very high, the probability of conflict achieves a constant value. This explains why in some instances a negotiation process does not emerge, even if it would be in the interest of the conflicting parties to initiate it. Although countries must bear certain costs every period the conflict lasts, they face even higher costs if negotiations emerge but no agreement is reached. The countries trade-off the conflict costs (today) against the expected outcome of negotiations, and therefore they make public statements that may involve the persistence of the conflict.

The utility reached at the disagreement point when negotiations break down is a crucial element to determine the probability of such negotiations to emerge. In our model, the threat point is parameterized by "n", the number of periods the conflict will last if no agreement is reached. The analysis made in Section 5 reveals that the social welfare depends negatively on "n" for any given conflict cost. It is then desirable that the threat point of a negotiation process is not too harmful for the parties in conflict. This implies that an important issue in the design of the rules of a negotiation process should be to prevent and limit the consequences of breaking negotiations. Specifically, before the bargaining over territory takes place, it would be welfare enhancing that the parties could commit to restart negotiations at an early date in case no agreement were reached. In general, any measure conducive to mitigate the negative outcome represented by the threat point would improve the likelihood of achieving a pacific settlement through negotiation.

If the countries could choose the intensity of the dispute, our model suggests that a moderate investment in resources devoted to the conflict proves optimal for a negotiation process to emerge. This conclusion is compatible with the approach that envisions conflict as part of a bargaining strategy, as in Sanchez-Pagés (2009). According to this view, the nations engaged in territorial disputes use limited confrontation as a way to convey information about their relative strength. The information revealed facilitates the emergence of a peaceful outcome. In our model, though, the line of explanation is a bit different. When conflict costs are positive, but not too high, the parties might find it beneficial to negotiate as long as the threat point in case of disagreement does not entail huge utility losses. However, if conflict costs become higher (for instance, if the countries engage in a nuclear arms race), then the threat point involves a final confrontation aimed to the absolute destruction of the enemy. We have shown that the territorial claims in this case tend to be more aggressive, thus lowering the probability of reaching a negotiated outcome.
Further research should focus on the public good nature of negotiation processes. Once the process is underway, no agent can be excluded from its benefits. Moreover, taking part in negotiations does not exclude other agents of being engaged in the process. Provision of the public good “negotiation” is clearly efficient, as long as the conflicting parties obtain a mutual benefit. However, contributing to this public good (i.e. making diplomatic efforts in the form of less demanding pre-bargaining claims) is a strategic decision plagued by the free rider problem. Interpreting the emergence of negotiations as a game of voluntary contributions, allow us to conclude that, in general, negotiations are under-provided. This view of the problem allows for a mechanism design solution to resolve international disputes over territory.

7. Acknowledgements

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8. References

“Social Welfare” offers, for the first time, a wide-ranging, internationally-focused selection of cutting-edge work from leading academics. Its interdisciplinary approach and comparative perspective promote examination of the most pressing social welfare issues of the day. The book aims to clarify some of the ambiguity around the term, discuss the pros and cons of privatization, present a range of social welfare paradoxes and innovations, and establish a clear set of economic frameworks with which to understand the conditions under which change in social welfare can be obtained.

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