Computer Aided Ore Body Modelling and Mine Valuation

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1. Introduction

Mine valuation can be defined as the process of determining the worth of a specific mineral deposit and capability of making a return by a prospective investment (SME, 2005). Although the definition is very brief and compact, actually, it has a very wide content. Determination of worth of an underground asset requires a plenty of works to explain physical, structural and economical properties of it (Kennedy, 1990).

Underground assets are invisible bodies whose shapes, quality compositions and quantities are unknown. Geological explorations and investigations aim at determining all these unknowns (Sinclair and Blackwell, 2004). At the beginning of process, topographical and lithological data are gathered and a database is generated. Depth, thickness and grade changes, overburden structure, ore volume, shape and extensions, footwall and hanging wall properties are determined by various mathematical approaches using this database. All numerical estimations and visual supports help bringing out ore body model (Singer and Menzie, 2010).

The most concrete data to define shape, location, quality and quantity of an ore body is drill hole cores. GPS data is mostly used to draw topographical maps and surfaces. Additionally, underground maps such as thickness and grade contours are drawn as well. When topographical coordinates are combined with stratigraphical information, a three dimensional data set is handled. Eventually, after following several mathematical techniques, three-dimensional model of ore body can be obtained (Hustrulid and Kuchta, 2006). Beside physical ore model, quality composition should also be known. This is crucial because further engineering activities have an economical aspect. Mine design and production schedule is fairly related to both physical structure and quality composition of ore (Hartman, 1992).

Surveying data include three-dimensional components $x$, $y$, $z$ (easting, northing, altitude/elevation) which enable surface modelling. Drill hole data including depth and layer information contribute to explain how geological structure is in the third dimension (Torries, 1998). Drill holes also carry the information of ore grade or calorific value. Geological interpretation of stratigraphical layers provides three-dimensional ore body model (Nieuwland, 2003).

Major instruments for computer aided mine valuation are;
- drill hole logs,
- contour maps,
- cross-sections,
surfaces (topography, thickness, grade, etc.),
- surface sections,
- solid models (three dimensional models), volume and reserve estimation.

Some of outputs of mine valuation are visual and some of them are numerical. Visual outputs help researchers see drill hole sections, how ore body is, how it extents, how its shape looks like, how ore body and overburden relation is, how quality and thickness of deposit changes thru axes. Numerical outcomes are generally, area and volume reports, drill hole lengths, survey coordinate sets, composite calculations, economical assessments, etc (Torries, 1998).

In the last several decades, many approaches have been developed to clear up geological modelling problems (Agoston, 2005). The purpose is to estimate unknown values regarding limited data in hand. The methods such as geostatistics and neural networks have complicated mathematical and statistical bases and are utilised to model topography, ore body, and grade distribution (McKillup and Dyar, 2010). Iterative structure of computations makes computer use necessary for many of recent methods. There are various specialised and expert software to support geological and mining engineers. Ore body modelling with computer aid is faster and more reliable. New information addition is simple and updating is much quicker. Different scenarios can be studied and better decisions can be made. General work plan and flowchart of computer aided mine valuation is shown in Fig. 1.

Once ore deposit is visually and numerically modelled, next step is mine design and production scheduling. Engineering economics and optimisation concepts are considered at this stage. Optimisation and simulation methods such as graph theory, dynamic programming, linear and goal programming, mixed integer programming, moving cones, genetic algorithm and network analyses are main techniques that can be counted. Scope of optimisation and simulation is generally open pit limits and production (Erarslan and Celebi, 2001).

Purpose of geological database generation is to describe physical and quality properties of ore bodies and preparation economical of database. Limited data should be optimally utilised and unknowns should be cleared. Last decades have shown great improvements in ore body visualisation and figuring. Buried underground body with so many unknowns can be visualised with animations and virtual reality applications by the help of computer software. Ore body is represented as a three dimensional entity laying under topography.

Modelling approaches utilise topographical information and drill hole database. Frequently and representatively recorded GPS values and aerial views help to visualise photo realistic views of topography. Next step is to define ore shape in space. Drill holes values are the most crucial and critical data set for this purpose. Problem at this stage is to determine what happens between sample points. Answer of the question requires two-dimensional and three-dimensional representations of ore body. This may look easy for regular structures. However, in most cases the picture is just reverse. Geological interpretation is highly needed. Computer systems help interpreters at this point. Computer programs, in general trend, provide two alternatives to interpreters for 3D determinations; geological parallel cross-sections and block models.

In this chapter, the basic ideas and some mathematical approaches of computer aided mine valuation are given. How GPS and drill hole data are used to figure out an entire ore body model in visual and numerical aspects are explained.
Fig. 1. Mine valuation work flow.

2. Database processes

Database is the base of every further study. Health of projects entirely depends on health of samples. GPS records are very crucial to model topography. However, the most concrete data that can be taken from field is drill hole cores. Mine valuation process starts with database building. It is composed of spatial coordinates of drill holes, geological formations that they intersect, depths of formations and their assay values. Surface determinations are, almost entirely related to GPS data.
### 2.1 Database structure

Input material for mine valuation systems is mainly topographical surveying data and drill hole cores. It is possible to categorise this database as; i- collar data, ii- survey data, iii- stratigraphical/lithological data, iv- assay values/grade analyses.

Collar data keeps \((x, y, z)\) coordinates of drill ring. Survey database may include also physical coordinates of drill holes, depth information, azimuth and bearing angles. Stratigraphy database contains the information of geological formations through each hole. Assay database has the quality values of ore formations. Drill hole log/stamp is like an identity card of them (Fig. 2).

![Drill hole log and drill holes in three dimensions](image)

#### Fig. 2. A single drill hole log and drill holes in three dimensions (Erarslan, 2007).

Surveying data may also include three-dimensional components \(x, y, z\) (easting, northing, altitude/elevation) which enable surface modelling. However, GPS data that have been taken with frequent intervals enables photo realistic models. If data can take representatively, topography model looks like the field itself. Drill hole data including depth and layer information contribute to explain how its shape is in the third dimension. Drill holes also carry the information of ore grade or calorific value. Geological interpretation of stratigraphical layers provides three-dimensional ore body model.

### 2.2 Drill hole compositing

Drill holes cut intersect downwards successive layers. Mechanical properties of waste layers and quality values of ore layers are determined by applying several tests on hole cores. Generally, mineral formations are not monolithic and single piece bodies. Inter-burden layers may intersect mass or mineralisation may occur with waste layers in alternating forms. In other words, valuable mineral layers may exist in different thickness and quality amounts. During numerical calculations, a single thickness and grade value may be needed. Some classical reserve estimation techniques such as triangulation and polygon methods need composite values. In that case, what is the net thickness of valuable part and its quality? Here, compositing computation is applied to have total thickness and a unique grade (quality) value for each drill hole. Regarding a cut-off grade, ore thicknesses are...
summed up to give ore thickness. On the other hand, grade value is the thickness-weighted average (Hustrulid and Kuchta, 2006).

\[ t_{total} = \sum_{i=1}^{n} t_i \]  

(1)

\[ g_{comp} = \frac{\sum_{i=1}^{n} g_i t_i}{\sum_{i=1}^{n} t_i} \]  

(2)

where, \( t_{total} \) is total ore thickness, \( g_{comp} \) is composited grade, \( t_i \) is thickness and \( g_i \) is grade of \( i^{th} \) core piece within \( n \) core pieces. After obtaining a single thickness and grade a value for each drill holes, volumes and reserve amounts in superimposed triangles or polygons can be calculated.

Another type of compositing is called as bench or level compositing. Ore field under investigation is divided into parallel horizontal levels and parts of drill holes corresponding to those levels are composited instead of compositing entire hole (Fig. 3).

![Fig. 3. Bench/Level Compositing](image)

Level elevations may be same with bench elevations of prospective open pit. Such a compositing process gives ability and base to further block modeling and bench/level reserve estimation. Bench reserves help for production planning in further stages.

### 3. Data extension on a network

Drill hole data looks like spotting points in bird’s eye view. Problem is to know how structure changes between these sample points. Sample points, where several parameters
such as thickness and grade are already known, should be used to estimate these parameters at points where no sampling is available. This process can be named as data extension. Data can be extended thru two-dimensional planes or three-dimensional space. During extension process, square or rectangular grids, wireframes are imposed onto area. Triangulation is another type of artificial net, applied on field. Then, extension methods such as inverse distance square, geostatistics and artificial neural networks are applied.

3.1 Gridding

Although there are several methods to figure underground treasures such as aerial and seismic surveys, drill holes are still the most reliable and decisive data givers. In plan view, they look as irregularly distributed point data. Grid is a regular network formed by triangle, square or rectangles generally. It is the result of a regular mesh need. Irregularly distributed sample points get a regular form if node points are assigned parameter values such as thickness and grade. In other words, thickness and grade values are estimated at node points, which results in a regular data structure (Fig. 4). Plenty of node points are generated by superimposing a wireframe grid onto a field (Knudsen, 1990, Parker, 1990).

$$p(i,j) = f(x_i, y_j)$$ (3)
where, \( x_i \) is east coordinate of \( i^{th} \) column and \( y_j \) is north coordinate of \( j^{th} \) row. Then \( x_i \) and \( y_j \) can be calculated as;

\[
x_i = s_x + i \cdot \Delta x \quad (4)
\]
\[
y_j = s_y + j \cdot \Delta y \quad (5)
\]

where, \( \Delta x \) and \( \Delta y \) are spacing thru \( x \) and \( y \) lines and \( s_x \) and \( s_y \) are origin coordinates for east and north respectively. After computing \( x \) and \( y \) coordinates of all nodes, next step is calculating the third coordinate which may be thickness, grade, elevation or anything else. Third coordinate/parameter comes from surrounding sample points by carrying/extending drill hole parameters to these node points. For this purpose, there are several methods such as inverse distance square, geostatistics and neural networks. Once a grid system is built, parametric values like thickness and grade are assigned. Then by means of rotation around an axis, third dimension can be sensed (Fig. 7).

### 3.2 Data extension in 2D

Data extension is a result of need to interpret how ore body behaves between present samples. Major two-dimensional extension applications of drill holes are triangular and polygonal area generation and of course, contour maps. The purpose is to carry pointwise data (drill hole) values into areas by certain mathematical methods and acceptations.
Fig. 7. Rotated grid system referring origin \((s_x, s_y)\).
Additionally, in order to get a regular data structure, a grid is superimposed onto area. Each node point is assigned parametric values after computations. By this way, each grid node may act as a sample point.

### 3.2.1 Triangles and triangulation

Triangles are generated by joining drill holes. Area enclosed between three adjacent drill holes is calculated. Next step is to calculate volume \((V - m^3)\) by multiplying triangle area \((A - m^2)\) with average thickness \((t - m)\) of three drill hole composited thicknesses.

\[
V = A \cdot t_{\text{ave}}
\]

\[
t_{\text{ave}} = \frac{t_1 + t_2 + t_3}{3}
\]

It is also possible to find triangular reserve \((R - \text{ton})\) if tonnage factor \((f - \text{ton/m}^3)\) and thickness weighted average grade \((g - \%)\) of three composited grades is known.

\[
R = V \cdot f \cdot g_{\text{ave}}
\]

\[
g_{\text{ave}} = \frac{g_1 \cdot t_1 + g_2 \cdot t_2 + g_3 \cdot t_3}{t_1 + t_2 + t_3}
\]

Triangle method is a classical and rough estimation one with certain acceptations and could be employed only for horizontally and regularly bedded fields. Sedimentary type fields such as coal may be applied. Each triangle corner is a drill hole and composite thickness and grade values are used for computations (Fig. 8).

Fig. 8. Triangular grid by drill holes and local reserve.
Triangulation method can also be applied to produce sub-triangles and find coordinates of sub-triangles’ corners. DeLaunay, Voronoi and Thiessen approaches can be employed during triangulation (Sen, 2009). New triangular network is assigned parametric values by several methods such as inverse distance square, geostatistics and neural network. It can be employed not only for reserve estimation but also for surface drawing. During node assignments, a question may arise; which sample points should be included in calculations? There should be a limitation for drill holes and answer is radius of influence. Samples within radius of influence area are used in calculation of weighted averages. Radius of influence may be result of trial-error attempts or may be geostatistical variogram range, which will be explained later.

3.2.2 Polygonal gridding
Polygon method is based on linear influence area concept. They are geometrically defined by the perpendicular bisectors of the lines between all points (Sen, 2009). Influence distances between drill holes are at their mid-points (Fig. 9). Therefore, joint points of lines perpendicular to mid-points form a polygon and polygonal area can be calculated. Thickness and grade values within the area are assumed same with that drill hole’s values. Volume of polygon is multiplication of area and thickness and reserve is product of volume, specific gravity and grade.

4. Assignment methods
Grid or mesh generation is followed by assignment stage. Nodes in 2D and blocks in 3D are assigned parametric values. This structure is the base for 2D and 3D models. 2D and 3D ore body modelling can be accomplished by several approaches. Joining cross-sections taken thru ore body and block modelling techniques are mostly used methods. The models do not only give the shape of ore body but also provide volume and reserve amount. Either triangular or rectangular mesh generation requires an assignment method. Here, commonly used mathematical approaches are;

Fig. 9. Polyonalisation (Sen, 2009) and Voronoi polygons (BGIS, 2011).
i. Inverse distance,
ii. Geostatistics,
iii. Artificial Neural Networks.

4.1 Inverse distance method

Inverse distance method is actually a weighted average method. However, weights are calculated inversely. As distance of a sample point (drill hole) to an assignment point (node) gets more distant, its contribution on the average result gets less. In other words, closer sample point means more effect on assigned value. The basic idea is providing effect of a drill hole to point on which assignment is to be performed, is inversely related with the distance between them. This is realised by taking weighted average of parametric values by distances as shown below:

\[ z(i,j) = \frac{\sum_{k=1}^{K} \frac{z_k}{(\Delta_{k(i,j)})^\alpha}}{\sum_{k=1}^{K} \frac{1}{(\Delta_{k(i,j)})^\alpha}} \]  \[ (10) \]

where,
- \( z(i,j) \) = assigned value (i.e. grade, thickness) at node point on \( i^{th} \) column and \( j^{th} \) row.
- \( z_k \) = parameter value carried by \( k^{th} \) drill hole (sample).
- \( \Delta_{k(i,j)} \) = distance between \( k^{th} \) drill hole and \( (i,j) \) node.
- \( \alpha \) = power of inverse distance process (\( \alpha=2 \) in general and method is called as inverse distance square).

Distance between drill hole and node points can be calculated simply by;

\[ \Delta_{k(i,j)} = \sqrt{(\delta x(i,j) - D_{nx})^2 + (\delta y(i,j) - D_{ny})^2} \]  \[ (11) \]

where,
- \( \delta x(i,j) \) = x co-ordinate of node \( (i,j) \).
- \( D_{nx} \) = x co-ordinate of \( n^{th} \) drill hole.
- \( \delta y(i,j) \) = y co-ordinate of node \( (i,j) \).
- \( D_{ny} \) = y co-ordinate of \( n^{th} \) drill hole.

This method has an acceptation that geological structure has a linear behaviour. In case of sudden thickness or grade changes, inverse distance method may fail in determining a healthy result. On the other hand, its application is not only limited to 2D calculations but it can also be employed in 3D computations.

4.2 Geostatistical methods

Basic concept of geostatistics is regional variability of parameters (Matheron, 1971, 1963; Krige, 1984). Not only deterministic and descriptive manner but also probability and statistics take place in calculations with geostatistics (Mallet, 2002). If there is mathematically explainable structure of an ore body within a limited region, it can be possible to state that behaviour by equations and use them for estimation (Sarma, 2009). When variability cannot be formulated by mathematical models then it is decided that sample values show random behaviour and probabilistic methods of statistics can be applied there.
Geostatistics do not only consider distances between sample points and assignment points but also their position and direction with each other (Webster and Oliver, 2007). Additionally, during estimation, not only variances between sample points and assignment points are taken into account but also variances of samples within themselves are regarded (David, 1977, Davis, 1973). This means, geostatistics regards not only distance relation between sample points and node point but also variation between them in relation with distance and direction (David, 1988). Main two stages of geostatistical process are variogram modelling and kriging.

4.2.1 Variogram modelling
Main tool of geostatistics is variogram model (Diggle and Ribeiro, 2007). It can be defined as the graph representing the relation between distance and parametric variance in a certain direction (Deutsch and Journel, 1998). In estimation stage, kriging interpolation, geostatistics utilises co-variances between sample points and point/area/volume on which assignment is to be performed, and co-variances between sample points affecting that point for extending a value (Kitanidis, 2003).

Initially, a variogram model representing the relation between distance and variance is defined. Variance is:

\[
\gamma(h) = \frac{1}{2n} \sum_{i=1}^{N} (z - z')^2
\]  

where,

- \( \gamma(h) \) = variance of sample pairs located \( h \) distance (lag) apart.
- \( z \) = value at sample \( z \).
- \( z' \) = value at sample, \( h \) distance (lag) apart from \( z \).
- \( n \) = total number of pairs located \( h \) distance (lag) apart.

For each different \( h \) distance (lag), a variance \( \gamma(h) \) is calculated (Fig. 10). According to distribution of \( h-\gamma(h) \) pairs, a mathematical model is tried to fit. General trend is to use one of predefined variogram models such as, Spherical, Exponential, Gaussian, Linear, etc., (Davis, 1973; Krige, 1978).

![Fig. 10. Variance values marked for \( h \) distanced (lag) sample pairs.](image-url)
For this purpose, sample pairs are found for a certain trigonometric direction $\theta^\circ$ and lag distances $h, 2h, 3h, \ldots, dh$. As it may not be possible to find pairs having direction and distance conditions absolutely, a tolerance distance $\pm \Delta h$ and direction (angle) $\pm \Delta \theta^\circ$ is considered. Parameters, such as thickness and grade carried by sample points, are used in the formula and variance value is calculated. After marking variances on $h-\gamma$ graph, experimental variogram is drawn (Leuangthong and Deutsch, 2004). Then, variogram is a graph explaining variance versus distance relation.

Experimental variogram points show a trend and needs interpretation for deciding which model best fits. Spotted points may have a mathematical determination in equation form, which is also called variogram model. It is similar to regression modelling. (Fig. 11). In the graph $C_o$ is called as nugget effect, $C$ is sill and $a$ is range. Nugget is the variance value at zero distance. Normally it has to be zero however due to sampling problems, tolerances in pairing and structural inconsistencies, nugget gets over zero. Best graph and its mathematical equation (model) for $h-\gamma(h)$ pairs are decided by visual interpretation regarding several predefined models such as spherical model (Matheron), exponential model, De-Wisjian, model, linear model.

![Variogram model](image)

**Fig. 11.** Variogram model fitted for spotted points showing distance-variance; $h-\gamma(h)$ relation.

Models are classed as models with sill and without sill. Sill is defined as the platform where variance gets parallel to $h$ axis. Distance where graph reaches at sill level is called as range. Range may also be considered as radius of influence. Models are given below;

**Spherical Model**

$$\gamma(h) = \begin{cases} c \left( \frac{3h}{2a} - \frac{h^3}{3a^3} \right) + c_0 & h \leq a \\ c + c_0 & h > a \end{cases}$$

(13)

**Linear Model**

$$\gamma(h) = a \cdot h$$

(14)

**Exponential Model**

$$\gamma(h) = c \left( 1 - e^{-h/a} \right)$$

(15)
where,
\[ c = \text{Sill value (constant value where variances get parallel to lag axis).} \]
\[ c_0 = \text{Nugget effect (variance where distance is zero).} \]
\[ a = \text{Range (distance where variance reaches at sill value).} \]

Another concept in geostatistics is direction and variability of variograms according to directions. If variogram models developed for different directions can be accepted tolerably identical to each other, then ore body is called isotropic and anisotropic vice versa. Same variogram model can be used during kriging estimations for all directions if existence of an isotropy is decided.

### 4.2.2 Kriging

In order to extend sample data and use it for the prediction of unknown values, kriging is applied. Kriging is an extension and estimation method where parameters are assigned to assignment points by means of surrounding sample points. Aim is to calculate a weight \( a_n \) for each sample point.

Aim of developing a variogram model is to state variance as a function of distance and determination of mathematical equation that will be used to compute covariance matrix terms that will be used in kriging stage. Kriging process, developed by Krige (1966), is an estimation method regarding covariances between assignment point-sample points and within sample points mutually as well. In equation and matrix form after derivations, respectively;

\[
z^*(i,j) = \sum_{k=1}^{n} w_k \cdot z(x_k)
\]

\[
\sum_{k=1}^{n} w_k = 1
\]

where,
\[ z^*(i,j) = \text{assigned value to (i,j) node point regarding surrounding n sample points (drill holes)} \]
\[ z(x_k) = \text{parametric value of x_k sample point} \]
\[ w_k = \text{weight of sample point k}. \]

Simple representation of matrices is below:

\[
[\sigma_{xx}] \cdot [w_x] = [\sigma_{x*}]
\]

where,
\[ [\sigma_{xx}] = \text{matrix including variances between sample points mutually} \]
\[ [\sigma_{x*}] = \text{matrix including variances between assignment point and sample points.} \]
\[ [w_x] = \text{weight matrix} \]

Weight matrix is computed and weights are used to estimate unknown parameter at assignment point. \( \sigma \) variance values in matrices are calculated by putting distance amounts between sample points mutually \( (\Delta_{xx}) \) and sample-to-assignment points \( (\Delta_{x*}) \) into \( h \) term in variogram model.

Geostatistics has a wide application field in geological modelling. Main problem of the method is visual evaluation and interpretation of experimental variogram during model
development, which may be controversial and relative. Number of samples should be an acceptable amount; may be twelve and mostly over thirty (Sen, 2007).

One of the advantages of geostatistics is error estimation ability;

\[ e^2 = (Z' - Z)^2 \]  

(19)

where, \( e^2 \) is estimation error (variance), \( Z' \) is estimated value, \( Z \) is actual value (Sinclair and Blackwell, 2004).

4.3 Artificial neural network method

One of most recent methods used to extend sample data is artificial neural networks (Rabuñal and Dorado, 2006; Haupt, et.al., 2008 ). Initially, the neural system is trained to learn the structure of samples. Each sample is assigned a random weight and total error is recorded. Iteration by iteration, error is distributed to approximate actual sample values. An acceptable error level is succeeded and weights of sample points are calculated (Fig 12).

![Typical artificial neural network structures.](image)

Fig. 12. Typical artificial neural network structures.

Then accordingly, a value is assigned to any desired location through area or space. In formulated form;

\[ y = w_1x_1 + w_2x_2 + \ldots + w_nx_n = \sum_{j=1}^{n} w_jx_j \]  

(20)

where, \( y \) is value to be assigned, \( w_j \) is the weight assigned to \( j \)'th sample and \( x_j \) is value of \( j \)'th sample. As explained earlier, calculation of \( w_j \)'s requires an iterative process, which is called as training. A random weight assignment is followed by measurement of error between present input and desired output. An error above acceptable limit causes propagation of error through weights. During "squashing" the limit, an activation function, called as sigmoidal-function is employed;

\[ f(x) = \frac{1}{1 + e^{-x}} \]  

(21)

New weights are tested if they fulfil condition of error limit. When the system fits error level, weights can be used for assignment (Fausett, 1994).
Total error gets less and less at each iteration and process continues as expected error level is reached. In order to achieve minimum error, hundred thousands, even millions of iterations may be needed. At that stage, it is decided that system has learned database structure and ready for further interpretations and estimations. Any more, \( w_i \) coefficients are available for assignments (Freeman and Skapura, 1991; Dowla and Rogers, 1995).

5. Two-dimensional visual and numerical processes

After generation of triangular and rectangular nets, they can be used for different purposes such as volume and reserve estimation, surface representation and forming a contour database. Another very basic visual instrument to show behaviour of ore through area is contour map. Sample data is extended to area as isolines.

5.1 Contour maps

Contour maps are generally based on triangulated or gridded networks, which are superimposed on mine area to represent topography in computerised environment. Node points on network wire are assigned several values such as topographical elevation, compositied thickness and grade, ore seam upper or bottom surface elevations, etc. (Watson, 1992). Main idea and purpose is to estimate several parametric values at node points using sample values obtained by drill holes. Here, inverse distance square, geostatistics and artificial neural networks are applied to assign node values. Thereafter topographical, thickness and grade contour maps can be drawn (Fig. 8). Bézier curves, B-Splines and Cubic Splines are primarily applied mathematical techniques (Mortenson, 1999; Comnino, 2006; Foley et al., 1990; Vince, 2005; Vince 2010). Points on grid nodes with \((x,y)\) coordinates get third dimension coordinate as well after assigning a parametric value as \(z\) coordinate such as thickness, grade, etc. Then, point data set is ready for contour (curve) fitting.

Bézier curves employ Bernstein polynomials (Vince, 2010). Its general equation is given below:

\[
B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}
\]

where \( \binom{n}{i} \) is shorthand for the number of selections of \( i \) different items from \( n \) distinguishable items when the order of selection is ignored and the coordinates of any point on the circumference in terms of some parameter \( t \). Bézier curves may also be in quadratic and cubic Bernstein polynomials form (Vince, 2010).

B-splines also use polynomials to form a curve segment. However, B-splines use a series of control points determining the curve’s local geometry. This feature provides and ensures that only a small portion of the curve is changed with movement of a control point (Vince, 2010). Splines can be classed as uniform and non-uniform and also rational and non-rational.

Cubic splines enable continuity between segments, which puts them one-step away from simple quadratics. Cubic splines can also be considered as piecewise polynomials (Fig. 13).
Here, curve segment $S_i$ is under influence of points $P_i$, $P_{i+1}$, $P_{i+2}$, $P_{i+3}$, and curve segment $S_{i+1}$ is related to points $P_{i+1}$, $P_{i+2}$, $P_{i+3}$, $P_{i+4}$. There exist $(m+1)$ control points and $(m-2)$ curve segments. Hence, a particular segment $S_i(t)$ of a B-spline curve is defined by

$$S_i(t) = \sum_{r=0}^{3} p_{i+r}b_r(t) \quad \text{for} \quad [0 \leq t \leq 1]$$

(24)

where,

$$B_0(t) = \frac{-t^3 + 3t^2 - 3t + 1}{6} = \frac{(1-t)^3}{6}$$

$$B_1(t) = \frac{3t^3 - 6t^2 + 4}{6}$$

$$B_2(t) = \frac{-3t^3 + 3t^2 + 3t + 1}{6}$$

$$B_3(t) = \frac{t^3}{6}.$$  

(25)

These are the basic functions of cubic splines (Rogers and Adams, 1990). Finally, topography, ore seam upper and bottom contours, thickness (isopach), grade (isograde), etc., can be drawn (Fig. 14).

Curve fitting methods are modified for also surface fitting in three-dimensional environments. Eventual aim of computer systems is to display ore body in 3D space.

6. Three dimensional operations

3D ore body modelling can be accomplished by several approaches. Joining cross-sections taken thru ore body and block modelling techniques are mostly used methods. The models do not only give the shape of ore body but also provide volume and reserve amount. Drill hole cores explain researchers definite coordinates of underground layers. Topographical information is combined with depths to yield three-dimensional coordinates.
Fig. 14. Contour maps; topography, thickness and grade maps, respectively (Erarslan, 2007).

Major instruments for 3D computer aided mine valuation are;
- surfaces
- cross-sections,
- solid models (three-dimensional models).

6.1 Surfaces
Surfaces are important visual outputs of computer aided mine valuation systems. Surface can be prepared for any parameter assigned to grid system. \((x,y)\) coordinate pairs of each node can be calculated easily. Fig. 15 shows how third dimension is sensed after rotation of 2D grid around y-axis.

Third coordinate may be topographical elevation, thickness, grade or anything else. Surface will be named according to its third parameter such as topographical surface, thickness surface, grade surface, etc. 3D visualisation of surfaces helps researcher imagine parametric changes.

Fig. 15. Three-dimensional surface model (Pirsa, 2011).
Surface fitting is applied also on triangular or rectangular grid data. Assignment of third coordinate to \((x,y)\) pairs of grid nodes is realised by already mentioned methods. After obtaining \((x,y,z)\) coordinate set, bicubic, planar surface patch, quadratic Bézier surface patch and cubic Bézier surface patch, B-Spline surface approaches can be utilised (Hill, 1990).

Surface can be in wireframe (fishnet) or rendered form. During rendering action, several materials such as soil, several rock types, etc., and, also sky views for background can be imposed (Fig. 16). By this way, photo realistic appearances can be obtained. Some virtual reality program supports such as OpenGl and GlView allow users walk or fly over developed model (RealTech, 2011; GlView, 2011).

Fig. 16. 3D surface model; i- wireframe appearance (Erarslan, 2003), ii- rendered with texture (Golden, 2011).

### 6.2 Geological parallel cross-sections

Drill hole sections aligned thru or near a cross-section line are used to draw geological section view. Softwares show users hole lithologies and let them make interpretation on them. Closed polygons regarding stratigraphy determine ore body cross-section thru that section line (Figure 17).
6.3 Three-dimensional ore body models
Three dimensional ore body cross-sections do not only give an idea about structure of deposit but also prepare a base for three-dimensional ore model. Successive and parallel cross-sections are interpreted along ore body. Software can later on combine parallel cross-sections to form a 3D ore body model. Some software is also capable of combining cross-section, which are not parallel, and maybe intersecting each other. This type of sections are visualised on so-called fence diagrams (Fig. 18). Parallel cross-sections of ore body are fundamental tools for 3D modelling. They can be horizontal or vertical (Fig. 18). Computer programs help researchers display sections and join them to build a 3D appearance. Each closed polygon enables calculation of section area and by means of average areas method, volume of ore body can be calculated. Gauss-Green area formulation can be employed to estimate sectional area:

$$A = \frac{1}{2} \sum_{i=1}^{n-1} (y_{i+1} - y_{i-1})x_i$$

(26)

where, $A$ is polygonal area, $x_i$ and $y_i$ terms are $x$ and $y$ coordinates of polygon nodes.
6.4 Block models
Another very frequently used ore body modelling method is block models. Block models could be thought as three-dimensional forms of 2D gridding. Field is divided into blocks, physical properties and quality composition are represented by this geometric form (Fig. 19). Centre and corner coordinates of each block can be calculated as \((x, y, z)\) data sets. Reference/origin point is at top or bottom. In some cases, prospective open pit benches define height between levels \((h)\). Regarding block width, length and height and referring to origin, coordinate computations are performed. Determination of physical position of each block is followed by thickness and grade assignments. Block assignments are performed by several mathematical approaches and geological block model is generated. Similar to calculations carried out on grid nodes, process is repeated for each elevation level. Geostatistics and neural networks are most advanced estimation models. Inverse distance square method can also be applied for more regular geological structures such as sedimentary beddings. According to assigned grade, blocks are coloured in software systems to enable observing quality tableau better.

6.5 Volume and reserve estimation
Volume and reserve of deposits is crucial subject of mine valuation as well as the visual aspect. There are various methods for numerical estimations. However, recent methods are computer dependent. Several approaches for ore volume and reserve estimations are given below (Hartman, 1992):

i. The area enclosed by ore limits are multiplied by the average thickness. The volume is also multiplied with average tonnage factor to give inventory. Percent grade gives how much of this inventory is ore; that is reserve.

ii. Triangular net is used to calculate total reserve. Each triangular area is found and average thickness is multiplied by area to calculate volume of ore in triangle. Product of volume and weighted averages of tonnage factor and grade give triangular reserve. Summation of reserve of triangles yields total reserve.

iii. Field is divided into grids. Node points are assigned thickness, grade, etc. by inverse distance square, geostatistics and neural network methods. By this way, corner vertices of each node cell are assigned a value. Multiplication of cell area that is enclosed by grid
Fig. 19. 3D Ore body block model and Solid Model of Zambujal Ore body (Lundin, 2007).

iv. Areas of parallel geological sections are calculated by Gauss-Green formula. Volumes between successive sections are estimated by average areas method.

v. Ore body block model gives also numerical results as well as visual. Each block reserve can be calculated. Total reserve is the reserve total of all blocks. Block volume is simply multiplication of length, width and height of it. However, after thickness assignment to a block, thickness of ore should be considered rather than its geometrical height. Tonnage factor and ore grade give block reserve. Total reserve is the summation of all block reserves (Taylor, 1994, 1993).

\[
\text{Block reserve (t) = } \text{block volume (m}^3\text{)} \cdot \text{tonnage factor (t/m}^3\text{)} \cdot \text{grade (٪)}
\]  

(27)

Volume inside the open pit and ore volume bench by bench can later be estimated (Taylor, 1992). After determining open pit limits, border of each bench can be thought as a polygon and by means of Gauss-Green formula pit volume is computed. Stripping volume can also be calculated to find stripping ratio (Taylor, 1991).
An important factor in reserve estimation is cut-off grade, which can be described as the grade where excavated material is classed as ore or waste (Taylor, 1986, 1985). It may be considered as breakeven point as well (Taylor, 1972). So many researches and models have been developed to determine accurate cut-off grade. However, this economically crucial subject is special and unique to each field and it should be studied particularly (Wellmer et al., 2008).

6.5.1 Isopach maps for volume calculation

Beside triangle and polygon methods, a special technique using isopach contours is employed for volume computation. Contour maps can be thought as two-dimensional extension and interpretation of drill holes. Isolines are drawn for several parameters provided by holes. Thickness and grade values are very crucial to determine ore body. It is possible to see where thickness increases and decreases or where ore is rich in grade and less valuable. Isopach maps, which are isolines for thickness, can also be used for volume calculation; volume under isopach maps are ore volume itself (Fig. 20).

![Fig. 20. Cross-section of isopach maps and volume calculation by using average areas rule.](image)

Areas, at each thickness level are estimated and volume between each successive area pairs is calculated by using average areas rule:

\[
V_j = \frac{A_j + A_{j+1}}{2} \cdot h
\]

where, \( V_j \) is volume at level \( j \), \( A_j \) and \( A_{j+1} \) are successive areas of thickness level \( j \) and \( h \) is height (depth difference) between thickness isolines.

6.6 Mine design, production planning and mineral economics

Final stage of mine valuation is to decide if an ore deposit is worth making investment. Geological structure provides a database for economical assessment. Regarding physical and geological outputs of computerised systems, mine can be designed and production planning can be accomplished (Fig. 21). Possible investment, annual costs and incomes are considered to estimate and foresee how that underground asset can be extracted optimally. Present worth, future worth, annual worth and rate of return of the net cash flows are calculated and reported. During calculations, straight line, double declining balance and sum of the years’ digits methods are used for depreciation (Steiner, 1992). Taxes, salvage values, royalty costs, etc. are all taken into account.
Optimisation is another wide application branch of mine valuation. Geological block models are used to generate \textit{economical block models} by using unit costs and income (Erarslan, 2001). As volume of a block, thickness and grade of ore at each particular block is known, then it becomes possible to convert this information to economical aspect. Multiplication of volume, tonnage factor and grade give block reserve. Unit production cost and expected income are considered and an economical value is assigned to each block. Economical block models have visual and numerical results. 3D appearances of them give an idea where ore body is rich and how quality changes. Mine design and optimisation applications such as optimum pit limit and optimum production planning are comprehensive numerical assessment methods (Erarslan, 2001).

7. Computer software for ore body modelling and mine valuation

Generally, size of the database may be too bulky to manage studies with hand effort. Hence, numerical algorithms and mathematical approaches necessitate computer applications to overcome huge computational time and processes. Today, many software and computer aided systems serve for geological modelling and mine valuation in this sense. The accuracy and speed of computers enable evaluation of various scenarios within reasonably short times. Commercial softwares in general have robust database management capability. After building a healthy database structure, computer programs are ready for ore body modelling. Many mathematical models and approaches in literature take place in software packages to determine shape, location and quality composition of the entire body. Visual appearance of geological body is supported by numerical data such as ore reserve amount and quality composition, which are vital parameters for mine design and scheduling. There are several integrated commercial packages for this purpose (MineSight, 2011; Gemcom, 2011; GDM, 2011; Techbase, 2011; Datamine 2011; Lynx Mining, 2011; RockWorks, 2011). They can successfully handle real cases, which may be fairly complex structures. Their processing and graphical capabilities and utilities have improved year and year (Fig. 22). Computer systems make engineering designs, project preparation and management easier. Raw data such as GPS and drill hole cores are converted to contour maps, three-dimensional solid structures, volume and reserve reports, open and underground mine plans, economical assessments and production schedules. Various scenarios are examined quickly. Eventually, underground asset with so many unknowns becomes visual. Its properties thru space are clear. Million dollars are invested regarding this tableau. Thus, computer systems are vital partners of geological and mining engineers.
Fig. 22. Computer software functions for ore body modelling (RockWorks, 2011).
8. Conclusion

As mining is an industry requiring millions of dollars for investment and further operations, mine valuation is a crucial stage, which provides basic information for future stages. Before deciding on a mining investment, the preliminary process includes exploration, data gathering and valuation, determining geological structure, ore body modelling, mine design and planning.

In the last several decades, a number of mathematical and computational approaches have been developed to give the most accurate information related to ore bodies under investigation. In parallel, many computer programs have been developed to accomplish these complicated processes. This means that all investments are dependent on physical and economical characteristics of ore deposit. Similarly, profitability of the investment is strictly related to mine design and planning (Hartman, 1992). Eventually, data evaluation and ore body modelling is a very critical and basic process and mining operations are based on its results (Kennedy, 1990).

Drill hole database, geometrical and numerical analyses, contour maps, surfaces, sections, three dimensional ore body block models, volume and reserve calculations, economical assessment, classical valuation methods, triangulation, polygons, gridding, geostatistical approaches, neural network method, etc. are headings and instruments of mine valuation. Computer aid has become inevitable and vital for contemporary ore body modelling and mine valuation applications. Long and complicated process starting with data base constitution and ending with mine design and production planning requires software support due to iterative mathematical structure of computations. Drill hole composites, triangulation, polygonal and grid nets, contouring, cross-sections, three-dimensional ore models, various volume and reserve estimations, geological and economical block models, mine design and production scheduling, optimisation and simulation works are highly computer dependent. At each stage of the processes, many developed mathematical approaches are utilised. One who deals with computer aided ore body modelling is faced with many and many studies and methods. However, geostatistics, neural networks, inverse distance methods are very basic subjects to be comprehended. Scientists work to improve present methods and develop new ones as well. Additionally, many commercial software are under a continuous development for a better service.

9. References


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The studies of Earth’s history and of the physical and chemical properties of the substances that make up our planet, are of great significance to our understanding both of its past and its future. The geological and other environmental processes on Earth and the composition of the planet are of vital importance in locating and harnessing its resources. This book is primarily written for research scholars, geologists, civil engineers, mining engineers, and environmentalists. Hopefully the text will be used by students, and it will continue to be of value to them throughout their subsequent professional and research careers. This does not mean to infer that the book was written solely or mainly with the student in mind. Indeed from the point of view of the researcher in Earth and Environmental Science it could be argued that this text contains more detail than he will require in his initial studies or research.

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