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Robust Networked Control

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1. Introduction

Most of integrated industrial control systems adopt a multilevel, vertical control hierarchy. Logically, such a system (Fig. 1) is structured in three levels: the direct (device) control level, the supervisory level and the management level (Grega, 2010, Tatjewski, 2007, Grega et al., 2009).

The basic task of the direct (device) control level is to maintain the process states at the prescribed set values. The device controller level provides an interface to the hardware, either separate modules or microprocessors incorporated in the equipment to be controlled. Here, mainly PID digital control algorithms are implemented – in some cases these are more advances control methods such as multivariable control or adaptive functions. A number of embedded control nodes and Programmable Logical Controllers (PLC) are used as the front-ends to take the control tasks. High speed networks and fieldbuses are implemented at the direct control level to exchange in real time the information between front-ends and the device controllers and, vertically, with the supervisory control level. This architecture has the advantage of locating the hard real-time activities as near as possible to the equipment.

Fig. 1. Multilevel structure of an industrial control system
The supervisory level comprises workstations and industrial PCs providing high-level control support, database support, graphic man-machine interface, network management and general computing resources. Classically, the supervisory level calculates set points for controllers according to the defined criteria. For this purpose more complex mathematical models of the process are employed at this level to find the optimal steady-state, by solving optimisation and identification tasks. Due to the rapid development of control technology, there is growing scope for more advanced close-loop algorithms (predictive control, repetitive control) located at this level. However, increasing computational efficiency of PLCs at the device level supported by high performance networks transferring data and control signals vertically gives more flexibility to the designer. The control loops can be handled by local, device-level controllers, and also by the supervisory controllers (Fig.1). For example, a predictive control algorithm can be handled by a supervisory workstation as well as by a local PLC. It should be noted that upper level loops usually offer shorter computational time due to the higher efficiency of the workstations.

Feedback control systems wherein the control loops are closed through a communication network are referred to as Distributed Control Systems (DCS). They are distributed in the sense that their sensors, actuators and controllers (referred as “nodes”) communicate via a shared data transmission network. The behaviour of a networked control system depends on the performance parameters of the underlying network, which include transmission rate and access method to the network transmission medium.

Communication networks were introduced in control in the 1970s. They can be grouped into fieldbuses (e.g. CAN, Profibus, Modbus) and general purpose networks (e.g. IEEE standard LANs), (Zurawski, 2005). Each type of network has its own protocol that is designed for a specific range of applications. Fieldbuses are intended for real-time applications. The most important feature of these industrial networks is that they guarantee bounded transmission delays. More and more popular is application of general-purpose networks, inexpensive and easy to maintain. Ethernet is a solution, which seems to become an industrial standard in the near future (Felsner, 2005).

The advantages of data transmission channels integration into control system are obvious, such as reducing wiring costs and increasing flexibility. Thanks to these important benefits, typical applications of these systems range over various fields, such as automotive, mobile robotics, advanced aircraft, and so on. However, introduction of communication networks in the control loops makes the analysis and synthesis of distributed control systems more complex.

DCS can be considered a special case of digital control systems, as data is sent through the network periodically, in units called packages. Therefore, any signal continuous in time must be sampled to be carried over the network. Real-time assumptions are as important for DCS as for any other computer controlled systems. Hence, there are similarities between DCS and real-time digital control systems due to sampling effects. The most challenging problem with DSC that needs to be properly addressed are time delays. A network induced delays occurs while sending data among nodes connected to the shared data transmission medium of limited throughput. Network-induced delays may vary depending on the network load and Medium Access Protocol (MAC). Lack of access to the communication network is an important constraint compared to lack of computer power or time errors of the real-time operating system. It is well known that time delays can degrade the performance of the control system or even destabilize the system. Especially, the following effects are observed in DCS:
variable computation-induced delays,
- variable network induced delays,
- data loss, caused by packet dropouts.
resulting in:
- violation of the assumption that sampling/actuation intervals are evenly spaced,
- violation of the causality principle.

From the point of view of control theory networked control often introduces some additional dynamics and temporal non-determinism. Therefore, novel methodologies should be developed for stability analysis of DCS and optimise the performance. An integrated approach is necessary, that combines data transmission issues (modelling of variable communication delays), sampling theory and control theory.

The notion of robustness of various DCS properties (especially stability) plays an important role in design of control systems, as confirmed by extensive literature discussion (Walsh et al, 2002, Gupta and Chow, 2010). Very general formulation of robustness for DCS is illustrated in Fig. 2. As it was mentioned before, DCS can be considered as a special case of digital control systems. Therefore, it is sensitive to the sampling period $T_0$ variations. For non-networked digital control system the quality of control generally increases while $T_0$ is getting shorter. This must not be true for DCS. Increasing network traffic results in longer and variable network-induced delays, and leads to the deterioration of control quality. In this case robust design means shifting the DCS quality characteristic as close as possible to the characteristics of digital (non-network) control system.

During last 20 years various methods have been developed to maintain the stability and the performance of DCS with delay problems. In order to enhance robustness of DCS against network induced delays appropriate methods of control theory are supplemented by some methods of network traffic engineering. Therefore, two main research approaches can be distinguished (Gupta and Chow, 2010).

Fig. 2. Control quality versus sampling period
Study and research on communications and networks to make them suitable for real-time DCS, e.g. routing control, real-time protocols, congestion reduction, real-time protocols, codesign of networking and controllers are referred as Control of network.

Developing of control strategies and control systems design over the network to minimize the effect of adverse network parameters on DCS performance, such as network delay is referred as Control over network. The main advantage of this approach is its simplicity: the designer of DCS can exploit standard control algorithms and make them robust against effects of networking.

Following the Control of network approach, effects of the network configuration on the performance of the control system have been studied and different improvements have been proposed. At the physical level the network topology cannot be chosen freely but is subject to many practical constraints such as cost and reliability considerations. For example, the real-time performance of industrial Ethernet network depends strongly on the way the devices are allocated to the individual switches in the network. Therefore, the problem of optimal device allocation in industrial Ethernet networks with real-time constraints remains an important topic (Georges et al., 2006).

Another concept was to modify scheduling methods and communication protocols in such a way that data delays are minimized. Several solutions have been proposed. The most interesting of these involve:

- a new scheduling strategies based on a time division (Al-Hammouri et al., 2006),
- obtaining a maximum allowable delay bound for DCS scheduling (Walsh et al., 2002),
- adjustment of the network parameters (link quality measures) to the control quality,
- measures, by studying impact of frames priorities (Juanole et al., 2006).

Desire to incorporate a real-time element into some popular single-network solution has led to the development of different real-time Industrial Ethernet solutions, called Real-time Ethernet.

If the second approach is implemented (Control over network), the network is considered as a passive component of feedback loop, modeled in a simplified way. In most cases the control theory of delayed systems can be applied to compensate the effects of communication in order to guarantee the Quality of Control (QoC), (Hirai, 1980).

Network delays can be modeled and analyzed in various ways. They can be modeled as a constant delay (timed buffers), independent random delay and delay with known probability distribution, governed by Markov chain model.

One of the first applications taking the randomness of the network into account, either as a constant probability function or as a Markov chain together with time stamping was thesis of Nilsson (Nilsson, 1998). Later, the optimal stochastic methods approached the problem as a Linear-Quadratic-Gaussian (LQG) problem where the LQG gain matrix is optimally chosen based on the network delay statistics (Nilsson et al., 1998).

One simple idea is that constant delay in the control loop is better than variable delay. Introducing buffers reduces temporal dependency of the individual components of the close-loop model. The data package is delivered as soon as possible, but is hold in the buffer and is implemented to the process in the next sampling intervals. By this way, synchronisation of the control loop is achieved. Constant delay can be compensated using a standard approach, e.g. Smith predictor. It must be noted that constant delay buffer usually creates conservative controller gains. Better solutions give applications of switched or variable delay buffer. The stability analysis of the switched buffer model can be reduced to the problem of stability of the Asynchronous Dynamical Systems (ASD), (Hassibi, 1999).
Smith Predictor-based approach was proposed by several authors (Vatanski et al., 2009) for the control in the case when accurate delay measurements are accessible. In contrast to the robust control-based approach when only the estimate of the upper-bound end-to-end delays are available (Grega, 2002).

Other concept is to increase network utilization by modification of the transmission pattern – by samples grouping. The samples from sensor are transferred through network, however they are grouped together into $M$-element packages before they enter the network. Grouping effects can be compensated by an approximate model of the process (“observer”) at the controller side, and by control signal estimator (output to actuators) for some range of the sampling period and modelling errors (Grega and Tutaj, 2007).

Finally, network observers and state observes can be applied. The idea is that the communication delays between the sensor and the controller can be compensated by an approximate (non-exact) model of the process at the controller side, for some range of the sampling period and modelling errors. The performance of the method greatly depends on the model accuracy (Montestruque et al., 2003).

An intelligent control was proposed using fuzzy logic to adaptively compensate network induced time delay in DCS applications (Cao and Zhang, 2005). The advantage of the fuzzy logic compensator is that the existing PI controller needs not to be redesigned, modified, or interrupted for use on a network environment.

2. Control of the network

2.1 Optimizing protocols

The idea is to implement communication protocols and network topology that minimise data delays. Current communication systems for automation implement different protocols. This is a substantial disadvantage, leading to the need to use vendor-specific hardware and software components, which increase installation and maintenance costs. Moreover, presently used fieldbus technologies make vertical communication across all levels of the automation systems difficult. Gateways need to be used to establish connections between different kinds of fieldbus systems used in the lower levels, and Ethernet used in the upper level.

The evolution of industrial communication has moved to Industrial Ethernet networks replacing the proprietary networks (Larson, 2005, ARC Advisory Group, 2007). Ethernet provides unified data formats and reduces the complexity of installation and maintenance, which, together with the substantial increase in transmission rates and communication reliability over the last few years, results in its popularity in the area of industrial communications.

Ethernet, as defined in IEEE 802.3, is non-deterministic and, thus, is unsuitable for hard real-time applications. The media access control protocol, CSMA/CD can not support real-time communication because back-off algorithm for collision resolution is used. With CSMA/CD it can not be determine in advance how long the collision resolution will take. It was explained before, that delays and irregularities in data transmission can very severely affect real-time system operation. Therefore, various techniques and communication protocol modifications are employed in order to eliminate or minimise these unwanted effects and make the data transmission system time invariant.

To employ Ethernet in an industrial environment, its deterministic operation must first be assured. Coexistence of real-time and non-real time traffic on the same network infrastructure remains the main problem. This conflict can be resolved in several ways by:
• embedding a fieldbus or application protocol on TCP(UDP)/IP – the fieldbus protocol is tunneled over Ethernet, and full openness for “office” traffic is maintained,
• using a special Data Link layer for real-time devices – dedicated protocol is used on the second OSI Layer, implemented in every device. The real-time cycle is divided into slots, one of which is opened for regular TCP/IP traffic, but the bandwidth available is limited,
• using application protocol on TCP/IP, direct MAC addressing with prioritization for real-time, and hardware switching for fast real-time.

All these specific techniques allow a considerable improvement in terms of determinism. Different real-time Industrial Ethernet solutions were proposed, called Real-time Ethernet, such as PROFINET, EtherCAT, Ethernet/IP and many more (CoNet, 2011). The conditions for the industrial use of Ethernet are described by international standard IEC 61 784-2 Real Time Ethernet (See Fig. 3). IEC stands for International Electrotechnical Commission.

The following parameters are covered by the network performance metrics:
• latency (delay) – the amount of time required for a frame to travel from source to destination,
• jitter – a measure of the deviation of the latency from its average value,
• loss rate – the probability that an individual packet is lost (dropped) during the transmission,
• throughput – the amount of digital data transferred per time unit.

![Fig. 3. Classification of industrial Ethernet (IEC 61 784-2)](image)

Class 1 describes the use of standard Ethernet TCP/IP as it is. In this case the different real time protocols and the best-effort protocols, like HTTP, SNMP, FTP etc., uses the services of the TCP/IP protocol suite. This includes examples such as CIP Sync (Ethernet/IP, ModBus/TCP). The class 1 has the largest conformity to the Ethernet TCP/IP standard and can thereby use standard hardware and software components.
Class 2 introduces optimizations, whereby the realtime data bypasses the TCP/IP stack and thus considerably reduces the latency time and increases the achievable packet rate. In Classes 1 and 2, the priority support described by IEEE 802.1Q can also be used depending on the approach. In Class 3 the scheduling on the MAC level is again modified through the introduction of a TDMA method. Class 3 can be used in applications that require maximum latency in the range 1ms and maximum jitter below 1microsec. In this class there are strong restrictions for the use of standard hardware components or the necessity for special components, like dedicated switches. Generally, conformance with the Ethernet standard decreases when ones increase the Class number, while the achievable real-time performance increases.

2.2 Robust codesign

2.2.1 Dynamics of distributed control system

The basic model of the DCS is shown in Fig. 4. The process outputs are measured and control signals are applied through the distance I/O devices. The I/O devices are integrated with A/D and D/A converters.

The communication to and from the controller node is supported by a network. From a digital control point of view, it is natural to sample the process with an equal period $T_0$ and to keep the control delay as short as possible. This suggests that the sensor and actuator (A/D and D/A) converters are time-triggered (sampling period $T_0$), while the controller is event-triggered, which means that they are triggered by the arrival of the new data. The main complication of this control architecture is the presence of variable time delays. The additional dynamics observed in distributed control system depends on the performance parameters of the underlying network, which include transmission rate and transmission medium access method. Under certain circumstances the network-induced delays can be consider constants, but generally they might be varying from transfer to transfer (Fig.4). Thus, the introduction of a network in the feedback loop violates conventional control theory assumptions such as non-delayed sensing and actuation. This can degrade the performance of the control system or even can destabilise the system.
Fig. 5. Example: wireless network data transfer times and histogram of delays
2.2.2 Co-design
Computer implementation of distributed control systems, real-time algorithms, data transmission models and digital control theory methods cannot be developed separately because an unexpected control system performance may occur. Three parameters need particular attention from the distributed control design perspective: sampling and actuation tasks period, controller task period and network parameters (latency and jitter). Due to the close relationships between the network and control parameters the selection of the best sampling period will be a compromise. In this section we will demonstrate the construction of a networked control design chart, which can be used to select proper design parameters.

2.2.3 Sampling and actuation task
We will assume that the control algorithm design is based on correctly identified: model of the process and the model of disturbances (referred to as “nominal models”). We assume that it is possible for the nominal models to estimate a maximal, admissible sampling period, which would guarantee acceptable control performance.

One accepted rule is (Aström and Wittenmark, 1997) that the control task period should be \( a \) \( (a > 1, a \in N) \) times smaller than the period of the cut-off frequency, approximated in some reasonable way for the nominal process model. This upper bound of \( T_0 \) is denoted as \( T_0^u \) (Fig. 6).

For the design purpose we assume that performance of the closed-loop control system is a strictly monotonic function of \( T_0 \): any sampling (actuation) period \( T_0 < T_0^u \) improves the control performance. For \( T_0 < T_0^l \) improvement is not observed. Finally, the sampling (actuation) task period can be estimated as \( T_0 \in [T_0^l, T_0^u] \).

2.2.4 Controller task period
The applied control platforms (processor, peripherals hardware and operating systems) are characterized by a closed-loop execution time, estimated as \( \delta_s \in [\delta_s^l, \delta_s^u] \), where \( \delta_s^l \) - is the lower bound of the execution time for simple control algorithms, \( \delta_s^u \) - is the execution time of complex control algorithms.

The control algorithm is classified as "simple", if pseudocode of the controller task includes no more than 5-10 operations (loops are excluded). Examples of "simple" algorithms are: incremental PID or state feedback controller. If the pseudocode of the controller includes more than 10 operations or loops are included then the algorithm is classified as "complex".

2.2.5 Network parameters
Presence of networks introduces communication delays and limits the amount of data that can be transferred between nodes. In some cases not all samples from sensor or to actuator (produced with period \( T_0 \)) can be sent, because the network requires intervals longer than \( T_0 \) between the transfers of two consecutive packets. Therefore, constraints on the process data availability, introduced by the communication channel are defined.

The average communication delay between the sensor node and the controller node is denoted as \( \tau^{sc} \), \( \tau^{ca} \) is average communication delay between the controller node and the actuator node, \( \Delta(k) \) represents a total jitter in the feedback loop, \( k \) – is the number of the control step.
Actually, the communication delays and jitters can be added to the controller execution time creating an estimation of delays and uncertainty in the control loop. The total delay in the control loop is

\[ \tau(k) = \tau_{sc} + \tau_{ca} + \delta_s + \Delta(k) \]

It will also be assumed that the jitter is bounded by \( 0 \leq \Delta(k) \leq \Delta^u \).

### 2.2.6 Robust codesign

In the previous section we have introduced a number of parameters that need special attention from the perspective of real-time digital control: \( T_0 \) - sampling period defining the temporal granularity related to the process dynamics, \( \delta_s \) - execution time describing the efficiency of the hardware and software application platform and \( \tau_{sc}, \tau_{ca}, \Delta \) - communication delays and jitter. Now, we will demonstrate, how these parameters interacts one to another, how to select the application platforms and how to set closed-loop execution times in such a way, that process dynamics and communication network properties are balanced.

![Fig. 6. Distributed control system design chart](https://www.intechopen.com)

The operating point of the distributed control system should be located in the area between \( T_0^l \) and \( T_0^u \) in Fig. 6. The operating must lie below the line separating “time critical” solution, which simply means that control loop execution time must be less than sampling period. Points A, A’ in Fig. 6 also represent a situation where the design is robust against possible variations (jitter) of the task execution and data transfer times (shadowed area in Fig. 6).

Let us assume that Ethernet network is implemented. Computational delay of the controller \( \delta_s \) is fixed, but for Ethernet network the transmission time delay increases linearly with increasing load - in same case exponentially, when the load on the network exceeds 35 - 40%.
It means, that a faster sampling rate for guaranteeing better control performance will saturate the network traffic load, and eventually increase the data transmission time. For the example given in Fig. 6, the best operating point for Ethernet network is A’ and is constrained by the process data availability introduced by transmission time delays of the communication channel.

If communication can be supported by high-speed real-time network, e.g. Profinet, Class 2 (Amiguet et al. 2008) the constraint of this kind is not active. However, another constraint becomes active and critical. Control loop execution time can not be longer than the sampling period (A’’ in Fig.6), including the jitter \( \Delta(k) \). The reason is that cycles of the control loop do not accept intervals between transfers of the two consecutive packets shorter than \( N_1 \). The time diagram for this situation is given in Fig. 7. For the model from Fig.6 we must assume that

\[
\tau_{sc} + \tau_c + \tau_{sc} + \Delta(k) \leq T_0 = N_1
\]

It means, that the operating point (A’’) must be located below the line separating “time-critical” zone, including the jitter zone (Fig.6).

![Timing model](image)

**Fig. 7. Timing model that can be used for a regularly sampled process**

### 3. Control over the network: Increasing the robustness

One commonly used approach to increase the robustness of DCS stability with respect to the network effect is extension of the standard control algorithms by new components.

#### 3.1 Buffering

The idea is to reduce temporal dependency of the individual parts of the model from Fig. 4 by introducing buffers at the actuator (Tutaj, 2006). Buffering can be easily implemented using PLCs’ or embedded controller at the device level. In digital control this operation can be handled by use of a zero-order holds on the control signal.

First approach presented in this section incorporates one-step buffer introduced at actuator side to compensate variable time delays. Let \( \tau \) be the overall delay (round trip latency time, \( \tau = \tau_{sc} + \tau_{ca} + \tau_{sc} \)). The controlled process model is assumed to be linear, in the form

\[
\frac{dx}{dt} = Ax(t) + Bu(t - \tau), \quad x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^1
\]

For applied delayed linear control law

\[
u(t - \tau) = Kx(t - \tau)
\]
the closed loop model takes the form

$$\frac{dx}{dt} = Ax(t) + BKx(t - \tau) = A_1(t - \tau)$$

The maximum tolerable time delay for given $K$ (or bounds for $K$ under some assumptions on $\tau$) can be computed from the solution of LMI optimisation problem. We should notice, that generally the network induced delays are different from the process delays, because they are time varying and unknown. One solution proposed in (Yi and Hang, 2002) determines condition for exponential stability of system (1) for $\tau(t) \in C_0^0$ - nonnegative, continuous and bounded at $[0, +\infty)$

$$\frac{A^T + A}{2} + \sqrt{\lambda_{\text{max}}}(A_1A_1^T)I < 0$$

where $\lambda_{\text{max}}$ - is the maximum eigenvalue.

Several authors have pointed out (Fujioka, 2009) that the above stability condition is usually conservative.

Assuming that:

- signal transmission is with a single packet (or frame),
- the sensor and actuator are time driven, the controller is event driven. The clocks operate at time period $T_0$ and are synchronized,
- the process dynamics is controllable,

then discrete time model can be introduced. For brevity in the ensuing text notation $x(k)$ will be used in place of $x(kT_0)$.

If the actuation period is selected as $T_0$, than $u(t - \tau)$ is piecewise constant over the actuation period and only changes value at $(kT_0 + \tau)$. Integration of (1) over the sampling period gives a discrete-time, finite dimensional approximation of the delayed model (1)

$$x[k+1] = \Phi_0 x[k] + \Gamma_1 u[k - q - 1] + \Gamma_0 u[k - q]$$

where

$$\tau = qT_0 + \gamma, \quad q \geq 1, \quad \Gamma_1 = \int_{T_0 - \gamma}^{T_0} e^{As}Bds, \quad \Gamma_0 = \int_0^{T_0} e^{As}Bds, \quad \Phi_0 = e^{AT_0}$$

We define new state variables

$$z_1[k] = u[k - q - 1]$$
$$z_2[k] = u[k - q]$$
$$\vdots$$
$$z_{q+1}[k] = u[k - 1]$$

For the assumed considered timing method and the condition on total network delay

$$\tau(k) \leq T_0$$  \hspace{1cm} (2)
fulfilled, the model

\[
\begin{bmatrix}
    x(k+1) \\
    z(k+1)
\end{bmatrix} = \begin{bmatrix}
    \Phi_0 & \Gamma_1(\tau) \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z(k)
\end{bmatrix} + \begin{bmatrix}
    \Gamma_0(\tau) \\
    1
\end{bmatrix} u(k)
\]

(3)

\[y(k) = \begin{bmatrix}
    1 & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z(k)
\end{bmatrix}\]

describes behaviour of closed-loop system.

It is known, that for a discrete linear system with time-varying parameters location of the system eigenvalues in a stable region for all admissible values of the parameters does not imply stability of the system. The buffer can be used at actuator side to eliminate the delay variability in the loop, thereby enabling more effective use of delay compensation algorithms (e.g. Smith predictor). Generally, the buffered control loop can take advantage of more deterministic loop delay, and in consequence the controller can be design more “aggressively” - if only a good process model is available.

The augmented state model with one-step, constant length buffer is obtained in the form

\[
\begin{bmatrix}
    x(k+1) \\
    z_1(k+1)
\end{bmatrix} = \begin{bmatrix}
    \Phi_0 & \Gamma_1 \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z_1(k)
\end{bmatrix} + \begin{bmatrix}
    0 \\
    1
\end{bmatrix} u(k)
\]

The data package is delivered as soon as possible to the actuator, but is hold in the buffer and is implemented to the process in the next sampling intervals. As long as (2) is fulfilled, the “buffered” loop delay is constant and is equal to the buffer length (\(\tau_B = T_0\)).

If the control strategy is assumed as linear feedback

\[u(k) = -\begin{bmatrix}
    K_x & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z_1(k)
\end{bmatrix}, \quad u(k) \in \mathbb{R}^1\]

(4)

the closed-loop system can be written as

\[
\begin{bmatrix}
    x(k+1) \\
    z(k+1)
\end{bmatrix} = \begin{bmatrix}
    \Phi_0 - \Gamma_0(\tau)K & \Gamma_1(\tau) \\
    -K & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z(k)
\end{bmatrix}
\]

(5)

If the condition (2) is not fulfilled for some \(kT_0\), the two-step, constant length buffer can be applied (\(\tau \leq 2T_0\)), Fig.8. For this case the model takes the form (\(q = 1, \tau_B = 2T_0\)).

\[
\begin{bmatrix}
    x(k+1) \\
    z_1(k+1) \\
    z_2(k+1)
\end{bmatrix} = \begin{bmatrix}
    \Phi_0 & \Gamma_1 & 0 \\
    0 & 0 & 1 \\
    -K_x & 0 & 0
\end{bmatrix} \begin{bmatrix}
    x(k) \\
    z_1(k) \\
    z_2(k)
\end{bmatrix}
\]

(6)

If the loop delay \(\tau(k)\) is time varying between \([0, 2T_0]\), it is reasonable to switch between \(T_0\) and \(2T_0\) buffers. The stability analysis of this model is the problem of stability of the Asynchronous Dynamical Systems (ASD) (Hasibi et al, 1999).

The model (5) can be rewritten in the equivalent form, as
\[
\begin{bmatrix}
  x(k+1) \\
  z_1(k+1) \\
  z_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
  \Phi_0 & 0 & \Gamma_1 \\
  0 & 0 & 0 \\
  -K_c & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  x(k) \\
  z_1(k) \\
  z_2(k)
\end{bmatrix}
\]

The following result applies in this case (Zhang, 2001). If for the linear DCS model

\[ w((k+1)) = \Phi_{s(k)} w(k), \quad w(k) = \begin{bmatrix} x(k) \\ z_1(k) \\ z_2(k) \end{bmatrix}, \quad s(k) = (1,2) \]

for a given rate \( r \) of the frames transmission there exists the Lyapunov function such that

\[ V(w(k)) = w^T(k) P w(k) \]

and scalars \( \alpha_1, \alpha_2 \) such that

\[ \alpha_1^r \alpha_2^{1-r} > 1 \]

\[ \Phi_1^T P \Phi_1 \leq \alpha_1^{-2} P, \quad \Phi_2^T P \Phi_2 \leq \alpha_2^{-2} P \]

(7)

than the system is exponentially stable. The rate \( r \) represents the fraction of time that each discrete state transition matrix \((\Phi_1, \Phi_2)\) occurs. Assuming the transmission rate, the problem (7) can be solved as the LMI problem.

Fig. 8. Time diagram of buffering for \( \tau_B = 2T_0 \)

Clearly, adding any delay to a closed-loop system generally degrades the performance. Therefore, once must investigate:

- proper buffer length for assumed model of delay distribution,
- design of controller that takes advantage of an effectively more deterministic loop delay.

A natural extension of this approach is application of variable length (adaptive) buffers (Tutaj, 2006). It is assumed that frames order can not be changed, frames are not lost or
doubled. The initial length of the buffer is $T_0$. The buffer length is adapted according to the following formula:

$$
\tau_{\text{b}}(k) = T_0 + \alpha T_0 (p - \varphi(k))
$$

where:
- $\alpha$ - adaptation parameter, $\alpha > 0$
- $p$ - assumed rate of frames delivered to the buffer in time (during the time interval no longer than $\tau_{\text{b}}(k)$), $0 < p < 1$

$$
\varphi(j) = \begin{cases} 
1 & \text{if the frame was delivered in time} \\
0 & \text{otherwise}
\end{cases}
$$

If the frame is not delivered in time (at $t = kT_0$ the buffer is empty) than $\varphi(k)$ is set to 1. First frame delivered to the buffer is released immediately.

After $k+1$ steps the buffer length can be calculated as (Tutaj, 2006)

$$
\tau_{\text{b}}(k + 1) = T_0 + \alpha T_0 \sum_{j=0}^{k} (p - \varphi(j))
$$

Such a model implements a kind of “filtration” of delays effects (Fig. 9).

Fig. 9. Example of adaptive filter operation (Tutaj, 2006): a) $\alpha = 0.2; p = 0.9$, b) $\alpha = 0.002; p = 0.9$, c) $\alpha = 0.05; p = 0.3$, d) $\alpha = 0.05; p = 0.9$ (black – after buffer, grey – before buffer)
3.2 Robust stability of the buffered DCS

Application of variable length buffer simplifies analysis of DCS. It can be assumed that the control delays are constant but not exactly known. In this case the problem of stability analysis of the DCS can be formulated as a parametric robust control problem. This allows using the mapping theorem (Bhattacharyya et al. 1995) to develop an effective computational technique to determine robust stability. The advantage of this approach over the stochastic method is that it is not necessary to identify the stochastic model of the delay.

3.2.1 Time-invariant delays in DCS

We assume that the total delay is slowly varying and known only with some precision

\[ \tau(k) = \tau, \quad \tau_{\text{min}} \leq \tau \leq \tau_{\text{max}} \]

In such a case we could design a controller stable for some range of slowly varying delay. The solution of this problem gives answer to the basic question “how much delay can the system tolerate”?

The state matrix of the closed loop system (5) can be next rewritten in the form

\[ M(\Psi) = \begin{bmatrix} \Phi_0 - \Gamma_0(\tau)K & \Gamma_1(\tau) \\ -K & 0 \end{bmatrix} = M_0 + M_1(\varphi_1) + M_2(\varphi_2) + ... + M_n(\varphi_n) = \]

\[ = M_0 + \varphi_1(\tau)\hat{M}_1 + \varphi_2(\tau)\hat{M}_2 + ... + \varphi_n(\tau)\hat{M}_n \tag{8} \]

where

\[ M_0 = \begin{bmatrix} \Phi_0 + A^{-1}BK & A^{-1}\Phi_0B \\ -K & 0 \end{bmatrix} \]

\[ \hat{M}_i = \begin{bmatrix} 0_1 & 0_2 \\ -K & -1 \\ 0 & 0 \end{bmatrix}, \quad \text{dim}0_1 = (i-1) \times 1, \quad \text{dim}0_2 = (i-1) \times 1 \tag{9} \]

The uncertain delay enters affinely into the state matrix of the closed loop system. If \( \tau_{\text{min}} \leq \tau \leq \tau_{\text{max}} \), then we could obtain the boundaries

\[ \Psi_{\text{min}} \leq \Psi(\tau) \leq \Psi_{\text{max}} \tag{10} \]

The following stability problem is important for the model formulated above: determine if matrix (9) remains Schur-stable as \( \varphi_i \) parameters ranges over the bounds given by (10)? The structure of the closed loop state matrix (8) is a special case of the interval matrix family and we are free to use results of the robust theory solutions for checking stability (Bhattacharyya et al. 1995). Under the assumption \( \text{rank}(M_i) = 1 \) for \( i = 1..n \) the coefficients of the
characteristics polynomial of $M(\Psi)$ are multilinear function of $\varphi$. The following theorem applies in this case:

Let the matrix $M_0$ be Schur stable. If the rank($\hat{M}_i$) = 1 for $i = 1..n$, than the family of the matrices $M(\Psi)$, $\Psi \in \Theta$ defined by (8)-(9) is robust Schur-stable if the testing function

$$F(y) > 0 \quad \forall y \in Y,$$

where the testing function is defined as

$$F(y) = \pi - \alpha(y)$$ (11)

$$\alpha(y) = \max\{\arg(\hat{p}_r(f(y)) - \arg(\hat{p}_k(f(y)))\},$$

$$r, k = 1, 2, K, \quad r \neq k$$

$$\hat{p}_k(z) = \frac{p_k(z)}{w_0(z)}, \quad k = 1, 2, ..., K$$

$$p_k(z) = \det(zI - M(\varphi_k))$$

$$w_0(z) = \det(zI - M_0) \quad K = 2^n.$$  

The function $f(y) = \exp(j\pi y), \quad y \in Y = [0, 2]$ is a parametric description of the unit circle, $M(\varphi_k)$ is a vertex matrix calculated for each $\varphi_k$ - the vertex of the set $\Theta$, $K$ is the number of the vertex matrices. The testing function (11) checks the maximal phase differences of the vertex polynomials over parameter box corresponding to the vertices given by (10).

### 3.2.2 Example: Distributed control of a tank system

Let us consider a problem of distributed control of a tank system. The process consists of the upper tank having constant cross section and the lower cylindrical tank, so having variable cross section. Liquid is pumped into the top tank by DC motor driven pump. The liquid outflows of the tanks only due to gravity. The orifices $C_1$ and $C_2$ determine the outflow of the liquid. The general objective of the control is to reach and stabilise the level in the lower tank by adjustment of the pump operation. The levels in the tanks are measured with pressure transducers (S). The appropriate interfaces (I) enabling distance transmission of the control signals to the pump were installed, creating a distributed control system from Fig.10.

If levels in the tanks are introduced as the states variables, the nonlinear model could be linearized at $H^0 = [H_1^0, H_2^0]^T$ giving finally (Grega, 2002)

$$\frac{d}{dt} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u$$

where: $H_1 = H_1^0 + h_1$, $H_2 = H_2^0 + h_2$, $q = q^0 + u$, $b_1 = \frac{1}{S}$, $a_4 = \frac{-C_1}{2S\sqrt{H_1^0}}$.  

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\[ a_3 = \frac{C_1 \sqrt{H_1^0}}{4H_1^0 \cdot w \sqrt{r^2 - (r - H_2^0)^2}}, \quad a_4 = \frac{-C_2 \sqrt{H_2^0}}{4H_2^0 \cdot w \sqrt{r^2 - (r - H_2^0)^2}} \]

Fig. 10. Distributed control of tank system

For an assumed sampling period \( T_0 \) the equivalent discrete model is \((\tau = 0)\)

\[
\begin{bmatrix}
    h_1(k+1) \\
    h_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
    e^{a_5 T_0} & 0 \\
    a_4 (e^{a_5 T_0} - e^{a_4 T_0}) & e^{a_4 T_0}
\end{bmatrix}
\begin{bmatrix}
    h_1(k) \\
    h_2(k)
\end{bmatrix} +
\begin{bmatrix}
    b_1 (e^{a_1 T_0} - 1) \\
    b_1 a_3 \left( \frac{e^{a_1 T_0} - e^{a_4 T_0}}{a_4} - (e^{a_4 T_0} - 1) \right)
\end{bmatrix} u(k)
\]

Linear feedback control law is in the form

\[ u(k) = -K h(k) \]

It was assumed that the controller has been design ignoring the network, hence the state matrix of (3) is stable for \( \tau = 0 \). The assumed parameters of the tank model were: \( C_1 = 10, \quad C_2 = 15, \quad T_0 = 80s \), giving the LQ controller gains: \( K_1 = 0.7167 \quad K_2 = 3.0950 \). Fig. 11 demonstrates the LQ optimal output of the model (simulation). Figure 13 illustrates observed perturbation of data transmission times, when Ethernet protocol was applied and some additional traffic in the network was generated.
The delay in the control loop reduces the stability margin of the system. Figure 12 shows how the fixed feedback delay ($\tau = 80s$) degrades the performance of the tank system control. Notice that this is equivalent to implementation of the fixed size buffer ($\tau_B = T_0$). So, to increase the stability margin and improve stability it is necessary to tune the feedback gains.
If the variable-length buffer is introduced, stability of the distributed digital control system for the assumed controller gains and $0 \leq \tau \leq \tau_{\text{max}}$ can be verified using the methodology described above. It is assumed now, that the control delays are constant but not exactly known. The LQ optimal robust gains (giving the stable matrix $M_0$) were calculated as: $K_1 = 0.3745 \quad K_2 = 0.3420$.

Fig. 13. Network delays – perturbation of data transmission times, $\tau \in [0,100]$ s

Fig. 14. Maximum phase differences of vertex matrices
The next step is verification of the testing function $F(y)$. The appropriate testing function is given in Fig. 14. The maximum phase difference over all vertices at each $\omega \in [0, 2\pi)$ is less than $180^0$. Figure 15 shows operation of the LQ controller for the above set of controller parameters and network delays, as given in Fig.13.

![Levels in tanks: 0 ≤ τ ≤ 80, robust gains: K=[0.3745, 0.3420] (h1, h2)](image.png)

Fig. 15. Robust, LQ – optimal control of the tank system

### 4. Final remarks

The introduction of networks, limited throughput of data transmission channels, combined with non-optimised hardware and software components introduce non-determinism in the distributed control system. For multilevel industrial systems this problem becomes even more complex. Some control loops can be handle by local, device – level controllers, but also by the supervisory controllers all them implementing data transmission networks. Special care must be taken when the communication channel generates sampling – actuation jitters or other kinds of run time violation of the closed-loop timing assumptions. It means that the introduction of data transmission networks into the feedback loop in many cases violates conventional control theories assumptions such as non-delayed or evenly spaced sampling and actuation. It is now reasonable to redesign controllers improving the temporal robustness of the distributed control system.

Control engineers do not care very much about real-time or distributed control implementations of control algorithms. In many cases they do not understand control timing constraints. The typical solutions proposed are: “buy a faster computer” or “install a more efficient data transmission network”. Basic control theory does not advise them on how to redesign controllers to take network limitation into account.

It was demonstrated in this chapter, that robust design it is not only a proper selection and tuning of control algorithms, but also study on communications protocols and networks, to make them suitable for real-time DCS.
We have proposed an integrated design approach combining several components: process dynamics, controller parameters and network constraints, and resulting in better quality of control systems.

Finally, it was shown how the extension of the standard controller with a buffer improves robustness of distributed control system. The model was formulated as variable parameter linear discrete-time model, where variability of parameters was introduced by the time varying delays. The variable length buffer was used at actuator side to eliminate high speed delay variability in the loop, thereby enabling more effective use of delay compensation algorithms. A water tank control example has shown how implementation of variable-length buffer algorithm and application of some results of interval matrices theory increases robustness of the control loop.

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6. References


The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.

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