Collaborative Environmental Monitoring with Hierarchical Wireless Sensor Networks

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1. Introduction

In the last decade, advances in wireless communication and micro-fabrication have motivated the development of large-scale wireless sensor networks (Akyildiz et al., 2002; Yick et al., 2008). A large number of low-cost sensor nodes, equipped with sensing, computing, and communication units, organize themselves into a multi-hop network. The wireless sensor network takes measurements from the environment, processes the sensory data, and transmits the sensory data to end-users. Beginning from the seminar work in (Estrin et al., 1999; 2002), the wireless sensor network technology has been well recognized as a revolutionary one that transforms everyday life. Typical applications of wireless sensor networks include military target tracking and surveillance (Simon et al., 2004; He et al., 2006), precise agriculture (Langendoen et al., 2006; Wark et al., 2007), industrial automation (Gungor and Hancke, 2009), structural health monitoring (Li and Liu, 2007; Ling et al., 2009), environmental and habitat monitoring (Zhang et al., 2004; Corke et al., 2010), to name a few.

1.1 Network infrastructure

To organize the large amount of sensor nodes and enable efficient data collection, a wireless sensor network generally adopts one of the following three infrastructures: centralized, decentralized, and hierarchical. In the centralized infrastructure, sensor nodes transmit the sensory data to the fusion center via multi-hop communication. In the decentralized infrastructure, each sensor node firstly refines the sensory data through collaborative and decentralized in-network processing with the neighboring sensor nodes, and secondly transmits the refined data to the fusion center. While in the hierarchical infrastructure, sensor nodes are divided into multiple clusters, and sensor nodes within one cluster send their sensory data to the cluster head. These cluster heads either transmit the collected sensory data to the fusion center, or collaboratively process them and transmit the refined one to the fusion center. These two different implementations of the hierarchical infrastructure, centralized processing and decentralized collaboration, are depicted in Figure 1.

In deploying a wireless sensor network, the choice of its infrastructure is decided by several key factors: energy, bandwidth, robustness, etc. Sensor nodes are often equipped with batteries and recharging is difficult. Since wireless data transmission is the main source of energy consumption of a sensor node (Sadler, 2005), the network infrastructure
should guarantee that each sensor node has low data transmission rate while successfully accomplishing the data collection task. Bandwidth is also a kind of precious resource in wireless environment; over-competition of wireless channels leads to frequent retransmission and hence consumes more energy. Further, sensor nodes are often fragile due to being out of batteries or other physical damages. The network infrastructure should be carefully designed such that the failure of few sensor nodes shall not result in the malfunction of the whole network.

When the network size is small, the centralized infrastructure is an acceptable choice. Take a volcano monitoring network containing 3 sensor nodes (Werner-Allen et al., 2005) as an example, these sensor nodes directly connect to a fusion center which collects sensory data and transmits them to the end-user. Later on the network is extended to the scale of 16 sensor nodes (Werner-Allen et al., 2006), and the sensor nodes communicate with the fusion center via multi-hop relays. However, for GreenOrbs (Liu et al., 2011), a large-scale forest monitoring network composed of up to 330 sensor nodes, experiments demonstrate that sensor nodes within some “hot areas” may face higher competition for bandwidth, consume more energy, and be more sensitive to the failure of sensor nodes. The decentralized infrastructure, on the other hand, has great potential to reduce the total amount of transmitted data and hence improve the energy efficiency via in-network collaboration; further, it also enhances robustness of the network since all sensor nodes play equal roles (Ling and Tian, 2010). Nevertheless, collaboration of the sensor nodes brings more difficulty to network coordination, and is subject to the limited processing and communication capabilities of sensor nodes. For this reason, the decentralized infrastructure is still far from practical applications. To the best of our knowledge, most large-scale wireless sensor networks are deployed with the hierarchical infrastructure. Following we give some examples: ExScal, an intrusion detection network with more than 1000 sensor nodes and more than 200 backbone nodes (Arora et al., 2005); VigilNet, a military surveillance network with 200 sensor nodes (He et al., 2006); Trio, a target tracking network with 557 solar-powered sensor nodes.
Collaborative Environmental Monitoring with Hierarchical Wireless Sensor Networks (Dutta et al., 2006); SenseScope, an environmental monitoring network consisting of from 3 to 97 sensor nodes (Bareneatea et al., 2008). In view of this fact, we will focus on the design of a hierarchical wireless sensor network.

1.2 Our contributions
In some hierarchical wireless sensor networks such as ExScal (Arora et al., 2005), the cluster heads are specifically designed, having better data processing and wireless communication abilities than general sensor nodes, and equipped with stronger or even uninterruptible power sources. These cluster heads can directly transmit the collected data to a remote fusion center, without introducing any collaborative processing among cluster heads. However, in most wireless sensor networks, cluster heads are elected from sensor nodes to simplify design, deployment, and maintenance. For example, in the LEACH protocol (Heinzelman et al., 2002), sensor nodes autonomously elect cluster heads, aiming at evenly distributing energy consumption among all sensor nodes so that there are no overly-utilized sensor nodes that will run out of energy before the others. In this case, how to process the collected sensory data in the cluster heads is a critical problem to accomplishing the data collection task while maximizing the network lifetime.

This chapter addresses this problem; specifically, we study a generalized environmental monitoring model with large-scale hierarchical wireless sensor networks, and focus on two questions: for cluster heads in a hierarchical network, should they collaborate or not collaborate and how can they collaborate. Our contributions are two-fold.

First, through theoretical analysis and simulation validation, we make the following recommendations on whether to collaborate or not: when each cluster head has a large amount of data to process (namely, each cluster contains a large number of sensor nodes) and multi-hop relay is necessary to communicate with a fusion center (namely, cluster heads have limited communication range), decentralized data processing among cluster heads is more efficient; otherwise centralized decision-making with the aid of a fusion center can be advantageous.

Previous work, such as (Rabbat and Nowak, 2004; Aldosari and Moura, 2004), has suggested similar network design principles in the context of decentralized infrastructures: when each sensor node collects a large amount of data or the size of the network is large, collaborative processing is more efficient than centralized decision-making. This paper extends the conclusions to hierarchical networks, and compares decentralized versus centralized processing among cluster heads rather than among all sensor nodes.

Second, we develop a decentralized collaborative algorithm for decision making among the sub-network of cluster heads, after they have collected sensory data from local sensor nodes within their individual clusters. Particularly, we study a typical environment monitoring application, in which a large-scale hierarchical wireless sensor network is deployed to monitor sparsely occurring phenomena over a large sensing field. The monitoring problem is formulated as a non-negative quadratic program, which optimizes a sparse decision vector depicting the spatial map of the phenomena of interest. An optimal iterative algorithm, in which cluster heads iteratively exchange information and make decisions, is proposed based on the alternating direction method of multipliers (ADMM) (Bertsekas and Tsitsiklis, 1997). Our development is permeated with the benefits of compressive sensing (Donoho et al., 2006). Exploiting the sparse nature of the unknown phenomena, we allow the number of sensor nodes to be much smaller than what would have been required in a traditional scheme for
sensing at high spatial resolution over a large field. In this sense, our proposed algorithm is also applicable to other compressive sensing problems in distributed systems.

1.3 Chapter organization
The rest of this chapter is organized as follows. We first give a brief survey on the applications of wireless sensor networks in environmental monitoring. Second, we study a generalized environmental monitoring model with large-scale hierarchical wireless sensor networks and develop a decentralized collaborative algorithm for decision making among the cluster heads. Finally we discuss the design consideration, namely, to collaborate or not to collaborate, based on theoretical analysis and simulation results.

2. A brief survey
In this section, we give a brief survey on the applications of wireless sensor networks in environmental and habitat monitoring. Though this overview is far from complete, it reflects the promising future of the wireless sensor network technology in helping us understand and protect natural environment.

For environmental and habitat monitoring applications, one of the first known practical wireless sensor networks was deployed by a group at Berkeley in 2002, on Great Duck Island on the coast of Maine, USA. Two networks with a total of 147 sensor nodes collect data to study the ecology of the Leach’s Storm Petrel (Szewsczyk et al., 2004). Later on, the Macroscope system which contains 33 sensor nodes, also developed at Berkeley, was used for microclimate monitoring of a redwood tree (Tolle et al., 2005). Another notable application is ZebraNet, which used GPS technology to record position data in order to track long term animal migrations. In the prototype system, researchers deployed 7 sensor nodes on zebras in Kenya (Zhang et al., 2004). Energy harvesting technologies have also attracted much research interest to address the challenge of energy supply in remote environmental monitoring applications. One successful example is LUSTER, which was developed at University of Virginia, featuring a specifically designed hybrid multichannel energy harvesting device (Selavo et al., 2007). Accompanied with the unprecedented data collection opportunities, data processing also emerges as a new challenge in the wireless sensor network technology. The data processing task is indeed application-oriented. For example, an ellipsoids-based anomaly detection algorithm was designed to monitor unusual and anomalous behaviors in a particular marine ecosystem (Bedzek et al., 2011). The network was deployed in 2009 at the Heron Island, Australia, as part of the Great Barrier Reef Ocean Observation System.

One significant advantage of wireless sensor networks over traditional data collection techniques is that they can be applied in harsh environments. For example, in the GlacsWeb system, researchers at University of Southampton deployed 9 sensor nodes inside a glacier (Martinez et al., 2004). The sensor nodes monitored pressure, temperature, and tilt, in order to monitor subglacial bed deformation. Even on active volcanos, which are often forbidden areas for data collection, wireless sensor networks can still work well. In the work of (Werner-Allen et al., 2005; 2006), one small sensor network with 3 sensor nodes was deployed on Vlcan Tungurahua in Ecuador as a proof of concept in 2004; then in 2005, the network size was extended to 16 sensor nodes. Wireless sensor networks are also fit for aquatic environmental monitoring applications. In (Alippi et al., 2011), a robust, adaptive, and solar-powered network was developed in 2007 for such an application. The network was deployed in Queensland, Australia, for monitoring the underwater luminosity and temperature, information necessary to derive the health status of the coralline barrier. At
the same time, sensory data can be used to provide quantitative indications related to cyclone formations in tropical areas.

However, applying wireless sensor networks in environmental monitoring is still a challenging task when the network size is large. When the number of sensor nodes increases, difficulties emerge for system integration (creating an end-to-end system that delivers data to the end-user), performance (reliability, accuracy, and calibration), productivity (how well the sensory data assists the end-user and how to reduce the total cost in implementing the wireless sensor network), etc (Corke et al., 2010). One negative example is reported in (Langendoen et al., 2006), in which researchers at Delft University of Technology deployed a large-scale network in a potato field to improve the protection of potatoes against disease. The application was not successful due to unanticipated issues; nevertheless, the lessons are precious, such as software, hardware, and even team coordination. A systematic discussion, named as “the hitchhiker’s guide”, is provided in (Barenetxea et al., 2008). Based on the deployment of a wireless sensor network on a rock glacier located at a mountain in the Swiss Alps, this guide covers almost all stages of a project, from hardware and software development, testing and preparation, to deployment. One of the recent efforts to investigate the practical implementations of large-scale wireless sensor networks is the GreenOrbs system (Liu et al., 2011). The network with 330 sensor nodes was deployed in Tianmu Mountain, China, aiming at all-year-around ecological surveillance in the forest. It is shown that many traditional design guidelines for small-scale wireless sensor networks can be questionable for large-scale applications.

3. Problem formulation

In this chapter, we focus on a generalized event detection model for environmental monitoring applications. Let us consider a large-scale wireless sensor network randomly deployed in a two-dimensional area for monitoring sparsely occurring events. The network has a set of \( L \) sensor nodes, denoted as \( \mathcal{L} = \{v_l\}_{l=1}^{L} \). Sensor nodes are divided into \( I \) clusters, each having one cluster head in the set \( \mathcal{I} = \{c_i\}_{i=1}^{I} \). Sensor nodes within a cluster are able to directly transmit measurements to the cluster head, and the cluster head is aware of the positions of all sensor nodes within its cluster. Further, the cluster heads have a common communication range \( r_C \) such that the sub-network of cluster heads is bi-directionally connected.

3.1 Basic assumptions

Suppose that at each sampling time, multiple phenomena may occur in the sensing field. Our objective is to detect and identify the source locations and estimate their amplitudes from sensory measurements. We make the following basic assumptions for the sensing problem of interest, similar to those in (Bazerque and Giannakis, 2010):

(A1): The sensing field is viewed through a spatial grid with \( K \) grid points denoted by \( \mathcal{K} = \{g_k\}_{k=1}^{K} \), whose locations are known to the corresponding cluster heads. Each event can occur only at a grid point, indicating the spatial resolution offered by this sensor network. The amplitude of an event occurring at grid point \( g_k \) is \( x_k \).

(A2): The influence of a unit-amplitude event at grid point \( g_k \) on a sensor point \( v_l \) is \( f_{kl} \). Generally speaking, \( f_{kl} \) is decided by the distance \( d_{kl} \) between \( g_k \) and \( v_l \).

(A3): The measurement of one sensor node is the linear superposition of the influences of all phenomena plus random noise. Mathematically, the measurement \( b_l \) of sensor node \( v_l \) is hence \( b_l = \sum_{k=1}^{K} f_{kl} x_k + e_l \) in which \( x_k \) is the amplitude of event at \( g_k \in \mathcal{K} \) and \( e_l \) is measurement noise.
Fig. 2. The sensor nodes denoted as solid squares are uniformly randomly deployed in the monitoring area. The candidate positions for phenomena are grid points denoted as solid dots. Phenomena denoted as pentagrams occur at the current snapshot, and the shadow regions illustrate the influence of phenomena.

These assumptions are depicted in Figure 2. The sensor nodes denoted as solid squares are uniformly randomly deployed in the monitoring area. The candidate positions for phenomena are grid points denoted as solid dots. Phenomena denoted as pentagrams occur at the current snapshot, and the shadow regions illustrate the influence of phenomena.

The assumption \((A1)\) simplifies the recovery problem by confining the sources of phenomena to grid points. Without this assumption, an alternative way is to use positions and amplitudes of the sources as decision variables and formulate a least squares problem. However, this formulation is highly nonlinear and intractable, since the number of decision variables is even unknown. Based on \((A1)\), we can formulate the otherwise nonlinear problem as recovering the vector \(x = [x_1, \ldots, x_K]^T\) from linear measurements \(b_l, \forall l\). Entries in \(x\) with nonzero values reveal the locations and amplitudes of the multiple phenomena of interest. This assumption approximately holds when the grid points are dense; namely, the density of the grid points decides the spatial resolution of the recovery algorithm.

The assumption \((A2)\) describes the influence of one event on the entire sensing field. For example, in target tracking or nuclear radioactive detection, the influence of a source decreases polynomially as the distance increases. Without loss of generality, we define the influence function as \(f_{kl} = \exp\left(-\frac{d_{kl}^2}{\sigma^2}\right)\) for grid point \(g_k\) and sensor point \(v_l\), where \(\sigma\) is a common constant. This Gaussian-shaped function well approximates the influence of many practical events.

Based on the assumption \((A3)\), we readily have the following least squares formulation for recovering \(x\):

\[
\min \sum_{l \in \mathcal{L}} (b_l - \sum_{k=1}^{K} f_{kl}x_k)^2,
\]

or equivalently in a matrix form:

\[
\min ||Fx - b||_2^2.
\]

Here \(b = [b_1, \ldots, b_L]^T\) is the measurement vector and \(F\) is the \(L \times K\) influence matrix with its \(l\)-th row given by \([f_{1l}, \ldots, f_{Kl}]\).
Nevertheless, the least squares formulation (2) ignores the sparsity of the vector $x$. Notice that when the grid is dense, the number of events is generally much smaller than the number of grid points; hence the vector $x$ is a sparse vector with a large amount of zero entries. Without considering this prior knowledge, the least squares formulation (2) leads to a non-sparse solution, which means a non-neglectable number of false alarms. The sparsity of a signal vector can be measured by its $\ell_1$ norm (Donoho et al., 2006). Exploiting the sparse nature of $x$ to alleviate false alarms, we formulate the following $\ell_1$ regularized least squares problem (Kim et al., 2007):

$$\min \frac{1}{2} ||Fx - b||^2 + ||x||_1. \quad (3)$$

Here $\lambda$ is a non-negative weight.

### 3.2 Decentralized optimization

In a centralized setting, the $\ell_1$ regularized least squares problem (3) has been extensively studied in both signal processing and numerical optimization communities (Donoho et al., 2006; Figueiredo et al., 2007). However, in a large-scale wireless sensor network, centralized processing is not efficient in terms of energy consumption and communication overhead. In contrast, collaborative signal processing among cluster heads is preferred, leading to a robust and scalable network.

We address this issue by developing a collaborative sparse signal recovery algorithm in the chapter. Sensor nodes or cluster heads do not necessarily exchange information with a fusion center; rather, sensor nodes only need to transmit measurements to their cluster heads, and cluster heads iteratively optimize the decision vector $x$ via exchanging information with their neighboring cluster heads.

For each cluster head $c_i$, let us collect the local measurements $\{v_l\}$ within this cluster and their corresponding $l$-th rows in the measurement matrix $F$ into a sub-vector $b_l$ and a sub-matrix $F_l$, for all sensor nodes $v_l$ whose cluster head is $c_i$. Per assumptions (A1) and (A2), each cluster head knows all sensor node locations and grid point locations within its cluster, which means that $F_l$ is known to $c_i$. Hence the problem boils down to the following one: *suppose that the local measurement vector $b_l$ and corresponding measurement matrix $F_l$ are available to each cluster head $c_i$, $\forall v_l$, how can we design a decentralized algorithm to recover the signal $x$ via collaboration among the cluster heads?*

Let $x_i$ denote the local copy of the decision vector $x$ at $c_i$, $\forall c_i \in I$. Meanwhile, given the communication range $r_C$, the set of neighboring cluster heads of $c_i$ is denoted by $N_i$, with cardinality $|N_i|$. The formulation in (3) can be transformed to the following consensus optimization problem:

$$\min \sum_{l=1}^{L} \left( \frac{1}{2} ||F_l x_i - b_l||^2 + \frac{1}{2} ||x||_1 \right), \quad s.t. \quad x_i = x_j, \quad \forall c_i \in I, c_j \in N_i. \quad (4)$$

The $K \times 1$ all-one vector $[1, 1, ..., 1]^T$ is denoted as $\mathbf{1}$. Here, cluster heads optimize their own local copies of $x$ separately, and these decision vectors are forced to be equal via the consensus constraints. An alternative formulation is to force $x_i$ to consent with the average of its neighboring decisions, as follows:

$$\min \sum_{l=1}^{L} \left( \frac{1}{2} ||F_l x_i - b_l||^2 + \frac{1}{2} \mathbf{1}^T x_i \right), \quad s.t. \quad x_i = \frac{1}{|N_i|} \sum_{c_j \in N_i} x_j, \quad \forall c_i \in I. \quad (5)$$

It has been proved that if the sub-network of the cluster heads is bi-directionally connected, then (4) and (5) are equivalent to (3) (Zhu et al., 2007). Both (4) and (5) can be solved similarly, as below.
4. Collaborative environmental monitoring algorithm

We now apply an optimal algorithm, the alternating direction method of multipliers (ADMM) (Bertsekas and Tsitsiklis, 1997), to solve (4).

4.1 Algorithm development

To solve (4) with the ADMM, we first introduce a new block of auxiliary variables. Then (4) can be rewritten as:

\[
\min_{\{x_i\}, \{z_{ij}\}} \sum_{i=1}^{l} \left( \frac{1}{2} ||F_i x_i - b_i||^2 + \frac{1}{2} 1^T x_i \right),
\]

subject to

\[
x_i = z_{ij}, x_j = z_{ij}, \quad \forall c_i \in L, c_j \in N_i.
\]

Here \(z_{ij}\) is an auxiliary vector attached to \(x_i\) and \(x_j\). The augmented Lagrangian function of (6) is:

\[
L_a \left( \{x_i\}, \{z_{ij}\}, \{\beta_{ij}\}, \{\gamma_{ij}\} \right) = \sum_{i=1}^{l} \left( \frac{1}{2} ||F_i x_i - b_i||^2 + \frac{1}{2} 1^T x_i \right) + \sum_{c_i \in N_i} \beta_{ij}^T (x_i - z_{ij}) + \sum_{c_j \in N_j} \gamma_{ij}^T (x_j - z_{ij}) + \frac{\lambda}{2} \sum_{i=1}^{l} \sum_{c_i \in N_i} ||x_i - z_{ij}||^2
\]

in which \(\{\beta_{ij}\}\) and \(\{\gamma_{ij}\}\) are Lagrange multipliers and \(d\) is a positive constant. At time \(t\), the ADMM optimizes the augmented Lagrangian function as follows:

**Step 1: Optimizing the local copies \(\{x_i\}\):**

\[
\{x_i(t+1)\} = \arg \min_{\{x_i\}} L_a \left( \{x_i\}, \{z_{ij}(t)\}, \{\beta_{ij}(t)\}, \{\gamma_{ij}(t)\} \right).
\]

Notice that the objective function is separable, \(x_i(t+1)\) can be updated as:

\[
x_i(t+1) = \arg \min_{x_i} \left( \frac{1}{2} ||F_i x_i - b_i||^2 + \frac{1}{2} 1^T x_i \right) + \sum_{c_i \in N_i} \beta_{ij}^T (x_i - z_{ij}) + \sum_{c_j \in N_j} \gamma_{ij}^T (x_j - z_{ij}) + \frac{\lambda}{2} \sum_{i=1}^{l} \sum_{c_i \in N_i} ||x_i - z_{ij}||^2.
\]

**Step 2: Optimizing the Auxiliary Variable \(\{z_{ij}\}\):**

\[
\{z_{ij}(t+1)\} = \arg \min_{\{z_{ij}\}} L_a \left( \{x_i(t+1)\}, \{z_{ij}\}, \{\beta_{ij}(t)\}, \{\gamma_{ij}(t)\} \right).
\]

Here the objective functions is also separable. Therefore:

\[
z_{ij}(t+1) = \arg \min_{z_{ij}} -\beta_{ij}^T (t) z_{ij} - \gamma_{ij}^T (t) z_{ij} + \frac{\lambda}{2} \sum_{i=1}^{l} \sum_{c_i \in N_i} ||x_i - z_{ij}||^2 + \frac{\lambda}{2} \sum_{i=1}^{l} \sum_{c_j \in N_j} ||x_j - z_{ij}||^2.
\]

It has an explicit solution:

\[
z_{ij}(t+1) = \frac{1}{2} \left( x_i(t+1) + x_j(t+1) \right) + \frac{1}{2d} \left( \beta_{ij}(t) + \gamma_{ij}(t) \right).
\]

**Step 3: Updating the Lagrange Multipliers \(\{\beta_{ij}\}\) and \(\{\gamma_{ij}\}\):**

\[
\beta_{ij}(t+1) = \beta_{ij}(t) + d \left( x_i(t+1) - z_{ij}(t+1) \right),
\]

\[
\gamma_{ij}(t+1) = \gamma_{ij}(t) + d \left( x_j(t+1) - z_{ij}(t+1) \right).
\]
The updating rules of (9), (11), and (13) can be further simplified. Substituting (12) to (13) yields:

\[
\begin{align*}
\beta_{ij}(t + 1) &= \beta_{ij}(t) + \frac{d}{2} \left( x_i(t + 1) - x_j(t + 1) \right) - \frac{1}{2} (\beta_{ij}(t) + \gamma_{ij}(t)), \\
\gamma_{ij}(t + 1) &= \gamma_{ij}(t) + \frac{d}{2} \left( x_i(t + 1) - x_j(t + 1) \right) - \frac{1}{2} (\beta_{ij}(t) + \gamma_{ij}(t)).
\end{align*}
\]

(14)

Since we often set \( \beta_{ij}(0) = \gamma_{ij}(0) = 0 \) where 0 denotes a \( K \times 1 \) all-zero vector \([0, 0, ..., 0]^T\), (14) implies that \( \beta_{ij}(t) = -\gamma_{ij}(t) = \gamma_{ji}(t) \). Then (12) becomes:

\[
\begin{align*}
\mathbf{z}_{ij}(t + 1) &= \frac{1}{2} \left( x_i(t + 1) + x_j(t + 1) \right).
\end{align*}
\]

(15)

and (13) becomes

\[
\begin{align*}
\beta_{ij}(t + 1) &= \beta_{ij}(t) + \frac{d}{2} \left( x_i(t + 1) - x_j(t + 1) \right) = \gamma_{ji}(t + 1), \\
\gamma_{ij}(t + 1) &= \gamma_{ij}(t) + \frac{d}{2} \left( x_i(t + 1) - x_j(t + 1) \right) = \beta_{ji}(t + 1).
\end{align*}
\]

(16)

Summarizing the three sides of (16) and define a new Lagrangian multiplier \( \alpha_i = \frac{2}{|N_i|} \sum_{c_j \in N_i} \beta_{ij} = \frac{2}{|N_i|} \sum_{c_j \in N_i} \gamma_{ji} \), the updating rule for \( \alpha_i \) is:

\[
\alpha_i(t + 1) = \alpha_i(t) + dx_i(t + 1) - \frac{d}{|N_i|} \sum_{c_j \in N_i} x_j(t + 1).
\]

(17)

Substituting (15) to (9), we have the updating rule for \( x_i \):

\[
\begin{align*}
x_i(t + 1) &= \arg \min_{x_i} \sum_{i=1}^I \left( \frac{d}{2} || F_i x_i - b_i ||_2^2 + \frac{1}{2} \lambda^T x_i \right) \\
&+ |N_i| \alpha_i^T(t) x_i + d \sum_{c_j \in N_i} || x_i - \frac{1}{2} (x_j(t) + x_j(t)) ||_2^2.
\end{align*}
\]

(18)

Iteratively solving (18) and updating (17) leads to the optimal solution of (4). It should be noted that the problem we are discussing is indeed a special case of compressive sensing (Donoho et al., 2006). When the number of sensor nodes is smaller than the number of grid points, down-sampling is achieved via exploiting the sparse nature of the signal \( x \). From this viewpoint, the proposed decentralized sparse signal recovery algorithm is also applicable to other compressive sensing problems, in which distributed sensor nodes hold parts of measurement matrices as well as measurement vectors, and collaboratively make decisions.

4.2 Algorithm outline

The collaborative sparse signal recovery algorithm is summarized as follows. The algorithm is fully decentralized, requiring only collaboration between neighboring cluster heads.

Algorithm: Collaborative environmental monitoring

Step 1: Initialization. At each sampling point, each cluster head collects position information and measurements from sensors within its cluster. Hence cluster head \( c_i \) knows the partial measurement matrix \( F_i \) and the partial measurement vector \( b_i \). The number of cluster heads \( I \), the non-negative constant \( \lambda \), and the positive constant \( d \) are also known.

Step 2: Communication. At iteration \( t + 1 \), each cluster head \( c_i \) broadcasts to its neighboring
cluster heads to acquire intermediate decision vectors $x_j(t)$ of iteration $t$, $c_j \in N_i$.

**Step 3: Optimization.** At iteration $t + 1$, each cluster head $c_i$ updates its Lagrange multiplier $\alpha_i(t + 1)$ and decision vector $x_i(t + 1)$ according to (17) and (18).

**Step 4: Iteration.** Repeat Step 2 and Step 3 until convergence.

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5. **Performance analysis**

In this section we will briefly discuss the impact of parameter settings on the performance of the algorithm, as well as the design choice of a hierarchical wireless sensor network. By performance we are mainly concerning: 1) quality of recovery, which includes the number of false alarms and the gap between the true and estimated amplitudes; and 2) convergence rate, which directly decides the communication burden of the cluster heads.

### 5.1 Parameter settings

The role of the non-negative weight $\lambda$ in (3) has been extensively discussed in compressive sensing literature, such as (Donoho et al., 2006; Figueiredo et al., 2007). There is a constant $\lambda_{\text{min}} = 1/||F^T b||_\infty$ such that if $\lambda \leq \lambda_{\text{min}}$ the optimal solution is $0$. When $\lambda$ goes to infinity, the optimal solution has the minimum $\ell_1$ norm among all points that satisfy $F^T (Fx - b = 0)$, if these points exist. Hence if $b$ is noise-free and $F$ is a full rank square matrix, then the optimal solution goes to the true signal. However large $\lambda$ generally leads to a non-sparse solution under the existence of measurement noise.

In Step 1 of the collaborative environmental monitoring algorithm, we need to know $I$, the number of cluster heads. This procedure requires multi-hop communications if the cluster heads are not directly connected with each other. However, accurate knowledge of $I$ is not necessary since it is the product $AI$ that decides the optimal solution.

The proposed algorithm converges for any given positive constant $d$; however, the value of $d$ influences the convergence rate, and thus the communication burden. It is possible to dynamically increase the value of $d$ to infinity to improve the convergence rate during the iterative optimization process (Bertsekas and Tsitsiklis, 1997). Due to the extra burden of updating $d$, we simply choose $d$ as a constant.

One of the most important advantages of the hierarchical infrastructure is its flexibility to different application scenarios. By setting $I = 1$, the infrastructure turns to be centralized; while with $I = L$ and sensors being cluster heads, the network is a fully decentralized one. This flexibility enables the network to adapt to different application scenarios.

### 5.2 To Collaborate or not to collaborate

Now we revaluate the order of the required communication load in a hierarchical network without any fusion center. First, at the data acquisition stage, cluster heads need to collect measurements from sensor nodes and construct the local measurement matrix, at communication cost on the order $\sim O(L)$. Second, at the optimization stage, one cluster head needs to transmit its decision vector to each neighboring cluster head at each iteration. The lengths of decision vectors are all $K$; the average number of neighboring cluster heads varies from $\sim O(1)$ (multi-hop communications) to $O(I)$ (one-hop communications) depending on the communication range $r_C$ of cluster heads. Denote the number of iterations as $T$, which is in general influenced by the choices of $d$ and $\lambda$, as well as the topology of the network. Therefore the overall communication load ranges from $O(KIT)$ to $O(KI^2T)$.
For comparison, we also consider the communication load of a hierarchical network in the presence of fusion center. The communication load at the data acquisition stage is the same as before. Then each cluster head needs to transmit its own part of the measurement vector and the measurement matrix to the fusion center. Transmission of the measurement matrix is necessary since the wireless sensor network is often subject to node failure, node displacement, etc; hence the measurement matrix is generally a dynamic one. Suppose that sensor nodes are evenly divided into clusters; as a result, each cluster head has $1/I$ of the entire measurement matrix with $KL$ entries and the entire measurement vector with $L$ data. Depending on the communication range $r_C$ of cluster heads, average communication load of sending one packet to the fusion center ranges from $O(1)$ (one-hop communications) to $O(I)$ (multi-hop communications). Thus, the overall communication load is from $O(KL)$ to $O(KIL)$.

The analysis provides us guideline on whether to use a fusion center or to implement decentralized collaborative information processing among cluster heads. When the number of sensor nodes within each cluster is large (large amount of data) and when the cluster heads are subject to multi-hop communications (limited communication range), the collaborative algorithm is superior in terms of energy efficiency. For a large-scale wireless sensor network, each cluster generally contains a large number of sensor nodes and the range of the sensing area is much larger than the communication range of cluster heads. Therefore, decentralized collaboration among cluster heads is preferred. In addition, collaborative information processing offers the benefits of robustness to node failure, obviation of multi-hop routing, and alleviated level of congestion.

6. Simulation results

Let us consider a $100 \times 100$ sensing area with length from 0 to 100 and width from 0 to 100. The area is divided into 100 squares with 121 grid points. Sensor nodes are uniformly randomly deployed in the sensing area. Two sources of events, one with amplitude 1 and another with amplitude 0.5, occur at grid points $(20, 80)$ and $(50, 50)$ respectively. We assume the influence function to be $f_{kl} = \exp(-d_{kl}^2/\sigma^2)$, with $\sigma = 20$. Two parameters of the decentralized algorithm are set to $\lambda = 100$ and $d = 1$.

First, we consider 100 sensor nodes, which are divided into 5 clusters with 20 sensor nodes in each cluster. The cluster heads are bi-directionally connected to each other by properly setting the communication range $r_C$. As an ideal case, the measurements are assumed to be noise-free. The convergence property of one cluster head is depicted in Figure 3. The decision variables corresponding to the two events converge to the optimal values, while the amplitudes of other grid points converge to 0. Due to the consensus constraints, decision vectors of different cluster heads converge to the same solution.

Next, we study the influence of the number of cluster heads $I$ on the convergence time. By convergence time we mean the minimum iteration number with which the differences between all decision variables and their optimal values are within 0.01. According to Figure 4, it is not surprising that the fully centralized infrastructure ($I = 1$) achieves the best convergence rate while the fully decentralized one ($I = 100$) converges slowly. Figure 4 indicates that the convergence time is $\sim O(I^2)$ for the decentralized algorithm.

Further, to compare the communication load between the two schemes, we assume all sensor nodes and cluster heads have a common communication range $r_C = 10$. In the centralized scheme, one sensor node is chosen as the fusion center. In the decentralized scheme, each cluster head is supposed to have at least one neighboring cluster head, since
Fig. 3. Convergence of the proposed algorithm.

Fig. 4. Number of cluster heads $I$ versus convergence time.

the connectivity of the sub-network of cluster heads generally cannot be satisfied when the number of cluster heads is small. The communication loads are depicted in Figure. 5. It is shown that when the cluster heads collect a large amount of data, collaborative optimization is energy-efficient. When the number of cluster heads increases, the communication load for consensus optimization dominates, leading to poor efficiency. This fact suggests us to properly select the cluster size and number of clusters in order to be energy efficient. It also points out an important but challenging research topic for future work, namely, improving the energy-efficiency of hierarchical networks via accelerating convergence rate of the decentralized collaborative algorithm.

Finally we simply discuss the compressive ratio of the proposed algorithm, namely, the ratio of the number of sensor nodes versus the number of grid points. We maintain the parameter settings in the previous simulation; the number of cluster heads is not necessary since any
settings will lead to global convergence. The relationship between the compressive ratio and the probability of successful recovery is shown in Figure 6. The simulation is repeated for 100 times with randomly deployed sensor nodes for each time. When the number of sensor nodes is larger than nearly half of the number of grid points, the recovery is successful with high probability.

7. Conclusion

This chapter discusses the design of hierarchical wireless sensor networks for environmental monitoring applications. Specifically, we focus on a generalized event detection model which is able to discover sparse events based on sensory data. Both positions and amplitudes of the events can be recovered from a convex program. Then we elaborate on an optimal decentralized algorithm which requires no fusion center but only collaboration of neighboring
cluster heads. Through theoretical analysis and simulation experiments, we suggest when the cluster heads need to collaborate and when not; this provides a design guideline for hierarchical wireless sensor networks.

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9. References


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