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# Time Evolution of a Modified Feynman Ratchet with Velocity-Dependent Fluctuations and the Second Law of Thermodynamics

Jack Denur

*Electric & Gas Technology, Inc., Garland, Texas  
U.S.A.*

## 1. Introduction

### 1.1 The Zhang formulation of the second law of thermodynamics, and a velocity dependent modified Feynman-ratchet model

The Zhang [1] formulation of the second law of thermodynamics (second law) states that no spontaneous momentum flow is possible in an isolated system. By *spontaneous*, it is meant [1]: not merely (a) *sustaining*, i.e., permanent; but also (b) *robust*, i.e., capable of withstanding dissipation, of surviving disturbances, and of generating (regenerating) itself if initially nonexistent (if destroyed). The Zhang [1] formulation of the second law implies that, *at thermodynamic equilibrium (TEQ)*, not even *merely sustaining* momentum flow is possible, i.e., that *no* systematic motion — most generally, *no* systematic process — is possible *at TEQ*: Systematic processes generated and maintained *spontaneously despite TEQ* violate the second law; by contrast, systematic *merely sustaining*, i.e., *nonrobust and nondissipative* — and hence *nonspontaneous* — processes do not violate the second law, but merely imply that TEQ has *not been completely realized* [1,2]. [Given any irreversibility (e.g., friction), (*nonspontaneous*) *merely sustaining* processes lose *even their sustainability* — they become *nonrobust and dissipative* — their negentropy (and hence free energy) is lost, and then TEQ *is completely realized* [1,2].] Corollary: The Zhang [1] formulation of the second law implies that, within a system maintained at *non-TEQ*, spontaneous momentum flow and hence systematic motion — most generally systematic process — is *certain*. Said *certainty* may be *actualized* or *in potentiality*: Example: a gas constrained to within less than the total volume of its container is at *non-TEQ* and has the *potential* for systematic motion — expansion — which is *actualized* upon release of the constraint.

Feynman's classic ratchet and pawl [3] elucidates the Zhang [1] formulation of the second law: In the original classic "Ratchet and Pawl" chapter [4], it is stated, as the upshot concerning Feynman's classic system,

"In spite of all our cleverness of lopsided design, if the two temperatures are exactly equal there is no more propensity to turn one way than the other. The moment we look at it, it may be turning one way or the other, but in the long run it gets nowhere. The fact that it gets nowhere is really the fundamental deep principle on which all of thermodynamics is based."

Recently, various formulations of the second law have been challenged, in both the quantum [5–8] and classical [5,8–10] regimes.

In this chapter [11], we show that velocity-dependent fluctuations (but not fluctuations in general) challenge the second law in the classical regime. [A digression concerning limited aspects of the quantum regime is provided in Sect. 6. Otherwise, except for the last four paragraphs of Sect. 3 (and a few very brief mentions elsewhere), this chapter deals only with the classical regime.] Our challenge is most self-evident with respect to the Zhang [1] formulation of the second law, but (as will be discussed in the last four paragraphs of Sect. 3): The Zhang [1] formulation of the second law is *maximally* strong — no other formulation thereof can be stronger [although some other(s) may be equally strong]. [Classically (with one exception [6gg] that is not applicable insofar as this present chapter is concerned) — all formulations of the second law are equivalent — but not so quantum-mechanically [6s–6ff].] Hence: A challenge to the Zhang [1] formulation of the second law is also a challenge to *all* other formulations thereof.

In this chapter [1], Feynman’s ratchet [3] is modified to the minimum extent necessary to ensure that velocity-dependence of fluctuations can spontaneously break the randomness of its Brownian motion at TEQ — spontaneously superposing a *nonrandom* walk on its Brownian motion and hence challenging the second law. This minimally-modified Feynman ratchet, illustrated in Fig. 1, will now be described.

In the right-handed Cartesian coordinate system of Fig. 1, the  $+X$ ,  $+Y$ , and  $+Z$  directions are to the right, into the page, and upwards, respectively. The Brownian motion of the disk 1 of mass  $m'$  (shown edge-on in Fig. 1) is constrained to be  $X$ -directional by the frictionless guide 2. The pawl 3 of mass  $m$  (whose lower tip protrudes below the disk in Fig. 1) is in a vertical groove within the  $+X$  disk face, wherein — in addition to its  $X$ -directional Brownian motion in lockstep with the disk as part of the combined disk-and-pawl system (DP) — it also has  $Z$ -directional Brownian motion relative to the disk per se. The DP’s total mass is  $M = m' + m \gg m$ . Each peg 4 is of  $Z$ -directional height  $H$ , and is separated from adjacent pegs by  $X$ -directional distance  $L$ . The pawl’s altitude  $Z$  is the vertical distance of its undersurface above the  $Z = 0$  level at the floor of the peg row 4, and is restricted to  $Z \geq Z_{\min}$  ( $0 < Z_{\min} < H$ ) by a stop within the  $+X$  disk face. (A simple design for the stop: Let the vertical groove that accommodates the pawl have thinner slots extending in the  $+Y$  and  $-Y$  directions. These slots accommodate pins extending from the pawl in the  $+Y$  and  $-Y$  directions, respectively. The floors of these slots preclude  $Z$ -directional motion of the pins below the pin/slot-floor contact level, thereby restricting the pawl’s altitude to  $Z \geq Z_{\min}$ .) The *net* peg height is thus  $H_{\text{net}} \equiv H - Z_{\min}$  ( $0 < H_{\text{net}} < H$ ). The entire system, including the DP, is at TEQ with equilibrium blackbody radiation (EBR) at temperature  $T$ .  $L$  is, for simplicity, taken to be large compared with the combined pawl-plus-peg  $X$ -directional thickness; yet  $L$  can easily still be small enough so that changes in the DP’s  $X$ -directional Brownian-motional velocity  $V$  occur, essentially, *only at* pawl-peg bounces, and *not* via DP-EBR  $X$ -directional momentum exchanges *between* pawl-peg bounces [12]. (The frictionless guide 2, of course, has no effect on  $V$ .) A uniform gravitational field  $g$  is attractive downwards (in the  $-Z$  direction). The  $V = 0$  rest frame — wherein (a) the frictionless guide 2 and peg row 4 are fixed and (b) the EBR at temperature  $T$  is isotropic — is (for simplicity) taken as that of  $g$ ’s source [of mass  $\gg M$  (or even  $\gg\gg M$ )]. Except for the EBR, our system is nonrelativistic: i.e., all speeds (except of EBR photons) are  $\ll c$ , and all pertinent differences in gravitational potential (e.g.,  $gH$ ) are  $\ll c^2$ .

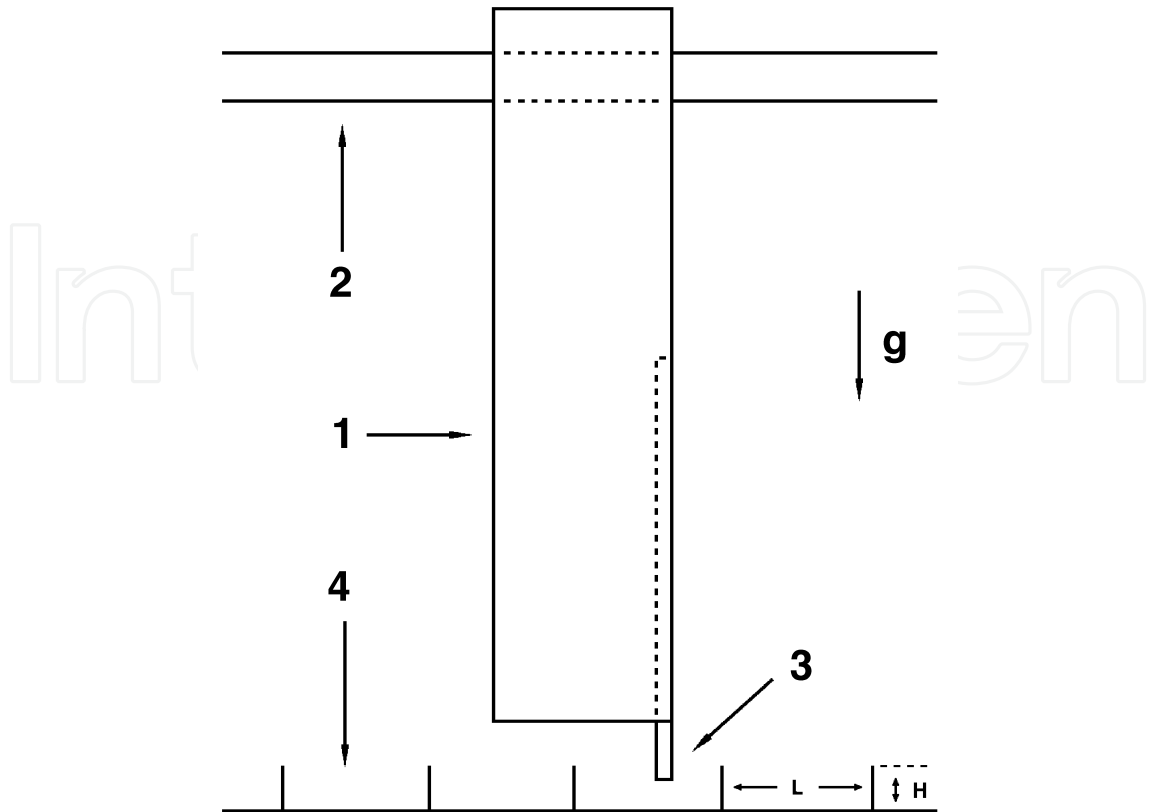


Fig. 1. Modified Feynman ratchet with velocity-dependent fluctuations

The right-handed Cartesian coordinate system described in the immediately preceding paragraph is the most appropriate one given *linear* X-directional DP Brownian motion. For transformation to *circular* X-directional DP Brownian motion, said right-handed Cartesian coordinate system can be transformed into a right-handed cylindrical coordinate system by (a) curving the X-directional axis into a circle, and (b) letting the +X, +Y, and +Z directions be counterclockwise, radially outwards from the center of this circle, and upwards, respectively. Corresponding to X-directional Brownian-motional velocity  $V$  of the DP, to first order in  $V/c$ , Doppler-shifted EBR at temperature [13]

$$T_{\pm}(V, \alpha) = T \left( 1 \pm \frac{V \cos \alpha}{c} \right) \quad (1)$$

impinges on the  $\pm X$  disk face at angle  $\alpha$  from the  $\pm X$  direction — at a rate proportional both to the differential solid angle  $2\pi \sin \alpha d\alpha$  and, by Lambert’s cosine law, to  $\cos \alpha$  [13]. {The pawl, being in the +X disk face, “sees” EBR impinging — as per the immediately preceding sentence [including (1)] with the + signs — *only* from directions with +X components (except for its lower tip — of *negligible* size compared with the *entire* pawl even at maximum tip protrusion, i.e., even at  $Z = Z_{\min}$  — when said tip protrudes below the disk).} Averaging over the range  $0 \leq \alpha \leq \pi/2$  [13],

$$T_{\pm}(V) = \langle T_{\pm}(V, \alpha) \rangle = \frac{\int_0^{\pi/2} T \left( 1 \pm \frac{V \cos \alpha}{c} \right) \sin \alpha \cos \alpha d\alpha}{\int_0^{\pi/2} \sin \alpha \cos \alpha d\alpha} = T \left( 1 \pm \frac{2V}{3c} \right). \quad (2)$$

The DP's thermal response time is sufficiently short that  $T_+(V)$  [ $T_-(V)$ ] is the temperature, corresponding to  $V$  having a given value, of the  $+X$  disk face (including the pawl) *itself* [12] [of the  $-X$  disk face *itself* [12]] — *not* merely of Doppler-shifted EBR “seen” thereby [13].

The stop *within the  $+X$  disk face* — and hence *itself* [12] at temperature, corresponding to  $V$  having a given value, of  $T_+(V)$  [12,13] — restricts the pawl's altitude to  $Z \geq Z_{\min}$ : this prevents mechanical thermal contact [although not radiative thermal contact (which is *negligible*)] between the floor of the peg row — at elevation  $Z = 0$  and temperature  $T$  — and the pawl's undersurface. (Except when the pawl's undersurface protrudes below the disk, the  $+X$  disk face shields it from EBR impinging from directions with  $-X$  components — and, in any case, the pawl's undersurface area is *negligible* compared with that of the *entire* pawl.) The pawl's thermal isolation within the  $+X$  disk face is thereby improved — helping to ensure that  $T_+(V)$  is the temperature, corresponding to  $V$  having a given value, of the pawl *itself* [12], *not* merely of Doppler-shifted EBR “seen” thereby [13].

In accordance with the Boltzmann distribution, and applying (2) with the  $+$  signs, the *conditional* probability [14]  $P(Z > H|V)$  that the pawl, of weight  $mg$ , can attain sufficient altitude  $Z > H$  to jump the pegs — and hence *not* to impede the DP's  $X$ -directional Brownian motion — given  $V$ , is

$$\begin{aligned} P(Z > H|V) &= \exp[-mg(H - Z_{\min})/kT_+(V)] \\ &\equiv \exp[-mgH_{\text{net}}/kT_+(V)] \\ &= \exp\left\{-mgH_{\text{net}} / \left[kT \left(1 + \frac{2V}{3c}\right)\right]\right\} \\ &\equiv \exp\left[-A / \left(1 + \frac{2V}{3c}\right)\right] \\ &= \left(1 + \frac{2AV}{3c}\right) e^{-A}. \end{aligned} \quad (3)$$

The second step of (3) restates the definition (initially given near the middle of the paragraph immediately following Fig. 1)

$$H_{\text{net}} \equiv H - Z_{\min}, \quad (4a)$$

the third step of (3) is justified by (2) with the  $+$  signs, the fourth step of (3) defines

$$A \equiv mgH_{\text{net}}/kT, \quad (4b)$$

and the last step of (3), which is correct to first order in  $V/c$ , is justified because  $V$  is nonrelativistic, with  $|V| \ll c$  for all values of  $|V|$  that have nonnegligible probabilities of being equaled or exceeded.

By (3),  $P(Z > H|V)$  is slightly greater when  $V > 0$  than when  $V < 0$ . Hence, *despite TEQ*, the *velocity-dependence* of  $P(Z > H|V)$  spontaneously superposes a *nonrandom* walk (spontaneous momentum flow [1]) in the  $+X$  (Forward) direction on the DP's Brownian motion — challenging the second law.

Note that  $T_{\pm}(V, \alpha)$ ,  $T_{\pm}(V)$ ,  $Z$ , and  $P(Z > H|V)$  manifest velocity-dependent fluctuations. By contrast,  $T$ ,  $H$ ,  $Z_{\min}$ ,  $H_{\text{net}} \equiv H - Z_{\min}$ ,  $L$ ,  $m'$ ,  $m$ ,  $M = m' + m \gg m$ ,  $g$ , and  $A \equiv mgH_{\text{net}}/kT$  are parameters, fixed in any one given (thought) experiment.

## 2. Markovian time evolution and challenges to the second law

The derivation of our system's time evolution will be easiest if we first consider, in (5) – (22) and the associated discussions [except in defining notation in the third paragraph of this Sect. 2, and in the last step of (7)], *only* occasions when  $|V|$  happens to have any *one* given value, i.e., when  $V = \pm |V|$ . Subsequently, we will average over *all*  $\pm |V|$  pairs, i.e., over *all*  $|V|$ .

By (3) and (4), we have, to first order in  $|V|/c$ , for the *conditional* probabilities [14]  $F$  and  $R$  of  $Z > H$  obtaining given DP Brownian motion in, respectively, the Forward or  $+X$  direction at  $V = +|V|$  and Reverse or  $-X$  direction at  $V = -|V|$ ,

$$F \equiv P(Z > H|V = +|V|) \equiv P(>|+) = \left(1 + \frac{2A|V|}{3c}\right) e^{-A} \quad (5)$$

and

$$R \equiv P(Z > H|V = -|V|) \equiv P(>|-) = \left(1 - \frac{2A|V|}{3c}\right) e^{-A}, \quad (6)$$

respectively. The states  $Z > H$ ,  $Z < H$ ,  $V = +|V| > 0$ , and  $V = -|V| < 0$  are denoted as  $>$ ,  $<$ ,  $+$ , and  $-$ , respectively. [Since  $Z$  and  $V$  are *continuous* random variables, the *point* values  $Z = H$  and  $V = |V| = 0$  each has *zero* probability measure [14a] of occurrence — and hence does not finitely contribute to any quantity integrated or averaged over any *finite* range of  $Z$  and  $V$ , respectively (e.g., over *all*  $Z$  and over *all*  $V$ , respectively).] Given  $V = \pm |V|$ , immediately preceding any pawl-peg interaction, the DP is in *one* of the four states  $> +$ ,  $> -$ ,  $< +$ , or  $< -$ ; the former two states implying that this interaction will be a pawl-over-peg jump, and the latter two that it will be a pawl-peg bounce. Immediately following a jump (bounce),  $\text{sgn } V$  is unchanged (reversed).

We now study our system's time evolution, given  $V = \pm |V|$ , in discrete time-steps of  $\Delta t = L/|V|$  that separate consecutive pawl-peg interactions, with time  $N$  immediately preceding the  $(N + 1)$ st pawl-peg interaction. If a quantity  $Q$  or an average thereof is time-dependent, then its value at time  $N$  is indicated via a subscript  $N$ . Let  $\langle Q \rangle_N$  ( $\langle\langle Q \rangle\rangle_N$ ) denote the expectation value at time  $N$  of a quantity  $Q$  over any *one* given  $\pm |V|$  pair ( $\langle\langle Q \rangle\rangle_N$  *itself* subsequently averaged over *all*  $|V|$ ). {Notes: (a) All averages in this chapter are, in this wise, either over any *one* given  $\pm |V|$  pair or over *all*  $|V|$ , except: (i) the average  $\langle T_{\pm}(V, \alpha) \rangle$  over  $\alpha$  in (2) (denoted via enclosure within single angular brackets), and (ii) some of the averages in Sect. 6, and in the Footnotes. (b) Consistently with the fifth-to-the-last sentence (especially the last clause thereof) of the paragraph immediately following Fig. 1: The combined pawl-plus-peg  $X$ -directional thickness is  $\ll L$ ; hence, the  $X$ -directional spatial, and temporal, intervals separating consecutive pawl-over-peg jumps are only negligibly greater [by said thickness, and (said thickness)/ $|V|$ , respectively] than those separating consecutive pawl-peg bounces (jump preceded or followed by bounce being the intermediate case).}

TEQ, i.e., *maximum initial total entropy*, implies that *initially*, at  $N = 0$ ,

$$\begin{aligned} P(+)_0 = P(-)_0 &= \frac{1}{2} \\ \iff \langle V \rangle_0 = |V| [P(+)_0 - P(-)_0] &= 0 \implies \langle\langle V \rangle\rangle_0 = 0. \end{aligned} \quad (7)$$

The expression in (7) for  $\langle V \rangle_0$  is true for *all*  $\pm |V|$  pairs, hence implying that for  $\langle\langle V \rangle\rangle_0$ . For all  $N \geq 0$ ,

$$\begin{aligned}\langle V \rangle_N &= |V| [P(+)_N - P(-)_N] \\ \Leftrightarrow P(\pm)_N &= \frac{1}{2} \left( 1 \pm \frac{\langle V \rangle_N}{|V|} \right).\end{aligned}\quad (8)$$

The second line of (8) is justified by  $P(+)_N + P(-)_N = 1$  and by (7).

Given  $V = \pm |V|$  and  $P(+)_N + P(-)_N = P(> |+) + P(< |+) = P(> |-) + P(< |-) = 1$ , said time evolution is a two-state discrete-time Markov chain [15] with (a) states + and -; and (b) the following *conditional* transition probabilities:

$$P[(+)_N | (+)_{N-1}] = P(> |+) = F, \quad (9a)$$

$$P[(-)_N | (-)_{N-1}] = P(> |-) = R, \quad (9b)$$

$$P[(-)_N | (+)_{N-1}] = P(< |+) = 1 - F, \quad (9c)$$

and

$$P[(+)_N | (-)_{N-1}] = P(< |-) = 1 - R. \quad (9d)$$

Note that (9) – (18) are correct not only for the *specific*  $F$  and  $R$  given by the rightmost terms of (5) and (6), respectively, but also for *general*  $F$  and  $R$  that are *at most* functions of  $|V|$  *only* — and hence *constant* for any *one* given  $|V|$ . [Of course, (1), (2), (7), and (8) are correct *independently* of any mention of  $F$  and  $R$ .]

Applying (9a), (9d), and  $P(+)_N + P(-)_N = 1$ , we obtain, for all  $N \geq 0$ , [15]

$$\begin{aligned}P(+)_N &= FP(+)_N + (1 - R)P(-)_N \\ &= FP(+)_N + (1 - R)[1 - P(+)_N] \\ &= (F + R - 1)P(+)_N + 1 - R \\ &= (F + R - 1)[(F + R - 1)P(+)_N + 1 - R] + 1 - R \\ &= (F + R - 1)\{(F + R - 1)[(F + R - 1)P(+)_N + 1 - R] \\ &\quad + 1 - R\} + 1 - R \\ &= (F + R - 1)^N P(+)_0 + (1 - R) \sum_{j=0}^{N-1} (F + R - 1)^j \\ &= (F + R - 1)^N \left( \frac{1}{2} \right) + (1 - R) \frac{1 - (F + R - 1)^N}{2 - F - R} \\ &= \frac{2(1 - R) - (F - R)(F + R - 1)^N}{2(2 - F - R)}\end{aligned}$$

$$\begin{aligned}\Rightarrow P(-)_N &= 1 - P(+)_N \\ &= \frac{2(1 - F) + (F - R)(F + R - 1)^N}{2(2 - F - R)}\end{aligned}$$

$$\Rightarrow P(\pm)_N = \frac{1}{2} \left\{ 1 \pm \frac{(F - R)[1 - (F + R - 1)^N]}{2 - F - R} \right\}. \quad (10)$$

The second step and third-to-the-last step of (10) are justified by  $P(+)_N + P(-)_N = 1$ . In the third through sixth lines of (10), a recursion relationship is developed via repeated substitution. In the seventh step of (10), we applied the first line of (7) and standard summation of the geometric series in the sixth line of (10). [If  $N = 0$ , then: (i) This geometric series contains no terms and hence vanishes. (ii)  $(F + R - 1)^0 = 1$  is true throughout the range  $-1 \leq F + R - 1 \leq 1$  of  $F + R - 1$ , with possible difficulty only at the *point* value  $F + R - 1 = 0$ . But, since  $(F + R - 1)^0 = 1$  remains true even as  $F + R - 1 \rightarrow 0^\pm$  *infinitesimally closely* (from both above and below) — by *continuity* we take  $(F + R - 1)^0 = 1$  even at the *point* value  $F + R - 1 = 0$ . Note that, among indeterminate forms, perhaps  $x^0$  alone is so well-behaved, maintaining a fixed well-defined unique finite value (1) even as  $x \rightarrow 0^\pm$  infinitesimally closely (from both above and below) — by contrast, for example,  $\frac{x}{0} \rightarrow \pm\infty$  as  $x \rightarrow 0^\pm$ . Hence,  $\lim_{x,y \rightarrow 0} x^y = \lim_{x,y \rightarrow 0} (e^{\ln x})^y = \lim_{x,y \rightarrow 0} e^{y \ln x} \xrightarrow{|y \ln x| \ll 1} 1 + y \ln x \xrightarrow{|y \ln x| \ll 1} 1$ : if  $x = F + R - 1$  and  $y = 0$ , then the last two steps immediately preceding yield *exactly* 1 — *not* merely a limiting value of 1. For perhaps the most general approach pertinent to (10) of  $F + R - 1$  to 0 that is consistent with  $(F + R - 1)^0 = 1$  even at the *point* value  $F + R - 1 = 0$ , let  $x = a(F + R - 1)$  and  $y = b(F + R - 1)^n = b\left(\frac{x}{a}\right)^n = \frac{b}{a^n}x^n$ , where  $a$ ,  $b$ , and  $n$  are arbitrary positive constants. Then  $\lim_{x,y \rightarrow 0} x^y = \lim_{x \rightarrow 0} x^{\frac{b}{a^n}x^n} = \lim_{x \rightarrow 0} (e^{\ln x})^{\frac{b}{a^n}x^n} = \lim_{x \rightarrow 0} e^{\frac{b}{a^n}x^n \ln x} = \lim_{x \rightarrow 0} \left(1 + \frac{b}{a^n}x^n \ln x\right) = 1 + \frac{b}{a^n} \lim_{x \rightarrow 0} x^n \ln x = 1 + 0 = 1$  (the last four steps immediately preceding being justified because  $\lim_{x \rightarrow 0} x^n \ln x = 0$  by L'Hospital's Rule).]

Applying the first line of (8) and the last line of (10) yields, for all  $N \geq 0$ ,

$$\begin{aligned} \langle V \rangle_N &= |V| [P(+)_N - P(-)_N] \\ &= |V| (F - R)[1 - (F + R - 1)^N] / (2 - F - R). \end{aligned} \tag{11}$$

By (11),  $\langle V \rangle_N$  is antisymmetric in  $F$  and  $R$ ; hence, *without* loss of generality, we *always* take  $F \geq R \implies \langle V \rangle_N \geq 0$  — e.g., as obtains for the *specific*  $F$  and  $R$  given by the rightmost terms of (5) and (6), respectively. The equality  $F = R \implies \langle V \rangle_N = 0$  obtains *only* given: (a) the *point* value  $V = |V| = 0$ , which has *zero* probability measure of occurrence; and/or (b)  $N = 0$ . Our challenge to the second law requires the *strict* inequality  $F > R \implies \langle V \rangle_N > 0$  *despite* TEQ, which obtains given  $|V| > 0$  and  $N \geq 1$ .

Direct calculation of  $P(+)_N$  and  $P(-)_N$  via (10) can be cumbersome. However, applying the second line of (11) — and then the antisymmetry of  $\langle V \rangle_N$  as per the paragraph immediately following (11) — to the last line of (10) further simplifies the already simpler expression given by the second line of (8) [restated in the first line of (12)]:

$$\begin{aligned} P(\pm)_N &= P(V = \pm |V|)_N = \frac{1}{2} \left(1 \pm \frac{\langle V \rangle_N}{|V|}\right) \\ \implies P(V)_N &= \frac{1}{2} \left(1 + \frac{\langle V \rangle_N}{V}\right) = P(V)_0 \left(1 + \frac{\langle V \rangle_N}{V}\right). \end{aligned} \tag{12}$$



The further simplification as per the second line of (12) [wherein  $P(V)_0 = \frac{1}{2}$  and  $V = \pm |V|$  insofar as (5) – (22) and the associated discussions are concerned] is justified by said antisymmetry.

By (11), the final steady-state value of  $\langle V \rangle_N$ , i.e.,

$$\langle V \rangle_\infty = |V| (F - R) / (2 - F - R), \quad (13)$$

is reached at  $N = 1$  if  $F + R - 1 = 0 \iff 2 - F - R = 1$ ; i.e.,

$$\begin{aligned} \langle V \rangle_1 &= |V| (F - R) \text{ for all } 0 \leq F, R \leq 1 \iff -1 \leq F + R - 1 \leq 1 \\ &= \langle V \rangle_\infty \text{ if } F + R - 1 = 0 \iff 2 - F - R = 1. \end{aligned} \quad (14)$$

Hence,  $P(V)_N$  of (12) manifests similar behavior. The completion of time evolution at  $N = 1$  if  $F + R - 1 = 0 \iff 2 - F - R = 1$  obtains for *all* quantities studied in this chapter. [In Sect. 4, we will show that, while allowing time evolution to  $N \rightarrow \infty$  *does* maximize  $\langle V \rangle_N$  and  $\left| P(V)_N - \frac{1}{2} \right|$ , it does *not* correspond to maximizing the force that tends to accelerate the DP in the +X direction, or to our *primary* objective of maximizing its power output and hence its time rate of negentropy production.]

Now, define

$$\begin{aligned} \langle V \rangle_{N+\frac{1}{2}} &\equiv \frac{1}{2} (\langle V \rangle_N + \langle V \rangle_{N+1}) \\ &= |V| (F - R) [2 - (F + R)(F + R - 1)^N] / [2(2 - F - R)] \end{aligned} \quad (15)$$

and

$$\begin{aligned} \langle \Delta V \rangle_{N+\frac{1}{2}} &\equiv \langle V \rangle_{N+1} - \langle V \rangle_N \\ &= |V| (F - R)(F + R - 1)^N. \end{aligned} \quad (16)$$

Let  $f$  be the force that tends to accelerate the DP in the +X direction. By Newton's second law and (16), at the  $N \rightarrow N + 1$  transition, i.e., at the  $(N + 1)$  st pawl-peg interaction, we have

$$\begin{aligned} \langle f \rangle_{N+\frac{1}{2}} &= M \langle \Delta V \rangle_{N+\frac{1}{2}} / \Delta t \\ &= M \langle \Delta V \rangle_{N+\frac{1}{2}} / (L / |V|) \\ &= (MV^2 / L)(F - R)(F + R - 1)^N. \end{aligned} \quad (17)$$

The second step of (17) is justified because consecutive pawl-peg interactions are separated in time by  $\Delta t = L / |V|$ . Let  $P^*$  be the DP's power output (not to be confused with probability  $P$ ). Applying (15) and (17), at the  $N \rightarrow N + 1$  transition, i.e., at the  $(N + 1)$  st pawl-peg

interaction, we have

$$\begin{aligned} \langle P^* \rangle_{N+\frac{1}{2}} &= \langle fV \rangle_{N+\frac{1}{2}} \\ &= \langle f \rangle_{N+\frac{1}{2}} \langle V \rangle_{N+\frac{1}{2}} \\ &= \frac{M|V|^3}{2L} \frac{(F-R)^2(F+R-1)^N [2-(F+R)(F+R-1)^N]}{2-F-R}. \end{aligned} \quad (18)$$

The second step of (18) is justified because  $\langle V \rangle_{N+\frac{1}{2}}$  of (15) is independent of which of the four DP states ( $> +$ ,  $> -$ ,  $< +$ , or  $< -$ ) — and hence of the corresponding  $(N+1)$ st pawl-peg interaction (jump or bounce) — that happens to occur at the  $N \rightarrow N+1$  transition [14].

For the specific  $F$  and  $R$  given by the rightmost terms of (5) and (6), respectively; (11), the second lines of (12), (17), and (18), respectively become

$$\langle V \rangle_N = (2V^2/3c)A[1 - (2e^{-A} - 1)^N]/(e^A - 1), \quad (19)$$

$$\begin{aligned} P(V)_N &= \frac{1}{2} \left\{ 1 + \left( \frac{2V}{3c} \right) \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1} \right\} \\ &= \frac{1}{2} + \left( \frac{V}{3c} \right) \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1}, \end{aligned} \quad (20)$$

$$\langle f \rangle_{N+\frac{1}{2}} = (4M|V|^3/3Lc)Ae^{-A}(2e^{-A} - 1)^N, \quad (21)$$

and

$$\langle P^* \rangle_{N+\frac{1}{2}} = \left( \frac{8M|V|^5}{9Lc^2} \right) \frac{A^2 e^{-A} (2e^{-A} - 1)^N [1 - e^{-A} (2e^{-A} - 1)^N]}{e^A - 1}. \quad (22)$$

Now, consider *all*  $\pm |V|$  pairs, i.e., *all*  $V$ , and hence also *all*  $|V|$ ; especially, consider fluctuations of  $V$  among *all* possible values. TEQ, i.e., *maximum initial total entropy*, implies that *initially*, at  $N = 0$ ,

$$\begin{aligned} P(V)_0 &= P(V)_{\text{mw}} \\ &= (M/2\pi kT)^{1/2} e^{-MV^2/2kT} \\ \implies P(|V|)_0 &= P(|V|)_{\text{mw}} \\ &= 2P(V)_0 = 2P(V)_{\text{mw}} \\ &= (2M/\pi kT)^{1/2} e^{-MV^2/2kT}, \end{aligned} \quad (23)$$

$P(V)_{\text{mw}}$  [ $P(|V|)_{\text{mw}}$ ] being the one-dimensional Maxwellian probability density of  $V$  [ $|V|$ ]. Letting  $P(V)_0 = \frac{1}{2} \rightarrow P(V)_0 = P(V)_{\text{mw}}$  and  $V = \pm |V| \rightarrow$  continuous  $V$  in the second line

of (12), and then applying (19), we obtain

$$\begin{aligned}
 P(V)_N &= P(V)_0 \left( 1 + \frac{\langle V \rangle_N}{V} \right) \\
 &= P(V)_{\text{mw}} \left( 1 + \frac{\langle V \rangle_N}{V} \right) \\
 &= P(V)_{\text{mw}} \left\{ 1 + \left( \frac{2V}{3c} \right) \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1} \right\}. \quad (24)
 \end{aligned}$$

Any  $\langle Q \rangle_N$ , e.g.,  $\langle V \rangle_N$ ,  $\langle f \rangle_{N+\frac{1}{2}}$ , or  $\langle P^* \rangle_{N+\frac{1}{2}}$ , is defined for a given  $\pm |V|$  pair, i.e., for a given  $|V|$  — it is *undefined* and cannot even be calculated given *only* a single value of  $V$ , e.g., given *only*  $+|V|$  alone or given *only*  $-|V|$  alone.  $\langle Q \rangle_N$  can be written in the more detailed form  $\langle Q(|V|) \rangle_N$ ; by contrast, the expression  $\langle Q(V) \rangle_N$  is meaningless. Since (20) and (24) are correct to first order in  $V/c$ , by (24),  $P(|V|)_N = P(+|V|)_N + P(-|V|)_N = P(|V|)_0 = P(|V|)_{\text{mw}}$  to first order in  $|V|/c$ ; hence, to first order in  $|V|/c$ , any average  $\langle \langle Q \rangle \rangle_N$  over  $P(|V|)_{\text{mw}}$  equals that over  $P(|V|)_N$  itself. [Of course, *initially*, at  $N = 0$ ,  $P(|V|)_N = P(|V|)_0 = P(|V|)_{\text{mw}}$  *exactly*, not merely to first order in  $|V|/c$ ; hence, any average  $\langle \langle Q \rangle \rangle_0$  over  $P(|V|)_{\text{mw}}$  is *exact*.] The following five averages over  $P(|V|)_{\text{mw}}$  will be useful:  $\langle \langle |V| \rangle \rangle_{\text{mw}} = (2kT/\pi M)^{1/2}$ ,  $\langle \langle V^2 \rangle \rangle_{\text{mw}} = kT/M$ ,  $\langle \langle |V|^3 \rangle \rangle_{\text{mw}} = [2(kT/M)^3]^{1/2}$ ,  $\langle \langle V^4 \rangle \rangle_{\text{mw}} = 3(kT/M)^2 = 3 \langle \langle V^2 \rangle \rangle_{\text{mw}}^2$ , and  $\langle \langle |V|^5 \rangle \rangle_{\text{mw}} = 8[2(kT/M)^5]^{1/2}$ . [Of course, *numerically*, these five averages are identical whether taken over  $P(|V|)_{\text{mw}}$  or over  $P(V)_{\text{mw}}$ . But, *conceptually*, as per the first two sentences of this paragraph — and anticipating the next paragraph — they are more correctly taken over  $P(|V|)_{\text{mw}}$ .] Averaging over any *one* given  $\pm |V|$  pair to obtain  $\langle Q \rangle_N$  first, and subsequently averaging over *all*  $|V|$  to obtain  $\langle \langle Q \rangle \rangle_N$ , is preferable to attempting to obtain  $\langle \langle Q \rangle \rangle_N$  directly because, e.g.: (a) the former procedure is easier, (b) *both*  $\langle Q \rangle_N$  and  $\langle \langle Q \rangle \rangle_N$  are thus obtained, and (c) the  $|V|$ -dependence of  $F - R$  is thus accounted for — e.g., as per application of (5) and (6) to the last terms of (11), (17), and (18) in order to obtain (19), (21), and (22), respectively. Averaging  $V^2$ ,  $|V|^3$ , and  $|V|^5$  in (19), (21), and (22), respectively, over  $P(|V|)_{\text{mw}}$  (as per the immediately preceding paragraph) yields (25), (27), and (28), respectively. So that (25) – (28) are a complete set of equations, we restate the last line of (24) as (26). Thus, we obtain

$$\begin{aligned}
 \langle \langle V \rangle \rangle_N &= \frac{2 \langle \langle V^2 \rangle \rangle_{\text{mw}}}{3c} \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1} \\
 &= \frac{2kT}{3Mc} \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1}, \quad (25)
 \end{aligned}$$

$$P(V)_N = P(V)_{\text{mw}} \left\{ 1 + \left( \frac{2V}{3c} \right) \frac{A[1 - (2e^{-A} - 1)^N]}{e^A - 1} \right\}, \quad (26)$$

$$\begin{aligned} \langle\langle f \rangle\rangle_{N+\frac{1}{2}} &= \frac{4M \langle\langle |V|^3 \rangle\rangle_{\text{mw}}}{3Lc} A e^{-A} (2e^{-A} - 1)^N \\ &= \frac{4}{3L} \left[ \frac{2(kT)^3}{Mc^2} \right]^{1/2} A e^{-A} (2e^{-A} - 1)^N, \end{aligned} \quad (27)$$

and

$$\begin{aligned} \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} &= \left( \frac{8M \langle\langle |V|^5 \rangle\rangle_{\text{mw}}}{9Lc^2} \right) \frac{A^2 e^{-A} (2e^{-A} - 1)^N [1 - e^{-A} (2e^{-A} - 1)^N]}{e^A - 1} \\ &= \frac{64}{9Lc^2} \left[ \frac{2(kT)^5}{M^3} \right]^{1/2} \frac{A^2 e^{-A} (2e^{-A} - 1)^N [1 - e^{-A} (2e^{-A} - 1)^N]}{e^A - 1}. \end{aligned} \quad (28)$$

Note that specification to  $P(V)_{\text{mw}} = \frac{1}{2}P(|V|)_{\text{mw}}$  is *not* required for the validity of our analyses: For example,  $P(V)_{\text{mw}}$  is *specifically* applied in the last two lines of (24) and in (26); and  $P(|V|)_{\text{mw}}$  in (25), (27), and (28). By contrast, for example, (7) – (22) and the first line of (24) are valid not only for  $P(V)_0 = P(V)_{\text{mw}} = \frac{1}{2}P(|V|)_{\text{mw}}$ , but for *any*  $P(V)_0$  — whether continuous, discrete, or possessing both continuous and discrete ranges — that is symmetrical about  $V = 0$ . {Of course, (1) – (6) are valid for *any*  $P(V)$  whatsoever [including, e.g., a Dirac  $\delta$ -function, in which case either (5) or (6) but (unless  $V = |V| = 0$ ) not both would obtain.]} For any *one* given value of  $|V|$ , i.e., for any *one* given  $\pm|V|$  pair, velocity-dependent fluctuations behave identically — and break the randomness of Brownian motion identically — given *any*  $P(V)_0$  that is symmetrical about  $V = 0$ . Note, in particular, as per (12), (20), and the first line of (24), that, for any given  $V$ , the bias of  $P(V)_N$  from  $P(V)_0$  is identical given *any*  $P(V)_0$  that is symmetrical about  $V = 0$ . Considering *all*  $V$ , and hence also *all*  $|V|$ ,  $P(V)_{\text{mw}} = \frac{1}{2}P(|V|)_{\text{mw}}$  is the symmetrical velocity probability density — indeed, *the* velocity probability density — corresponding to maximum entropy. Hence,  $P(V)_{\text{mw}} = \frac{1}{2}P(|V|)_{\text{mw}}$  is employed in this chapter — but with the view that generalization is possible to *any*  $P(V)_0$  that is symmetrical about  $V = 0$ .

### 3. Negentropy production, and formulations of the second law

Positive values (however small) of  $\langle V \rangle_N$ ,  $|P(V)_N - \frac{1}{2}|$ ,  $\langle f \rangle_{N+\frac{1}{2}}$ ,  $\langle P^* \rangle_{N+\frac{1}{2}}$ ,  $\langle\langle V \rangle\rangle_N$ ,  $|P(V)_N - P(V)_{\text{mw}}|$ ,  $\langle\langle f \rangle\rangle_{N+\frac{1}{2}}$ , and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  *despite* TEQ challenge the second law. A positive value of  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  *despite* TEQ is our *primary* challenge thereto, because if  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  overcomes a conservative resisting force, equal in magnitude but opposite in direction to  $\langle\langle f \rangle\rangle_{N+\frac{1}{2}}$ , then there obtains an uncompensated *negative* time rate of change in *total* entropy  $S$ :

$$\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} > 0 \implies dS/dt = - \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} / T < 0. \quad (29)$$

Perhaps the simplest such conservative resisting force,  $Mg \sin(-\theta) = -Mg \sin \theta \doteq -Mg\theta$ , is obtained by sloping our system very slightly upwards towards the  $+X$  direction — as per Fig. 1 and the two immediately following paragraphs, very slightly upwards towards the right given a right-handed Cartesian coordinate system, or into a very gentle counterclockwise upward spiral given a right-handed cylindrical coordinate system — at a very small slope angle  $\theta$  ( $0 < \theta \ll 1$  rad; also,  $0 < \theta \ll mgL/kT = AL/H_{\text{net}}$ ), such that  $\langle\langle f \rangle\rangle_{N+\frac{1}{2}} = Mg \sin \theta \doteq Mg\theta$ . (If, instead, the resisting force is frictional and hence *nonconservative*, then it can be overcome at steady state indefinitely — frictional dissipation being *recycled* into power  $P^*$  — *despite TEQ*.) Generation — or regeneration via recycling — of power  $P^*$  *despite TEQ* entails spontaneous momentum flow [1] in challenge of the Zhang formulation [1] of the second law (and hence, as per the last two paragraphs of this Sect. 3, of *all* formulations thereof). Of course, (29) is true for *all* DP power outputs, e.g., (29) is also true for the DP's Carnot-engine [17] power outputs.

But, in view of recent work concerning limitations of validity of certain formulations — especially, of *entropy*-based formulations — of the second law in the quantum regime [6s–6ff], the employment of the *entropy*-based (29) requires justification.

In the classical regime, (a) the Zhang [1] formulation of the second law (no spontaneous momentum flow in an isolated system  $\implies$  no systematic motion — most generally, no systematic process — at TEQ), and (b) Thomson's formulation thereof (no extractable work at TEQ), are equivalent to (c) the formulation thereof stating that total entropy (total negentropy) can never decrease (increase), and, indeed, to (d) all other formulations of the second law. But, in the quantum regime, entropy (or, equivalently, negentropy — and hence free energy) is a difficult, non-uniquely-defined concept — as opposed to heat, and especially to work [6s–6ff]. Hence, in the quantum regime, (a) and (b) immediately above are preferable to (c) [and (d)] immediately above. This present chapter deals only with the classical regime — except for the last four paragraphs of this Sect. 3, a digression concerning limited aspects of the quantum regime in Sect. 6, and a few very brief mentions elsewhere. This present chapter is based primarily on (a) immediately above — which implies (b) immediately above *always*, and, apart from difficulties in the quantum regime [6s–6ff], also (c) [and (d)] immediately above. Nevertheless: Insofar as this present chapter is concerned, certainly outside of Sect. 6 — and, owing to the *limited* nature of said quantum aspects, probably even in Sect. 6 — (c) immediately above [which justifies the employment of *entropy* in (29)], and also (d) immediately above, still retain validity. [As an aside, note that the usual statement of (b) immediately above — no extractable work *via cyclic processes* at TEQ — is too restrictive. If a system is capable of doing work *even only on a one-time basis via a noncyclic process* — e.g., via a *one-time* isothermal expansion of a gas initially constrained to within less than the total volume of its container — then it is *not initially* at TEQ: it is at TEQ only *after* the gas has expanded to occupy the total volume of its container and hence is no longer capable of doing work. Thus, deleting “via cyclic processes” yields a more general statement as per (b) immediately above, and in accordance with the first two paragraphs of Sect. 1.]

In *both* the classical *and* the quantum regimes — but primarily in the quantum regime (particularly for *finite* quantum systems [6ee]), wherein different formulations of the second law can be *inequivalent* [6s–6ff]: The Zhang [1] formulation of the second law [(a) immediately above] is a *maximally* strong formulation thereof, i.e., as strong a formulation thereof as is possible {some other formulation(s), e.g., the Thomson formulation [(b) immediately above], may be equally strong [6s–6dd], but no other formulation can be stronger}: Thus: A challenge

to the Zhang [1] formulation of the second law is a challenge to *all* formulations [6s–6ff] thereof — and hence a challenge to *the* second law [6dd]. By contrast (particularly in the quantum regime [6s–6ff]), a challenge to any other formulation(s) [6s–6ff] of the second law (i) may or (ii) may not be a challenge to the Zhang [1] formulation thereof, and hence to *all* formulations, thereof — and hence may be a challenge, respectively, (i) to *the* second law [6dd] or (ii) merely to *a* second law [6dd]. And a *true* challenge must be to *the* — not merely to *a* — second law.

There has recently been discovered a *classical* situation [6gg] wherein the minimal-work-principle formulation of the second law can be *invalid*. [The minimal-work-principle formulation of the second law has previously been investigated in the quantum regime (where it also can be *invalid*) [6v,6w].] But this is not applicable insofar as this present chapter is concerned, and in any case does not alter the *maximally strong status* of the Zhang [1] formulation of the second law.

#### 4. Details of Markovian time evolution, and maximization of challenges to the second law

Time evolution is complete at  $N = 1$  if  $F + R - 1 = 0 \implies A = \ln 2$ . This corresponds to an *overall* probability (considering *both* Forward *and* Reverse DP Brownian motion) of  $\frac{1}{2}$  (correct to first order in  $|V|/c$  for all  $N \geq 1$  and exact at  $N = 0$ ) that any given pawl-peg interaction is either a jump or a bounce, i.e., to  $P(> |F + R - 1 = 0 \implies A = \ln 2)_N = P(< |F + R - 1 = 0 \implies A = \ln 2)_N = \frac{1}{2}$  (correct to first order in  $|V|/c$  for all  $N \geq 1$  and exact at  $N = 0$ ). As  $F + R - 1 \rightarrow 1 \implies A \rightarrow 0$ , pawl-peg bounces become ever rarer, and hence time evolution becomes ever slower. As  $F + R - 1 \rightarrow -1 \implies A \rightarrow \infty$ , pawl-over-peg jumps become ever rarer, and hence time evolution becomes ever slower.

Time evolution of  $\langle V \rangle_N$  and  $P(V)_N - \frac{1}{2}$ , and likewise of  $\langle \langle V \rangle \rangle_N$  and  $P(V)_N - P(V)_{mw}$ , towards final steady-state values as  $N \rightarrow \infty$  is monotonic and asymptotic if  $0 < F + R - 1 < 1 \implies \ln 2 > A > 0$ , diminishing-oscillatory if  $-1 < F + R - 1 < 0 \implies \infty > A > \ln 2$ , and complete at  $N = 1$  if  $F + R - 1 = 0 \implies A = \ln 2$ .

For general  $F$  and  $R$  that [as per the sentence immediately following (9d)] are *at most* functions of  $|V|$  *only*, and hence *constant* for any *one* given  $|V|$  — not merely for the *specific*  $F$  and  $R$  given by the rightmost terms of (5) and (6), respectively — the functional form of any  $\langle Q \rangle_N$  with respect to  $F$ ,  $R$ , and  $N$  (and hence with respect to  $A$  and  $N$ ) is *independent* of  $|V|$ . Thus, the values of  $F$ ,  $R$ , and  $N$  (and hence of  $A$  and  $N$ ) yielding maximization of any  $\langle Q \rangle_N$  are also *independent* of  $|V|$  — and thus likewise also yield maximization of the corresponding  $\langle \langle Q \rangle \rangle_N$ .

By inspection of (10) – (14), (19), (20), (25), and (26),  $\langle V \rangle_N$  and  $|P(V)_N - \frac{1}{2}|$ , and likewise  $\langle \langle V \rangle \rangle_N$  and  $|P(V)_N - P(V)_{mw}|$ , are maximized by maximizing  $(F - R)[1 - (F + R - 1)^N]/(2 - F - R) \implies$  maximizing  $A[1 - (2e^{-A} - 1)^N]/(e^A - 1)$ : maximization obtains given  $0 < A \ll 1$  — implying maximization of  $(1 - \frac{1}{2}A)[1 - (1 - 2A)^N]$  — which is maximized at unity by letting  $A \rightarrow 0$  and  $N \rightarrow \infty$ , but with  $N \rightarrow \infty$  sufficiently faster than  $A \rightarrow 0$  such that  $(1 - 2A)^N \rightarrow 0$ . We thus obtain the *absolute* maxima

$$\begin{aligned} \langle V \rangle_{N,\max} &= \langle V \rangle_\infty |(A \rightarrow 0) = 2V^2/3c \\ \implies \langle \langle V \rangle \rangle_{N,\max} &= \langle \langle V \rangle \rangle_\infty |(A \rightarrow 0) = 2 \langle \langle V^2 \rangle \rangle_{mw} / 3c = 2kT/3Mc \end{aligned} \quad (30)$$

and

$$\begin{aligned}
 \left| P(V)_N - \frac{1}{2} \right|_{\max} / \frac{1}{2} &= \left| P(V|A \rightarrow 0)_\infty - \frac{1}{2} \right| / \frac{1}{2} \\
 &= |P(V)_N - P(V)_{\text{mw}}|_{\max} / P(V)_{\text{mw}} \\
 &= |P(V|A \rightarrow 0)_\infty - P(V)_{\text{mw}}| / P(V)_{\text{mw}} \\
 &= 2|V| / 3c. \tag{31}
 \end{aligned}$$

If  $F + R - 1 \rightarrow 1 \implies A \rightarrow 0$ , then time evolution becomes infinitely slow — requiring  $N \rightarrow \infty$  — because then pawl-peg bounces become infinitely rare. But any “practical” time evolution is limited to at most a *large but finite* number  $N$  of pawl-peg interactions. Hence,  $\langle V \rangle_N$  and  $|P(V)_N - \frac{1}{2}|$ , and likewise  $\langle\langle V \rangle\rangle_N$  and  $|P(V)_N - P(V)_{\text{mw}}|$ , attain “practical” maxima — corresponding to *small but not infinitesimal*  $1 - (F + R - 1) \implies$  *small but not infinitesimal*  $A$  and to *large but not infinite*  $N$  — that are almost but not quite as large as the *absolute* maxima given in (30) and (31) corresponding to  $F + R - 1 \rightarrow 1 \implies A \rightarrow 0$  and to  $N \rightarrow \infty$ . [This is especially true because, if pawl-peg bounces are *extremely* rare, then the DP has sufficient time *between* pawl-peg bounces so that its  $X$ -directional momentum exchanges with the EBR are no longer (as is assumed in our analyses) negligible compared with its  $X$ -directional momentum exchanges *at* pawl-peg bounces [12].]

For *small*  $N \geq 1$ , maximizing  $\langle V \rangle_N$  and  $\langle\langle V \rangle\rangle_N$  with respect to  $A$  by setting  $\partial \langle V \rangle_N / \partial A = 0 \implies \partial \langle\langle V \rangle\rangle_N / \partial A = 0$  yields maxima at *moderate*  $A$ , because *small*  $N \geq 1$  implies only *one or a few* pawl-peg interactions — *not* the *many* pawl-peg interactions that would be required to compensate (or overcompensate) for the *small* probability of pawl-peg bounces corresponding to *small*  $A$ . For example,  $\langle V \rangle_1$  and  $\langle\langle V \rangle\rangle_1$  are maximized at  $A = 1$ , with  $\langle V \rangle_{1,\max} = \langle V \rangle_1 | (A = 1) = 4V^2 / 3ec \implies \langle\langle V \rangle\rangle_{1,\max} = \langle\langle V \rangle\rangle_1 | (A = 1) = 4 \langle\langle V^2 \rangle\rangle_{\text{mw}} / 3ec = 4kT / 3Mc$ . Note that these maxima lie in the diminishing-oscillatory regime, as per the second paragraph of this Sect. 4, and hence are larger [by a factor of  $2(1 - e^{-1})$ ] than  $\langle V \rangle_\infty | (A = 1) = 2V^2 / [3c(e - 1)] \implies \langle\langle V \rangle\rangle_\infty | (A = 1) = 2 \langle\langle V^2 \rangle\rangle_{\text{mw}} / [3c(e - 1)] = 2kT / [3Mc(e - 1)]$ , [obtained by putting  $N = \infty$  and  $A = 1$  into (19) and (25)] — but smaller (by a factor of  $2/e$ ) than the *absolute* maxima as per (30). [Similar results obtain for  $|P(V)_1 - \frac{1}{2}| / \frac{1}{2} = |P(V)_1 - P(V)_{\text{mw}}| / P(V)_{\text{mw}}$ , whose maximization at  $A = 1$  yields  $|P(V)_1 - \frac{1}{2}|_{\max} / \frac{1}{2} = |P(V)_1 - P(V)_{\text{mw}}|_{\max} / P(V)_{\text{mw}} = |P(V|A = 1)_1 - \frac{1}{2}| / \frac{1}{2} = |P(V|A = 1)_1 - P(V)_{\text{mw}}| / P(V)_{\text{mw}} = 4|V| / 3ec$ .] For another example,  $\langle V \rangle_2$  and  $\langle\langle V \rangle\rangle_2$  are maximized at  $A = \frac{1}{2}$  ( $< 1$  as expected), but at the same values as  $\langle V \rangle_1$  and  $\langle\langle V \rangle\rangle_1$ , respectively; and similarly for  $|P(V)_2 - \frac{1}{2}| / \frac{1}{2} = |P(V)_2 - P(V)_{\text{mw}}| / P(V)_{\text{mw}}$ .

By contrast, via inspection of (17), (21), and (27),  $\langle f \rangle_{N+\frac{1}{2}}$  ( $\langle\langle f \rangle\rangle_{N+\frac{1}{2}}$ ) are maximized by: (a) first, setting  $\langle f \rangle_{N+\frac{1}{2}} = \langle f \rangle_{\frac{1}{2}}$  ( $\langle\langle f \rangle\rangle_{N+\frac{1}{2}} = \langle\langle f \rangle\rangle_{\frac{1}{2}}$ ), thereby maximizing  $(F + R - 1)^N$  at unity  $\implies$  maximizing  $(2e^{-A} - 1)^N$  at unity; and (b) then, setting  $d(Ae^{-A}) / dA = 0 \implies A = 1 \implies (Ae^{-A})_{\max} = e^{-1}$ . We thus obtain the *absolute* maxima

$$\begin{aligned}
 \langle f \rangle_{N+\frac{1}{2},\max} &= \langle f \rangle_{\frac{1}{2}} | (A = 1) = 4M|V|^3 / 3ecL \\
 \implies \langle\langle f \rangle\rangle_{N+\frac{1}{2},\max} &= \langle\langle f \rangle\rangle_{\frac{1}{2}} | (A = 1) = 4M \langle\langle |V|^3 \rangle\rangle_{\text{mw}} / 3ecL = \frac{4[2(kT)^3 / M]^{1/2}}{3ecL}. \tag{32}
 \end{aligned}$$

Time evolution of  $\langle f \rangle_{N+\frac{1}{2}}$  and  $\langle\langle f \rangle\rangle_{N+\frac{1}{2}}$  towards 0 as  $N \rightarrow \infty$  is monotonic and asymptotic through positive values for all  $N \geq 0$  if  $0 < F + R - 1 < 1 \implies \ln 2 > A > 0$ , diminishing-oscillatory through positive (negative) values at all even (odd)  $N \geq 0$  if  $-1 < F + R - 1 < 0 \implies \infty > A > \ln 2$ , and complete at  $N = 1$  if  $F + R - 1 = 0 \implies A = \ln 2$ . Thus, as per (18) and (28), in order to maximize  $\langle P^* \rangle_{N+\frac{1}{2}}$  ( $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$ ) and hence, by Sect. 3,  $-dS/dt$  — our *primary* challenge to the second law — we should *not* allow the Markovian time evolution of our DP to approach as closely as is “practical” (as per the fifth paragraph of this Sect. 4) to its *final* steady state  $N \rightarrow \infty$ . Rather, we should allow this time evolution to proceed *only* to  $N_{\text{opt}} + 1$ , where  $N_{\text{opt}}$  is the *optimum finite* value of  $N$ ; also set  $A$  at its corresponding *optimum finite* value  $A_{\text{opt}}$  (*not*  $A \rightarrow 0$ ); and then let the DP do work against a conservative resisting force equal in magnitude but opposite in direction to  $\langle f \rangle_{N_{\text{opt}}+\frac{1}{2}}$  ( $\langle\langle f \rangle\rangle_{N_{\text{opt}}+\frac{1}{2}}$ ) in this *imposed* steady state:  $N_{\text{opt}}$  and  $A_{\text{opt}}$  will be derived shortly. {If, instead, the resisting force is *nonconservative*, e.g., friction, then  $\langle P^* \rangle_{N+\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  are still thereby maximized, even though  $-dS/dt$  is then dissipated as fast as it is (re)generated via recycling [recall the last two sentences of the paragraph containing (29)].} In special cases, particular optimizations may correspond to the immediately aforementioned general ones, e.g., setting  $\theta = \theta_{\text{opt}}$  (with  $\theta_{\text{opt}} > 0$  and finite) if  $\langle\langle f \rangle\rangle_{N+\frac{1}{2}} = Mg \sin \theta \doteq Mg\theta$  as per the sentence immediately following (29).

As an aside, note that the DP’s Carnot-engine [17] power outputs, *are* maximized (at  $\theta = 0$ ) if the Markovian time evolution of our DP *is* allowed to approach as closely as is “practical” (as per the fifth paragraph of this Sect. 4) to its *final* steady state, with  $A \rightarrow 0$  and  $N \rightarrow \infty$  as per the first sentence of the paragraph containing (30) and (31) [17], corresponding (as closely as “practical”) to the *absolute* maxima given by (30) and (31).

Maximizing  $\langle P^* \rangle_{N+\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  with respect to  $N$  at given fixed  $F + R - 1 \implies$  given fixed  $A$ , by setting

$$\partial \langle P^* \rangle_{N+\frac{1}{2}} / \partial N = 0 \implies \partial \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} / \partial N = 0, \quad (33)$$

yields, for the optimum value of  $N$ ,

$$N_{\text{opt}} = -\ln(F + R) / \ln(F + R - 1) \implies N_{\text{opt}} = -\ln(2e^{-A}) / \ln(2e^{-A} - 1). \quad (34)$$

Obviously, (34) is valid only if  $0 \leq F + R - 1 < 1 \implies \ln 2 \geq A > 0$ . Also, obviously, if (34) yields a non-whole-number value for  $N_{\text{opt}}$ , then the *actual* value of  $N_{\text{opt}}$  equals the whole-number value either immediately smaller or immediately larger than the non-whole-number value yielded by (34). If  $-1 < F + R - 1 \leq 0 \implies \infty > A \geq \ln 2$ , and also if  $0 \leq F + R - 1 < 1 \implies \ln 2 \geq A > 0$  but with  $F + R - 1$  sufficiently close to 0  $\implies A$  sufficiently close to  $\ln 2$  such that (34) yields  $N_{\text{opt}}$  sufficiently close to 0 as opposed to 1, then  $N_{\text{opt}} = 0$ .

By (34) and the three immediately following sentences: As  $F + R - 1$  increases from 0 to 1  $\implies A$  decreases from  $\ln 2$  to 0,  $N_{\text{opt}}$  increases monotonically from 0 to  $\infty$ . (By the fifth paragraph of this Sect. 4, infinitesimally small  $1 - (F + R - 1) \implies$  infinitesimally small  $A$  and infinitely large  $N$  are not “practical”.) By (18), (22), and (28), the immediately preceding paragraph, and anticipating the two immediately following paragraphs: (a) If  $F + R - 1 = 0 \implies A = \ln 2$ , then  $\langle P^* \rangle_{\frac{1}{2}} > 0$ ,  $\langle\langle P^* \rangle\rangle_{\frac{1}{2}} > 0$ , and, for all  $N \geq 1$ ,  $\langle P^* \rangle_{N+\frac{1}{2}} = \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} = 0$ .



(b) If  $-1 < F + R - 1 < 0 \implies \infty > A > \ln 2$ , then  $\langle P^* \rangle_{N+\frac{1}{2}} \rightarrow 0$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} \rightarrow 0$  via ever-diminishing oscillations as  $N \rightarrow \infty$ , being positive (negative) at all even (odd)  $N \geq 0$ .  
(c) If  $0 < F + R - 1 < 1 \implies \ln 2 > A > 0$ , then  $\langle P^* \rangle_{N+\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  may reach their maxima at any  $N \geq 0$ .

Only for  $N = 0$  can  $\langle P^* \rangle_{N+\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  be maximized analytically with respect to  $A$  at given fixed  $N$  by setting

$$\partial \langle P^* \rangle_{N+\frac{1}{2}} / \partial A = 0 \implies \partial \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} / \partial A = 0. \quad (35)$$

For all  $N \geq 1$ , (35) must be solved numerically. [We neglect the trivial analytical solution of (35), which yields  $A = 0$  and corresponds to  $\langle P^* \rangle_{N+\frac{1}{2}} = \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} = 0$  for all  $N \geq 0$ .] Solving (35) analytically for  $N = 0$  yields, for the corresponding optimum value of  $A$ ,

$$A_{\text{opt}}|(N=0) = 1, \quad (36)$$

and, for the corresponding maximum values of  $\langle P^* \rangle_{\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{\frac{1}{2}}$ ,

$$\langle P^* \rangle_{\frac{1}{2}, \text{max}} = \langle P^* \rangle_{\frac{1}{2}} | (A=1) = 8M |V|^5 / [(3ec)^2 L] \quad (37)$$

and

$$\langle\langle P^* \rangle\rangle_{\frac{1}{2}, \text{max}} = \langle\langle P^* \rangle\rangle_{\frac{1}{2}} | (A=1) = \frac{8M \langle\langle |V|^5 \rangle\rangle_{\text{mw}}}{(3ec)^2 L} = \frac{64[2(kT)^5 / M^3]^{1/2}}{(3ec)^2 L}, \quad (38)$$

respectively. Note that (36) is consistent with the third sentence following (34). Equal and/or higher maxima — if any exist — of  $\langle P^* \rangle_{N+\frac{1}{2}}$  and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  for  $N \geq 1$  [corresponding to  $A_{\text{opt}}(N)$  in the range to be given shortly by (39)], can be found numerically.

By (34) and the three immediately following sentences, corresponding to *all* maxima of  $\langle P^* \rangle_{N+\frac{1}{2}}$  [ $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$ ] for *all*  $N \geq 1$  {whether or not any of these maxima equal or exceed  $\langle P^* \rangle_{\frac{1}{2}, \text{max}}$  of (37) [ $\langle\langle P^* \rangle\rangle_{\frac{1}{2}, \text{max}}$  of (38)]},

$$0 < A_{\text{opt}}|(N \geq 1) \leq [A_{\text{opt}}|(N=1)]_{\text{max}} < \ln 2, \quad (39)$$

where  $[A_{\text{opt}}|(N=1)]_{\text{max}}$  is, as per the third sentence following (34), the *maximum* value of  $A_{\text{opt}}(N)$  that corresponds to  $N = 1$  rather than to  $N = 0$ .  $A_{\text{opt}}(N)$  decreases monotonically and asymptotically towards 0 as  $N \rightarrow \infty$ . (But note the “practical” limits as per the fifth paragraph of this Sect. 4.)

We conclude this Sect. 4 by considering [assuming, for simplicity, the *specific*  $F$  and  $R$  given by the rightmost terms of (5) and (6), respectively], the quantity

$$\begin{aligned} F - R &= 4Ae^{-A} |V| / 3c \\ \implies \langle\langle F - R \rangle\rangle_{\text{mw}} &= 4Ae^{-A} \langle\langle |V| \rangle\rangle_{\text{mw}} / 3c = 4Ae^{-A} (2kT / \pi M)^{1/2} / 3c, \end{aligned} \quad (40)$$

an important measure of the degree to which the randomness of our DP's Brownian motion is broken — and maximization thereof. [Considering any *one* given  $\pm |V|$  pair,  $F - R$  does not involve an average, and hence is not enclosed within single angular brackets in (40) and (41).]

Maximizing  $F - R$  and  $\langle\langle F - R \rangle\rangle_{\text{mw}}$  with respect to  $A$ , i.e., setting  $d(Ae^{-A})/dA = 0 \implies A = 1 \implies (Ae^{-A})_{\text{max}} = e^{-1}$ , yields

$$\begin{aligned} (F - R)_{\text{max}} &= 4|V|/3ec \\ \implies \langle\langle F - R \rangle\rangle_{\text{mw,max}} &= 4\langle\langle |V| \rangle\rangle_{\text{mw}}/3ec = 4(2kT/\pi M)^{1/2}/3ec. \end{aligned} \quad (41)$$

Given (41), *all* other measures of the degree to which the randomness of our DP's Brownian motion is broken are also maximized immediately following the *first* step of time evolution, i.e., *all* other  $\langle Q \rangle_N$  and  $\langle\langle Q \rangle\rangle_N$  are also maximized at  $N = 1$  (if, e.g.,  $Q = f$  or  $Q = P^*$ , at the  $N = 0 \rightarrow N = 1$  transition, i.e., at the 1st pawl-peg interaction) and at  $A = 1$ . If  $Q = f$ , and possibly if  $Q = P^*$ , these maxima are absolute, i.e., not equaled or exceeded for any transition ending at  $N > 1$  — in contrast with maximization if, e.g.,  $Q = V$ . For  $N > 1$  (and for transitions ending at  $N > 1$ ), (41) does not, in general, correspond to maximization of  $\langle Q \rangle_N$  and  $\langle\langle Q \rangle\rangle_N$  (whether absolute or merely for the given  $N$ ). [Note that, in contrast with (41) — which corresponds to maximization at  $N = 1$  (or at the  $N = 0 \rightarrow N = 1$  transition, i.e., at the 1st pawl-peg interaction) given  $A = 1$  — completion of time evolution at  $N = 1$  corresponds to  $F + R - 1 = 0 \implies A = \ln 2$ .]

## 5. Scaling

Assuming uniform scaling and the validity of (28) and (38), DP size  $\propto L$  and  $M \propto L^3$ , so  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}} \propto L^{-11/2}$ ; and power density  $\propto \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}/L^3 \propto \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}/M \propto L^{-17/2}$ . Thus,  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}$  is maximized by minimizing system size, and  $\langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}/L^3 \propto \langle\langle P^* \rangle\rangle_{N+\frac{1}{2}}/M$  is maximized even more strongly by both minimizing system size and maximizing the number of systems operating in parallel [18]. Also, both power and power density scale as  $T^{5/2}$ . As per (29), maximizing power (power density) also maximizes the time rate of the associated total negentropy production (total negentropy production density).

In correction of a previous error [19],  $|T_+(V) - T_-(V)| = 4T|V|/3c$ , the magnitude of the temperature difference between the  $+X$  and  $-X$  disk faces corresponding to  $V = \pm|V|$  [as per (2), the two immediately following sentences, and the paragraph immediately thereafter], cannot be reduced via *diffraction* of EBR around the disk, not even if the disk's diameter and thickness are small (linear dimensions  $\lesssim hc/kT$ ) or even very small (linear dimensions  $\ll hc/kT$ ) compared with the wavelength of a typical EBR photon at temperature  $T$  ( $\approx hc/kT$ ) [20]. An EBR photon approaching the disk from, e.g., the  $+X$  direction cannot, say, be diffracted into a "U-turn" path, thence impinging on the disk from the  $-X$  direction: this requires (forbidden) backwards propagation of Huygens' wavelets [20b,20c] and violates conservation of momentum [20c]! (Diffraction can, of course, occur "around a corner", but *not* into a U-turn path [20b,20c].) *Diffraction* is thereby forbidden from reducing the opacity of a small (linear dimensions  $\lesssim hc/kT$ ) or even of a very small (linear dimensions  $\ll hc/kT$ ) disk and hence from degrading  $|T_+(V) - T_-(V)|$  [20b,20c].

Nevertheless, *diffraction* aside, a small (linear dimensions  $\lesssim hc/kT$ ) disk, and especially a very small (linear dimensions  $\ll hc/kT$ ) disk, typically *does* suffer from small opacity. For a typical very small (linear dimensions  $\ll hc/kT$ ) disk, the efficiency of absorption/(re)radiation of EBR *per unit of DP volume* (and hence, assuming uniform scaling, also *per unit of DP mass*) is *independent* of DP size [21] — thereby rendering a very thin (thickness  $\ll hc/kT$ ) disk

highly transparent. Said transparency degrades DP performance by (a) reduced probability of absorption/(re)radiation of any given EBR photon, and (b) rendering the pawl almost as likely to be impinged on by an EBR photon that *is* absorbed/(re)radiated emanating from the  $-X$  direction as by one emanating from the  $+X$  direction. Hence, typically, for such a pawl,  $T_+(V) = T \left(1 + \frac{2\gamma V}{3c}\right)$ , where  $0 < \gamma \ll 1$ , i.e.,  $|T_+(V) - T| = \frac{1}{2} |T_+(V) - T_-(V)|$  is seriously degraded in comparison with (2) — and thus DP performance *is also* seriously degraded. Since small DP size (and mass) *without* appreciable degradation of DP performance is necessary for significant — or even measurable — power and negentropy production densities, the question arises as to whether or not said degradation in small (linear dimensions  $\lesssim hc/kT$ ), and even very small (linear dimensions  $\ll hc/kT$ ), DPs can be overcome.

Perhaps, it can be overcome via a DP possessing one or more of the following *untypical* properties: (a) Overlapping resonances: If the DP is comprised of atoms and/or molecules whose resonances overlap to significantly “cover” the Planck spectrum corresponding to  $T$ , then the DP might be highly opaque even if it is very small (linear dimensions  $\ll hc/kT$ ) [22]. (b) An internal reflective shield: A nonreflective [purely absorptive/(re)radiative] nonresonant material cannot be both thin ( $\lesssim hc/kT$ ) and opaque to EBR corresponding to  $T$  [23]. But a reflective material (even if *nonresonant*) can be [24]. Therefore, a thin reflective (if *also* resonant, so much the better) midsection comprising the “center slab” of the disk separates its absorptive/(re)radiative  $+X$  and  $-X$  faces not only spatially, but — more importantly — thermally. [Of course, whether or not such a reflective shield is present, the  $+X$  and  $-X$  disk faces *themselves* must be absorptive/(re)radiative — not reflective: any purely reflective material obviously can never (re)thermalize!] (c) Alternatively, perhaps a nonrelativistic positive-rest-mass thermal background medium at temperature  $T$  might be made preponderant over the EBR [25], in which case  $c \rightarrow |U|$ , with  $|U|$  on the order of a typical thermal or sonic molecular speed in said medium, rather than the speed of light in vacuum — which yields the advantage, for any given DP size and mass, of  $|U| \ll c \implies |V|/|U| \gg |V|/c$  [25]. A further advantage obtains if DP size and mass given a nonrelativistic positive-rest-mass thermal background medium being preponderant over the EBR can be smaller than those given the EBR being the *sole* thermal background medium. (Even given a nonrelativistic positive-rest-mass thermal background medium being preponderant over the EBR, there seems to be no advantage in “excluding” the EBR corresponding to  $T$ : such “exclusion” begins to obtain if the DP is enclosed within, say, a conducting shell of diameter  $\lesssim hc/kT \approx$  wavelength of typical EBR photon at temperature  $T$ , and obtains *strongly* if said diameter  $\ll hc/kT$ .)

## 6. A digression concerning limited aspects of DP operation in the quantum regime

For brevity in notation in this Sect. 6, we first define, in the *classical* regime,

$$F \equiv \frac{1}{2}(\mu + \epsilon) \quad (42)$$

and

$$R \equiv \frac{1}{2}(\mu - \epsilon). \quad (43)$$

Note that (42) and (43) imply, and are implied by,

$$\mu \equiv F + R \tag{44}$$

and

$$\epsilon \equiv F - R. \tag{45}$$

Also, for simplicity, in this Sect. 6, we consider any *one* given  $\pm |V|$  pair, i.e., any *one* given  $|V|$ ; except for the last paragraph thereof, wherein averages over *all*  $\pm |V|$  pairs, i.e., over *all*  $|V|$ , are briefly mentioned.

Applying (44) and (45) in the *classical* regime, we can rewrite the last line of (10), (11), (15), (16), (17), and (18), respectively, as

$$P(\pm)_N = \frac{1}{2} \left\{ 1 \pm \frac{\epsilon[1 - (\mu - 1)^N]}{2 - \mu} \right\}, \tag{46}$$

$$\langle V \rangle_N = |V| \frac{\epsilon[1 - (\mu - 1)^N]}{2 - \mu}, \tag{47}$$

$$\langle V \rangle_{N+\frac{1}{2}} = |V| \frac{\epsilon[2 - \mu(\mu - 1)^N]}{2(2 - \mu)}, \tag{48}$$

$$\langle \Delta V \rangle_{N+\frac{1}{2}} = |V| \epsilon(\mu - 1)^N, \tag{49}$$

$$\langle f \rangle_{N+\frac{1}{2}} = \left( \frac{MV^2}{L} \right) \epsilon(\mu - 1)^N, \tag{50}$$

and

$$\langle P^* \rangle_{N+\frac{1}{2}} = \left( \frac{M|V|^3}{2L} \right) \frac{\epsilon^2(\mu - 1)^N [2 - \mu(\mu - 1)^N]}{2 - \mu}. \tag{51}$$

The  $\epsilon$ -,  $\mu$ -, and  $N$ -functionalities are mutually *independent*. At constant  $\mu$  and  $N$  [except for the trivial case  $N = 0$  in (46) and (47), corresponding to  $|P(\pm)_0 - \frac{1}{2}| = 0$  and to  $\langle V \rangle_0 = 0$ , respectively], all  $\langle Q \rangle_N$ , as per (46) – (51), are  $\propto \epsilon$  (except  $\langle P^* \rangle_{N+\frac{1}{2}} \propto \epsilon^2$ ).

We now explore limited aspects of DP operation in the *quantum* regime, considering the pawl's quantum-mechanical tunneling through [26a] (or quantum-mechanical — as opposed to classical — jumping over [26b]) pegs when it would classically bounce, and its quantum-mechanical reflection or bouncing from pegs when it would classically jump [26]. For simplicity, we assume in this Sect. 6 — as we do throughout this chapter — (in addition to our nonrelativistic assumptions as per the last sentence of the paragraph immediately following Fig. 1): (a) that  $m \langle |v| \rangle_{\text{mw}} (Z - Z_{\text{min}})_{\text{scale}} = m(2kT/\pi m)^{1/2}(kT/mg) = (2/\pi m)^{1/2}(kT)^{3/2}/g \approx (kT)^{3/2}/m^{1/2}g \gg h$ , so that the pawl's  $Z$ -directional thermal (Brownian) motion can still be treated classically; (b) that  $M \langle |V| \rangle_{\text{mw}} L = M(2kT/\pi M)^{1/2}L = (2kTM/\pi)^{1/2}L \approx (kTM)^{1/2}L \gg h$ , so that the DP's  $X$ -directional thermal (Brownian) motion can still be treated classically; and (c) that the combined pawl-plus-peg  $X$ -directional thickness is  $\ll L$ , so that quantum-mechanically (as well as classically)  $\Delta t$  is only negligibly affected thereby.

*Classically*, given DP Brownian motion at  $V = +|V|$ , the pawl's altitude at pawl-peg interaction is  $Z > H$  ( $Z_{\text{min}} \leq Z < H$ ) with probability  $F$  ( $1 - F$ ), corresponding — with *certainty* — to the interaction being a jump (bounce). By contrast, in the *quantum* regime,

the pawl can, with *nonzero* probability, tunnel through [26a] (or quantum-mechanically — as opposed to classically — jump over [26b]) a peg if  $Z_{\min} \leq Z < H$ , and reflect or bounce from a peg if  $Z > H$  [26]. Within approximations (a) and (b) as per the immediately preceding paragraph, quantum-mechanically — as classically — given DP Brownian motion at  $V = +|V|$ , the pawl's altitude at pawl-peg interaction is  $Z > H$  ( $Z_{\min} \leq Z < H$ ) with probability  $F$  ( $1 - F$ ). But, quantum-mechanically [26]: (i) if  $Z_{\min} \leq Z < H$ , then the pawl will jump with probability  $(1 - F)\tau_+$  and bounce with probability  $(1 - F)(1 - \tau_+)$ , where  $\tau(+|V|) \equiv \tau_+$  is the quantum-mechanical probability of tunneling given  $V = +|V|$ , integrated over the range  $Z_{\min} \leq Z < H$ . And, (ii) if  $Z > H$ , then the pawl will jump with probability  $F(1 - \rho_+)$  and bounce with probability  $F\rho_+$ , where  $\rho(+|V|) \equiv \rho_+$  is the quantum-mechanical probability of reflection or bouncing given  $V = +|V|$ , integrated over the range  $Z > H$ . This paragraph obviously also obtains given DP Brownian motion at  $V = -|V|$ , as per the substitutions  $V = +|V| \rightarrow V = -|V|$ ,  $F \rightarrow R$ ,  $\tau(+|V|) \equiv \tau_+ \rightarrow \tau(-|V|) \equiv \tau_-$ , and  $\rho(+|V|) \equiv \rho_+ \rightarrow \rho(-|V|) \equiv \rho_-$ . Explicitly, applying (3) – (6) with  $H_{\text{net}} \equiv H - Z_{\min} \rightarrow Z - Z_{\min}$ , to first order in  $|V|/c$ ,

$$\begin{aligned} \tau_{\pm} \equiv \tau(\pm|V|) &= \langle \tau(\pm|V|, Z) \rangle = \frac{\int_{Z_{\min}}^H \tau_{\pm}(Z) P(Z|V = \pm|V|) dZ}{\int_{Z_{\min}}^H P(Z|V = \pm|V|) dZ} \\ &= \frac{\int_{Z_{\min}}^H \tau_{\pm}(Z) \left(1 \pm \frac{2mg(Z - Z_{\min})|V|}{3kTc}\right) e^{-mg(Z - Z_{\min})/kT} dZ}{\int_{Z_{\min}}^H \left(1 \pm \frac{2mg(Z - Z_{\min})|V|}{3kTc}\right) e^{-mg(Z - Z_{\min})/kT} dZ} \end{aligned} \quad (52)$$

and

$$\begin{aligned} \rho_{\pm} \equiv \rho(\pm|V|) &= \langle \rho(\pm|V|, Z) \rangle = \frac{\int_H^{\infty} \rho_{\pm}(Z) P(Z|V = \pm|V|) dZ}{\int_H^{\infty} P(Z|V = \pm|V|) dZ} \\ &= \frac{\int_H^{\infty} \rho_{\pm}(Z) \left(1 \pm \frac{2mg(Z - Z_{\min})|V|}{3kTc}\right) e^{-mg(Z - Z_{\min})/kT} dZ}{\int_H^{\infty} \left(1 \pm \frac{2mg(Z - Z_{\min})|V|}{3kTc}\right) e^{-mg(Z - Z_{\min})/kT} dZ}. \end{aligned} \quad (53)$$

Applying (52) qualitatively [26], for any  $|V| > 0$ ,  $\tau_+ > \tau_-$ , because, given  $Z_{\min} \leq Z < H$ ,  $Z$  is [as per (3) – (6) with  $H_{\text{net}} \equiv H - Z_{\min} \rightarrow Z - Z_{\min}$ ] more probably closer to  $H$  if  $V = +|V|$  than if  $V = -|V|$ . Similarly, applying (53) qualitatively [26], for any  $|V| > 0$ ,  $\rho_- > \rho_+$ , because, given  $Z > H$ ,  $Z$  is [as per (3) – (6) with  $H_{\text{net}} \equiv H - Z_{\min} \rightarrow Z - Z_{\min}$ ] more probably closer to  $H$  if  $V = -|V|$  than if  $V = +|V|$ .

Thus, letting the subscript “q” denote “quantum-mechanical”, we have {as the quantum-mechanical analog of (5), (9a), and (42) [within said approximations (a) and (b)]}, for the *overall* probability, integrated over *all*  $Z \geq Z_{\min}$ , given DP Brownian motion at  $V = +|V|$ , of a pawl-over-peg jump in the  $+X$  direction,

$$F_q = F(1 - \rho_+) + (1 - F)\tau_+. \quad (54)$$

Similarly, {as the quantum-mechanical analog of (6), (9b), and (43) [within said approximations (a) and (b)]}, the *overall* probability, integrated over *all*  $Z \geq Z_{\min}$ , simply via DP Brownian motion at  $V = -|V|$ , of a pawl-over-peg jump in the  $-X$  direction is

$$R_q = R(1 - \rho_-) + (1 - R)\tau_- \quad (55)$$

All *classical* results in this chapter are modified in the *quantum* regime — within said approximations (a) and (b) — simply via the substitutions  $F \rightarrow F_q$  and  $R \rightarrow R_q$ . Also, note that the correspondence principle is obeyed: *purely* classical behavior is recovered in the limits  $\tau_+ \rightarrow 0$ ,  $\tau_- \rightarrow 0$ ,  $\rho_- \rightarrow 0$ , and  $\rho_+ \rightarrow 0$ .

For still greater brevity in notation, we define, for the remainder of this Sect. 6,

$$\Sigma\tau \equiv \tau_+ + \tau_-, \quad (56)$$

$$\Sigma\rho \equiv \rho_- + \rho_+, \quad (57)$$

$$\Delta\tau \equiv \tau_+ - \tau_- > 0, \quad (58)$$

and

$$\Delta\rho \equiv \rho_- - \rho_+ > 0. \quad (59)$$

The inequalities in (58) and (59) are justified by the two sentences immediately following (53). Applying (42) – (45) and (54) – (59), we have, as the quantum-mechanical analogs — within said approximations (a) and (b) — of (44) and (45), respectively,

$$\begin{aligned} \mu_q &\equiv F_q + R_q \\ &= F(1 - \rho_+) + (1 - F)\tau_+ + R(1 - \rho_-) + (1 - R)\tau_- \\ &= F + R - F(\rho_+ + \tau_+) - R(\rho_- + \tau_-) + \tau_+ + \tau_- \\ &= \mu - \frac{1}{2}(\mu + \epsilon)(\rho_+ + \tau_+) - \frac{1}{2}(\mu - \epsilon)(\rho_- + \tau_-) + \tau_+ + \tau_- \\ &= \mu - \frac{\mu}{2}(\Sigma\rho + \Sigma\tau) + \frac{\epsilon}{2}(\Delta\rho - \Delta\tau) + \Sigma\tau \end{aligned} \quad (60)$$

and

$$\begin{aligned} \epsilon_q &\equiv F_q - R_q \\ &= F(1 - \rho_+) + (1 - F)\tau_+ - [R(1 - \rho_-) + (1 - R)\tau_-] \\ &= F - R - F(\rho_+ + \tau_+) + R(\rho_- + \tau_-) + \tau_+ - \tau_- \\ &= \epsilon - \frac{1}{2}(\mu + \epsilon)(\rho_+ + \tau_+) + \frac{1}{2}(\mu - \epsilon)(\rho_- + \tau_-) + \tau_+ - \tau_- \\ &= \epsilon + \frac{\mu}{2}(\Delta\rho - \Delta\tau) - \frac{\epsilon}{2}(\Sigma\rho + \Sigma\tau) + \Delta\tau. \end{aligned} \quad (61)$$

As per the sentence containing (46) and the two sentences immediately following (55), the *classical* (46) – (51) are modified in the *quantum* regime — within said approximations (a) and (b) — simply via the substitutions  $\epsilon \rightarrow \epsilon_q$  and  $\mu \rightarrow \mu_q$ .

We now consider the simplest nontrivial special case, which obtains at  $N = 1$  in (46) and (47), and at the  $N = 0 \rightarrow N = 1$  transition in (48) – (51). [The trivial case is:  $N = 0$  in (46) and (47): recall the second sentence following (51).] In this simplest nontrivial special case,  $\epsilon$ -dependence alone obtains [recall the paragraph immediately following (51)]. Can  $\epsilon_q > \epsilon$

obtain, i.e., can a quantum DP [within said approximations (a) and (b)] outperform a classical DP in this simplest nontrivial special case? Applying (45) and (61) yields

$$\begin{aligned}\epsilon_q > \epsilon &\implies \frac{\mu}{2}(\Delta\rho - \Delta\tau) - \frac{\epsilon}{2}(\Sigma\rho + \Sigma\tau) + \Delta\tau > 0 \\ &\implies \frac{\epsilon(\Sigma\rho + \Sigma\tau)}{\Delta\tau} + \mu \left(1 - \frac{\Delta\rho}{\Delta\tau}\right) < 2.\end{aligned}\quad (62)$$

Owing to algebraic difficulty, it is unclear whether or not (62) can be fulfilled for any physically realistic values of quantities appearing therein, let alone whether or not  $\epsilon_{q,\max} > \epsilon_{\max}$ . The second-simplest nontrivial special case — entailing the (mutually independent)  $\epsilon$ - and  $\mu$ -functionalities — obtains in the limit  $N \rightarrow \infty$  (within “practical” limits as per the fifth paragraph of Sect. 4) in (46) – (48). Applying (44), (45), (60), and (61) yields, in this second-simplest nontrivial special case, as the requirement for a quantum DP [within said approximations (a) and (b)] to outperform a classical DP,

$$\begin{aligned}\frac{\epsilon_q}{2 - \mu_q} &> \frac{\epsilon}{2 - \mu} \\ &\implies \frac{\epsilon + \frac{\mu}{2}(\Delta\rho - \Delta\tau) - \frac{\epsilon}{2}(\Sigma\rho + \Sigma\tau) + \Delta\tau}{2 - [\mu - \frac{\mu}{2}(\Sigma\rho + \Sigma\tau) + \frac{\epsilon}{2}(\Delta\rho - \Delta\tau) + \Sigma\tau]} > \frac{\epsilon}{2 - \mu} \\ &\implies (2 - \mu) \left[\epsilon + \frac{\mu}{2}(\Delta\rho - \Delta\tau) - \frac{\epsilon}{2}(\Sigma\rho + \Sigma\tau) + \Delta\tau\right] \\ &> \epsilon \left\{2 - \left[\mu - \frac{\mu}{2}(\Sigma\rho + \Sigma\tau) + \frac{\epsilon}{2}(\Delta\rho - \Delta\tau) + \Sigma\tau\right]\right\} \\ &\implies \left[\mu - \frac{1}{2}(\mu^2 - \epsilon^2)\right] (\Delta\rho - \Delta\tau) + (2 - \mu)\Delta\tau > \epsilon\Sigma\rho \\ &\implies \mu \left(1 - \frac{1}{2}\mu\right) (\Delta\rho - \Delta\tau) + (2 - \mu)\Delta\tau > \epsilon\Sigma\rho.\end{aligned}\quad (63)$$

The last step of (63) is justified because [given the *specific*  $F$  and  $R$  as per the rightmost terms of (5) and (6), respectively, and applying (42) – (45)]  $\epsilon^2 \ll \mu^2$ . It is even less clear — owing to greater algebraic difficulty — whether or not (63) can be fulfilled for any physically realistic values of quantities appearing therein, let alone whether or not  $[\epsilon_q/(2 - \mu_q)]_{\max} > [\epsilon/(2 - \mu)]_{\max}$  can obtain.

Owing to still greater algebraic difficulty, we will not specifically consider the completely general case — entailing all three (mutually independent)  $\epsilon$ -,  $\mu$ -, and  $N$ -functionalities.

By averaging over *all*  $\pm|V|$  pairs, i.e., over *all*  $|V|$  — similarly as for our classical DP [as per the third paragraph and last four paragraphs of Sect. 2, and in light of the two sentences immediately following (55) and that immediately following (61)] — *overall* quantum DP behavior [within said approximations (a) and (b)] can be similarly derived.

## 7. Conclusion

In the original classic “Ratchet and Pawl” chapter [4], Feynman’s upshot concerning the “Ratchet and Pawl” elucidates the Zhang [1] formulation of the second law:

“In spite of all our cleverness of lopsided design, if the two temperatures are exactly equal there is no more propensity to turn one way than the other. The moment we

look at it, it may be turning one way or the other, but in the long run it gets nowhere. The fact that it gets nowhere is really the fundamental deep principle on which all of thermodynamics is based.”

Feynman’s ratchet (which manifests *only non-velocity-dependent* fluctuations) can operate — albeit *not* in violation of the second law — *if* negentropy (and hence free energy) is supplied thereto, i.e., at *non-TEQ*; e.g., (a) if the two temperatures are unequal, or (b) if the pawl — assumed to have a sufficiently short thermal response time — is heated (cooled) whenever the wheel turns in the Forward (Reverse) direction.

But Feynman’s classic ratchet and pawl manifests asymmetry, i.e., lopsidedness, *only geometrically*, i.e., *only in coordinate space alone*. By contrast, *velocity-dependent* fluctuations — which *spontaneously* break the randomness of our DP’s Brownian motion at TEQ — manifest asymmetry, i.e., lopsidedness, in *velocity or momentum space*, hence allowing operation *despite TEQ*: Velocity-dependent fluctuations execute procedure (b) as stated in the immediately preceding paragraph, but *spontaneously*, i.e., *without cost* in negentropy (and hence in free energy). Of course, our DP is *also* asymmetrical geometrically: geometrical (coordinate-space) asymmetry plays an auxiliary yet necessary role in the case of our DP — and perhaps in general — to momentum-space-asymmetrical velocity-dependence of fluctuations in the spontaneous breaking of the randomness of Brownian motion at TEQ. But geometrical (coordinate-space) asymmetry *alone* does not *imply* the velocity- or momentum-space asymmetry that seems to be the *central* requirement for this spontaneous randomness-breaking.

If the impedance to Brownian-motional velocity  $V$  is thus an asymmetrical function of  $V$  *itself*, then can the randomness of  $V$  be spontaneously broken, thereby challenging the Zhang [1] formulation — and hence, as per the last four paragraphs of Sect. 3, *all* formulations — of the second law?

The correspondence principle requires that the following necessary — albeit, of course, not sufficient — condition must be satisfied by any *valid* new and/or generalized scientific theory: the new and/or generalized theory must reduce to any more restricted theory that is a special case thereof within the more restricted theory’s (narrower) range of validity. Our theory challenging the second law via *velocity-dependence* of fluctuations *is* in accordance with the correspondence principle: it predicts that the second law and all results based thereon remain inviolate if *velocity-dependence* of fluctuations vanishes.

Of course, the five immediately preceding paragraphs do *not* preclude *unrelated* challenges to the second law that are *not* based on velocity-dependence of fluctuations [5–10]. Also, obviously, the five immediately preceding paragraphs do *not* imply that *every* velocity-dependent effect can spontaneously break the randomness of Brownian motion at TEQ — challenging the second law. For example, it seems unlikely that the Lorentz force — and hence, by extension, *any nondissipative* velocity-dependent force that acts perpendicularly to the velocity *itself* (i.e., to the direction of motion), and hence which can do no work — can thus challenge the second law [27]. [Note: The Zhang formulation of the second law, enunciated in Ref. [1a] (and restated in the first two paragraphs of Sect. 1, with further discussions in the last four paragraphs of Sect. 3, of this present chapter), is valid irrespective of whether or not (*nondissipative*) velocity-dependent forces acting perpendicularly to the velocity itself (i.e., to the direction of motion), and hence which can do no work — such as the Lorentz force — can challenge the second law.] Also, of course,



(in general, velocity-dependent) *dissipative* frictional forces do *not* challenge the second law — rather, friction (whether velocity-dependent or not) *manifests* the second law.

Questions that have not been addressed or answered in this present chapter imply the following: (i) resolving the challenge to the second law (pro or con) posed by our classical velocity-dependent DP model, i.e., posed by our DP per se [as developed initially in Ref. [3k] and more quantitatively both in this present chapter and in previous shorter versions [28] thereof], and posed by possible classical generalizations of our DP model; (ii) investigating alternative derivations relevant to our DP model; (iii) more thorough study of quantum-mechanical velocity-dependent models; (iv) possible generalization of source(s) of velocity-dependence — in both the classical and quantum regimes — to other than the Doppler effect, if such source(s) exist; (v) investigating whether or not geometrical asymmetry is *always* an auxiliary requirement to velocity-dependence of fluctuations for our challenge to the second law, and the issue of auxiliary requirement(s) in general (in both the classical and quantum regimes); (vi) understanding which manifestation(s) of velocity-dependence can or cannot challenge the second law in both the classical and quantum regimes — recall the last four sentences of the immediately preceding paragraph; and (vii) investigating relationship(s) to other (classical and quantum) challenges to the second law [5–10], including a search for unifying principle(s) behind challenges thereto — whether based on velocity-dependence of fluctuations or not (recall the immediately preceding paragraph).

Perhaps, in passing, it might be noted that there have also been challenges — albeit *unrelated* to this present chapter — to the first [29] and third [30] laws of thermodynamics.

## 8. Acknowledgments

Dr. Donald H. Kobe is gratefully acknowledged for many very thoughtful and extensive discussions and correspondences, hence contributing many improvements, directly concerning this present chapter and concerning the expoundment of previous shorter versions [28] thereof, as well as concerning ideas initially discussed in Ref. [3k] and in still earlier work [31] that are developed more quantitatively in this present chapter and in said shorter versions [28]. I am very thankful to Dr. Daniel P. Sheehan for insightful and detailed correspondences, including both a draft and the final version of relevant material from his new book [32], that provided an independent viewpoint concerning Refs. [3k] and [28] and hence helped to inspire new ideas for this present chapter (especially for Sect. 5); to Dr. Alexey V. Nikulov for interesting and insightful correspondences concerning the Zhang [1] formulation of the second law and various challenges thereto; to Dr. Paolo Grigolini for thought-provoking background discussions (especially those concerning temperature fluctuations and superstatistics [12]) and for helpful supplementary discussions (especially those concerning style and mathematical notation); to Dr. James H. Cooke for thoughtful and interesting discussions, correspondences, and constructive criticisms concerning Ref. [3k] but also pertinent for this present chapter; to Dr. Marlan O. Scully and to Dr. Th. M. Nieuwenhuizen for perceptive discussions concerning the second law, especially in the quantum regime; to Dr. Jacek M. Kowalski for interesting discussions concerning Markov processes; and to Dr. Richard McFee for helpful correspondences and discussions concerning Ref. [7a], and the second law in general; and to Dr. Bright Lowry, who first introduced me to Feynman's ratchet and pawl in 1972 during insightful discussions concerning the second law of thermodynamics. Grateful acknowledgments are expressed to Dr. Kurt W. Hess, Dr. Stan

Czamanski, Mr. S. Mort Zimmerman, and Mr. Robert H. Shelton for many interesting and thoughtful discussions, as well as numerous helpful suggestions and correspondences, both directly concerning this present chapter and also concerning background ideas pertaining thereto that were originally developed in Ref. [3k] and in still earlier work [31]. Additionally, thanks are expressed to Dr. Bruce N. Miller, Dr. Wolfgang Rindler, Dr. Roland E. Allen, Dr. Bruno J. Zwolinski, Dr. G. R. Somayajulu, Dr. Abraham Clearfield, Dr. Patricia H. Reiff, Dr. Paul Sheldon, Dr. Mauro Bologna, Dr. Baris Bagci, and Dr. Nolan Massey for helpful background discussions. I thank Mr. Charles M. Brown for informing me of Refs. [27e] and [27f], and (see Ref. [27g]) of a talk concerning therewith at the PQE-2006 conference (which was followed by another talk concerning therewith at the PQE-2007 conference); as well as for updating me concerning his own efforts in diode self-rectification. Finally, Dr. A. N. Grigorenko is gratefully acknowledged for very comprehensive, thoughtful, and interesting correspondences, and for constructive criticisms, concerning Ref. [3k] and related ideas.

### 9. List of symbols

thermodynamic equilibrium	TEQ
Cartesian coordinates in right-handed Cartesian coordinate system in 3-dimensional Euclidean space	
right-left directional axis, positive to right	X
into-page/out-of-page directional axis, positive into page	Y
vertical directional axis, positive upwards	Z
(Z-directional) height of pegs	H
Z is also used for the altitude of the pawl, which is constrained from below at $Z_{\min}$ . Thus, the net height of the pegs is $H_{\text{net}} \equiv Z - Z_{\min}$ .	
X-directional spatial separation between adjacent pegs	L
disk-and-pawl system	DP
mass of disk	$m'$
mass of pawl	$m$
mass of disk-and-pawl system	$M = m' + m$
X-directional velocity of disk-and-pawl system (DP)	V
gravitational acceleration	g
speed of light	c
Kelvin temperature	T
Boltzmann's constant	k
equilibrium blackbody radiation	EBR
dimensionless gravitational potential energy parameter of pawl corresponding to it just barely clearing the pegs	$A \equiv mgH_{\text{net}}/kT$
angle of impingement, away from the normal or perpendicular, of a given EBR photon on the right disk face (which includes the pawl)	$\alpha$
Kelvin temperature of EBR impinging on the right disk face, including the pawl	$T_+$
Kelvin temperature of EBR impinging on the left disk face	$T_-$

probability	$P$
probability of the pawl clearing a peg when the DP is moving in the forward or $+X$ direction	$F$
probability of the pawl clearing a peg when the DP is moving in the reverse or $-X$ direction	$R$
time	$t$
time interval between consecutive pawl-peg interactions (which could be either jumps or bounces)	$\Delta t$
time step in discrete Markovian time evolution (discrete time)	$N$
change in $V$ between consecutive pawl-peg interactions (which could be either jumps or bounces)	$\Delta V$
force exerted on DP by pegs	$f$
power output of DP	$P^*$
general quantity	$Q$
entropy	$S$
angle of incline (upwards towards the right, i.e., towards the $+X$ direction)	$\theta$
Planck's constant	$h$
speed of gas molecules surrounding DP if DP is not (as we usually assume) surrounded only by EBR in vacuum	$U$
$\mu \equiv F + R$	
$\epsilon \equiv F - R$	
quantum-mechanical probability of the pawl tunneling through a peg when it is not high enough to clear a peg classically, i.e., when $Z_{\min} \leq Z < H$	$\tau$
quantum-mechanical probability of the pawl bouncing from a peg when it is high enough to clear a peg classically, i.e., when $Z > H$	$\rho$

#### List of subscripts

minimum	min
net	net
maximum	max
optimum	opt
Maxwellian	mw
quantum	q

## 10. References

1. (a) Zhang, K.; Zhang, K. Mechanical models of Maxwell's demon with noninvariant phase volume. *Phys. Rev. A* 1992, 46, 4598–4605. [Reference [1a] is a classical rather than quantum-mechanical treatment; as is true of this present chapter (except for the last four paragraphs of Sect. 3, Sect. 6, and a few very brief mentions elsewhere, therein), and also of Refs. [3k], [28a], and [28b]; but, in contrast therewith, Ref. [1a] considers only (*nondissipative*) velocity-dependent forces that are perpendicular to the velocity

*itself* (i.e., to the direction of motion), and hence which can do no work (e.g., the Lorentz force).]

Reference [1a] was partially anticipated earlier by

- (b) Bridgman, P. W. Note on the Principle of Detailed Balancing. *Phys. Rev.* 1928, 31, 101–102; and, in extension, by
- (c) Davies, P. C. W. *The Physics of Time Asymmetry*. University of California Press: Berkeley, 1974; Sect. 6.4 (especially the last paragraph).

Reference [1a] is summarized in

- (d) Čápek, V.; Sheehan, D. P. *Challenges to the Second Law of Thermodynamics: Theory and Experiment*. Springer: Dordrecht, 2005 (Fundamental Theories of Physics Series, Vol. 146), Sect. 10.1.2.

- 2. Allahverdyan, A. E.; Nieuwenhuizen, Th. M. A mathematical theorem as the basis for the second law: Thomson's formulation applied to equilibrium. *Physica A* 2002, 305, 542–552. See the first and fifth paragraphs of Sect. 5.

Note: There is no contradiction between Ref. [1a], which forbids spontaneous momentum flow at TEQ, and Ref. [2], which allows persistent currents at TEQ. Persistent currents such as those allowed by Ref. 2 at TEQ are at *zero* resistance and hence do *not* qualify as spontaneous momentum flows, which must self-maintain *despite both* TEQ *and nonzero* (positive) resistance. An example of a persistent current such as is allowed by, and mentioned in, Ref. [2] is a quantum vortex, which exists in a quantum-mechanical ground state from which no energy can be extracted either dissipatively as heat or nondissipatively as work.

- 3. (a) Feynman's classic ratchet and pawl is discussed in Feynman, R. P.; Leighton, R. B.; Sands, M. *The Feynman Lectures on Physics: Definitive Edition*. Addison Wesley: Reading, Mass, 1963 (Pearson Addison-Wesley: San Francisco, Calif., 2006); Vol. I, Chap. 46.

More recent analyses of Feynman's classic system are developed in, for example:

- (b) Parrondo, J. M. R.; Español, P. Criticism of Feynman's analysis of the ratchet as an engine. *Am. J. Phys.* 1996, 64, 1125–1130.
- (c) Magnasco, M. O.; Stolovitzky, G. Feynman's Ratchet and Pawl. *J. Stat. Phys.* 1998, 93, 615–632.
- (d) Jarzynski, C.; Mazonka, O. Feynman's ratchet and pawl: An exactly solvable model. *Phys. Rev. E* 1999, 59, 6448–6459.
- (e) Sokolov, I. M. Reversible fluctuation rectifier. *Phys Rev. E* 1999, 60, 4446–4449.
- (f) Velasco, S.; Roco, J. M. M.; Medina, A.; Hernández, A. C. Feynman's ratchet optimization: maximum power and maximum efficiency regimes. *J. Phys. D: Appl. Phys.* 2001, 34, 1000–1006.
- (g) Pesz, K.; Gabrys, B. J.; Bartkiewicz, S. J. Analytical solution for the Feynman ratchet. *Phys. Rev. E* 2002, 66, 061103.
- (h) Munakata, T.; Suzuki, D. Rectification efficiency of the Feynman ratchet. *J. Phys. Soc. Japan* 2005, 74, 550–553.
- (i) Suzuki, D.; Munakata, T. Stationary probability flow and vortices for the Smoluchowski-Feynman ratchet. *Physica A* 2006, 368, 16–24.
- (j) Tu, Z. C. Efficiency at maximum power of Feynman's ratchet as a heat engine. *J. Phys. A - Math. Theor.* 2008, 41, Article Number 312003.
- (k) For a survey (as of 1989) anticipating our velocity-dependent DP, see Denur, J. Velocity-dependent fluctuations: Breaking the randomness of Brownian motion.

*Phys. Rev. A* 1989, 40, 5390–5399. Erratum. *Phys. Rev. A* 1990, 41, 3390. See Sect. I and Footnotes 1–5.

4. Reference [3a], the last paragraph of Sect. 46-2: This is the upshot of the discussions concerning Feynman's classic ratchet and pawl in Sect. 46-1 and 46-2. Said upshot is qualitatively justified on bases of (i) mechanics in Sect. 46-3 and (ii) special initial conditions in Sects. 46-4 and 46-5.
5. (a) Investigations of various formulations of the second law, various viewpoints concerning the absolute validity thereof, and challenges thereto in both the quantum and classical regimes, are given in Sheehan, D. P., Editor. *First International Conference on Quantum Limits to the Second Law, AIP Conference Proceedings (Vol. 643)*. AIP: Melville, New York, 2002. Various authors cited in Footnotes [6] – [10] also have papers published in this *AIP Conference Proceedings (Vol. 643)*. Specific topics and/or models are studied in Ref. [3k] and in the references cited in Footnotes [6] – [11] and [27]; with Refs. [31] and [3k] being precursors, and Refs. [28a] and [28b] being shorter versions, of this present chapter.

For more recent investigations concerning the topics mentioned in Ref. [5a], and related topics, see also the following book (Ref. [5b]) and three Special Issues of journals (Refs. [5c] – [5e]):

- (b) Čápek, V.; Sheehan, D. P. *Challenges to the Second Law of Thermodynamics: Theory and Experiment*. Springer: Dordrecht, 2005 (Fundamental Theories of Physics Series, Vol. 146)
  - (c) Nikulov, A. V.; Sheehan, D. P. (Guest Editors). *Entropy* March 2004, 6 (Issue 1). Special Issue on Quantum Limits to the Second Law of Thermodynamics.
  - (d) Nieuwenhuizen, Th. M.; Keefe, D. P.; Špička, V. (Guest Editors). Proceedings of the International Conference Frontiers of Quantum and Mesoscopic Thermodynamics FQMT'04 (Prague, Czech Republic, 26–29 July 2004). *Physica E* October 2005, 29 (Issues 1–2).
  - (e) Sheehan, D. P. (Guest Editor). *Found. Phys.* December 2007, 37 (Number 12). Special Issue: The Second Law of Thermodynamics: Foundations and Status.
  - (f) To be published: Sheehan, D. P., Editor. *The Second Law of Thermodynamics: Status and Challenges*. AIP Conference Proceedings (Volume number to be determined). AIP: Melville, New York, 2011.
6. Viewpoints concerning the second law in systems manifesting quantum-mechanical entanglement and/or coherence range from (I) that it can be violated: for example:
    - (a) Čápek, V.; Bok, J. Violation of the second law of thermodynamics in the quantum microworld. *Physica A* 2001, 290, 379–401.
    - (b) Čápek, V.; Frege, O. Violation of the 2nd law of thermodynamics in the quantum microworld. *Czech. J. Phys.* 2002, 52, 679–694.
    - (c) Čápek, V. Zeroth and second laws of thermodynamics simultaneously questioned in the quantum microworld. *Eur. Phys. J. B* 2002, 25, 101–113.
    - (d) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Breakdown of the Landauer bound for information erasure in the quantum regime. *Phys. Rev. E* 2001, 64, 056117.
    - (e) Nieuwenhuizen, Th. M.; Allahverdyan, A. E. Statistical thermodynamics of quantum Brownian motion: Construction of perpetual mobile of the second kind. *Phys. Rev. E* 2002, 66, 036102.

- (f) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Bath-generated work extraction and inversion-free gain in two-level systems. *J. Phys. A Math. Gen.* 2003, *36*, 875–882.
- (g) Čápek, V. Dimer as a challenge to the second law. *Eur. Phys. J. B* 2003, *34*, 219–223. Note: Reference [6g] was Dr. V. Čápek's last paper before he passed away. Therein, Dr. V. Čápek responds to Ref. [6l], and Dr. Daniel P. Sheehan has written a tribute to Dr. V. Čápek. There is also a tribute to Dr. V. Čápek in Ref. [5d]: Špička, V.; Nieuwenhuizen, Th. M.; Keefe, D. P.; Špička, V. (Guest Editors). In memoriam: Vladislav Čápek (1943–2002). Ref. [5d], vii–viii.
- (h) Nieuwenhuizen, Th. M. Thermodynamics and small quantum systems. *J. Mod. Optics* 2003, *50*, 2433–2441.
- (i) Reference [5b]; Chap. 3, Sect. 4.6.  
to (II) that it cannot: for example:
- (j) Scully, M. O. Extracting Work from a Single Thermal Bath via Quantum Negentropy. *Phys. Rev. Lett.* 2001, *87*, 220601.
- (k) Hoffmann, K. H. Quantum thermodynamics. *Ann. der Phys.* 2001, *10*, 79–88.
- (l) Novotný, T. Investigation of apparent violation of the second law of thermodynamics in quantum transport studies. *Europhys. Lett.* 2002, *59*, 648–654.
- (m) Linke, H. Coherent Power Booster. *Science* 2003, *299*, 841–842.
- (n) Scully, M. O.; Zubairy, M. S.; Agarwal, G. S.; Walther, H. Extracting Work from a Single Heat Bath via Vanishing Quantum Coherence. *Science* 2003, *299*, 862–864.
- (o) Scully, M. O.; Rostovtsev, Y.; Saryanni, Z-E.; and Zubairy, M. S. Using quantum erasure to exorcize Maxwell's demon: I. Concepts and context. Ref. [5d], 29–39.
- (p) Rostovtsev, Y.; Saryanni, Z-E.; Zubairy, M. S.; and Scully, M. O. Using quantum erasure to exorcize Maxwell's demon: II. Analysis. Ref. [5d], 40–46.
- (q) Saryanni, Z-E.; Rostovtsev, Y.; Zubairy, M. S.; and Scully, M. O. Using quantum erasure to exorcize Maxwell's demon: III. Implementation. Ref. [5d], 40–46.
- (r) A qualitative summary was given in Weiss, P. Breaking the Law. *Science News* Oct. 7, 2000 *158* (No. 15), pp. 234, 235, and 239.
- For recent studies concerning the second law in the quantum regime, see, for example:
- (s) Allahverdyan, A. E.; Balian, R; Nieuwenhuizen, Th. M. Maximum work extraction from finite quantum systems. *Europhys. Lett.* 2004, *67*, 565–571.
- (t) Allahverdyan, A. E.; Balian, R; Nieuwenhuizen, Th. M. Quantum thermodynamics: thermodynamics at the nanoscale. *Journal of Modern Optics* 2004, *51*, 2703–2711.
- (u) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Fluctuations of work from quantum subensembles: The case against quantum work-fluctuation theorems. *Phys. Rev. E* 2005, *71*, 066102.
- (v) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Minimal work principle: Proof and counterexamples. *Phys. Rev. E* 2005, *71*, 046107.
- (w) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Adiabatic processes need not correspond to optimal work. Ref. [5d], 74–81.
- (x) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Explanation of the Gibbs paradox within the framework of quantum thermodynamics. *Phys. Rev. E* 2006, *73*, 066119.
- General overviews of the second law in the quantum regime are given in, for example:
- (y) Reference [5d], 1–28.
- (z) Schröder, H.; Teifel, J.; Mahler, G. Work and work fluctuations in quantum systems. *Eur. Phys. J. Special Topics* 2007, *151*, 181–188.

(aa) Susskind, L.; Lindesay, J. *An Introduction to Black Holes, Information, and the String Theory Revolution: The Holographic Universe*. World Scientific: Singapore, 2005.

(bb) Scully, R. F. and Scully, M. O.: *The Demon and the Quantum*, Wiley-VCH: New York, 2007; especially Sects. 6.6 – 6.9 and Endnotes for Sects. 6.6 – 6.9.

The various formulations of the second law are not all of equal generality:

(cc) Various formulations of the second law are discussed in, for example: Ref. [5b], Sect. 1.2; and Ref. [6t].

(dd) Violations of *a* second law can fall short of violations of *the* second law — as per, for example: Ref. [6t], p. 2707; and Ref. [6y], the third paragraph of Sect. 1.1. According to Dr. Marlan O. Scully, private communications at the *36th Winter Colloquium on the Physics of Quantum Electronics (PQE-2006)*, January 2–6, 2006 (see Refs. [6j], [6n], [6o], [6p], and [6q]), by the time of said *Colloquium*, viewpoints seemed to be converging towards the conclusion that the work required to prepare systems with quantum-mechanical entanglement, correlations, and/or coherence that can then, say, surpass the (theoretical-maximum *classical*) Carnot efficiency, equals or exceeds the extra-Carnot-efficiency work that can thereby be obtained. (This was confirmed in later private communications from Dr. Marlan O. Scully.): Thus, the extra-Carnot-efficiency work *per se* violates *a* second law [specifically, as per Ref. [5b], pp. 4–5, the Carnot theorem formulation (and possibly also the Efficiency and/or Heat Engines formulations) of the second law], but the *overall* process — *including the preparation work* — does *not* violate *the* second law [i.e., the Zhang formulation of the second law, enunciated in Ref. [1a] (and restated in the first two paragraphs of Sect. 1, with further discussions in the last four paragraphs of Sect. 3, of this present chapter)]. Some questions concerning the validity of *the* second law — not merely *a* second law — in the quantum regime (e.g., for nonlinear systems coupled to a bath) which had been open have now been resolved in favor of the second law: See, for example: Kim, I; Mahler, G. Quantum Brownian motion and the second law of thermodynamics. *Eur. Phys. J. B* 2006, *54*, 405–414. Erratum, *Eur. Phys. J. B* 2007, *56*, 279. Also, Kim, I.; Mahler, G. “The second law of thermodynamics in the quantum Brownian oscillator at an arbitrary temperature”, *Eur. Phys. J. B* 2007, *60*, 401–408. Resolution in favor of the second law is discussed in: Kim, I.; Mahler, G. Clausius inequality beyond the weak-coupling limit: The quantum Brownian oscillator. *Phys. Rev. E* 2010, *81*, 011101-1–12. Also, Bandyopadhyay, M. Does the second law hold in the quantum regime?. *Phys. Scr.* 2010, *81*, 065004-1–7. And, Kim, I. Field-induced dynamics in the quantum Brownian oscillator: An exact treatment. *Phys. Lett. A* 2010, *374*, 3828–3837.

(ee) According to Th. M. Nieuwenhuizen, private communications at the Symposium: The Second Law of Thermodynamics: Foundations and Status on June 19–20, 2006 (part of the 87th Annual Meeting of the Pacific Division of the AAAS on June 18–22, 2006): All formulations of the second law of thermodynamics are equivalent in the classical regime [with one exception that is not applicable insofar as this present chapter is concerned]: see Ref. [6gg], and also for infinite systems in the quantum regime. However, they are not equivalent for a finite quantum system, even if in thermal contact with an infinite heat bath. (For a finite quantum system in thermal contact with a finite heat bath, the deviations from classical behavior are even larger.)

- (ff) Allahverdyan, A. E.; Johal, R. S.; Nieuwenhuizen, Th. M. Work extremum principle: Structure and function of quantum heat engines. *arXiv:0709.4125v1 [cond-mat-stat-mech]* 26 Sep 2007.
- (gg) Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Minimal Work Principle and its Limits for Classical Systems. *arXiv:cond-mat/0607579 v1* 22 Jul 2006. Also, Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Minimal-work principle and its limits for classical systems. *Phys. Rev. E* 2007, 75, 051124.
7. (a) Spontaneous rectification of thermal voltage fluctuations in diodes with very small capacitance at very low temperatures is studied in, for example, McFee, R. Self-Rectification in Diodes and the Second Law of Thermodynamics. *Am. J. Phys.* 1971, 39, 814–820.
- Spontaneous rectification based on the Little-Parks effect is investigated in, for example:
- (b) Nikulov, A. V. Quantum force in a superconductor. *Phys. Rev. B* 2001, 64, 012505.
- (c) Nikulov, A. V. One of Possible Applications of High-Tc Superconductors. *arXiv:physics/0106020 v1* 6 Jun 2001.
- (d) Nikulov, A. V. Perpetuum Mobile without Emotions. Ref. [5a], pp. 207–213.
- (e) Dubonos, S. V.; Kuznetsov, V. I.; Nikulov, A. V. Quantum voltage oscillations observed on segments of an inhomogeneous superconducting loop. *arXiv:cond-mat/0305337 v1* 15 May 2003.
- (f) Dubonos, S. V.; Kuznetsov, V. I.; Zhilyaev, I. N.; Nikulov, A. V.; Firsov, A. A. Observation of the External-ac-Current-Induced dc Voltage Proportional to the Steady Current in Superconducting Loops. *JETP Lett.* 2003, 77, 371–375.
- (g) Aristov, V. V.; Nikulov, A. V. Quantum Power Source. Putting in Order of a Brownian Motion without Maxwell's Demon. *arXiv:cond-mat/0310073 v1* 3 Oct 2003. In *Proceedings of SPIE 2003, Vol. 5128 "Quantum Informatics"*, 148–156.
- (h) Berger, J. Noise Rectification by superconducting loop with two weak links. *Phys. Rev. B* 2004, 70, 024524.
- (i) Nikulov, A. About Peretuum Mobile without Emotions. Ref. [5a], pp. 207–212.
- (j) Reference [5b], Sects. 4.2.3 and 4.4.
- (k) Nikulov, A. V. Why the persistent power can be observed in mesoscopic quantum system. *arXiv:cond-mat/0404717 v1* 29 Apr 2004.
- (l) Nikulov, A. V. Quantum limits to the second law and a breach of a symmetry. *Invited Lecture at the Conference: "Frontiers of Quantum and Mesoscopic Thermodynamics"*, 26–29 July 2004. Will be published In *Phys. Rev. E*. (E-print: Quantum limits to the second law and breach of symmetry. *arXiv:cond-mat/0505508 v1* 20 May 2005.) Two related papers: (i) Nikulov, A. V. Differences between application of some basic principles of quantum mechanics on atomic and mesoscopic levels. *arXiv:cond-mat/0506653 v1* 24 June 2005. (ii) Nikulov, A. Fundamental Differences between applications of some basic principles of quantum mechanics on atomic and mesoscopic levels. In Burra G. Sidearth, Antonio Alfonso-Faus, Marius J. Fullana, Editors. *Frontiers of Fundamental Physics: Eighth International Symposium. AIP Conference Proceedings (Vol. 905)*. AIP: Melville, New York, 2007; pp. 117–119. (E-print: Fundamental Differences Between Application of Basic principles of Quantum Mechanics on Atomic and Higher Levels. *arXiv:0707.0827 v1* 2007.) {Note: These two related papers investigate possible difficulties with the *purely*-dynamical law of conservation of (angular) momentum associated with (anti)*thermo*-dynamical second-law-violating



currents at *nonzero* resistance in Little-Parks-effect circuits. Of course, the Little-Parks (anti)*thermo*-dynamical second-law-violating effect could still obtain even given *strict* conservation of *purely*-dynamical *total* (angular) momentum: Generation of counter-rotation in the surroundings with which a Little-Parks current's electrons interact could *strictly* conserve *total* (angular) momentum at essentially infinitesimal cost — which might be paid for by the Little-Parks effect itself — in kinetic energy imparted to said surroundings, if said surroundings are extremely massive compared with the combined masses of these electrons. A possible coupling mechanism might entail leakage of these electrons' wave packets into classically forbidden regions and hence into said surroundings, perhaps similarly to the mechanism discussed in Ref. [15] cited in *arXiv:cond-mat/0506653 v1* 24 June 2005 [Hirsch, J. E. The Lorentz force and superconductivity. *Phys. Lett. A* 2003, 315, 474–479 (see especially, on p. 478, the second paragraph and Fig. 4)]. Even almost infinitesimally weak coupling might suffice, because the kinetic energy thus imparted to the surroundings also need be almost infinitesimal. This topic is further investigated in later papers, e.g.: Aristov, A. A.; Nikulov, A. V. Could the EPR correlation be in superconducting structures?. Possibility of experimental verification. *arXiv:cond-mat/0604566* 25 Apr 2006; Nikulov, A. V. About Essence of the Wave Function on Atomic Level in Superconductors. *arXiv:0803.1840v1 [physics.gcn-ph]* 12 Mar 2008. and Nikulov, A. V., "What is azimuthal quantum force in superconductor," *arXiv:1104.4856v1 [cond-mat.supr-con]* 26 Apr 2011.

(m) Spontaneous rectification in nanoscale RLC circuits is studied in, for example, Allahverdyan, A. E.; Nieuwenhuizen, Th. M. Testing the violation of the Clausius inequality in nanoscale electric circuits. *Phys. Rev. B* 2002, 66, 115309.

(n) Results are discussed and various viewpoints through 2005 are summarized in Berger, J. The Chernogolovka experiment. Ref. [5d], 100–103.

Examples of more recent work are:

(o) Nikulov, A. Fundamental Differences Between Application of Basic Principles of Quantum Mechanics on Atomic and Higher Levels. *arXiv:0707:0827* 2007. (Talk presented at the *Eighth International Symposium Frontiers of Fundamental Physics*, Madrid, October 17–19, 2006.)

(p) Gurtovoi, V. L.; Dubonos, S. V.; Karpil, S. V.; Nikulov, A. V.; Tulin, V. A. Contradiction between the Results of Observations of Resistance and Critical Current Quantum Oscillations in Asymmetric Superconducting Rings. *J. Exp. Theor. Phys.* 2007, 105, 262–267.

(q) Dubonos, S. V.; Gurtovoi, V. L.; Nikulov, A. V.; Tulin, V. A. Can dc voltage proportional to the persistent current be observed on segment of asymmetric mesoscopic ring? *arXiv:0707.0807v1 [cond-mat.supr-con]* 5 Jul 2007.

(r) Nikulov, A. V. The quantum challenge at the mesoscopic level. *arXiv:0707.4351v1 [cond-mat.supr-con]* 30 Jul 2007.

(s) Burlakov, A. A.; Gurtovoi, V. L.; Dubonos, S. V.; Nikulov, A. V.; Tulin, V. A. Little-Parks effect in a system of asymmetric superconducting rings. *JETP Letters* 2007, 86, 517–521.

- (t) Gurtovoĭ, V. L.; Dubonos, S. V.; Nikulov, A. V.; Osipov, N. N.; Tulin, V. A. Dependence of the Magnitude and Direction of the Persistent Current on the Magnetic Flux in Superconducting Rings. *J. Exp. Theor. Phys.* 2007, 105, 1157–1173.
- (u) Nikulov, A. V. About Essence of the Wave Function on Atomic Level and in Superconductors.  
*arXiv:0803.1840v1 [physics.gen-ph]* 12 Mar 2008.
- (v) Burlakov, A. A.; Gurtovoi, V. L.; Dubonos, S. V.; Nikulov, A. V.; Tulin, V. A. Observation of the Little-Parks Oscillations in a System of Asymmetric Superconducting Rings.  
*arXiv:0805.1223v1 [cond-mat.supr-con]* 8 May 2008.
- (w) Nikulov, A. V. Bohm's quantum potential and quantum force in superconductor.  
*arXiv:0812.4118v1 [quant-ph]* 22 Dec 2008.
- (x) Gurtovoi, V. I.; Il'in, A. I.; Nikulov, A. V.; Tulin, V. A. The dc voltage proportional to the persistent current observed on system of asymmetric mesoscopic loops.  
*arXiv:0910.5140v1 [cond-mat.supr-con]* 27 Oct 2009.
- (y) Gurtovoi, V. I.; Il'in, A. I.; Nikulov, A. V.; Tulin, V. A. Could dissipationless current be observed at non-zero resistance? *Proceedings of the 18th International Symposium "NANOSTRUCTURES: Physics and Technology"*, p. 277 (St. Petersburg, Russia, 2010). Also at *arXiv:1006.5346v1 [cond-mat.supr-con]* 28 Jun 2010.
- (z) Gurtovoi, V. I.; Il'in, A. I.; Nikulov, A. V.; et. al. Weak dissipation does not result in disappearance of the persistent current. *Low Temperature Physics* 36, 974–981 (2010). A challenge to the second law of thermodynamics entailing thermal evaporation of electrons in capacitors at sufficiently but not excessively high temperatures is explored in:
- (aa) D'Abramo, G. Thermo-charged capacitors and the Second Law of Thermodynamics. *Physics Letters A* 374, 1801–1805 (2010).
- (bb) D'Abramo, G. On the exploitability of thermo-charged capacitors. *Physica A: Statistical Mechanics and its Applications* 390, 482–491 (2011).
8. (a) Proposed violations of the second law in plasma systems can be studied classically. See, for example, Sheehan D. P.; Means, J. D. Minimum requirement for second law violation: A paradox revisited. *Phys. Plasmas* 1998, 5, 2469–2471; and references cited therein, especially those by Sheehan, D. P.
- (b) An alternative (classical) viewpoint given in Cruden, B. A. On the proposed second law paradox in a nonzero floating potential. *Phys. Plasmas* 2001, 8, 5323–5326.  
This alternative viewpoint is reviewed in
- (c) Reference [5b], Sect. 8.4.
- (d) However, quantum-mechanical analyses may be more rigorous. See, for example, V. Čápek; Sheehan, D. P. Quantum mechanical model of a plasma system: a challenge to the second law of thermodynamics. *Physica A* 2002, 304, 461–479; and references cited therein.
- (e) Reference [5b], Chap. 8.
9. Many proposed violations of the second law in low-density gas systems can be analyzed classically. Low-density gas-systems in gravitational fields are investigated in, for example:
- (a) Sheehan, D. P.; Glick, J.; Duncan, T.; Langton, J. A.; Gagliardi, M. J.; Tobe, R. Phase Space Portraits of an Unresolved Gravitational Maxwell Demon. *Found. Phys.* 2002, 32, 441–462.

- (b) Sheehan, D. P.; Glick, J.; Duncan, T.; Langton, J. A.; Gagliardi, M. J.; Tobe, R. Phase space analysis of a gravitationally-induced, steady-state nonequilibrium. *Phys. Scripta* 2002, 65, 430–437.
- (c) Sheehan, D. P. A Classical Gravitational Second Law Paradox. Ref. [5a], pp. 391–396.
- (d) Reference [5b], Chap. 6.  
A critique is given in
- (e) Wheeler, J. C. Resolution of a Classical Gravitational Second-Law Paradox. *Found. Phys.* 2004, 34, 1029–1062.
- This critique is reviewed in
- (f) Reference [5b], Sect. 6.2.5.  
Field-free low-density gas systems are investigated in, for example:
- (g) Miller, S. L. Insights into the Second-Law of Thermodynamics from Anisotropic Gas-Surface Interactions. Ref. [5e], 1660–1684.
- (h) Denur, J. Speed-Dependent Weighting of the Maxwellian Distribution in Rarefied Gases: A Second-Law Paradox? Ref. [5e], 1685–1706. [Note: Reference [9h] considers a *speed*-dependent second-law paradox, in contrast with the *velocity*-dependent second-law challenge considered in this present chapter.]
- (i) Crosignani, B. and Di Porto, P. Random Fluctuations of Diathermal and Adiabatic Pistons. Ref. [5e], 1707–1715.
10. Proposed violations of the second law in solid-state systems are understandable classically. See, for example:
- (a) Sheehan, D. P.; Putnam, A. R.; Wright, J. H. A Solid-State Maxwell Demon. *Found. Phys.* 2002, 32, 1557–1595.  
A concise summary of Ref. [10a] is given in
- (b) Weiss, P. Motor design flouts physical law. *Science News* Nov. 2, 2002, 162 (No. 18), 286.  
More recent research, concerning a p-n junction solid-state oscillating motor, is discussed in
- (c) Sheehan, D. P.; Wright, J. H.; Putnam, A. R.; and Perttu, E. K. Intrinsically biased, resonant NEMS-MEMS oscillator and the second law of thermodynamics. Ref. [5d], 87–99.
- (d) The system investigated in Ref. [10c] is analyzed with respect to fundamental second-law considerations in Sheehan, D. P.; Gross, D. H. E. Extensivity and the thermodynamic limit: Why size really does matter. *Physica A* 2006, 270, 461–482.
11. Reference [3k]. The analyses in this present chapter, as well as in previous shorter versions [28] thereof, are more quantitatively correct. The analyses in this present chapter (except for the last four paragraphs of Sect. 3, Sect. 6, and a few very brief mentions elsewhere, therein), as well as in Refs. [3k], [28a], and [28b], are classical.
12. (a) Reference [3k]. See especially the two paragraphs immediately following that containing Eq. (7), Appendixes A, B, and C, and Footnote 7. The references cited in Footnote 7 of Ref. [3k] provide further supplementation.  
An extension of the concept of temperature fluctuations is developed in, e.g.:
- (b) Beck, C. Dynamical Foundations of Nonextensive Statistical Mechanics. *Phys. Rev. Lett.* 2001, 87, 180601.
- (c) Beck, C. Superstatistics: Recent developments and applications. *arXiv:cond-mat/0502306 v1* 12 Feb 2005. Also in Zichichi, A. (Senior Editor). Beck, C.; Benedek, G.; Rapisarda, A.; Tsallis, C. (Editors). *Complexity, Metastability, and Nonextensivity*. World Scientific: Singapore, 2005; pp. 33–42.

- (d) Abe, S.; Beck, C.; Cohen, E. G. D. Superstatistics, thermodynamics, and fluctuations. *Phys. Rev. E* 2007, 76, 031102
13. See, for example:
- (a) Peebles, P. J. E., *Principles of Physical Cosmology*. Princeton University Press: Princeton, N. J., 1993; pp. 134–138, 151–158, 174, and 176–181.
- (b) Misner, C. W.; Thorne, K. S.; Wheeler, J. A. *Gravitation*. W. H. Freeman: New York, 1973; Sect. 22.6 [especially pp. 587–589 and most especially Exercise 22.17 (of Chap. 22) on pp. 588–589].
- (c) Peebles, P. J. E.; Wilkinson, D. T. Comment on the Anisotropy of the Primeval Fireball. *Phys. Rev.* 1968, 174, 2168.
- (d) Gill, T. P. *The Doppler Effect*. Academic: New York, 1965; Chap. VIII.
14. See, for example:
- (a) Kolmogorov, A. N. *Foundations of the Theory of Probability, Second English Edition*. Chelsea: New York, 1956.
- (b) Ghahramani, S. *Fundamentals of Probability with Stochastic Processes, Third Edition*. Pearson Prentice Hall: Upper Saddle River, N. J., 2005.
- (c) Kelly, D. G. *Introduction to Probability*. Macmillan: New York, 1994.
15. See, for example:
- (a) Cox, D. R.; Miller, H. D. *The Theory of Stochastic Processes*. Chapman and Hall: London, 1965 (1990 Printing); Sects. 3.1, 3.2, and 3.6.
- (b) Jones, P. W.; Smith, P. *Stochastic Processes, An Introduction*. Arnold: London, 2001; Chap. 4 (especially Sect. 4.3).
- (c) The recursion method used in (10) is similar to that given in Hoel, P. G.; Port, S. C.; Stone, C. J. *Introduction to Stochastic Processes*. Houghton Mifflin: Boston, 1972 (reissued by Waveland: Prospect Heights, Ill., 1987); pp. 1–2, Sects. 1.1, 1.2, and 1.4.2, and pp. 47–49.
16. See, for example, Nikulov, A. V. Ref. [7c] (see especially Sect. 4.2.); and Refs. [7d] and [71].
17. Reference [3k], Appendixes B and C.
18. See, for example, Leff, H. S.; Rex, A. F. in Leff, H. S.; Rex, A. F., Editors. *Maxwell's Demon 2: Entropy, Classical and Quantum Information, Computing*. Institute of Physics Publishing: Bristol, U. K., 2003; Sect. 1.8.4 and references cited therein. This book contains an overview of various Maxwellian demons, reprints of many important papers in the field, and much additional informative material.
19. Reference [3k], the paragraph immediately following that containing Eq. (23), and the third paragraph of Appendix D.
20. To address the error in cited Ref. [19]: *Diffraction* does not reduce the opacity even of a small (linear dimensions  $\lesssim hc/kT$ ) disk, or even of a very small (linear dimensions  $\ll hc/kT$ ) disk: Note, for example, in correction, that the temperature of Doppler-shifted EBR impinging on a body is unaffected by *diffraction*, even if size  $\lesssim hc/kT$  (or even  $\ll hc/kT$ ), as per:
- (a) Reference [13a], pp. 174 and 176–177.
- (b) Hecht, E. *Optics, Fourth Edition*. Addison-Wesley: New York, 2002; Sect. 4.4.2, pp. 444–446, Sects. 10.4–10.5. (See especially p. 105 in Sect. 4.4.2.) Also: Hecht, E. Why don't Huygens' wavelets go backwards?. *Physics Teacher* 1980, 18, 149.
- (c) Dr. Patricia H. Reiff, private communications, 1999.
21. See, for example:

- (a) Bohren, C. F.; Nevitt, T. J. Absorption by a sphere: a simple approximation. *Applied Optics* 1983, 22, 774–775.
- (b) van de Hulst, H. C. *Light Scattering by Small Particles*. John Wiley: New York, 1957 (corrected republication by Dover: New York, 1981); Chaps. 2, 6, and 14.
- (c) Bohren, C. F.; Huffman, D. R. *Absorption and Scattering of Light by Small Particles*. John Wiley: New York, 1983 (Professional Paperback Edition, 1998); Chap. 1, Chap. 5 (especially Sects. 5.1–5.2), and pp. 287–289.
- (d) Reference [13a]; pp. 151–158, 174, and 176–181 (especially p. 174, and pp. 176–181).
- (e) Some general discussions are given in Bohren, C. F.; Clothiaux, E. E. *Fundamentals of Atmospheric Radiation*. Wiley-VCH: Weinheim, 2006; Sects. 1.3, 1.4, 2.9, and 3.5.
22. See, for example:
- (a) Bohren, C. F. How can a particle absorb more than the light incident on it? *Am. J. Phys.* 1983, 51, 323–327.
- (b) H. Paul; R. Fisher. Comment on “How can a particle absorb more than the light incident on it?”. *Am. J. Phys.* 1983 51, 327.
- (c) Bohren, C. F.; Huffman, D. R. Ref. [21c]; Sect. 3.4 (especially p. 72), Sect. 4.7 (especially the last two paragraphs — which show that resonance can obtain if size  $\lesssim hc/kT$ ), and Chap. 12 (in Sect. 12.1.8, the model studied in Ref. [22a] is treated again).
- (d) Garbuny, M. *Optical Physics*. Academic: New York, 1965; pp. 100–101 and 268–269.
- (e) Dr. Roland E. Allen, private communications, 2003.
- (f) References [20a] – [22e] may further address the difficulty associated with DP size  $\lesssim hc/kT$  discussed qualitatively in Ref. [3k], the two paragraphs immediately following that containing Eq. (23) — noting (as per Footnote [20]), in correction, that the temperature of Doppler-shifted EBR impinging on a body is *unaffected* by *diffraction*, even if DP size  $\lesssim hc/kT$  (or even  $\ll hc/kT$ ).
23. See, for example, Ref. [21b]; Sect. 11.23 (especially pp. 182–183), and p. 269.
24. See, for example, Ref. [21b], Chap. 14.
25. Reference [3k], Appendix D.
26. See, for example:
- (a) Schiff, L. I. *Quantum Mechanics, Third Edition*. McGraw-Hill: New York, 1968; pp. 100–107.
- (b) Cohen, B. L. A Simple Treatment of Potential Barrier Penetration. *Am. J. Phys.* 1965, 33, 97–98.
27. (a) A challenge to the second law based on the Lorentz force was proposed in Fu, X. Y. An Approach to Realize Maxwell’s Hypothesis. *Energy Convsr. Mgmt.* 1982, 22, 1–3.
- This challenge was critiqued in:
- (b) Wang, Y. R. Fu’s Experiment and the Generalized Gibbs Distribution. *Energy Convsr. Mgmt.* 1983, 23, 185–191. {Note: Reference [27b] (see especially the first two paragraphs on p. 188) seems to imply that (in the *classical* regime) an increase in entropy is *not* equivalent to the Clausius-Heat formulation of the second law as stated in Ref. [5b], p. 4: “No process is possible for which the *sole* effect is that heat flows from a reservoir at a given temperature to a reservoir at a higher temperature.”: (“sole” is not italicized in original text). But: If the entropy increase associated with the second term of Y. R. Wang’s unnumbered equation [that immediately following his Eq. (12)] on p. 188 equals or exceeds in magnitude the entropy decrease associated with heat flowing “uphill” as per the first term thereof, then is not the Clausius-Heat

formulation of the second law obeyed *despite* such “uphill” heat flow — since it is then *not* the only (or even the largest) effect? Does Y. R. Wang neglect the word *sole*?

The Fu challenge seemed to be resolved in favor of the second law in:

- (c) Wang, Y. Z. The Origin of the Electric Current in Fu’s Experiment — An Analysis of Fu’s Work on Realizing Maxwell’s “Demon”. *Energy Convsr. Mgmt.* 1986, 26, 249–252.
- (d) Reference [1a]. Reference [1a] considers only (*nondissipative*) velocity-dependent forces acting perpendicularly to the velocity *itself* (i.e., to the direction of motion), and hence which can do no work — of which the Lorentz force is one example.

There have been more recent attempts to revive it:

- (e) Fu, X.; Fu, Z. Realization of Maxwell’s Hypothesis. *arXiv:physics/0311104* 2003.
- (f) Fu, X.; Fu, Z. Another Way to Realize Maxwell’s Demon. *arXiv:physics/0509111* 2005.
- (g) As per private communications at the 36th Winter Colloquium on the Physics of Quantum Electronics (PQE-2006), January 2–6, 2006: The Fu analyses (in Refs. [27a], [27e], and [27f]) through the time of this 2006 Colloquium neglected electrons that leave plate B towards the right in *both* of Fu’s Lorentz-force systems, as per *both* Fig. 1 and Fig. 2 of Ref. [27f]. This seemed, *at that time*, to resolve the Fu challenge in favor of the second law more strongly than does Ref. [27c]. But Fu met this challenge with a new Lorentz-force system [private communications at the 37th Winter Colloquium on the Physics of Quantum Electronics (PQE-2007), January 2–6, 2007]. Nevertheless, as per other private communications at this 2007 Colloquium, there may still be unresolved issues concerning even Fu’s new Lorentz-force system.
- (h) Note: The Zhang formulation of the second law, enunciated in Ref. [1a] (and restated in the first two paragraphs of Sect. 1, with further discussions in the last four paragraphs of Sect. 3, of this present chapter), is valid irrespective of whether or not (*nondissipative*) velocity-dependent forces acting perpendicularly to the velocity itself (i.e., to the direction of motion), and hence which can do no work — such as the Lorentz force — can challenge *the* second law (or even merely *a* second law): recall the last four paragraphs of Sect. 3, and Refs. [6s] – [6ff], especially Ref. [6dd].

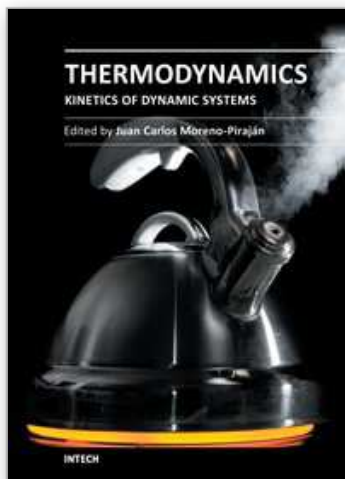
28. (a) Denur, J. Modified Feynman ratchet with velocity-dependent fluctuations. Ref. [5a], pp. 326–331. A revised version of Ref. [28a] is:

(b) Denur, J. Modified Feynman ratchet with velocity-dependent fluctuations. Ref. [5c], 76–86.

29. Sheehan, D. P.; Kriss, V. G. Energy Emission by Quantum Systems in an Expanding FRW Metric. *arXiv:astro-ph/0411299 v1* 11 Nov 2004. See also references cited therein, especially Ref. 20 cited therein: Harrison, E. R. Mining Energy in an Expanding Universe. *Astroph. J.* 1995, 446, 63–66. [It should be noted, however, that it may be possible to consider the loss in the cosmic background radiation’s kinetic energy associated with the cosmological redshift as being compensated for by the gain in its gravitational potential energy in the universe’s gravitational field as the universe expands: This is easiest to visualize in a positively-curved (curvature index = +1) closed universe (shell of 3-sphere) wherein said radiation is trapped. Letting  $\nu$  ( $\lambda$ ) be the frequency (wavelength) corresponding to the peak of the cosmic-background-radiation blackbody Planck distribution, and considering expansion of said 3-sphere from radius of curvature  $R_1$  to radius of curvature  $R_2$ ,  $\nu(R_2)/\nu(R_1) = \lambda(R_1)/\lambda(R_2) = R_1/R_2$ . This implies conservation of *total* — *kinetic plus gravitational-potential* — energy given a cosmological gravitational potential difference of  $\Delta\Phi = c^2 \ln \frac{R_2}{R_1}$ . (Of course, similarly, during a possible

- contraction phase, it may be possible to consider the gain in the cosmic background radiation's kinetic energy associated with the cosmological blueshift as being compensated for by the loss in its gravitational potential energy in the universe's gravitational field as the universe contracts.) See, for example, Rindler, W. *Relativity: Special, General, and Cosmological, Second Edition*. Oxford: Oxford, U. K., 2006; Sect. 1.6, Exercise 1.9 (of Chap. 1) on p. 28, Chap. 9, and Sects. 12.2 and 16.4.]
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Constructive criticisms, given in  
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and  
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hopefully have been addressed in Ref. [3k], and more completely in this present chapter and  
in previous shorter versions [28] thereof.
32. Reference [5b], Sect. 5.3.

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Phone: +86-21-62489820  
Fax: +86-21-62489821



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