1. Introduction

Product quality and reliability are essential in the medical device industry. In addition, predictable development time, efficient manufacturing with high yields, and exemplary field reliability are all hallmarks of a successful product development process. One challenge in electronic hardware development normally involves understanding the impact of variability in component and material properties and the subsequent potential impact on performance, yield, and reliability. Over-reliance on physical testing and characterization of designs may result in subsequent yield issues and/or post-release design changes in high volume manufacturing. Issues discovered later in the product cycle make development time unpredictable and do not always effectively eliminate potential risk. Using hardware testing to verify that the embedded system hardware and firmware work under the worst case conditions in the presence of variation is potentially costly and challenging. As a result, improving predictability early in design with a virtual environment to understand the influence of process corners and better control of distributions and tails in components procured in the supply chain is important. The goal is to ensure that design works in the presence of all specified variability and to ensure the component designed is appropriately controlled during purchasing/manufacturing. This is achieved by establishing a clear link between the variability inherent in the supply chain on the performance, yield, and reliability of the final design. This will lay the groundwork for managing expectations throughout the entire supply chain, so that each functional area is aware of its responsibilities and role in the overall quality and reliability of the product.

In this chapter, a methodology is outlined that utilizes electrical simulations to account for component variability and its predicted impact on yield and quality. Various worst-case circuit analysis (WCCA) methods with the advantages, assumptions and limitations are introduced in Section 2. A simulation based flow is developed in Section 3 to take advantage of the best qualities of each method discussed to understand design, reliability, and yield in relation to how the product is used and how the effects of variability in the supply chain influence the outcome. Furthermore, predictive yield estimation is enabled using a computationally efficient Monte Carlo analysis technique extending results of worst case analysis with actual component parameter distributions obtained from the supply chain is
discussed in Section 4. Transfer functions are built upon simulation-based design of experiments and realistic distributions applied to the various input parameters using statistically based data analysis. Building upon simulations to statistically predict real-world performance allows creating a virtual operations line for design yield analysis, which allows effective design trade-offs, component selection, and supply chain control strategies.

2. Worst-case circuit analysis methods

Worst-case circuit analysis (WCCA) is a method to ensure the system will function correctly in the presence of allowed/specified variation. WCCA quantitatively assesses the performance that takes into consideration the effect of all realistic, potential variability due to component and IC variability, manufacturing processes, component degradation, etc. so as to ensure robust and reliable circuit designs. Modeling and simulation-based worst-case circuit analysis enables corners to be assessed efficiently, and allows design verification at a rigorous level by considering variations from different sources.

2.1 Sensitivity analysis

An initial approach for understanding the primary sources of variability usually starts with a sensitivity analysis study which is a method to determine the effects of input parameter variation on the output of a circuit by systematically changing one parameter at a time in the circuit model, while keeping the other parameters constant (Figure 1). Sensitivity is defined as follows:

\[
\text{Sensitivity} = \frac{\Delta \text{output}}{\Delta \text{parameter}} \tag{1}
\]

Fig. 1. Sensitivity analysis: circuit output changes due to variation of the input

If the output variation is reasonably linear with the variation of the component parameter across its entire tolerance range, sensitivity can be multiplied by the tolerance range of the component parameter to determine the output variation due to this tolerance. Two important attributes in the sensitivity analysis are the magnitude and polarity/direction. When the input increases, the polarity/direction is positive if the output increases, and is negative if the output decreases. Because of the huge potential number of simulation variables (e.g. m components with n parameters each), sensitivity analysis can be used to investigate one factor at a time (OFAT) to provide an initial triage of those parameters.
requiring subsequent evaluation. For typical designs, there are multiple outputs that need to be understood, so separate sensitivity analysis and subsequent treatment is usually employed, which is discussed in Section 3 and Section 4. The real-world is rarely as simple as textbook-like examples.

In each case, as one parameter is varied, all others are held at their nominal conditions. This approach assumes that all variables are independent and there are no interactions among them. While this technique is much less sophisticated than other formal methods, it provides an effective means of reducing the subsequent analysis and complexity, potentially by several orders of magnitude. Figure 2 shows one example of a sensitivity analysis result.

A few top critical input factors that dominate output response are identified from sensitivity analysis with 74 parameters varied within the specified limits. A large number of other factors that are insignificant are eliminated from subsequent analysis by performing this important sensitivity analysis step. Subsequent simulations or physical testing can then focus limited resources on the factors with the greatest importance.

![Fig. 2. Top five critical factors identified from sensitivity analysis](image)

**2.2 Extreme value analysis (EVA)**

Extreme value analysis is a method to determine the actual worst case minimum or maximum circuit output by taking each component parameter to their appropriate extreme values. The EVA method decomposes the simulations into two steps for a circuit with \( n \) input variables.

First \( 2n \) sensitivity simulations are run, where each component parameter is simulated separately at its minimum and maximum (Figure 3). The results of the sensitivity simulations are analyzed, and the magnitude of change on the output due to each individual input
variation can be ranked in a Pareto chart (Figure 2). Parameters that make the most influence can be identified as critical factors. Knowing critical parameters from sensitivity analysis provides information to narrow down the list of variables and provides information for component selection and control in case needed.

![Graph showing sensitivity analysis](image)

Fig. 3. Sensitivity analysis: output sensitivity to all inputs (2n ‘quick’ simulations)

Next, for each output measurement, two simulations are run that combine critical input parameters at either low spec limit and/or upper spec limits. Thus, this method requires only 2n+2 simulations so this method can be efficiently used on large number of outputs.

EVA is a commonly used worst case analysis method and the easiest one to apply (Reliability Analysis Center, 1993). It is also a more conservative method compared to root-sum-squared analysis or Monte Carlo analysis. One limitation of EVA is the assumption that critical factors are independent of one another, and the polarity determined from sensitivity doesn’t change between the nominal and the worst case scenarios. EVA can be an effective and efficient way of performing worst case analysis. In other situations where interactions exist among input parameters or when the very conservative nature of EVA is too prohibitive for design, other methods such as design of experiments or circuit level Monte Carlo simulations can be used instead.

### 2.3 Root-sum-squared (RSS) analysis

As EVA targets the worst case corners which can be very conservative, Root-Sum-Squared analysis provides a statistically realistic estimation. Assuming an output Y can be approximated by n inputs x₁ to xₙ.

\[
Y = \sum_{i=1}^{N} a_i X_i
\]  

Variance of Y is

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\[
Var(Y) = \sum_{i=1}^{N} a_i^2 Var(X_i) + 2 \sum_{i<j} a_i a_j Cov(X_i, X_j)
\]  \hspace{1cm} (3)

Assuming correlation between inputs are zero, the covariance of \(x_i\) and \(x_j\) is zero. The variance of \(Y\) is

\[
Var(Y) = \sum_{i=1}^{N} a_i^2 Var(X_i)
\]  \hspace{1cm} (4)

When there are no interactions among input variables, a practical Root-Sum-Squared analysis method calculates the root-sum-square of the tolerances. Using RSS, the standard deviation of the output measurement is determined as follows (Reliability Analysis Center, 1993):

\[
\sigma_Y = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial Y}{\partial X_i} \sigma_{X_i} \right)^2}
\]  \hspace{1cm} (5)

where \(\sigma_{X_i}\) is the standard deviation of the \(i^{th}\) input parameter.

Assuming the output follows a normal distribution, the worst case performance limits of the output measurement can be approximated as mean ± \(m \sigma_Y\) depending on the worst case criteria. Compared to EVA, RSS provides more realistic, less conservative worst case predictions. The limitation is that it is not a “true” worst case analysis. In addition, it assumes component parameters have normal distributions that are described by a mean and a standard deviation. Due to these limitations, RSS is not as widely applied compared to EVA, especially in critical reliability applications.

2.4 Monte Carlo simulations

The Monte Carlo method is an algorithm that utilizes random sampling of input parameters to compute a statistical output result. Monte Carlo simulations in this Section refer to circuit simulations with electronic design software programs. Circuit level Monte Carlo analysis takes into account process variation, mismatch, and/or other design and component variables. For each iteration, a circuit is constructed by selecting a set of component parameters using statistical distributions, and then the circuit is simulated and the results are captured. After the simulations are completed, the result is a statistical distribution of the output. The Monte Carlo method requires a statistical distribution for each actual part tolerance distributions in the circuit, which is used to create the component model. The parameter distributions are not limited to normal distributions and can be extended to any real data distribution that can be described mathematically. This is particularly effective if the output variation is NOT linear with the variation of the component parameter across its entire tolerance range. Thus, the Monte Carlo simulation method is the most accurate method to provide a realistic variability evaluation. However, the Monte Carlo approach requires running a large number of analysis iterations, which is computationally expensive, especially when simulating complex circuit functionality consistent of many integrated circuits and discrete components. Due to this reason, it is challenging to use Monte Carlo simulations to provide a “true” worst-case result, and it is more practical to use it to
estimate mean and standard deviation of the output based upon a practical number of Monte Carlo samples. When using Monte Carlo simulations to estimate yield for cases where the probability of failure is small, the number of needed iterations can be very large. To obtain a yield estimate with (1−ε)100% accuracy and with (1−δ)100% confidence when the probability of failure is p, the required number of iterations is

\[ N(\varepsilon, \delta) \approx \frac{\log(\delta^{-1})}{p\varepsilon^2} \] (6)

Thus, for 90% accuracy (ε = 0.1) and 90% confidence (δ = 0.1), roughly 100/p samples are needed (Date et al., 2010; Dolecek et al., 2008). Other modified methods such as Importance Sampling, are developed for variance reduction and thus to accelerate convergence with reduced number of runs (Zhang & Styblinski, 1995). Nowadays many simulation platforms have built-in Monte Carlo algorithms and algorithms to facilitate variability analysis. Circuit level Monte Carlo simulations can be very time consuming. Due to the size and complexity of today’s systems, it is more practical and efficient to partition the Electrical systems into smaller functional blocks/circuits and perform simulation based WCCA or yield predictions on the circuit block level, or to perform simulations at the system level with abstract block behavioral models to improve speed.

2.5 Monte Carlo analysis based on empirical modeling

Instead of running circuit level Monte Carlo simulations requiring a large number of runs and computational expense, Monte Carlo analysis based on a transfer function that mathematically describes the relationship between the input variables and the outputs can be used. This transfer function can be an analytical design model or an empirical model generated from design of experiments (Maass & McNair, 2010).

\[ Y = f(x_1, x_2, \ldots, x_i) \] (7)

Using design of experiments (DOE) methodologies, factorial experiments are conducted and influence of input variables on outputs are analyzed from a statistical point of view. Furthermore, response surface methodology (RSM) focuses on optimizing the output/response by analyzing influences of several important variables using a linear function or (first-order model) or a polynomial of higher degree (second-order model) if curvature exists (Montgomery, 2009). One advantage of DOE and RSM is finding the worst case in situations where interactions exist among input variables, which sensitivity analysis and EVA may not take into account. In addition, Monte Carlo analysis based on transfer functions generated from DOE or RSM can greatly improve computation efficiency compared to Monte Carlo circuit simulations by replacing large number of random samples to a limited number of corner simulations. However, the accuracy of transfer functions is based on how well it represents real behavior. These methods work well if the assumptions are valid that a linear or quadratic function accurately describes the relationship between inputs and the output. Otherwise, circuit level Monte Carlo simulations for yield estimations are more accurate, though more
computationally expensive. In addition, when the number of factors is large, the number of runs required for a full factorial design could be too large to be realistic. In such cases, fractional factorial design can be used with fewer design points. However, design knowledge is needed to make judgment and assumptions, as some or all of the main effects could be confounded with interactions (Montgomery, 2009). Low resolution designs with fractional factorial design are thus more useful for screening critical factors rather than to be used to generate an empirical model.

3. Simulation flow for WCCA and yield predictions

As different methods have different assumptions, advantages, and limitations, a simulation based WCCA and yield prediction method has been utilized. The simulation-based WCCA flow, shown in Figure 4, describes how the methods discussed in Section 2 are used in different scenarios to estimate the worst case limits and to develop the transfer functions needed to understand design, reliability, and yield in relation to how the product is used.

Fig. 4. Simulation-based worst-case circuit analysis and yield prediction flow
and how the effects of variability in the supply chain impact design success. This method provides a flow to effectively narrow down critical factors and a conservative estimation of worst case limits, while taking advantage of the best qualities of different methods for the optimal accuracy and computational efficiency.

The process begins with the following key elements:

- Identify output signals to monitor and potential input factors to analyze
- Generate and validate circuit models (component and IC models) that support worst case analysis
- Determine component tolerances and ranges
- Determine worst case operating modes

WCCA requires that the components in the circuit have specifications that include the minimum and maximum for important component parameters, which are integrated into the component models needed to support WCCA simulations. Using component or subsystem specification limits as tolerance limits could be conservative, as the specification limits can be wider than actual distributions. This is mainly to ensure requirement consistency at different hierarchy levels. Setting worst-case limits at or beyond the specification limits helps ensure conservative simulations that are most likely to capture the worst-case behavior of the system.

Simulations start with sensitivity analysis to determine the impact of each component parameter variance on each output signal. At this step, 2n simulations are performed for sensitivity analysis in a circuit with n component parameters for each output. This is more efficient to screen and to identify critical factors if there are a large number of component parameters in the circuit that are suspected to impact the design outputs.

From the sensitivity analysis, k critical factors are identified according to the impact on the output changes. One example of identified critical factors is shown in a Pareto chart in Figure 2. In this example, 74 parameters were varied within the specified limits in the sensitivity analysis and the first a few top critical factors that dominate are identified from this screening and will be used in subsequent treatments. Note that it is possible that a potential critical parameter might be left out if the impact shown is negligible, as the sensitivity analysis is only performed with one parameter varied and others are held at their nominal conditions. In such cases, design knowledge may need to be applied and design of experiments can be used instead to screen and determine if the suspected parameters have critical impact on outputs.

With the critical factors identified for each corresponding output, worst case limits can be determined using the component specifications and other additions due to aging or environmental (e.g. radiation) exposure. If the critical factors are independent of one another based on design knowledge, EVA can be applied to determine the worst case design performance limits for that output. Two simulations are run with EVA that combine critical input parameters at either low spec limit and/or upper spec limits. If interactions among input parameters are not negligible, simulation corners can be designed based on DOE and RSM to address interactions. With a full factorial design of two-level k critical factors, 2^k simulations are run based on the worst-case limits for each of the critical parameters. A transfer function is then generated that describes the relationship between the output and critical inputs in a linear or quadratic equation. Worst case limits can be determined based on the generated empirical methods and simulations can be used to confirm the results.
The derived transfer function can be further used for yield estimates via Monte Carlo analysis, which is illustrated in details in Section 4. If an accurate transfer function is not easily derived and simulation speed is permitted, circuit level Monte Carlo simulation is preferred to estimate output distribution and yield.

One major application of worst-case circuit analysis is to determine design trending through sensitivity analysis, and determine design capability limits and design margin. Figure 5 illustrates the results of worst case analysis and predicted distributions.

Besides design verification, another major application for WCCA is to determine component level worst case electrical use conditions, which can only be driven by simulations with WCCA. Understanding worst case use conditions is critical in reliability engineering to assess component reliability relative to capability data obtained from critical component reliability testing and modeling.

Design for reliability approaches integrate reliability predictions into the hardware development process, thus improving design decisions and ensuring product reliability early in the life cycle. The objective is to capture quality / reliability issues earlier in the design cycle, and utilize quantitative reliability predictions based on simulated use conditions to drive design decisions. Use of simulations provides not only nominal use conditions, but also the variations in use conditions due to different operating modes and underlying component variability. Understanding use conditions related to design and variance is critical to create a virtual field use model for reliability predictions and to ensure design for reliability early in development. Based on the predictions, operating modes or component parameters that contribute to circuit overstress or premature wearout will be captured earlier to drive design and supply chain changes. On the other hand, some component parameters drift over time due to aging or exposure to certain environments (e.g. medical radiation), which may result in product failure at some point. Integrating these aging effects in simulations can help capture how the system functions when experiencing “faults.” This fault condition analysis helps to understand design capability limits, to prevent/alleviate certain failure mechanisms, and to help put the right controls in supply chain.
The simulation based WCCA / variability method developed in this chapter can be very conservative. The probability that all component parameters shift simultaneously to the worst case limits is extremely small. In addition, using the specification limits as tolerance limits make the results even more conservative, as some of the component specifications may have much wider limits than what the components actually perform to. However, the intent of using specification limits is to ensure that specification ranges at lower levels are consistent with higher level design requirements, and to highlight the potential risk and extracted critical component parameters if inconsistency exists. Using actual distribution data to start with will leave unanalyzed regions at risk if the distribution drift but still meet specification.

The fact that with a conservative WCCA method and data the circuit still meets requirements provides great confidence of the design quality. Otherwise, limits used in the component models can always be revisited, and more detailed analysis such as Monte Carlo analysis can be performed to get a better idea of the circuit behavior that includes variation. In general, WCCA should be performed early in the project, during the design phase of a project as an integral part of hardware verification. When the analysis results indicate the circuit does not work in the worst case, there are several options:

- Change the circuit design
- Select different components
- Change requirement for a component
- Screen critical component parameters in manufacturing
- Perform a less conservative WCCA and estimate the distribution and Cpk

If opportunities are found that critical component parameters need to have a tighter range, controls should be put in place to get the new component level requirements implemented in supply chain.

WCCA originated in the days when design was based on standard components and circuit boards. Thus design consisted of selecting the correct components and connecting them together correctly. The components were small ICs, discrete semiconductors, and passive components. The purpose of worst case circuit analysis was to ensure that the design would work correctly in the presence of all allowed variation, as specified in vendor datasheets of the standard parts. If the design didn’t work at the worst case scenario, a different component will be selected or the circuit design will be changed. With more custom or semi-custom components nowadays, design optimization (in terms of design margin) is more emphasized as part of the design process.

4. Application of computationally efficient Monte Carlo techniques

Worst-case circuit analysis (WCCA) provides confidence that designs are robust against all potential design and manufacturing variability, due in major part to the variation inherent in all electronic components and assemblies. WCCA evaluates the design against various performance and reliability metrics in the presence of this variation. WCCA is capable of understanding the effect of parametric variation on design performance, establishing quantified metrics that identify and quantify the critical features necessary for design success (and margin), and demonstrating performance at the extreme limits of variation. By successfully analyzing a circuit using the WCCA methodologies, a high level of confidence can be demonstrated that circuits will perform as anticipated, even under these extreme conditions.
conditions. To our knowledge, no experimental approach to design verification can make equivalent claims of design robustness relative to WCCA.

If a circuit is robust against these worst-case measures, it is safe to assume that high levels of design margin have been achieved. Yet, it is important to also understand more realistic levels of design margin, in order to further optimize designs that can trade-off design margin against other metrics such as performance or component cost. It is not always judicious to design for maximum design margin at the expense of these other metrics, after all, why pay extra for a ±1% resistor when a ±5% resistor will do just as well in a certain application. Rather, we would like to demonstrate a balance between design margin and other business and performance factors. Other analysis methods, such as Monte Carlo based simulation, can give more realistic estimates of real-world performance, yet it is hampered by two major tool limitations in a circuit simulation environment: 1) computational expense and 2) inflexibility in many simulation platforms in being able to accurately reflect real-world distributions using non-normal distribution functions.

In our improved methodology, called extended WCCA (EWCCA), we build the transfer functions based upon the WCCA methodology and apply more realistic distributions to the various input parameters using statistically based data analysis. This maintains the accuracy of circuit simulation while also providing the flexibility to evaluate various parametric distributions of critical inputs in a computationally efficient manner. The WCCA method provides the simulated design performance over a wide range of permitted (by specification) variability while the EWCCA method simply leverages those data to build transfer functions and utilize real-world distributions to make estimates of realistic performance. The results can be analyzed extremely rapidly using readily available software tools to virtually simulate the design performance of hundreds of thousands of units in a matter of seconds. This combination of accuracy and computational efficiency drives the real power in EWCCA towards predictive yield, real-world design margin, and reliability margin, while preserving the robust design analysis from the WCCA methodology.

4.1 Methodology
Extended worst-case circuit analysis (EWCCA) builds upon the WCCA simulation based approach where variability is simulated in order to predict performance and reliability margin as well as identify critical features for control. During the evaluation of a design under WCCA, all of the parameters are set at either a lower specification limit (LSL) or an upper specification limit (USL) and may also include variation due to aging or radiation exposure. By setting component (IC or discrete) specifications at their limits, a sensitivity of the relevant output parameters are observed via simulation. The parameters with the greatest influence on the outputs are quantified and captured as critical features. Once the top \( n \) critical features are identified, a simulation based design of experiments (DoE) is executed using the \( n \) critical features as experimental inputs while the simulation provides the virtual experimental output. Using a full \( 2^n \) factorial design based simulation set permits the development of a transfer function model between the inputs and the outputs as shown in Figure 6. Of course, design of experiments is capable of utilizing more efficient, smaller sample, data input combinations, such as central-composite or Box-Behken for example. Regardless of the design of experiments approach that is taken, the primary aim is to leverage the simulation capabilities to perform the experiment, rather than taking the time, expense, and energy to replicate the experiment using physical hardware.
Fig. 6. Variation in inputs (X1’s) leading to observable output (Y). Relationship between Xi’s and Y creates a transfer function F(x1,x2,…xn)

Using standard statistical analysis software, it is relatively straightforward to generate a linearized model that relates the observed outputs (Y’s) to the n critical design inputs (X’s). Since the simulated worst-case circuit analysis was built upon a 2^n factorial experiment, all of the pieces are available to develop a linearized model which can be used for rapid, and accurate, calculations suitable for predicting real-world circuit behavior. For each of the critical features identified during the WCCA, either the lower-specification limit (LSL) or the upper-specification limit (USL) was used in the 2^n factorial design. Here, for each input variable, the LSL is coded as a ‘-1’ and the USL is coded as a ‘+1’ during the model generation and analysis. Uncoded (actual) X_i values can also be used to generate models. In either situation, the end result should be the same, it’s simply a matter of how one arrives at the end state.

A first-order model assumes that only the critical parameters identified in the WCCA sensitivity analysis have a significant effect on the outputs, while ignoring the potential interactions between terms. In general, a first order model takes the form:

\[ Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_n X_n + \varepsilon \]  \hspace{1cm} (8)

where Y is the observed output given the various input parameters (X_i’s). The Y can be a performance metric, such as charge time, to assess design rigor or it may be a component use condition, such as dissipated power, that will be used to estimate reliability of the component. The \( \alpha_i \) terms are simply the model coefficients and \( \varepsilon \) is a term accounting for the
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Table 1. Design of experiments simulation analysis results showing transfer function of simulation output vs. model parameters and input variables ($X_i$)

residual error in the model. In general, for the WCCA results, a first order model provides a reasonably good prediction of the ‘true’ simulated outputs. It is relatively simple with software to create an improved version of the model that takes into account second-order effects, e.g. first-order interactions between all of the terms. While slightly more complex in form, it is a simple matter to generate such a second-order model and subsequently improve the predictive nature of the linearized model. The general form of a second-order model has the form:

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + ... \alpha_n X_n + \beta_{12} X_1 X_2 + ... \beta_{1n} X_1 X_n + ... \beta_{ij} X_i X_j + ... + \varepsilon$$  \hspace{1cm} (9)$$

where $Y$ is again the observed output given the various input parameters ($X_i$’s), the $\alpha_i$ terms reflect the first order model coefficients (which may be different than the $\alpha_i$’s generated using only the first order model, and the $\beta_{ij}$ terms relate to interaction terms between the respective $X_i$’s. By including the interaction terms, the model is better able to predict the ‘true’ response of the design. In Figure 7, a comparison of the predictive nature of both a first-order and a second-order model are shown relative to the ‘true’ simulated response of the predicted output of a hardware circuit block. While the first-order model demonstrates very good agreement, the second-order model improves the accuracy without making the model overly burdensome. The goal is to demonstrate that the 1st or 2nd-order models accurately reflect the more computationally expensive simulation output. In this example,
the critical factors for this design output \( Y \) identified via WCCA are \( x_1, x_2, x_3, x_4, x_5, \) and \( x_6 \). Recall that parameters \( x_7 \rightarrow x_{10} \) were determined to have only minor impact on the predicted output \( (Y) \), and are thus treated as part of the error term \( (\varepsilon) \) in equation (4) above.

Fig. 7. Comparison of a 1st-order and a 2nd-order model predictive relative to the ‘true’ simulation output result. In this case, a WCCA result predicting high voltage FET power dissipation is illustrated. While the 1st-order model shows good predictive behavior, the addition of the 2nd-order terms greatly improves the predictability of the model.

With a linearized model, it is now possible to leverage the computational efficiency of the approach and work to understand the predictive performance and yield of the design relative to real-world component variation. While the WCCA process was developed to guarantee performance at the limits of component specs many real-world distributions will not be at their worst-case limits, but will be represented more accurately by a statistical distribution. Many distributions are not accurately represented with the traditional normal distribution, but are rather more complicated.

4.2 Modeling distributions

There are many methods for modeling distribution functions in various statistical packages, and some very complicated distributions can be generated when the proper techniques are used. Not only can relatively standard normal, lognormal, and Weibull distribution functions be obtained, but models of bi- or multi-model distributions can be generated as well. Here, the algorithms necessary to select a random variable \( X \) from either a normal, lognormal, or Weibull distribution in Excel is shown in Table 2.

In order to create a data set corresponding to a particular distribution, the above functional models are repeatedly applied to create a data set. A flowchart for this method is shown in Figure 8.
### Table 2. Models for different distribution types suitable for use in the Excel spreadsheet program

<table>
<thead>
<tr>
<th>distribution</th>
<th>functional model</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal</td>
<td>= NORMINV(RAND(), μ, σ)</td>
<td>RAND() is the random number generator in Excel where (0 &lt; RAND() &lt; 1) and the expression NORMINV(probability, μ, σ) is a function that returns the inverse of the normal cumulative distribution function for the specified mean (μ) and standard deviation (σ)</td>
</tr>
<tr>
<td>lognormal</td>
<td>= EXP(NORMINV(RAND(), μ, σ))</td>
<td></td>
</tr>
<tr>
<td>Weibull</td>
<td>= α*[-LN(RAND())]1/β</td>
<td></td>
</tr>
</tbody>
</table>

### 4.3 Bi- and multi-modal distributions

It is relatively straightforward to create bi- and multi-modal distributions using these distribution functions and some additional random number manipulation. If one is dealing with a distribution that is bimodal, statistical properties for the two distributions can be separated and then combined again to form a random distribution of variables that, in turn, will recreate the statistical behavior of the original bimodal distribution during Monte Carlo modeling.

One example of a common bimodal distribution can be observed in many commercial off the shelf (COTS) components such as capacitors where tolerance levels are specified. Because a supplier can usually charge a little more for components with higher tolerances, it is not uncommon to order a batch of ±10% components and find that some of the ±5% components were removed from the distribution, and presumably sold as higher cost parts to another customer, as shown in Figure 9.

All of the ±10% capacitors meet the specifications, but there now becomes a more multi-modal distribution that should be understood. The different distributions can be separated
in order to parameterize the data set. An analysis of the multi-lot distribution shows that approximating the component parameters received from the supplier can be reflected with three separate distributions.

![Graph](image1.png)

Fig. 9. Incoming data for +/-10% caps where some of the +/-5% caps were removed and used for other applications. The resulting distribution of capacitance values is multi-modal.

The presence of bi- or multi-modal distributions should raise some level of speculation unless there is a clear underlying cause. These distributions imply that there is more than one type of behavior occurring in the overall population. These differences in behavior can be seen in the resulting distribution function, but they can also signify potential differences in failure modes, failure rates, and overall reliability of the component. With this disclaimer, it is very important to realize that multi-modal distributions are generally not desirable in a highly controlled, high reliability manufacturing environment, even if all parameters meet specification. Great care and a lot of work need to be performed to justify use of components with ‘odd’ behavior.

With that disclaimer, we will set out to replicate bi- (and by extension multi-) modal distributions for subsequent statistical Monte Carlo analysis. Decomposition of the full distribution shown in Figure 9 reveals the existence of about three separate distributions that can statistically describe the total distribution. The extracted distributions are:

At this juncture, perfect accuracy is not required. The intent is to be able to statistically model the distribution, not claim perfect equivalency. In order to create a model for this multi-modal distribution using our algorithms described above, we will make a data set of random variables of selected from each of the three populations. The process is outlined in Figure 10. Here, the three data sets (n=3) is assumed and each population is modeled per the parameters in Table 3. The repeated calling of the random variable \( \phi \) will select a randomly generated parameter from sub-population 1 45% of the time, from sub-population 2 30% of the time, and sub-population 3 the remaining 25% of the time. As the number of samples increases, the subsequent modeled population provides a statistical representation of the real-world distribution function, even in the case of complex, multi-modal distributions.
Table 3. Distribution parameters for the multi-model distribution seen in Figure 9

<table>
<thead>
<tr>
<th>distribution</th>
<th>type</th>
<th>mean</th>
<th>variance</th>
<th>fraction of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>sub-population 1</td>
<td>normal</td>
<td>0.933</td>
<td>0.01367</td>
<td>~0.45</td>
</tr>
<tr>
<td>sub-population 2</td>
<td>normal</td>
<td>1.003</td>
<td>0.02484</td>
<td>~0.30</td>
</tr>
<tr>
<td>sub-population 3</td>
<td>normal</td>
<td>1.064</td>
<td>0.0115</td>
<td>~0.25</td>
</tr>
</tbody>
</table>

Fig. 10. Algorithm for handling bimodal (extendable to multi-modal) distributions

The final modeled results are shown in Figure 11. The agreement between the original data distribution and the modeled data set shows the ability of this mixing algorithm to accurately model more complex distributions.

4.4 Monte Carlo analysis

Now that we have some basic tools for modeling distributions, we can link the methods together to perform Monte Carlo analysis of our transfer functions, obtained from the linearized models built upon the 2^n factorial simulation results of the WCCA, and real-world parametric distributions from incoming component variability analysis. The basic flow is illustrated in Figure 12. The linearized models were built upon simulations where the critical features were investigated at their upper and lower specification limits (USL and LSL, respectively.). Depending upon the type of model generated, coded vs. uncoded models... In the linearized model, these reflect input variables codes with permissible
Fig. 11. Comparison of original data set and the bi-modal modeled distribution using two independent distributions and a mixing ratio

![Graph showing comparison between original and modeled data.]

Fig. 12. Integrated extended worst-case circuit analysis flow

![Diagram illustrating the integrated extended worst-case circuit analysis flow.]
values ranging from (-1 at the LSL, 0 at nominal, and +1 at the USL). Any continuous value between -1 and +1 would reflect some point of the distribution that meets specification.
In this example, a predictive Monte Carlo run is repeatedly performed to estimate the effect of the randomly selected input variables (x₁₋ₙ) on the circuit output (Y). Additional studies could be taken to determine if six critical parameters was sufficient or if fewer parameters would still provide results with sufficient accuracy.
Based upon the WCCA, a linearized model using coded inputs based upon the 2ⁿ simulation results was extracted as:

\[ Y = 36.6 + 4.1x_1 + 1.1x_2 + 0.68x_3 - 0.81x_4 - 0.028x_5 - 0.03x_6 + 0.13x_1 \cdot x_2 \\
+ 0.077x_1 \cdot x_3 - 0.092x_1 \cdot x_4 - 0.0007x_1 \cdot x_5 - 0.005x_1 \cdot x_6 + 0.021x_2 \cdot x_3 \\
+ 0.021x_2 \cdot x_4 - 0.0026x_2 \cdot x_5 + 0.0003x_2 \cdot x_6 - 0.014x_3 \cdot x_4 - 0.0008x_3 \cdot x_5 \\
- 0.0002x_3 \cdot x_6 - 0.0018x_4 \cdot x_5 + 0.0012x_4 \cdot x_6 + 0.0004x_5 \cdot x_6 \]

The excellent fit between the linearized, second-order model and the more computationally expensive simulation results were shown in Figure 7.
For each of the critical component parameters, real distributions were obtained from in-house test or vendor supplied test data. A summary of the distributions is shown in Figure 13. The minimum and maximum values on each of the corresponding x-axes are the relevant LSL and USL for each of the distribution parameters.

Fig. 13. Modeled distributions for the critical parameters determined from the worst-case circuit analysis. All distributions reflect realistic distributions seen via the supply chain procurement process. The limits of each graph show the specification limits for the component parameter. Some components have very high Cpk, while others go through extensive screening to maintain in-spec compliance.
Using the distributions and the 2nd-order linearized model, a Monte Carlo run was performed using over 10,000 data points, essentially modeling the electrical performance of 10,000 circuits built in a high volume manufacturing facility. The results were summarized and statistically analyzed using Minitab software, and a summary of the results is shown in Figure 14. The real-world results, simulated from distributions in our Monte Carlo model, permit us to estimate the yield of this circuit to be an effective $C_{pk} = 5.2$ at the ±20% requirement level and 2.0 at a tighter ±10% level.

![Figure 14. Monte Carlo simulated output given the variability of the critical inputs.](https://www.intechopen.com)

Given these predicted output distributions, it is possible to not only demonstrate the design margin, but also to predict the yield of the design and process. Once this estimate is available, it becomes possible to compare the simulation results to end-of-line test data in order to determine the initial accuracy of the simulations. If deviations or differences are observed to be significant, it is suggested that the difference is understood in order to either improve the simulation accuracy (maybe requiring more accurate discrete component or integrated circuit simulation models) or look for the impact of test hardware or test execution.

One of the main benefits of having a statistical estimation of the critical output distribution is being able to understand how variations in incoming components and materials impact the end of line performance. This can drive appropriate control plans and monitoring strategies around the most critical parameters first and then expanding the scope of the incoming material control plans as time and resources allow. In addition, a statistical estimation of end of line performance is also crucial for being able to proactively control the quality and reliability of manufactured products. The simulation-based statistical model as well as the on-going test data collected for the purposes of statistical process control will help identify tested units that violate the statistical expectations for performance, even if they meet the end of line specification. Essentially, this means that even though a unit meets specification, if it does not fit the expectations for performance based upon the statistical
picture of the design and process, it should be suspected of potentially not meeting the same performance expectations over time compared to the statistically well-behaved units. This situation is illustrated in Figure 15, where a statistical distribution based upon both simulation (line) and end-of-line test data (symbols) are compared against a tested unit that meets specification but differs from the statistical model of the output distribution (the outlier near 31.5). An essential part of any control strategy, whether it is in incoming supply chain component and material procurement or end-of-line unit performance, should involve close scrutiny of statistical outliers in order to maintain the quality and reliability of products that the customers will see.

Fig. 15. Example of an in-spec, but out of control data point (at Y~31.5) compared to the simulated distribution prediction (line) and the cumulative end-of-line test data (symbols). This statistical anomaly should be treated as suspect unless convincing data proves otherwise.

Figure 16 shows one example of simulated worst case limits and distributions versus the actual manufacturing test data distribution of 305 samples. In this case, one output parameter for an Implantable Cardioverter Defibrillator (ICD) was simulated with models built for each IC, discrete components, and the tester. Simulations were first conducted at a smaller block level to sweep more than 70 initial component parameters with specified variations at a faster speed, compared to simulating the entire ICD. From the sensitivity analysis results 5 critical parameters were identified. Full factorial design is conducted to address potential interactions among the critical component parameters. Thirty-two simulations were run at the device level with models of all hardware included. A transfer function was built to describe the relationship between output and the 5 identified critical input parameters. Monte Carlo analysis was performed to generate the distribution of the output based on the transfer function. This way the computation efficiency is much higher compared to running Monte Carlo simulations for the entire parameter set of 70+ components for this ICD output. It is demonstrated from Figure 16 that the simulated distribution matches well with the actual product manufacturing data. In this particular
case, the simulated worst case limits are within the manufacturing test requirements, which indicate design margin. It also accurately showed that the distribution for this output is highly skewed toward the lower end of the requirement, which leaves less design margin at the lower limit side compared to the higher limit which is advantageous in this scenario.

Fig. 16. A device output distributions from Monte Carlo analysis and manufacturing test. Simulated worst case limits are shown in red dashed line

While the generalized method of EWCCA was demonstrated here using electrical circuit simulation, any experimental or simulation based analysis method can be treated in this fashion to understand both the worst-case anticipated variation in a design: electrical, thermal, or mechanical, etc. as well as realistic variation which can be modeled accurately and computationally efficiently using the methods described in this paper.

5. Conclusion

Increased focus on product quality is requiring electrical designers to more effectively understand design margin. Fully understanding design margin provides designers the data to effectively make design trade-offs. These trade-offs may include rationale for component selection and manufacturing yield. This requires better understanding the influence of corners and better control of distribution tails. However, assessing the impact of corners or parameter shift is difficult to achieve in lab testing. Furthermore, using hardware testing to verify that system hardware works under all conditions in the presence of variation is very challenging, as the number of units tested cannot represent all the possible variations. This information can be provided proactively through worst-case circuit analysis to ensure the design works correctly in the presence of all specified variability.
This chapter provides an overview of different WCCA/variability analysis methods with the pros and cons for each method introduced. In addition, a simulation based flow for WCCA and yield predictions is developed to address different scenarios to allow extended analysis for yield estimation.

Worst-case circuit analysis is a demonstrated method that provides clear understanding of design margin. The extended worst-case circuit analysis builds upon these findings to create mathematically simple transfer functions which can be used to simulate a virtual high volume manufacturing line that reflects real-world variability of incoming components and processes. The application of the EWCCA technique provides predictive yield and permits the use of realistic performance outputs, component stresses (use conditions) for use in subsequent reliability analysis, and helps create opportunities to balance design margin against a variety of other factors, including reliability and economic considerations.

The benefits of simulation-based WCCA and yield predictions include rigorous identification of critical features to properly select components and define control strategies, understanding component use conditions, evaluation of design and manufacturing trade-offs, enabling predictive reliability, implementing design for reliability and manufacturability, and establishing meaningful component limits based upon design capability. In instances where inconsistencies exist between component tolerance and higher level design requirements, early, proactive solutions can be implemented in design, component selection, control requirements, or test requirements.

In summary, with the disciplined use of simulation-based variability analysis and enabled predictive reliability analysis, product development can further improve time-to-market and reduce reliability issues, including those resulting from supply chain sources. Limits of circuit / component use conditions, insight into design margin, predictions on reliability and yield, and recommendations on critical control parameters can be provided to design and supply chain to improve design performance and yield. Identified critical features in simulations from a design for reliability and manufacturability perspective are used to drive supply chain decisions to build robust designs in an efficient way.

6. Acknowledgement

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7. References


Supply Chain Management (SCM) has been widely researched in numerous application domains during the last decade. Despite the popularity of SCM research and applications, considerable confusion remains as to its meaning. There are several attempts made by researchers and practitioners to appropriately define SCM. Amidst fierce competition in all industries, SCM has gradually been embraced as a proven managerial approach to achieving sustainable profits and growth. This book "Supply Chain Management - Applications and Simulations" is comprised of twelve chapters and has been divided into four sections. Section I contains the introductory chapter that represents theory and evolution of Supply Chain Management. This chapter highlights chronological prospective of SCM in terms of time frame in different areas of manufacturing and service industries. Section II comprised five chapters those are related to strategic and tactical issues in SCM. Section III encompasses four chapters that are relevant to project and technology issues in Supply Chain. Section IV consists of two chapters which are pertinent to risk managements in supply chain.

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